Capacitors

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Summary

- Capacitors are circuit elements that can store and release electrical energy
- Each terminal of a capacitor is connected to a metal plate that can hold charges $+Q$ and $-Q$
- The **capacitance** C is a quantity that lets us calculate how much charge Q will collect on the plates when a voltage V is applied across the terminals

  $$Q = C \times V$$

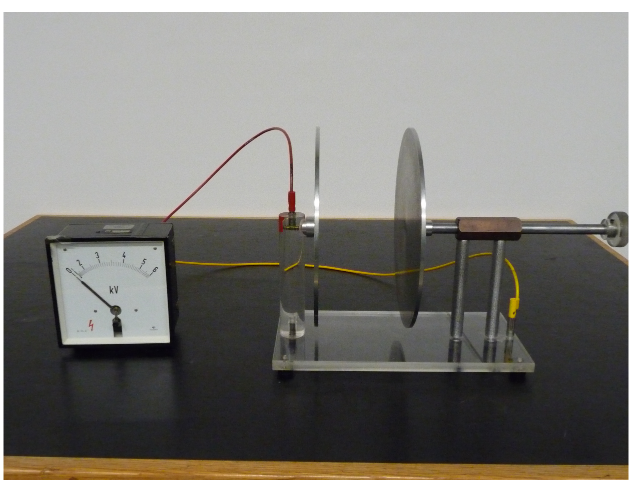
  The unit for capacitance is the Farad (F).
- The simplest type of capacitor involves two identical parallel plates with surface area A, separated by a distance d. The capacitance of such a parallel plate capacitor is

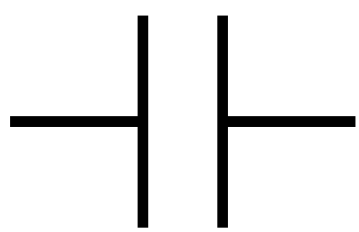
  $$C = \frac{\epsilon_0 A}{d}$$

- Capacitors help to smoothen out time-varying electrical signals

What is a capacitor?

A capacitor is a device that can store and release electrical energy. There are two terminals on a capacitor, and each is connected to a metal surface. The simplest version is when the two surfaces are identical and flat, and held parallel to one another. This is a **parallel-plate capacitor**

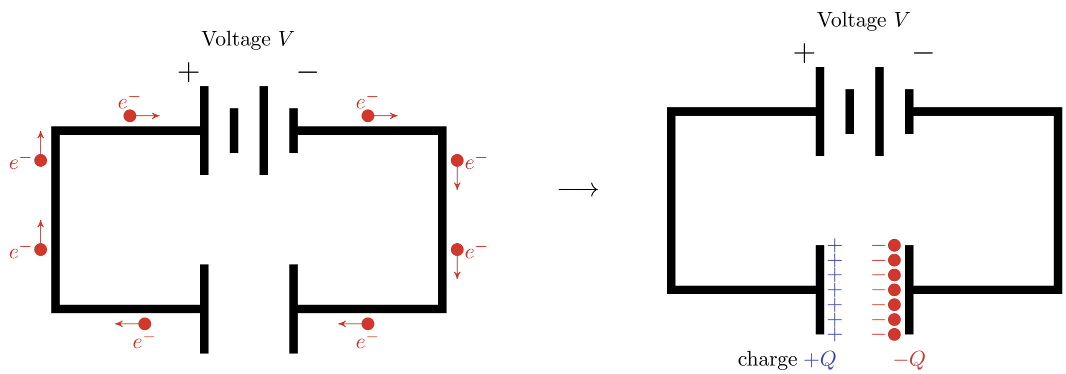




(Image credit: Washington University Physics Department) The left picture shows a capacitor made from circular metal plates, which are connected to wires on either side. We represent this using the diagram on the right, which is the standard symbol for a capacitor (whether or not it is made with parallel plates).

Connecting a battery to a capacitor

If we connect a battery to a capacitor, the battery will cause electrons to flow round the circuit, as shown on the left panel of the figure below. However, because the plates of a capacitor do not touch, there is no way for electrons to travel all the way around the loop of the circuit—instead, they start to accumulate on one of the plates



The battery has caused electrons to move from one of the plates to the other. This leaves a net positive charge on one plate, and a net negative charge on the other (see right figure above). But charge can't continue to build up and up for ever: Electrons repel each other so adding more of them onto the negative plate would require increasingly large amounts of energy. Because of this, the electrons will eventually stop flowing, at which point we will end up with a steady charge of $\pm Q$ on either plate.

The capacitance

How much charge will there be on the plates at equilibrium? This depends on:

- The voltage applied V: Higher potential differences will be able to push more charge onto the plates
- The geometry of the plates. For instance, a wider plate will accommodate more electrons, because they can be more spaced out and hence repel each other less

As we will see in an example later, in general the charge Q is directly proportional to the applied voltage V. The **capacitance** C for a particular capacitor is the coefficient of proportionality:

$$Q = C \times V \tag{1}$$
$$\text{charge} = \text{capacitance} \times \text{voltage}$$

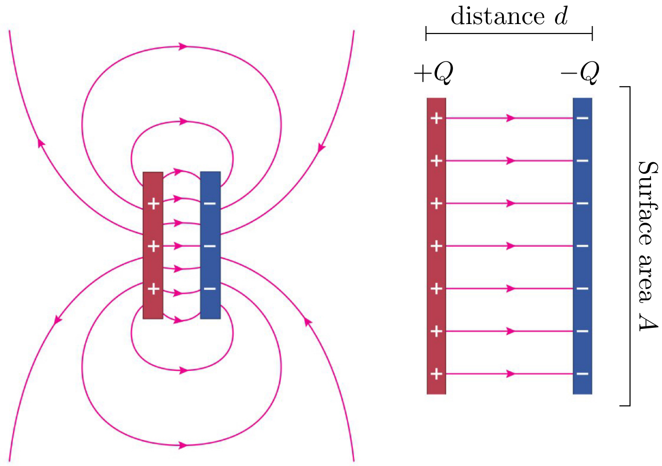
Since V is measure in units of Volts, and Q is measured in Coulombs, the unit for capacitance is

$$[\text{Volts}][\text{Coulomb}]^{-1} \equiv [\text{Farads}] \tag{2}$$

We *define* one Farad (F) as one Volt per Coulomb—this is the S.I. unit for capacitance. One Farad is actually a very large capacitance; in practice you will see capacitors measured in mircofarads $1\mu\text{F} = 10^{-6}\text{F}$.

The capacitance of a parallel-plate capacitor

With charges $\pm Q$ on the capacitor, let us look at what's going on in the region between the two plates. Due to the charges on each plate, there will be a strong electric field inside this space, directed from the positive to the negative charges



The left panel shows the actual electric field lines that one would get from two oppositely charged parallel plates separated by a distance d that is much smaller than the width of each plate. While there are some field lines that extend far away from the capacitor (these are called '*fringe fields*'), most of the field is concentrated in the region directly between the two plates. If d is indeed small, we can neglect the fringe fields and consider only the electric field inside the capacitor, as we see on the right figure. When we make this approximation, we say that we are considering an **ideal parallel-plate capacitor**. Because the electric field lines are all parallel to each other, their strength must be uniform everywhere inside the capacitor[1].

If we know the electric field E, we can work out how much energy it would take to move one extra electron from the positively charged to the negatively charged plate. Remember that the electric field strength lets us calculate the electrostatic force on a particle of charge q via $F = q \times E$. So, to move an electron of charge $(-e)$ from one plate to the other, we will need to do an amount of work equal to

$$W = \text{Force} \times \text{distance} = (eE) \times d \tag{3}$$

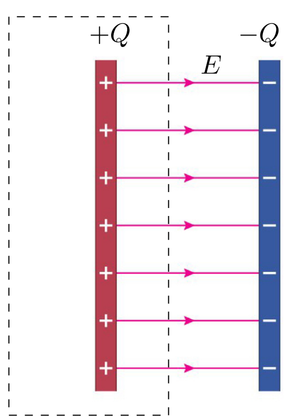
The battery can provide an energy of $e \times V$ to each electron. So, at equilibrium where the charge can no longer flow, we must have $W = eV$, i.e.

$$V = Ed \tag{4}$$

To calculate the electric field strength E, we need to recall Gauss's law:

> **Gauss's Law**: *The net electric flux through a closed surface is equal to the total charge contained within the surface, divided by the permittivity* ϵ_0

The surface that we choose encloses the positively charged plate, as shown by the dotted line here



[1]Think about why this has to be the case. Remember, every point on each plate is being held at the same electric potential.

In the idealised approximation, the electric field is nonzero only in the region directly between the two plates. So, to calculate the flux we need to multiply the field strength E by the area spanned by the electric field, which here is the same as the surface area of the plates A. Gauss' law gives us

$$\begin{aligned} \text{Electric flux} &= \text{Charge}/\epsilon_0 \\ \Rightarrow \quad EA &= Q/\epsilon_0 \end{aligned} \tag{5}$$

Taking Equations (5) and (4) together and rearranging, we find

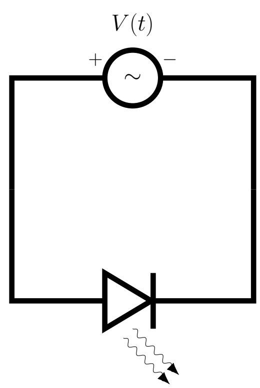
$$Q = \left(\frac{A\epsilon_0}{d} \right) V$$

which gives us the capacitance

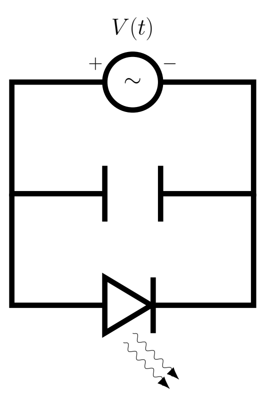
$$C = \frac{A\epsilon_0}{d} \tag{6}$$

What are capacitors used for?

One of the main practical uses for capacitors is to smooth out time-varying signals that would otherwise be 'jumpy', which could cause damage to electrical components. To see why, let's look at a very simple circuit with a time-dependent voltage source connected to a light source (in this case an LED)

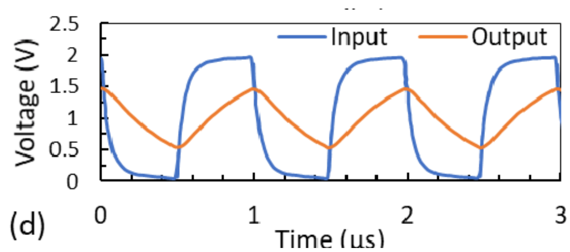


If the voltage $V(t)$ toggles rapidly between voltages 0 and $+V$, the LED will flicker on and off: It can only emit light while $V(t)$ is nonzero. Now let us add a capacitor in parallel:



When the voltage is on, the capacitor will charge up and the LED will emit light. But this time, when the voltage turns off abruptly, the charge in the capacitor will begin to rebalance, since the uneven distribution of charges results in a potential difference $V = Q/C$ across the capacitor. It can do so by flowing through the LED. So, even though the voltage source is turned off, current will continue to flow through the LED and the light will remain on, at least until all of the charge Q has rebalanced.

In general, the net effect of adding a capacitor into a circuit in this way is a smoothing out of the signal—this is known as a 'low-pass filter', reflecting the fact that high-frequency parts of the signal are removed, while the slow, low-frequency parts of the signal are preserved. Here is an example of this effect in action



(Image credit: Kashif Riaz) The curves show the original voltage profile as a function of time (blue), and the result of passing through this signal through a low-pass filter, constructed with a capacitor. See how the sudden jumps that appear in the input are transformed into more gradual variations in time.

Further Reading

- See Section 23.5 in Ref. [1] for additional discussion of the electric field in a parallel plate capacitor, and Chapter 28 therein for some more uses of capacitors in circuits.
- This video provides a quick and clear introduction to the fundamentals of capacitors and some of their uses `https://youtu.be/X4EUwTwZ110?feature=shared`

References

[1] R. Knight, *Physics for Scientists and Engineers: A Strategic Approach with Modern Physics* (Pearson Education, Incorporated, 2022).