Advanced Macroeconomics II, Summer Term 2023
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Assignment 1

Please hand in your problem set answers electronically (in the group allocation you have been assigned to) through the 'assignment module' of the WU Canvas platform, stating your names and student numbers on the problem set. Please submit your answers in one zip file. The submission should include one concise document in which you collect and present all your main answers to the assignments' question(s). Derivations can be hand written, as long as they are legible. Also include all data files that have been used to obtain your answers! Please do not spilt work on this problem set by assigning individual group members to individual questions, but work on all questions together as a group!

Important: Please include the following written statement of confirmation at the beginning of your assignment answers and sign these by all group team members:

We hereby declare that the answers to the given assignment are entirely our own, resulting from our own work effort only.

Our team members contributed to the answers of the assignment in the following proportions:

Name of group member 1: XX%
Name of group member 2: XX%
Name of group member 3: XX% (if appropriate)

Question 1) Business cycle stylized facts (4 points)

a) For a country of your choice go to the EuroStat website to "Database by themes\Economy and finance\ National accounts (ESA 2010) (na10)\Quarterly national accounts (namq_10)\Main GDP aggregates (namq_10_ma)\GDP and main components (namq_10_gdp)". Download quarterly time series, seasonally and calendar adjusted, in constant prices (chain linked volumes (2010) in million euros), for the maximum length of data availability, for: real GDP (Y, "Gross domestic product at market prices"), consumption (C, "Final consumption expenditure of households", investment (I, "Gross fixed capital formation"), government expenditure (G, "Final consumption expenditure of general government". You can also use another data source and/or another country if you wish.

- Construct and report time series plots of the original raw (growing) data for Y, C, I, and G. Report sample means of C/Y, I/Y, G/Y.

- Obtain the stationary (cyclical) component of these macroeconomic time series through a method of your choice (e.g., by HP-filtering the logged time series for Y, C, I, G). Construct and report time series plots of these cyclical components.

- For the full length of your downloaded data, construct and report a summary table on business cycle stylized facts (standard deviations, relative standard deviations and contemporaneous output correlations) of the cyclical component of these macroeconomic time series. Now do the same for the following two subsample splits: start of your data until 2007Q4, 2008Q1 until end. Discuss and interpret your results!

Question 2) A real business cycle model with energy price shocks (5 points)

Consider a benevolent social planner that aims to maximize lifetime expected discounted utility of the representative household:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \theta \log(1 - N_t) \tag{1}$$

subject to the following resource constraint:

$$C_t + K_{t+1} + P_t EN_t \leq A_t K_t^{\alpha} N_t^{\gamma} EN_t^{1-\alpha-\gamma} + (1-\delta) K_t \tag{2}$$

where, C_t, is consumption expenditure, N_t, is hours worked, K_t is the capital stock, EN_t is energy, and parameters β, σ, α, γ, δ, θ reflect the discount factor, the coefficient of relative risk aversion, the capital share, the labor share, the depreciation rate and a preference parameter on labor supply. In addition variables A_t and P_t (the energy price) are exogenously modeled as (log) autoregressive processes, capturing a shock to total factor productivity and an energy price shock:

$$\begin{aligned} \log A_{t+1} &= \rho_A \log A_t + \varepsilon_{A,t+1}, \\ \log P_{t+1} &= \rho_P \log P_t + \varepsilon_{P,t+1}, \end{aligned} \tag{3}$$

their steady state values being normalized to one, i.e., $\overline{A} = 1$, $\overline{P} = 1$.

The social planner maximizes, for all t, objective function (1) subject to the resource constraint (2), and non-negativity constraints ($C_t, K_{t+1} \geq 0$, K_0, A_0, P_0 given):

a) Set up the social planner's intertemporal optimization problem and, step by step, solve for the first order optimality conditions. List the variables whose optimal time paths are described by the optimality conditions and be sure you end up with as many equations (label them) as you have variables in your system. To define investment, realize it is governed by the capital law of motion. If you subsituted out for Lagrange multiplier λ_t, you should, this way, arrive at a system of 8 equations (forming a nonlinear expectational system of difference equations) in 8 variables: Y_t, C_t, I_t, K_t, N_t, EN_t, A_t, P_t.

b) Provide an economic interpretation for each of your first order and equilibrium conditions (that is, explain their meaning in words).

Question 3) Understanding impulse responses and model simulation *(total of 8 points)*

a. *(1.5 points)* Solve for the steady state. Take parameter values to be $\beta = 0.99$, $\sigma = 1$, $\theta = 3.48$, $\alpha = 0.3$, $\delta = 0.025$, $\overline{A} = 1$, $\overline{P} = 1$, $\rho_A = 0.95$, $\sigma_A = 0.007$, $\rho_P = 0.5$ $\sigma_P = 0.00001$. Given these, compute the steady state values of $\overline{C}$, $\overline{K}$, $\overline{N}$, $\overline{Y}$, $\overline{I}$, $\overline{EN}$, $\overline{A}$,$\overline{P}$. (Hint: To obtain the steady state values of $\overline{C}$, $\overline{K}$, $\overline{N}$, $\overline{EN}$, you need to obtain them from the steady-state versions of the system you found in question 2. This may appear difficult, so here is my recommendation for how you approach this: starting from the steady-state capital Euler equation, solve for the steady-state ratio $\frac{\overline{K}}{\overline{Y}}$, as a function of parameters only. Notice that from the FOC w.r.t. EN_t you directly can find the expression for steady state ratio $\frac{\overline{EN}}{\overline{Y}}$. Then, from the steady-state version of the resource constraint, solve for the steady-state ratio $\frac{\overline{C}}{\overline{Y}}$ as a function of only parameters and the (now known) steady-state ratio $\frac{\overline{K}}{\overline{Y}}$ and $\frac{\overline{EN}}{\overline{Y}}$. Finally, from the steady-state version of the consumption-labor equation solve for $\overline{N}$ as a function of only parameters and the (now known) ratio $\frac{\overline{C}}{\overline{Y}}$. From the steady state production function compute $\overline{Y}$. Once you have $\overline{Y}$, you can of course back out $\overline{C} = \frac{\overline{C}}{\overline{Y}}\overline{Y}$, etc., and compute all other steady state variables).

b. *(4 points)* Write a Dynare code that solves the RBC model you solved for in Question 2, so that you obtain a solution in terms of *percentage deviations from steady state* (i.e., variables in logs or in terms of hat variables). Code up the model so that including the relevant 8 equations that describe the behavior of the following 8 variables: Y_t, C_t, I_t, K_t, N_t, EN_t, A_t, P_t. Make sure you choose the correct timing for your state variables (in particular, recall that what appears as K_{t+1}(respectively, K_t) in the problem set or in the slides of lecture block one, needs to be entered as K (respectively K(-1)) in the dynare code. Report the solution you get from Dynare (the policy functions for your variables).

c. *(2 points)* Construct and plot impulse responses (for 20 periods) to a 1% productivity decrease (i.e. $\varepsilon_{A,t} = -0.01$ in period $t = 1$ and $\varepsilon_{A,t} = 0$ for all other $t \neq 1$), for variables $\widehat{Y}_t$, $\widehat{C}_t$, $\widehat{I}_t$, $\widehat{K}_t$, $\widehat{N}_t$, $\widehat{EN}_t$, $\widehat{A}_t$, $\widehat{P}_t$. Give a detailed interpretation of why you see **each** of the variables move in response to the shock as they do.

d. *(1 point)* b) Repeat the same for a 10 percent energy price shock in period 1 (i.e. $\varepsilon_{P,t} = 0.1$ in period $t = 1$ and $\varepsilon_{P,t} = 0$ for all other $t \neq 1$).

e. *(3.5 points)* Log-linearize all 8 first order and equilibrium conditions.

f. *(1.5 points)* Solve the above model by mapping them into matrix format and usinig a linear rational expectations solution algorithm like the one used in klein.m. Having solved for the steady state, and having log-linearized, map your resulting log-linear system into format: $AE_t z_{t+1} = Bz_t$. Write a code where you define these matrices and solve for the policy functions via the klein.m algorithm. Report the policy functions you obtain. If you want you can also derive impulse responses as in exercises check if your results are identical to question c. and d.