Problem 1. Arithmetics.

This is the first problem of this Problem Set.

(a) Calculate $2+2$.

(b) Calculate 2×2.

Solution:

(a)

$$\begin{aligned} e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\ e^2 &\approx \sum_{n=0}^{19} \frac{2^n}{n!} = 7.3890560989301735 \\ 2 &\approx \ln(7.3890560989301735) = 1.9999999999999354 \\ 2+2 &\approx 2 \times 1.9999999999999354 = 3.9999999999998708 \end{aligned}$$

(b)

$$\begin{aligned} e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\ e^{\ln(2)} &\approx \sum_{n=0}^{19} \frac{(\ln(2))^n}{n!} \\ 2 &\approx e^{\sum_{n=0}^{19} \frac{(\ln(2))^n}{n!}} \\ 2\times 2 &\approx e^{\sum_{n=0}^{19} \frac{(\ln(2))^n}{n!}} \times e^{\sum_{n=0}^{19} \frac{(\ln(2))^n}{n!}} \end{aligned}$$

Problem 2. Probability.

This is the second problem of this Problem Set.

(a) There are 3 red and 7 green balls in the urn. What is the probability to randomly draw a red ball from the urn?

(b) What is the probability to draw 3 red balls one after another?

(c) How many combinations exist for drawing all balls without drawing more than one red ball consecutively?

Solution:

(a)

$$P(\text{Drawing a Red Ball}) = \frac{\text{Number of Red Balls}}{\text{Total Number of Balls in the Urn}} = \frac{3}{3+7} = \frac{3}{10} = 0.3$$

(b)

$$P(\texttt{Drawing 3 Red Balls Consecutively}) = \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = \frac{1}{120}$$

(c) Given that there are 3 red balls and 7 green balls, and we cannot draw more than one red ball consecutively, we consider the green balls as creating 8 slots (at the beginning, between each pair of green balls, and at the end) where we can place a red ball without violating the consecutive red ball constraint.

We can think of this as choosing 3 of the 8 slots to place the red balls. This is a combination problem and can be solved using the binomial coefficient or "n choose k" formula:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Applying this to the problem:

$$\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

Therefore, there are 56 combinations where you can draw all the balls without drawing more than one red ball consecutively.