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Problem 1. Arithmetics.

This is the first problem of this Problem Set.

- (a) Calculate 2+2.
- (b) Calculate 2×2 .

Solution:

(a)

(b)

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \cdots$$

$$e^{\ln(2)} \approx \sum_{n=0}^{19} \frac{(\ln(2))^{n}}{n!}$$

$$2 \approx e^{\sum_{n=0}^{19} \frac{(\ln(2))^{n}}{n!}}$$

$$2 \times 2 \approx e^{\sum_{n=0}^{19} \frac{(\ln(2))^{n}}{n!}} \times e^{\sum_{n=0}^{19} \frac{(\ln(2))^{n}}{n!}}$$

Problem 2. Probability.

This is the second problem of this Problem Set.

- (a) There are 3 red and 7 green balls in the urn. What is the probability to randomly draw a red ball from the urn?
- (b) What is the probability to draw 3 red balls one after another?
- (c) How many combinations exist for drawing all balls without drawing more than one red ball consecutively?

Solution:

(a)

$$P({\rm Drawing~a~Red~Ball}) = \frac{{\rm Number~of~Red~Balls}}{{\rm Total~Number~of~Balls~in~the~Urn}} = \frac{3}{3+7} = \frac{3}{10} = 0.3$$

(b)

$$P(ext{Drawing 3 Red Balls Consecutively}) = rac{3}{10} imes rac{2}{9} imes rac{1}{8} = rac{1}{120}$$

(c) Given that there are 3 red balls and 7 green balls, and we cannot draw more than one red ball consecutively, we consider the green balls as creating 8 slots (at the beginning, between each pair of green balls, and at the end) where we can place a red ball without violating the consecutive red ball constraint.

We can think of this as choosing 3 of the 8 slots to place the red balls. This is a combination problem and can be solved using the binomial coefficient or "n choose k" formula:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Applying this to the problem:

$$\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

Therefore, there are 56 combinations where you can draw all the balls without drawing more than one red ball consecutively.