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Public

Finance through algorithmic lens

KNMF AGH Financial Mathematics Conference

Maxim Litvak

Model Risk Management & Control

April 22, 2017

Quants Connect

powered by UBS and PRMIA

Kraków, TAURON Arena
June 9th, 2017

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Our mission is:

- To support cooperation in quants environment
- To facilitate an exchange of experiences between representatives of business sector and university community
- To advise young professionals in choosing the right career path
- To share knowledge and new trends

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PRMIA
The Professional Risk Managers' International Association

Quants Connect – partners and speakers

Partners:

- PRMIA – Professional Risk Managers International Association
- KNMF – Scientific Association of Financial Modelling at AGH
- AGH University of Science and Technology
- Jagiellonian University in Krakow

Speakers:

- Dr **Manfred Plank** – Risk COO at UBS
- Dr **Andrzej S. Kulik** CFA, PRM – Head of Internal Audit at mBank, PRMIA
- Dr hab. **Lukasz Delong** – associate professor at Warsaw School of Economics, ex president of Polish Society of Actuaries
- Dr **Diana Kapsa** – Head of Risk Methodology Retail at UBS

Agenda

09:00 – 09:30	Opening Speech
09:30 – 10:15	**UBS senior speaker:** Dr Manfred Plank
10:15 – 10:30	Coffee Break
10:30 – 11:15	**PRMIA speaker:** Dr Andrzej S. Kulik
11:15 – 12:00	**Guest speaker:** Dr hab. Łukasz Delong
12:00 – 13:00	Lunch Break and networking session
13:00 – 13:45	**UBS speaker:** Dr Diana Kapsa
13:45 – 14:00	Coffee Break
14:00– 16:00	**Poster Session**
16:00 – 16:30	Coffee Break
16:30 – 18:00	**Panel Discussion**
18.00	Closing

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Finance through algorithmic lens

Maxim Litvak

2017-04-22

Outline

Outline

1st half - build algorithmic lens

- We'll get the fast overview of fundmentals of algortihms
- Take-aways: 1 picture, 2 paradoxes, a few example algorithms

2nd half - look through it on finance

- 3 practical cases
- Some theoretical insights
- Blockchain
- Auctions
- Non-monetary algorithms

Example: find an atom in the universe

- How fast can we find a specific atom in the universe?
- The current estimate is that there are 10^{80} ($\approx 2^{270}$) atoms in the universe.
- Sounds like a needle in a haystack at least ...

Example: find an atom in the universe

Algorithm

- Split the universe in 2 halves: in which half is the atom we need?
- Repeat

Result

- In 270 operations we'll find it
- Could be done by hand in less than 5 minutes . . .
- We've seen an algorithm being extremely fast on a large input

Example: Hanoi tower

Example: Hanoi tower

Task

Move all disks from one peg (tower) to another

Rules:

- 1. Move one disk at a time
- 2. Every disk must be bigger than the one below
- 3. You can move only the upper disk

Example: Hanoi tower

- 3 moves needed for 2 disks
- 7 moves are needed for 3 disks
- 15 moves are needed for 4 disks
- . . . million moves for 20 disks
- Number of moves goes up exponentially with number of disks!

Example: Hanoi tower

- We've seen a task with an algorithm that is fast on huge input (atom searching)
- We've seen a task with an algorithm that is slow on small input (Hanoi tower)

Picture to take away

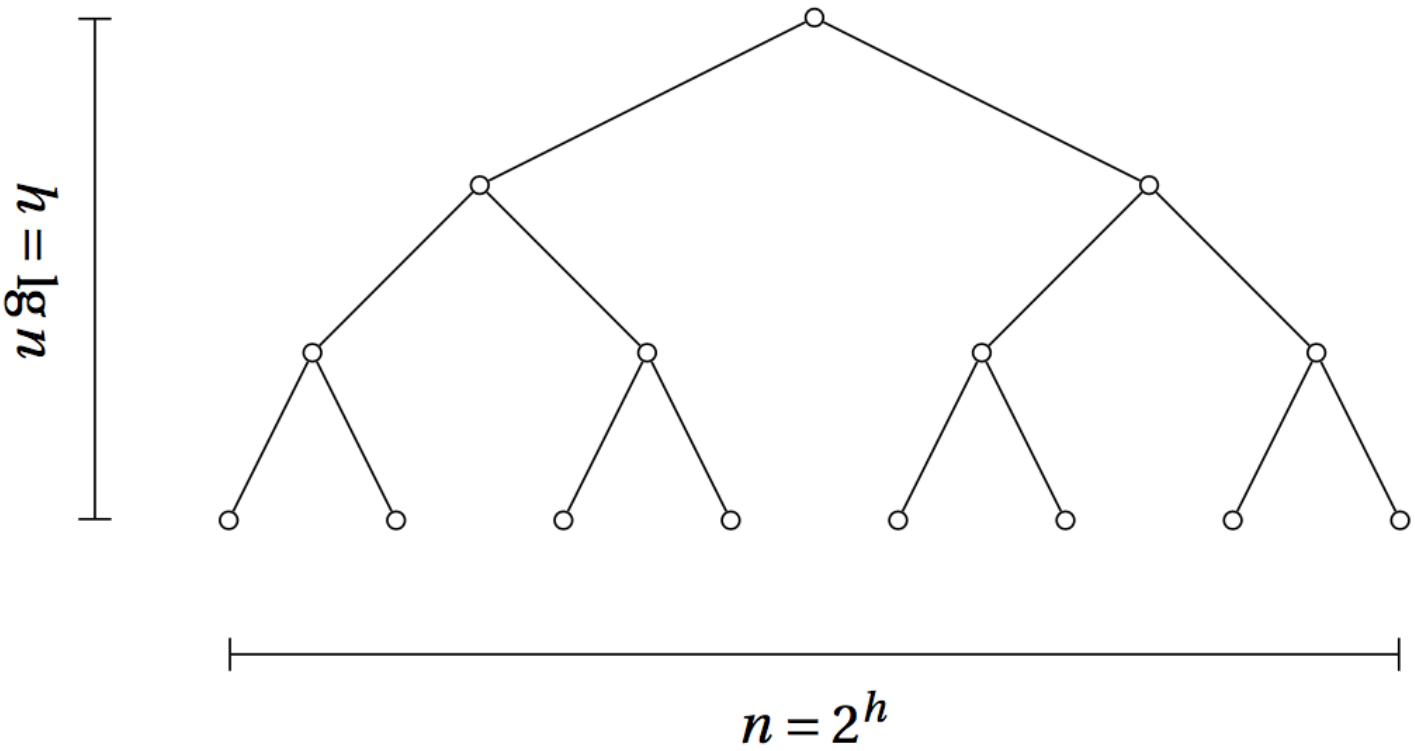
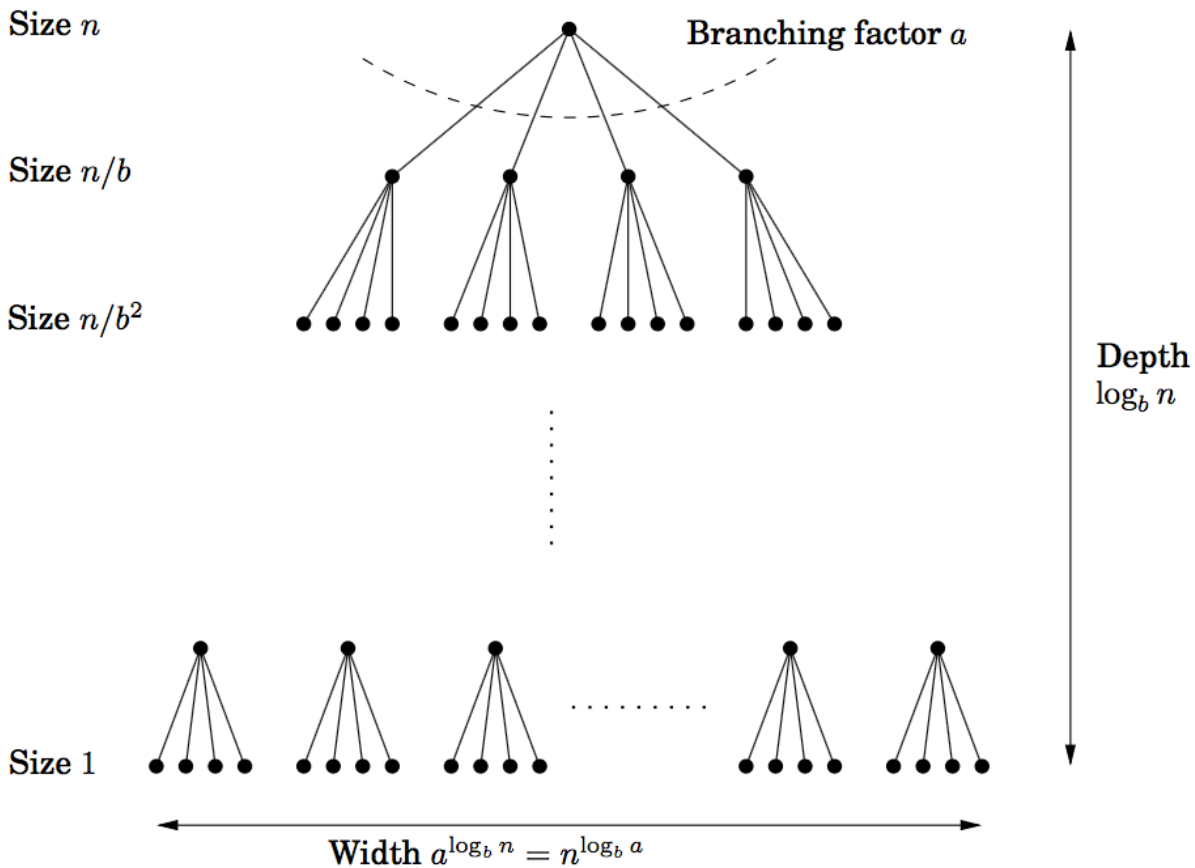


Figure 3-3. *The height and width (number of leaves) of a perfectly balanced binary tree*

- The height is log(n) (fast), the width is $n = 2^h$, the number of nodes is $2n - 1 = 2^{h+1} - 1$ (slow)

Picture to take away

Figure 2.3 Each problem of size n is divided into a subproblems of size n/b.

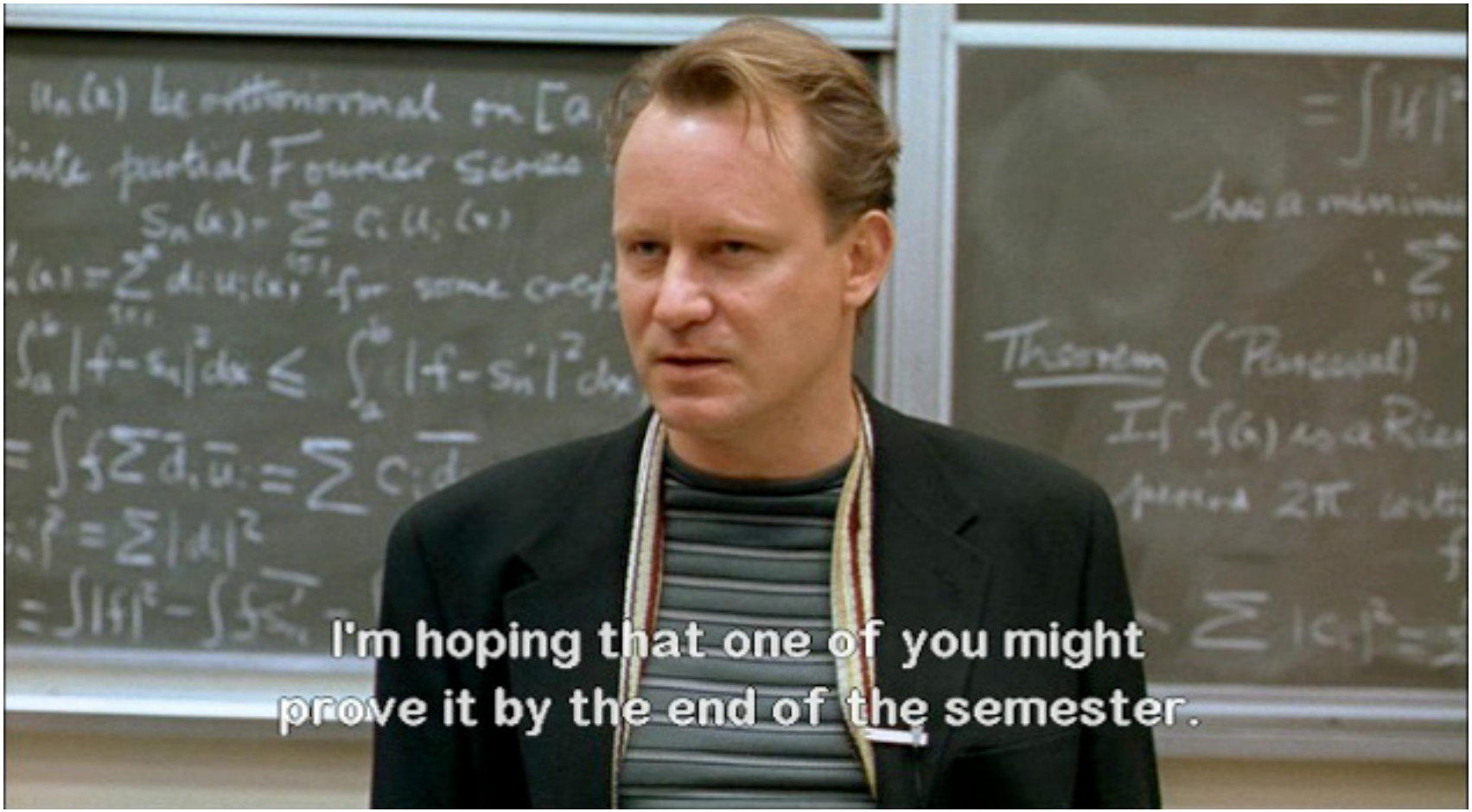


Paradoxes - Multiplication

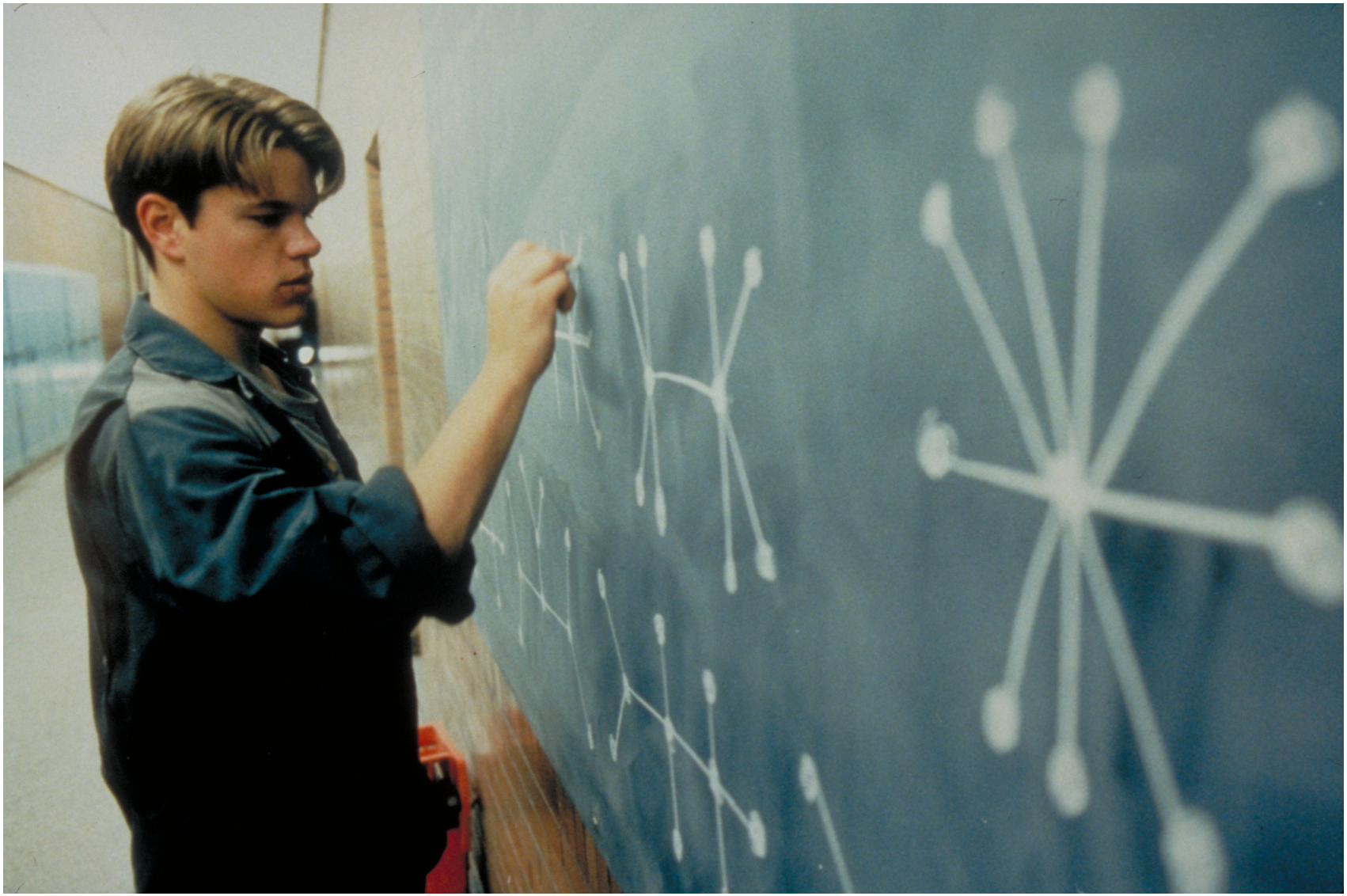
It's possible to multiply 2 numbers faster than the school method

- Andrey Kolmogorov in 1960 conjectured that there's no multiplication method faster than the school method (used by the humanity for thousands of years), and organized seminar to prove it
- A 17 years old student (Anatoly Karatsuba) found a faster method ...

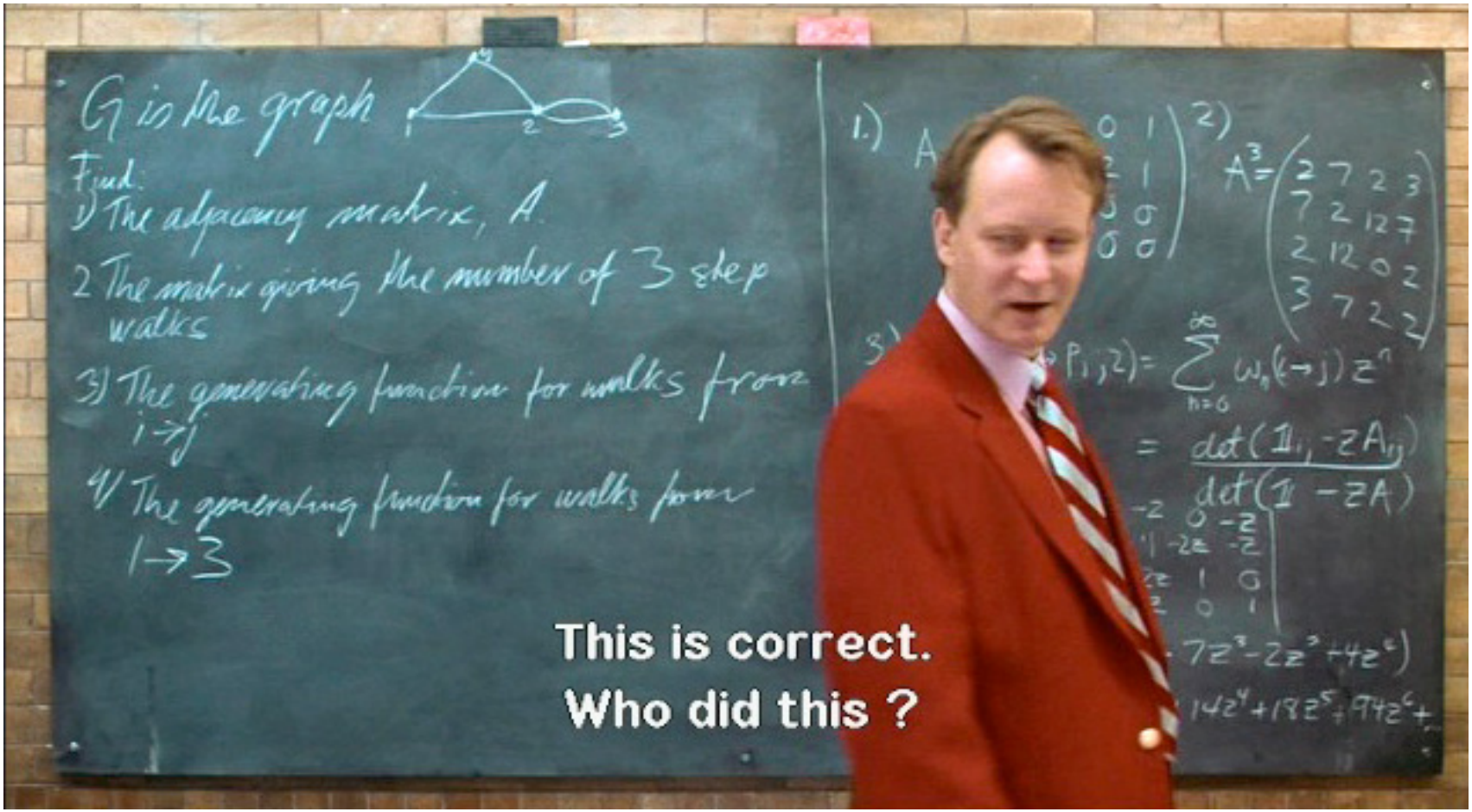
Paradoxes - Multiplication



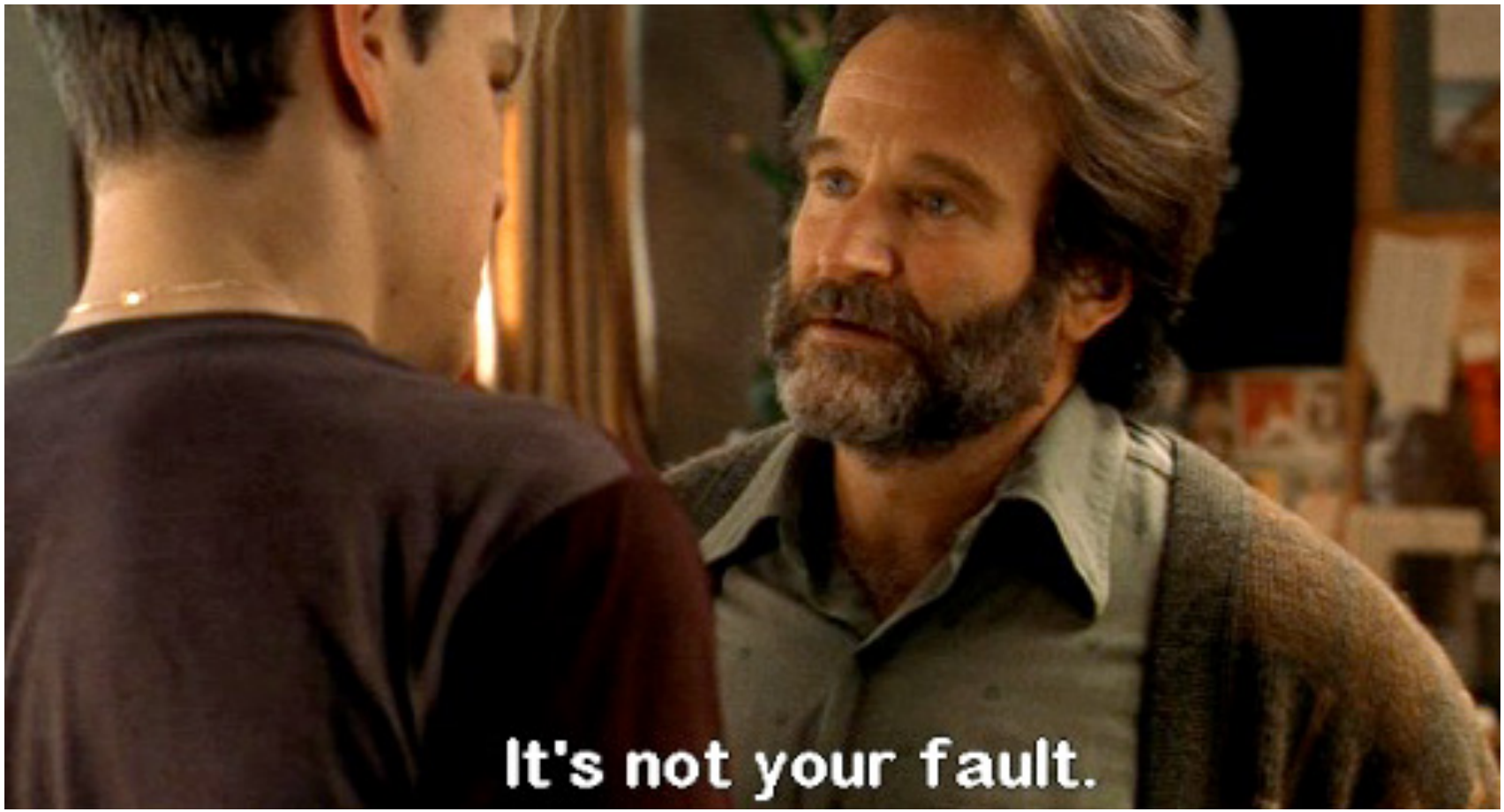
Paradoxes - Multiplication



Paradoxes - Multiplication



Paradoxes - Multiplication



Paradoxes - Multiplication

School

```
 25
x
 17
 ---
 175
+
 25
 ---
 425
```

$$(2\times10+5)(1\times10+7) = (2\times1)\times100+(2\times7+5\times1)\times10+(5\times7)\times1 = 425$$

Paradoxes - Multiplication

Karatsuba

```
(2*10 + 5)(1*10 + 7) = x*100 + y*10 + z*1 = ...
-----------------------
x = 2*1 = 2
z = 5*7 = 35
y = (2 + 5)*(1 + 7) - (x + z) = 56 - 37 = 19
-----------------------
... = 2*100 + 19*10 + 35*1 = 425
```

Paradoxes - Multiplication

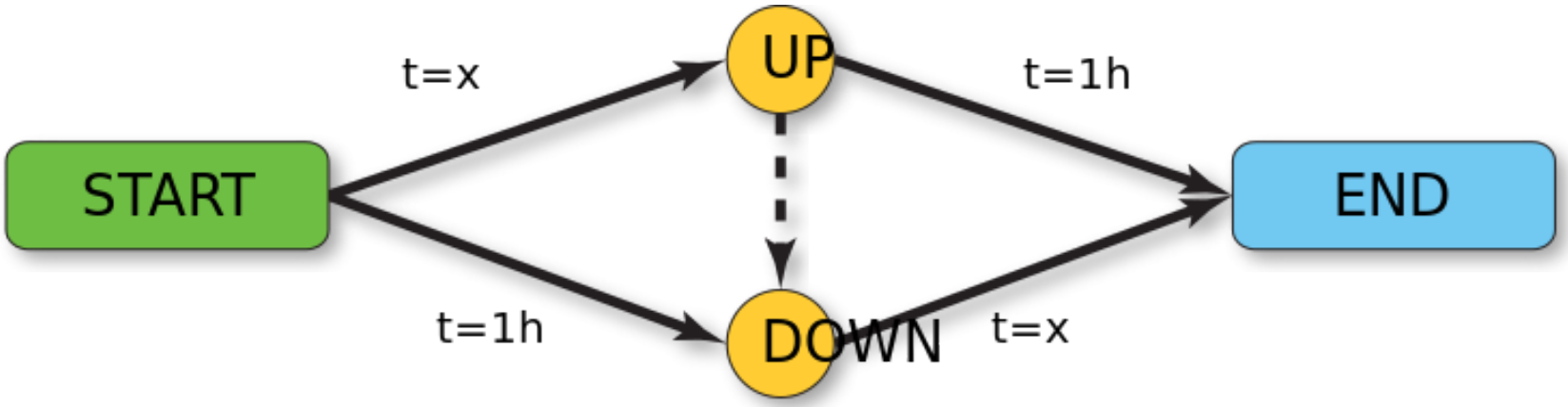
Difference

- School method - 4 multiplications and some additions
- Karatsuba method - 3 multiplications and some more additions
- Karatsuba is faster because the recursion can be deployed on multiplication

Paradoxes - Braess paradox

- Decreasing the number of opportunities might increase the efficiency
- In option pricing - adding an opportunity can't reduce the price

Paradoxes - Braess paradox

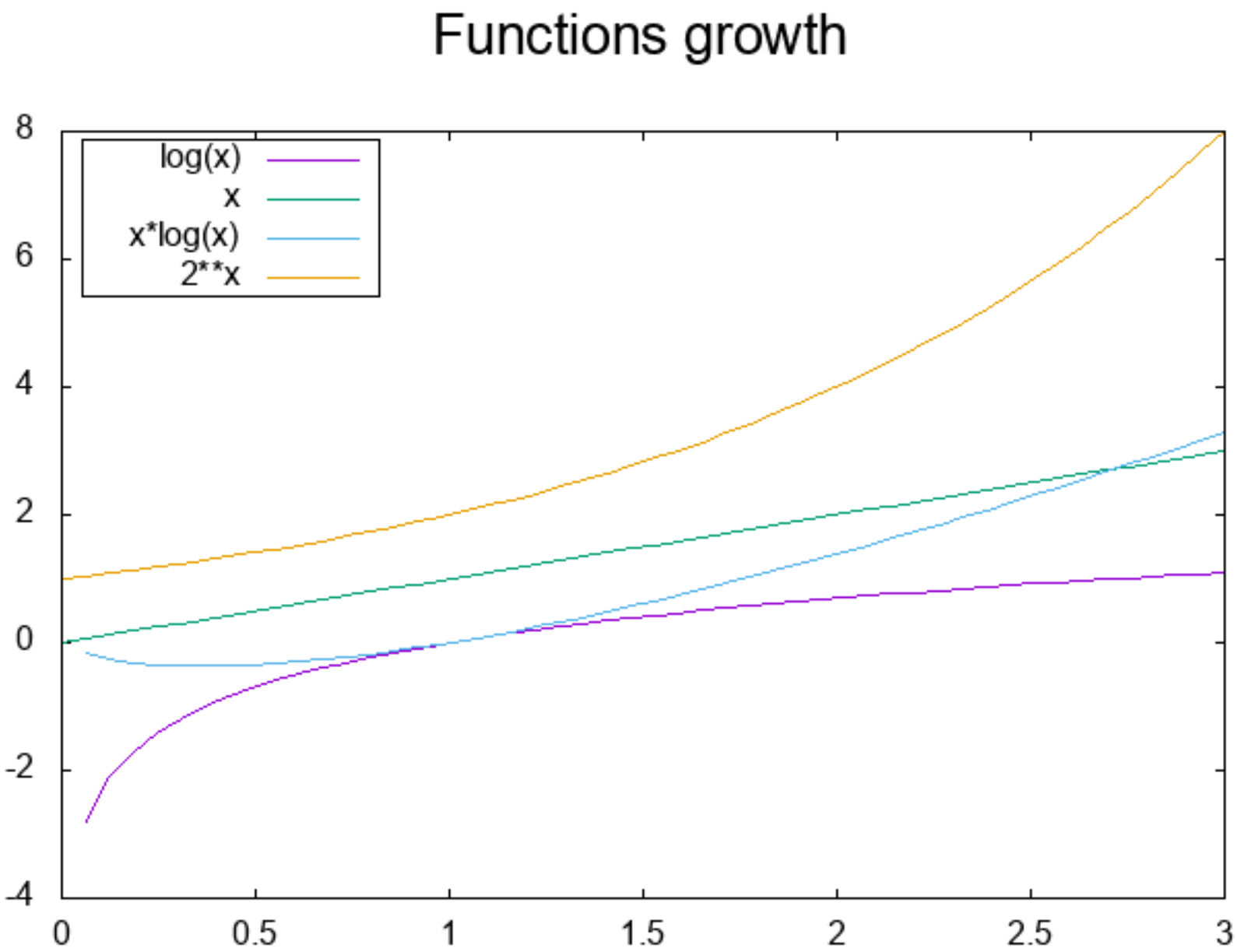


- The roads START-DOWN and UP-END take 1 hour, the roads S-up and Down-t depend on traffic (100% cars on the road - 1 hour, 50% - 30min)
- Initial equilibrium - 1.5h trip time (one half goes the upper way)
- Adding high-speed UP-DOWN highway (with 0h time to cross it) increases the time to 2h

What is "fast"

- From algorithmic point of view, under the "speed" is considered the asymptotic behaviour of the algorithm on input
- For a certain input, algorithm A might be faster than B, but "slower" in the asymptotic sense, i.e. with input big enough B outperforms A
- Big-O notation: f(n) = O(g(n)) if (up to a constant) for some n, g(n) catches up or outperforms f(n)

What is "fast"



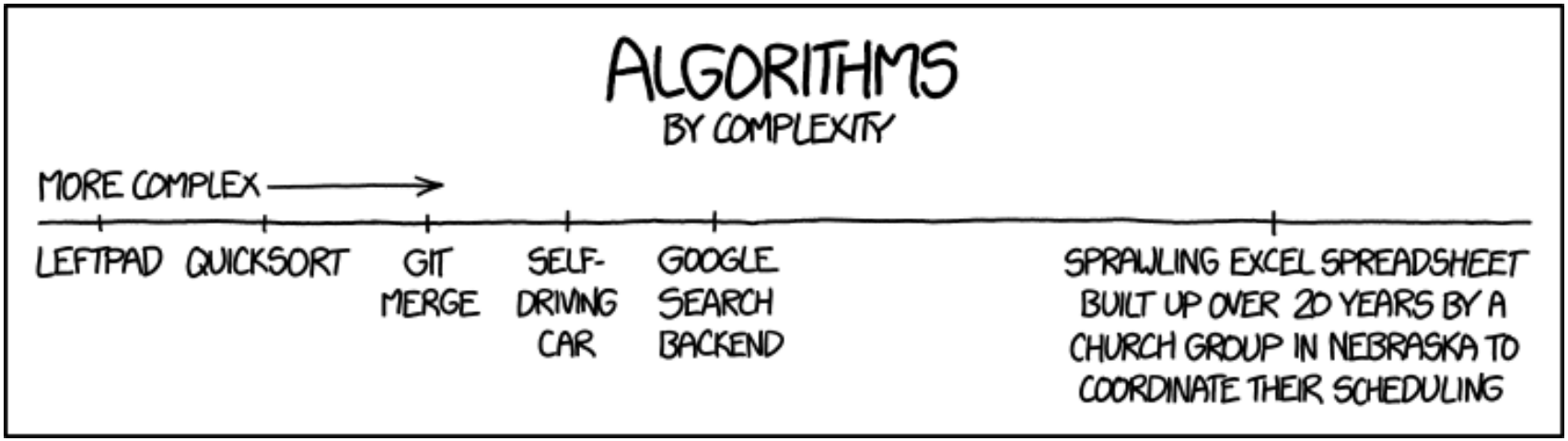
What is "hard" to compute

- As a rule of thumb, the algorithms that can be computed in polynomial time are considered as "easy" to compute
- ... and algorithms which computation time can't be bound by a polynom (e.g. exponential) are considered to be "hard"
- Sometimes computational "hardness" is a good thing, if you don't want something to be easily computed (e.g. cryptography)
- A lot of "hard" algorithms are considered to be "hard" since no faster solution is known (includes P = NP problem)

Examples of algorithm's speed (current stand)

- Sorting numbers - O(n log(n))
- Binary search - O(log(n))
- Satifiability (if some logical statement is true or not) - exponential
- Integer number factorization (important in cryptography) - exponential

Non-standard examples on complexity . . .



- xkcd.com

Algorithmic topics in finance

- Cases in practice of quantitative finance
- Theoretical insights
- Blockchain
- Auctions
- Non-monetary mechanisms

Case: generate normal r.v.

- We can generate a normally distributed r.v. in a few ways, not all of them are equal computationally

(Bad solution)

- Generate 1000 uniformly distributed r.v. and use the CLT
- Their sum (centered by mean and normalized by standard deviation) is (approximately) normally distributed
- However, it's computationally expensive to do it this way

Case: generate normal r.v.

Another solution:

- use inverse c.d.f.
- $\xi = \Phi^{-1}(\eta)$, s.t. $\eta \in U_{0,1}$
- Next question: how expensive is it to compute inverse c.d.f.?

Subproblem: computation of inverse c.d.f.

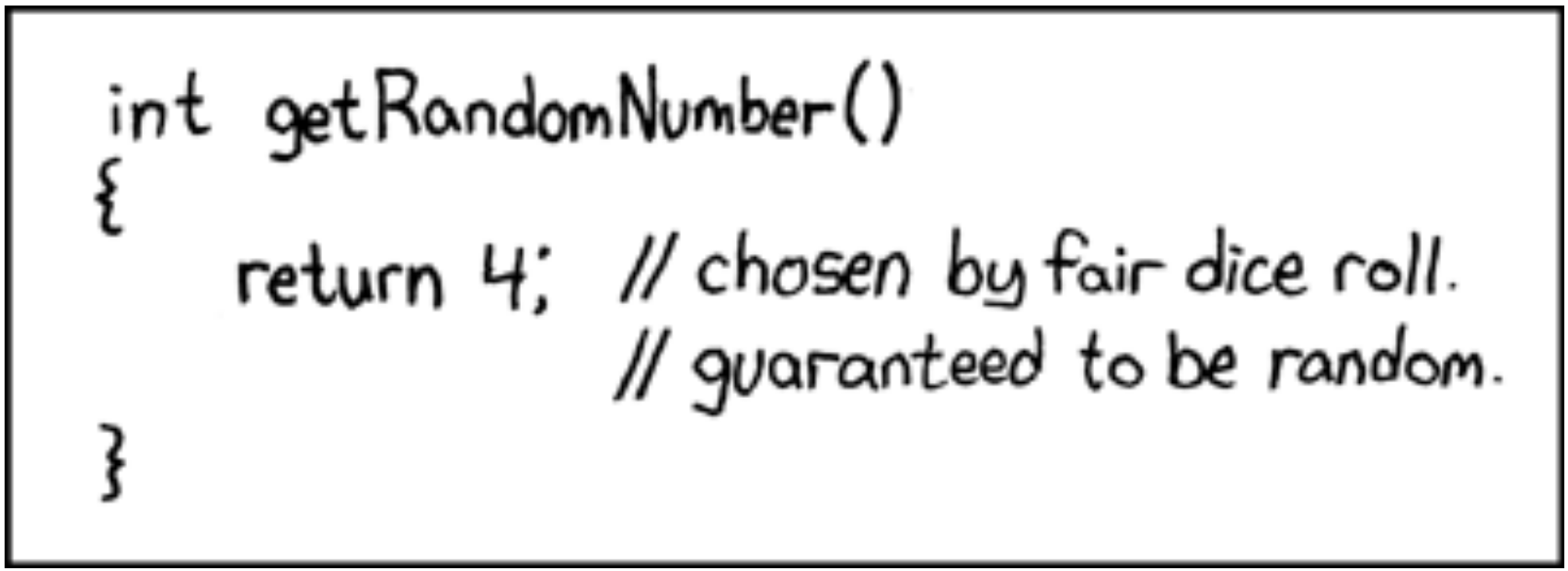
- Option: calculate using the root-finding procedure (on average 5-6 iterations, however, in this case the c.d.f. itself might be costly to compute)
- Option: use the Taylor expansion (well explored for normal distribution, might not be the case for other distributions)

Case: generate normal r.v.

- Other methods are known (Box-Miller procedure etc)
- However, the very 1st question: how costly is it to compute pseudo-random uniformly distributed r.v.?
- (guess) r.v. generators with better properties might be harder to compute

Case: generate normal r.v.

- Fast, but not so efficient generator . . . (xkcd.com)



Case: generate normal r.v.

- Example: middle square method
- Simple, but has different flaws
- Was enough for the simulations for the 1st nuclear bomb

middle square method

- Start with some n-digit number
- Square it
- Take the middle as the next number in the sequence
- Example: 6234^2 -> 38862756, i.e. next number is 8627
- Problem: 7600^2 -> 57760000, i.e. generator got stuck at the same number
- For 4 digits maximal period is 2^{12} (one of modern generators has the period 2^{19937})

Case: credit pricing

- A big bank re-calculated for controlling purposes each month its whole portfolio of credits (10^6 credits)
- For a lot of credits you need to do the same calculation (e.g. credits with the same conditions, but different principals)

Credit was priced (simplification) based on the following parameters:

- loan-to-value (LTV)
- r (interest rate)
- T (time to maturity)
- R (rating of the client)
- K (quality category of the principal)

Case: credit pricing

Idea

- why not to split all parameters into intervals (10 on average)
- pre-calculate all the possible situations (10^5)
- Then assign to each credit its price based on its parameters
- Rational: searching in the DB is faster than calculating

Case: credit pricing

- Some time later, a few improvements to the price engine were introduced
- instead of collateral category, there were introduced the mean and standard deviation of its value
- an option for extra payment (e.g. maximally 5% p.a. of the collateral)
- type of credit (most of the credits were annuities, but some were bullet loan credits etc)
- now, there were 10^8 credits in the database
- with a few other propositions to extend the number of parameters down the way ...

Case: credit pricing

- time complexity (linear on number of credits and polynomial on number of parameters) was exchanged for the space complexity (exponential on number of parameters)
- at some point of time exponential growth outperformed the polynomial . . .

Case: search for the best regression

- The frequent problem: given a number of factors, pick some of them, s.t. the dependend variable is explained in the "best" way
- The criteria can be e.g. some information criteria (e.g. Akaike)
- ... or some combined criteria (R^2 is high enough, p-values for coefficients are at least x%, p-value for normality test in residuals at least y% etc)

Case: search for the best regression

- Problem: the number of all combinations $\sum_{k=0}^{n} C_n^k = 2^n$, i.e. grows exponentially on number of factors
- e.g. for 50 factors, we would need to consider $2^{50} \approx 10^{15}$ regressions - too much

Case: search for the best regression

Attempt solution:

- constrain the number of possible factors (who needs regression with 100 factors?)
- e.g. for 50 possible factors, consider regressions with at most 4 factors, we'd need ca. 250 tsd. regressions - doable ...
- ... doable, but not scalable. Consider, we'd like also to consider the 1st lags (doubling the number of factors), then for 100 factors, we'd need to go through ca. 4 mln regressions

Case: search for the best regression

- The flaw of the attempt solution: it's faster, but it still grows exponentially
- p.s. $O(\sum_{k=0}^{k_{max}} C_n^k) = O(2^{nH(\frac{k_{max}}{n})})$, where H() is the binary entropy function

Case: search for the best regression

- Solutions in practice
- As the criteria (e.g. AIC) is chosen that assigns a number to a regression
- Deploy an optimal solution search method
- It usually has a polynomial complexity (however, not guaranted to find the global optimum)

Algorithms in practice of quantitative finance

- Different tasks require different speed
- Calculation of capital can easily take a few hours (or even days) longer
- With a client sitting in front of you, you want the program to calculate the conditions in 10-20-30 seconds
- In high-frequency trading you need to deploy the fastest algorithm, in the most efficient language on the fastest infrastructure

Theoretical insights

- Usual question in economic theory: does the equilibrium (e.g. price) exist?
- The algorithmic approach: how fast it can be computed?
- Nash-equilibrium is "hard" to compute
- One of ideas: markets are efficient if P = NP

Blockchain

- Blockchain is a distributed database of transactions
- "Hard to find, easy to check" principle used
- incentives used to ensure the fairness

Market capitalization as of 22.04.17

- Bitcoin $20b
- Ethereum $4.5b

Blockchain

- Some blockchain allows smart contracts (e.g. Ethereum)
- Would they reduce some risks (e.g. counterparty risk) and create the new ones? How to price them?
- Still an emerging field
- Cryptocurrencies are not exactly money

Money properties

- medium of exchange
- measure of value
- store of value

Auctions

- 2nd price auction principle - the highest bid wins, but the 2nd highest bid is paid
- Everytime you see an ad from google, an auction is run (i.e. millions auctions a day take place)

Non-monetary mechanisms

- not all tasks are solved with money, examples:
- matchings
- allocations

Matchings

- How to match different parties with different preferences
- e.g. stable marriage problem (match couples s.t. noone could improve their choice)

Examples

- In USA, medical students are assigned to hospitals in a similar way
- In France, teachers to public school

Allocations

- Allocate through exchange

Examples

- dormitory rooms exchange in China

Non-monetary mechanisms

- Simple matching/allocation algorithms are "easy to compute"
- Some not, e.g. hospital/residents problem with couples
- Hospital might admit multiple students, the constraint to assign a married couple to one hospital makes the problem "hard"

Thank you for your attention

- Thank you for your attention
- You can find the slides here:
  `https://github.com/maxlit/lectures/`
- Questions?

Material sources

- xkcd.com
- Dasgupta et al. "Algorithms"
- M.Hetland "Mastering Basic Algorithms in the Python language"

We are UBS

UBS

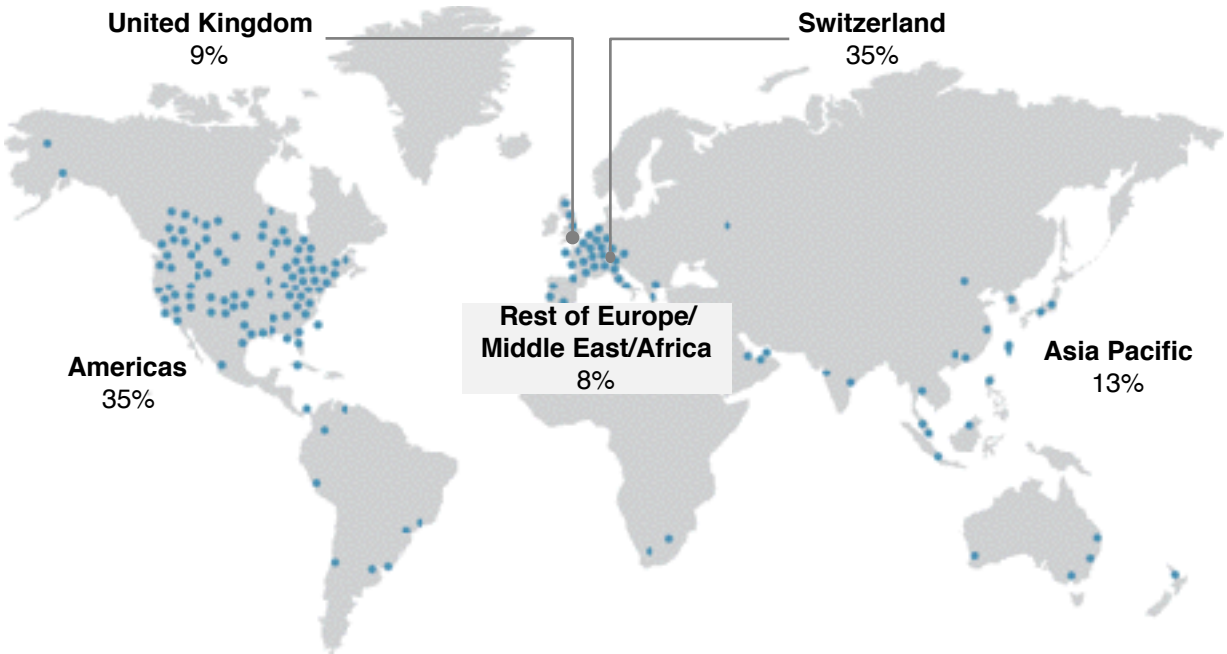
Get to know us

We are a premiere global financial services company.

We are well-established and thoroughly global

We're:

- a client-focused financial services firm with a 150-year history
- headquartered in Switzerland
- over 60,000 people in more than 50 countries
- committed to our wealth management businesses and our universal bank in Switzerland, along with our asset management and investment banking businesses.



What we do

We aim to be the world's leading wealth manager and the premier universal bank in Switzerland, enhanced by an asset manager and investment bank which are world-class in their chosen areas of focus.

Meeting our clients' diverse needs

Wealth Management and Wealth Management Americas
Financial services for wealthy individuals

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Asset management for institutional and other investors

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Client-centered investment banking services

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Banking for retail, corporate and institutional clients in Switzerland

Corporate Center
From operations, legal, IT and risk to HR, finance, communications and branding, Corporate Center functions provide governance and support to all business divisions

We are UBS Quants

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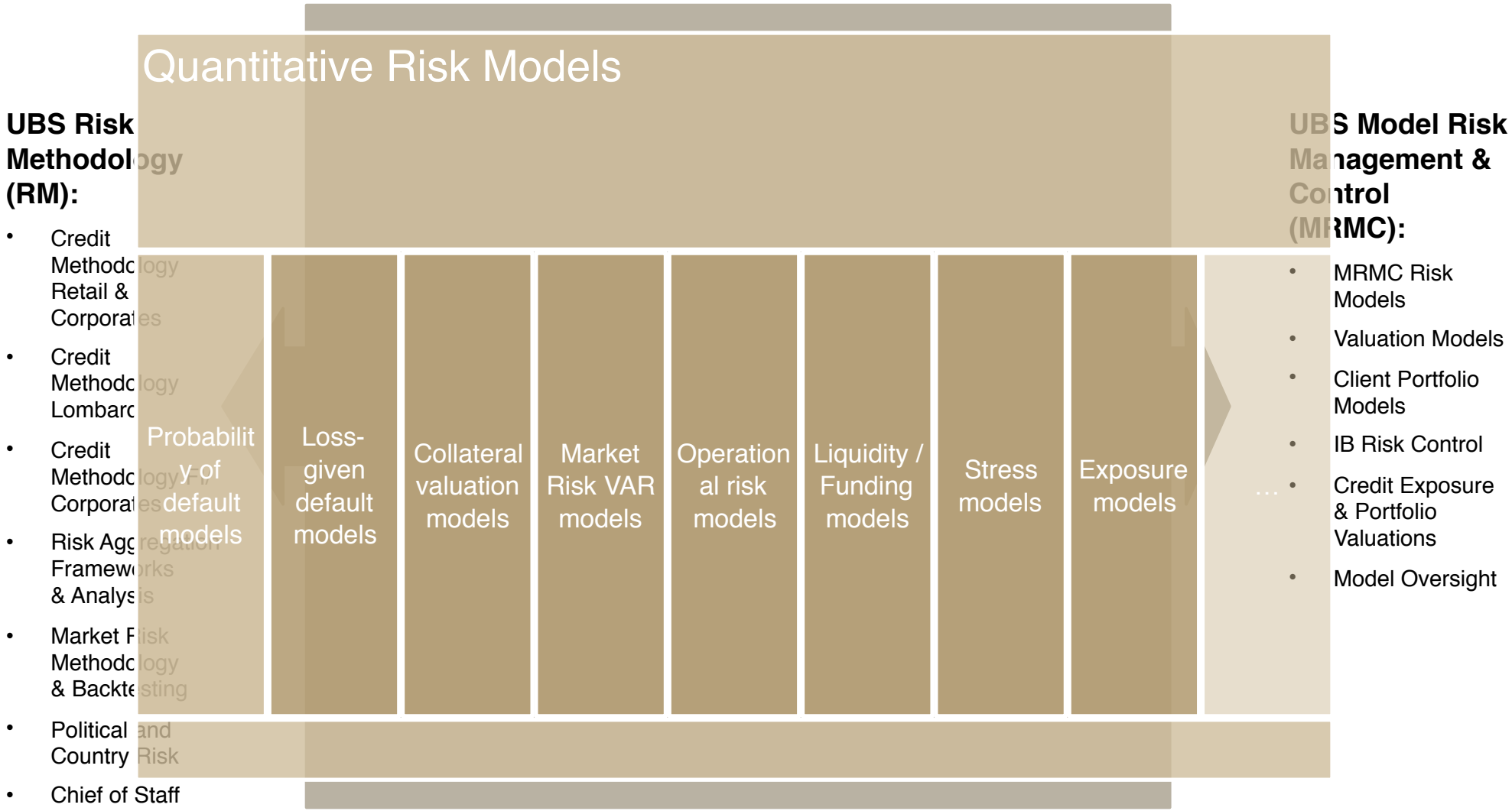
Mission Statement

Mandate	Build, confirm and validate the models which measure our credit, market, country and operational risks at an individual and aggregated level. In particular, we develop, validate and maintain the measures and models that: • Support the control of risk taking of UBS • Provide input to the calculation of UBS's regulatory capital
Setup	Existing teams in UBS in Zurich and LDN; Quant hub in UBS in Krakow
Job Requirements	1. Solid math / stat skills; 2. excellent programming skills; 3. motivated, self-directed and driven; 4. affinity to number and data; 5. communication skills
Clients	Divisional CROs and Credit Officers, Client Advisors, Regulators, Audit (internal and external), Quantitative Risk Controls, Banking Products, ...
Challenges	Regulatory requirements, industrialization / automation, Big Data, internal data completeness and quality, IT constraints, …

Different Risk classes and different models…

…and two big players



UBS Quant hub in Krakow

Being part of a team …

Intelligent	We are Mathematicians, Physicists, Statisticians, Econometricians, etc.
Integrated	We are integrated part of existing teams in Zurich, London, New York, etc;
International	We are coming from 17 countries: Poland, Switzerland, Germany, Italy, China, India, Brazil, Russia, etc;
Impactful	We are delivering and contributing to bank's most important projects: CCAR, IFRS9, ICAAP, etc

Questions?

Contact information

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ubs.com/polandcareers