* 1. *Solve and simulate the model with this new parametrization. Are the borrowing con-  
     straints binding? How did you reach this conclusion? Describe (qualitatively) the  
     consumption and wealth paths over the life cycle that you obtain. Then explain why,  
     given the theory of consumption that we have studied in class, you could have expected this result.*

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Automatisch generierte Beschreibung

According to the permanent income hypothesis the agent maximizes lifetime utility by achieving a flat consumption path. The agent knows her future income path, increasing monotonically and falling to zero after retirement. Thus, in the first periods the agent is smoothing consumption by borrowing and financing consumption up until period 4. In the optimum the agent then starts to save money for the final last two periods.

The borrowing constraints are not binding. This can be inferred from the flat consumption path and the asset path. The agent is borrowing, reaching a minimum of about -2 in asset holdings. Since the lower bound is set at -6 of the asset grid, and the asset path is not flat at any time, the borrowing constraint is not binding.

This pattern stems from the preference specification. Given the isoelastic utility function and the ….

The optimal consumption path would thus be to consume 2.2 units in every period. We do not obtain the exact same consumption path in python. This is because of the discretization of the asset grid by which the level of savings is determined in python. The steps of the asset grid (st.A) are not sufficient to calculate the optimal values exactly. However, the solution is reasonably close to the optimal outcome.

1.3)

|  |  |  |
| --- | --- | --- |
| r = 0 | Borrowing constraint | No borrowing constraint |
| Consumption | 21.9999999 | 21.999999999 |
| Utility | -4.9666748488885695 | -4.545496652604855 |

Consumption is equal in both scenarios since the interest rate is zero. In this case neither saving nor borrowing assets can changes the overall disposable income, which is solely determined by the income streams. Under the no borrowing constraint scenario, the agent is smoothing consumption by borrowing money in the first periods, yielding a flat consumption path. Due to the specification of the utility function with (u\_c > 0 AND u\_cc < 0), a stable consumption path maximizes lifetime utility. Under the borrowing constraint the agent has a lower level of consumption in the first periods, and a slightly higher level in the later periods. However due to the specification of the utility function, this leads to an overall lower level of welfare.

|  |  |  |
| --- | --- | --- |
| r = 0.05 | Borrowing constraint | No borrowing constraint |
| Consumption | 22.620842105263158 | 21.93684210526316 |
| Utility | -4.085138163715906 | -3.657731629454826 |

In the second case consumption is higher under the borrowing constraint. This is because of the interest rate, which affects the overall consumption over the lifetime in two ways. First, the interest rate increases the cost of borrowing. Secondly under the no borrowing constraint scenario the agent is saving slightly more assets, which will yield a higher rate of return. Both effects lead to the observed pattern, that the borrowing constraint increases the overall level of consumption. However, since the agent prefers smooth consumption, the overall level of utility is higher under the no borrowing constraint scenario as the agent can finance a higher level of consumption in the first periods.

1.4)

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Automatisch generierte Beschreibung

The lagrangian multiplier shows the marginal utility of relaxing the borrowing constraint by one unit. If mu is positive, the constraint is binding. The graph shows the trajectory over the lifetime of the agent. In the first period relaxing the borrowing constraint would bring large utility gains to the consumer as her income is low and consumption is low. The marginal utility of increasing consumption by one unit would therefore bring large benefits. In the subsequent periods the benefit declines exponentially. In period 3 the marginal benefit of relaxing the resource constraint is extremely small as income is already relatively high. From period four onwards the constraint is not binding.

1.5)

Comparing scenario, A and B, we find that discounting the future leads to a declining consumption path over the life cycle. As present consumption brings higher utility, the agent borrows heavily in the first four periods to finance consumption in earlier periods. In figure one the borrowing constraint becomes binding after four periods as assets reach the lowest level of -6. The interest rate is set to zero, which accounts for the fact that the agent is not incurring a cost for the high level of borrowing.

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Automatisch generierte Beschreibung

Figure 1 beta = 0.6, a\_min = -6

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Automatisch generierte Beschreibung

Figure 2 beta = 1, a\_min = -6

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Automatisch generierte Beschreibung

Figure 3 beta = 0.6 a\_min = 0

Comparing scenario B and C, we see that consumption tracks income growth over the first periods, as the borrowing constraint is binding. After seven periods the agent starts to save for retirement. Assets are increasing to finance the consumption over for the retirement period. Consumption declines after period six. The presence of discounting makes earlier consumption relatively more attractive than later consumption. Thus the optimal consumption path is declining over the lifetime of the consumer.

This scenario appears to track the empirically observed consumption and income patterns. We see the inverted U- (or in this case rather V shaped) consumption path. The presence of discounting necessitates a declining optimal consumption path, as earlier periods bring a higher level of utility. In period 6-8, when the agent starts to save for retirement, earnings exceed consumption. This can also be observed in the CES graphs. To summarize the graph reflects two distinctive empirical facts 1) the hump shaped consumption pattern, and 2) the drop of consumption with retirement.

*Problem 2. Please briefly describe the three empirical tests of the borrowing constraints*

*theory proposed by Zeldes (1989). In particular, make sure you mention: i) why he splits*

*the observations into two groups, ii) in which case and why each test supports the borrowing*

*constraints theory or the PIH.*

Zeldes (1989) tests the null hypothesis that households smooth consumption over their life cycle as predicted by the PIH against the alternative that agents are maximizing lifetime utility subject to a set of borrowing constraints. In contrast to other papers, he does not test the PIH against, either the alternative that the model does not fit the data or that consumers behave "Keynesian" (i.e. consumption equals a share of income). \

All tests are based on the Euler equation, derived from the maximization of expected lifetime utility. The equation implies that the marginal utility from consuming one unit today should be equal to the discounted expected marginal utility from what the agent would have received if the unit was invested and consumed in the future period. \

The first test is based on a regression of the growth of food consumption on a set of control variables and income. Zeldes (1989) splits the sample into two groups.

The sample restriction is based on whether the borrowing constraint is likely to be binding (Group 1) or not (Group 2). If a household at any given time period belongs to group 1, the Lagrangian multiplier on the constraint is necessarily positive. Relaxing the constraint would therefore increase expected lifetime utility. Likewise if a household belongs to group 2 current income should not have a positive effect on consumption growth. The split is based on the liquid financial assets to income ratio.

Zeldes (1989) uses a sample split because it is not possible to derive a closed-from expression of the Lagrangian multiplier. \\

The first test shows that for group 1, income reduces the growth of consumption statistically significantly, as predicted by the borrowing constraints theory. For group 2 income does not affect consumption growth significantly. The prediction of the Euler equation is thus violated for the borrowing constrained group 1, whilst in line for group 2.

The second test is based on the one-sided inequality of the Euler equation. The Lagrangian multiplier $\lambda\_{it}$ must be strictly positive. This is because households are borrowing, but not saving constraint. Zeldes (1989) tests the size and direction of the average Lagrangian multiplier for group 1 by constructing an estimate for the population level Lagrangian multiplier from group 2. He finds that there is a positive and statistically significant effect only at the $10\%$ level. Thus the borrowing constraint theory is somewhat supported in this case but the evidence is not strong.

The third test is based on whether the Lagrangian multiplier $\lambda\_{it}$ is negatively related to current real disposable income $y\_{it}$. The premise is that higher current income should lead to diminished utility from easing the borrowing constraints. Zeldes $(1989)$ uses the consistent estimate of group 1's average excess growth in consumption due to liquidity constraints, $\hat{x}\_{it+1}$. He finds a negative, albeit not statistically significant relationship between income and excess consumption. However this regression estimates a total derivative. Thus the results of the third test are theoretically and empirically not clear cut, since Zeldes cannot identify the partial derivative, with a ceteris paribus interpretation. Should increases in income signal an even higher anticipated future income stream as well, excess consumption growth might not be affected.