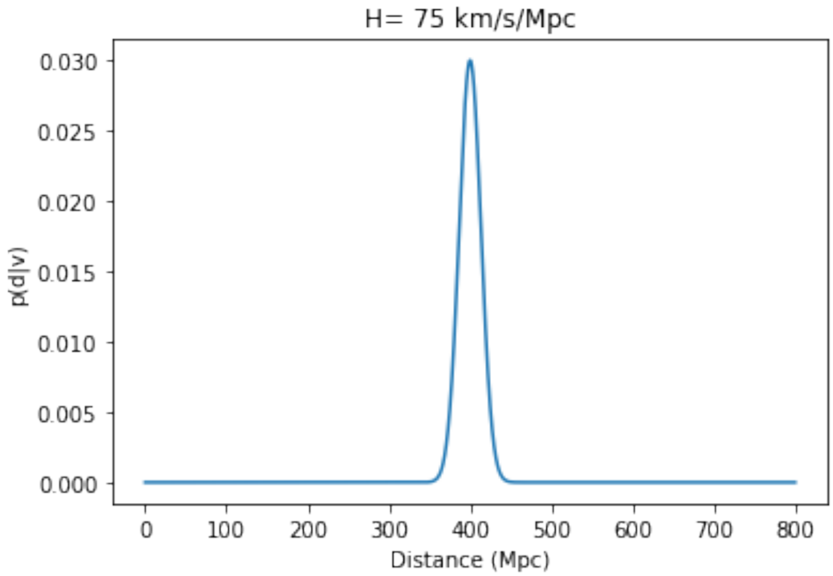
In [19]:
```
import math
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad, dblquad
```

In [354]:
```
#PART A

sig = 1e3
mu = 3e4
H = 75

A = 1/np.sqrt(2*np.pi*sig**2) #constant in front of gaussian

def num(x): #numerator to integrate
    return (A/800)*np.exp( -((H*x - mu)**2)/ (2*sig**2))

denom, err = quad(num,0,800) #integrate numerator (this is denominator
)
print(denom, err)

ys = lambda x: num(x)/denom

xpts = np.linspace(0,800,1e4) #make x points and plot
plt.plot(xpts,ys(xpts))
plt.xlabel('Distance (Mpc)')
plt.ylabel('p(d|v)')
plt.title('H= 75 km/s/Mpc')
```

```
1.666666666666735e-05 1.0792000544336891e-11
```

Out[354]:
```
Text(0.5, 1.0, 'H= 75 km/s/Mpc')
```



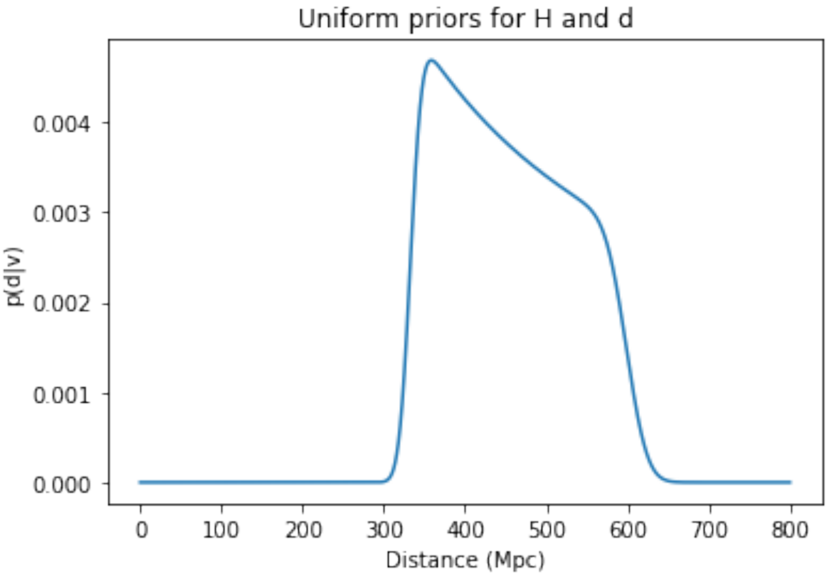
In [358]:
```
#PART B

def CDFa(x): #creating cumulative distribution function
    return quad(ys, 0,x)[0]

yBE = []
for i in xpts: #creating list of plotted points to find mode below
    yBE.append(ys(i))

for i in xpts: #Find point in CDF that corresponds to lowest 16% of di
st
    if CDFa(i) >= .16:
        print('Lower bound of 68% CI is: ',i,' Mpc')
        break

for i in xpts: #Same as above for highest 84% of dist
    if CDFa(i) >= .84:
        print('Upper bound of 68% CI is: ',i,' Mpc')
        break

m = yBE.index(max(yBE))#finding maximum of probability distribution
print('Best estimator is: Mode=',xpts.tolist()[m], ' Mpc')
```

```
Lower bound of 68% CI is:  386.75867586758676  Mpc
Upper bound of 68% CI is:  413.32133213321333  Mpc
Best estimator is: Mode= 399.9599959996  Mpc
```

In [359]:
```
#PART C

sig = 1e3
mu = 3e4

A = 1/np.sqrt(2*np.pi*sig**2)
Hpri = 1/40 #flat H prior
Xpri = 1/800 #flat d prior

def gauss(H,x): #integrand for denominator
    return Xpri*Hpri*A*np.exp( -((H*x - mu)**2)/ (2*sig**2))


denom, err = dblquad(gauss,0,800, lambda x: 50, lambda x: 90) #double integration
print(denom, err)


def numerator(x): #numerator as function of x
    func = lambda H: A*np.exp( -((H*x - mu)**2)/ (2*sig**2))*Xpri*Hpri
    num, err2 = quad(func, 50,90)
    return num


ypts = [] #making y points for prob(x|v)
for i in xpts:
    ypts.append(numerator(i)/denom)

plt.plot(xpts,ypts)
plt.xlabel('Distance (Mpc)')
plt.ylabel('p(d|v)')
plt.title('Uniform priors for H and d')
```

```
1.8368333332515972e-05 1.3185210470178896e-08
```

Out[359]:
```
Text(0.5, 1.0, 'Uniform priors for H and d')
```



In [362]:
```
#PART D

ys = lambda x: numerator(x)/denom #making cumulative distribution func
tion
def CDFc(x):
    return quad(ys, 0,x)[0]

for i in xpts: #Finding lower bound of CI
    if CDFc(i) >= .16:
        print('Lower bound of 68% CI is: ',i,' Mpc')
        break

for i in xpts: #Finding upper bound of CI
    if CDFc(i) >= .84:
        print('Upper bound of 68% CI is: ',i,' Mpc')
        break

weights= lambda x: x*numerator(x)/denom #finding expectation value
mean = quad(weights,0,800)

print('Best estimator is: Mean=',mean[0],' Mpc')
```

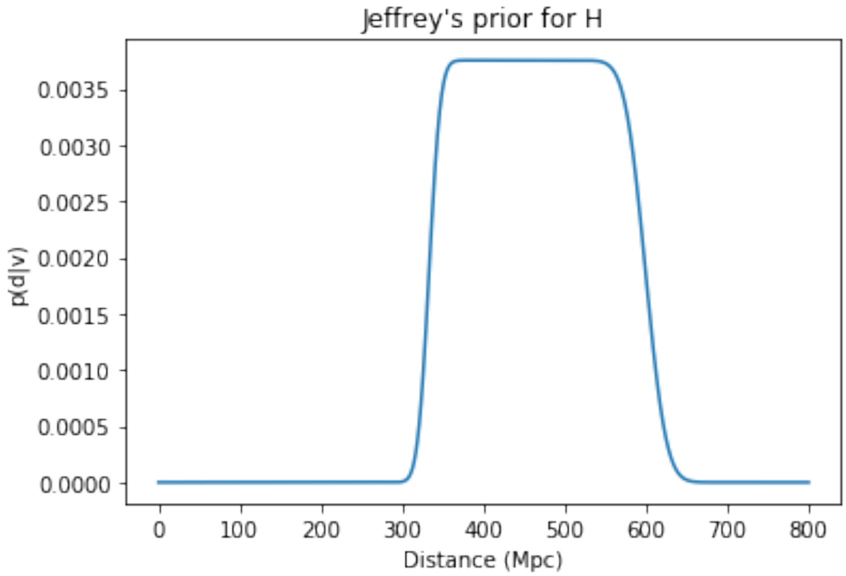
```
Lower bound of 68% CI is:  366.03660366036604  Mpc

/Users/maxmckie/opt/anaconda3/lib/python3.7/site-packages/ipykernel_
launcher.py:5: IntegrationWarning: The occurrence of roundoff error
is detected, which prevents
  the requested tolerance from being achieved.  The error may be
  underestimated.
  """

Upper bound of 68% CI is:  545.8945894589459  Mpc
Best estimator is: Mean= 453.67934123049673  Mpc
```

In [363]:
```
#PART E

sig = 1e3
mu = 3e4

A = 1/np.sqrt(2*np.pi*sig**2)
HHpri = 1/np.log(90/50) #Jeffrey's H prior multiplied by H
Xpri = 1/800 #still flat d prior

def gauss(H,x): #denominator
    return Xpri*HHpri*(1/H)*A*np.exp( -(1/2)*((H*x - mu)/sig)**2)


denom, err = dblquad(gauss,0,800, lambda x: 50, lambda x: 90) #double
integration of denominator
print(denom, err)


def numerator(x): #numerator as function of x
    func = lambda H: A*np.exp( -(1/2)*((H*x - mu)/sig)**2)*Xpri*(1/H)*
HHpri
    num, err2 = quad(func, 50,90)
    return num


ypts = [] #getting points for p(d|v)
for i in xpts:
    ypts.append(numerator(i)/denom)

plt.plot(xpts,ypts)
plt.xlabel('Distance (Mpc)')
plt.ylabel('p(d|v)')
plt.title("Jeffrey's prior for H")
```

```
1.890330595105745e-05 7.069192154215825e-09
```

Out[363]:

```
Text(0.5, 1.0, "Jeffrey's prior for H")
```



In [365]:
```
#PART F

ys = lambda x: numerator(x)/denom #creating CDF
def CDFe(x):
    return quad(ys, 0,x)[0]

for i in xpts: #Finding lower bound of CI
    if CDFe(i) >= .16:
        print('Lower bound of 68% CI is: ',i,' Mpc')
        break

for i in xpts: #Finding upper bound of CI
    if CDFe(i) >= .84:
        print('Upper bound of 68% CI is: ',i,' Mpc')
        break

weights= lambda x: x*numerator(x)/denom #calculating mean
mean = quad(weights,0,800)
print('Best estimator: Mean=',mean[0],' Mpc')
```

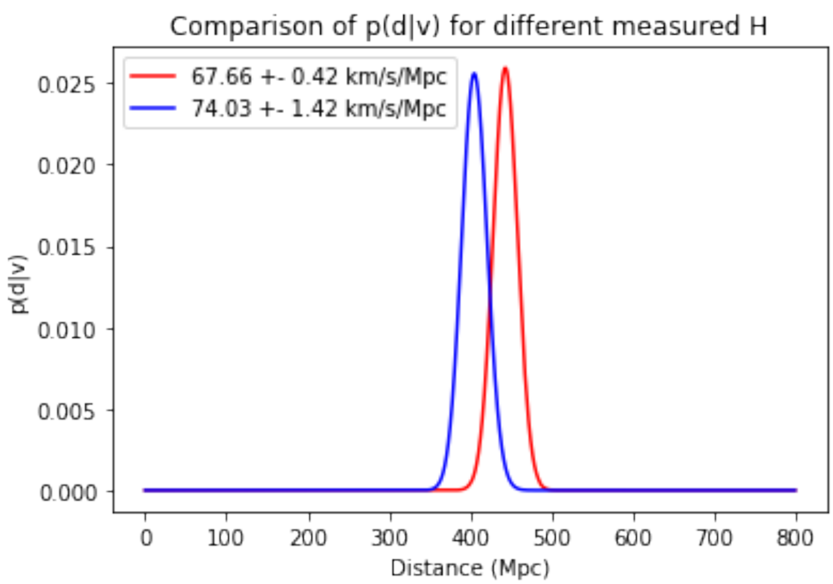
```
Lower bound of 68% CI is:  375.63756375637564  Mpc
Upper bound of 68% CI is:  556.8556855685569  Mpc
Best estimator: Mean= 466.66671175850985  Mpc

/Users/maxmckie/opt/anaconda3/lib/python3.7/site-packages/ipykernel_
launcher.py:18: IntegrationWarning: The occurrence of roundoff error
is detected, which prevents
  the requested tolerance from being achieved.  The error may be
  underestimated.
```

In [366]:
```
#PART G

sigv = 1e3
muv = 3e4

sigH1 = 1.42 #sigma and mu corresponding to measurements 1 and 2 in th
e
muH1 = 74.03 # problem statement
sigH2 = .42
muH2 = 67.66

A = 1/(sigv*np.sqrt(2*np.pi))  #constants in front of gaussian distrib
ution
B1 = 1/(sigH1*np.sqrt(2*np.pi))
B2 = 1/(sigH2*np.sqrt(2*np.pi))

Xpri = 1/800 #flat x prior (I used x instead of d)
```

```
def Hgauss1(H,x): #integrand for first denominator
    HHpri = B1*np.exp( -(1/2)*((H-muH1)/sigH1)**2)
    den1 = Xpri*HHpri*A*np.exp( -(1/2)*((H*x - muv)/sigv)**2)
    return den1


denom1, err1 = dblquad(Hgauss1,0,800, lambda x: 50, lambda x: 90) #dou
ble integral of 1st denominator
print(denom1)


def numerator1(x): #integrating H out of numerator so it is only funct
ion of x
    func = lambda H: A*B1*Xpri*np.exp( -(1/2)*((H*x - mu)/sig)**2) * n
p.exp( -(1/2)*((H-muH1)/sigH1)**2)
    num1 = quad(func, 50,90)[0]
    return num1


ypts1 = [] #making points to plot on y axis for 1st distribution
for i in xpts:
    ypts1.append(numerator1(i)/denom1)




def Hgauss2(H,x): #integrand for second denominator
    HHpri = B2*np.exp( -(1/2)*((H-muH2)/sigH2)**2)
    den2 = Xpri*HHpri*A*np.exp( -(1/2)*((H*x - muv)/sigv)**2)
    return den2


denom2, err2 = dblquad(Hgauss2,0,800, lambda x: 50, lambda x: 90) #dou
ble integral of 2nd denominator
print(denom2)


def numerator2(x): #integrating H out of numerator so it is only funct
ion of x
    func = lambda H: A*B2*Xpri*np.exp( -(1/2)*((H*x - mu)/sig)**2) * n
p.exp( -(1/2)*((H-muH2)/sigH2)**2)
    num2, err2 = quad(func, 50,90)
    return num2


ypts2 = [] #making points to plot on y axis for 2nd distribution
for i in xpts:
    ypts2.append(numerator2(i)/denom)

plt.plot(xpts,ypts2,'r',label = '67.66 +- 0.42 km/s/Mpc')
```

```
plt.plot(xpts,ypts1, 'b',label='74.03 +- 1.42 km/s/Mpc' )
plt.xlabel('Distance (Mpc)')
plt.ylabel('p(d|v)')
plt.title('Comparison of p(d|v) for different measured H')
plt.legend()
plt.show()
```

```
1.6891266951096314e-05
1.8474628402733928e-05
```



In [368]:
```
#PART H

ys1 = lambda x: numerator1(x)/denom1 #making CDFs for 1 and 2
def CDFg1(x):
    return quad(ys1, 0,x)[0]

ys2 = lambda x: numerator2(x)/denom2
def CDFg2(x):
    return quad(ys2, 0,x)[0]

#finding CI bounds for 1 and 2

for i in xpts:
    if CDFg1(i)>= .16:
        print('Lower bound of 68% CI for blue dist.: ',i,' Mpc')
        break

for i in xpts:
    if CDFg1(i)>= .84:
        print('Upper bound of 68% CI for blue dist.: ',i,' Mpc')
        break

for i in xpts:
    if CDFg2(i)>= .16:
        print('Lower bound of 68% CI for red dist.: ',i,' Mpc')
        break

for i in xpts:
    if CDFg2(i)>= .84:
        print('Upper bound of 68% CI for red dist.: ',i,' Mpc')
        break

weights1= lambda x: x*numerator1(x)/denom1 #Finding expectation values
for
weights2= lambda x: x*numerator2(x)/denom2 #  best estimators
mean1 = quad(weights1,0,800)
mean2 = quad(weights2,0,800)
print('Best estimator of red curve: Mean= ',mean2[0],' Mpc')
print('Best estimator of blue curve: Mean= ',mean1[0],' Mpc')
```

```
Lower bound of 68% CI for blue dist.:  390.03900390039007  Mpc
Upper bound of 68% CI for blue dist.:  421.0821082108211  Mpc
```

```
/Users/maxmckie/opt/anaconda3/lib/python3.7/site-packages/ipykernel_
launcher.py:9: IntegrationWarning: The occurrence of roundoff error
is detected, which prevents
  the requested tolerance from being achieved.  The error may be
  underestimated.
  if __name__ == '__main__':
```

```
Lower bound of 68% CI for red dist.:  428.52285228522857  Mpc
Upper bound of 68% CI for red dist.:  458.4458445844585  Mpc
Best estimator of red curve: Mean=  443.3897883265897  Mpc
Best estimator of blue curve: Mean=  405.539841809249  Mpc
```

```
/Users/maxmckie/opt/anaconda3/lib/python3.7/site-packages/ipykernel_
launcher.py:36: IntegrationWarning: The maximum number of subdivisio
ns (50) has been achieved.
  If increasing the limit yields no improvement it is advised to ana
lyze
  the integrand in order to determine the difficulties.  If the posi
tion of a
  local difficulty can be determined (singularity, discontinuity) on
e will
  probably gain from splitting up the interval and calling the integ
rator
  on the subranges.  Perhaps a special-purpose integrator should be
used.
```