Advanced Macroeconometrics – Assignment 4

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June 30, 2023

Contents

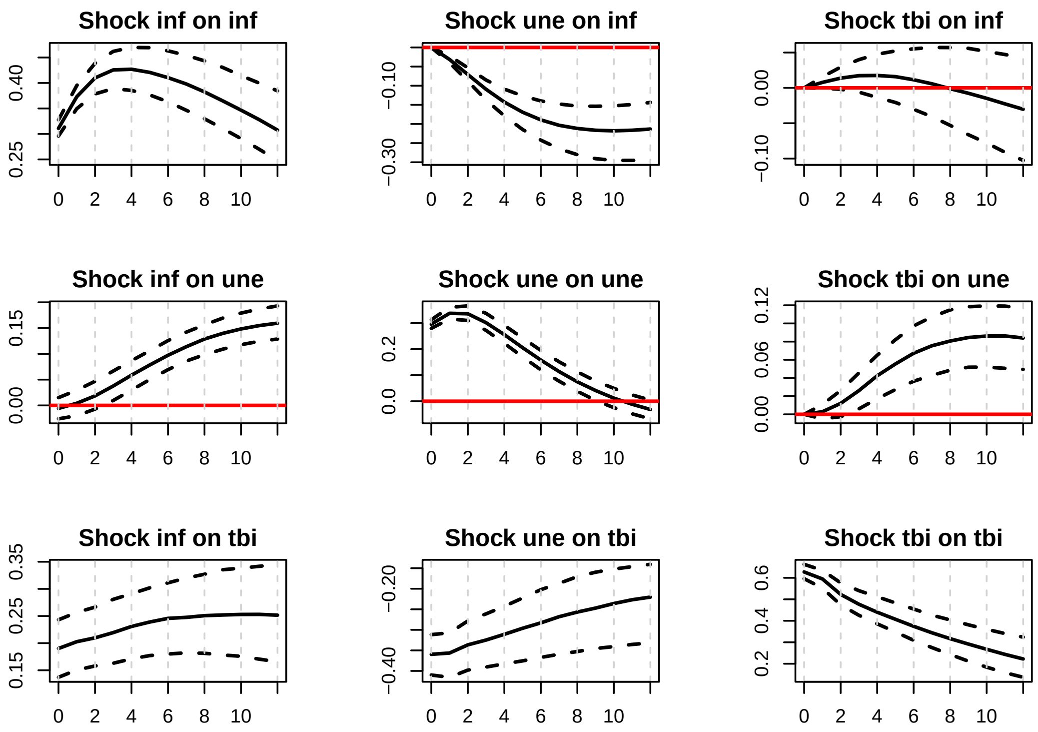
The executable code that was used in compiling the assignment is available on GitHub at https://github.com/maxmheinze/macrometrics.

Exercise 1 – Different Prior Values for the Variance

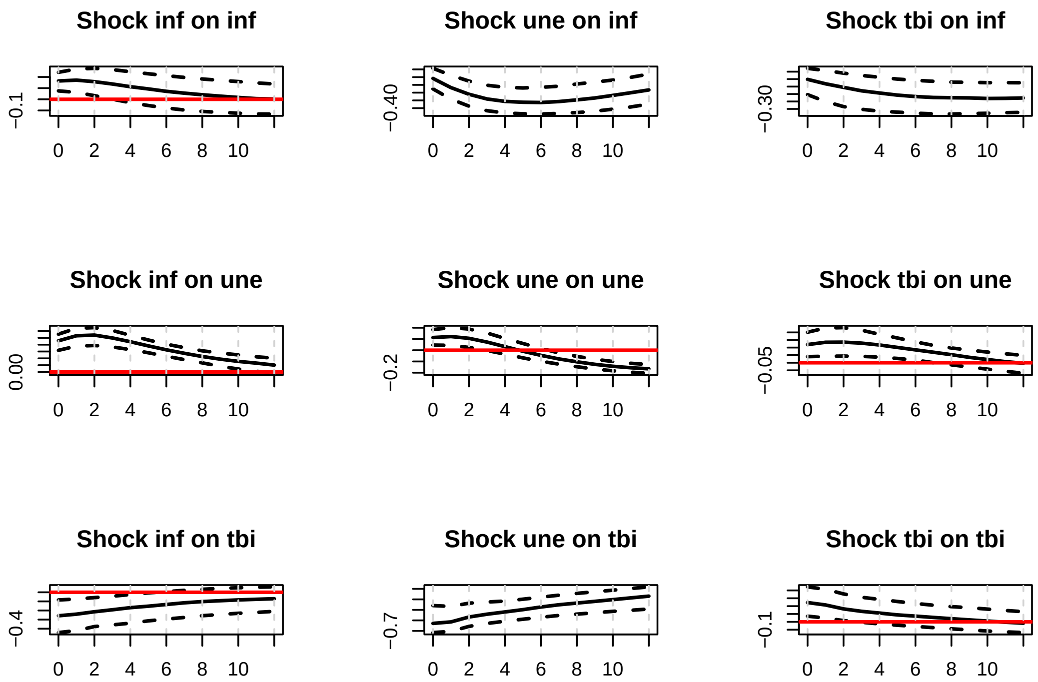
Using the sample code provided, we estimate the VAR using different λ_1 and λ_2 values for the Minnesota prior.

Default Lambda Values

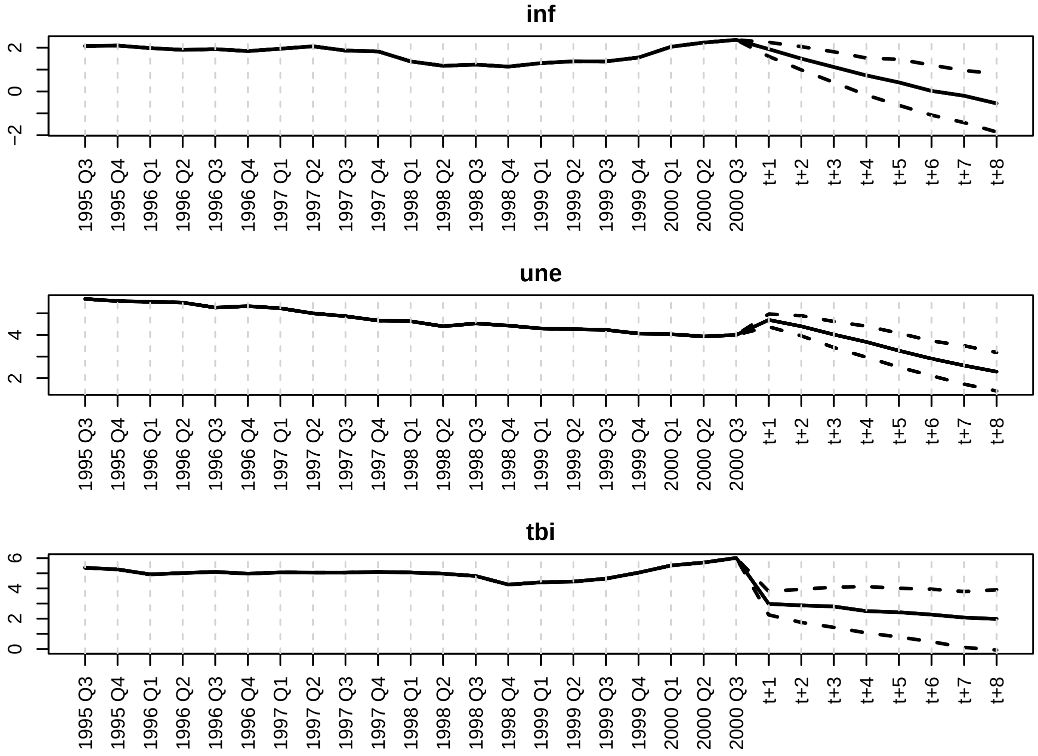
Sign IRFs



Cholesky IRFs



Predictions



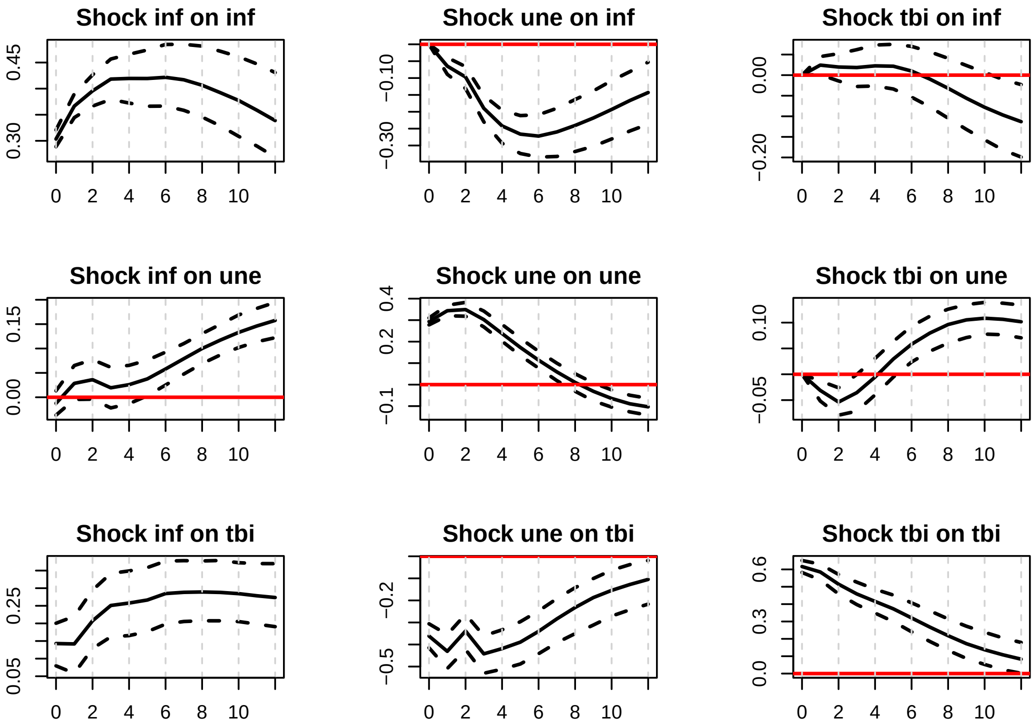
Coefficients

Coefficient means and standard deviations for the values $\lambda_1 = 0.1$ and $\lambda_2 = 0.5$ are as follows:

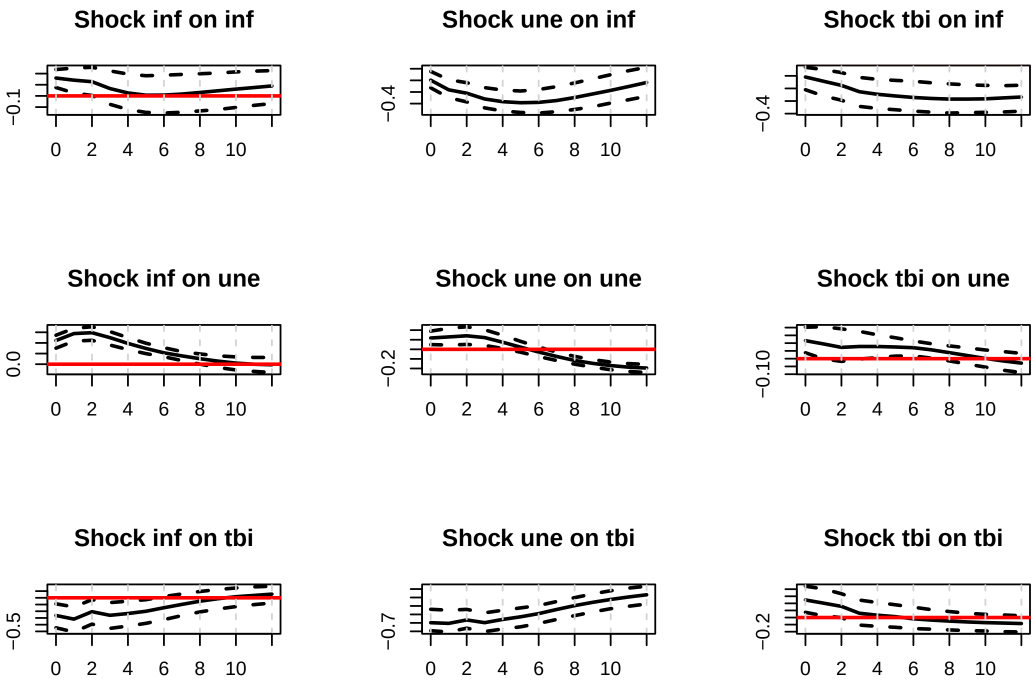
	inf	une	tbi
inf, 1	1.187 (0.0405)	0.0307 (0.0228)	0.0781 (0.0584)
une, 1	-0.0911 (0.0281)	1.1473 (0.0405)	-0.0578 (0.0658)
tbi, 1	0.012 (0.0128)	0.0046 (0.0126)	0.9455 (0.0446)
inf, 2	-0.1121 (0.0429)	-0.0018 (0.0205)	0.004 (0.0497)
une, 2	1e-04 (0.0235)	-0.1543 (0.0417)	0.0089 (0.0563)
tbi, 2	-0.0032 (0.0095)	0.0089 (0.0096)	-0.0622 (0.039)
inf, 3	-0.0577 (0.0284)	-4e-04 (0.014)	-0.0118 (0.0355)
une, 3	0.0094 (0.016)	-0.0747 (0.0275)	0.0148 (0.037)
tbi, 3	-0.0025 (0.007)	0.0073 (0.0063)	0.0348 (0.028)
inf, 4	-0.0275 (0.0218)	0.0037 (0.0108)	-0.0043 (0.0266)
une, 4	0.0123 (0.0124)	-0.0112 (0.0203)	0.0252 (0.0285)
tbi, 4	-0.0018 (0.0052)	0.0037 (0.0046)	0.0025 (0.0224)
const	0.4208 (0.0976)	0.2928 (0.0921)	0.2584 (0.2247)

Variant 1: $\lambda_1 = 0.1$, $\lambda_2 = 100$

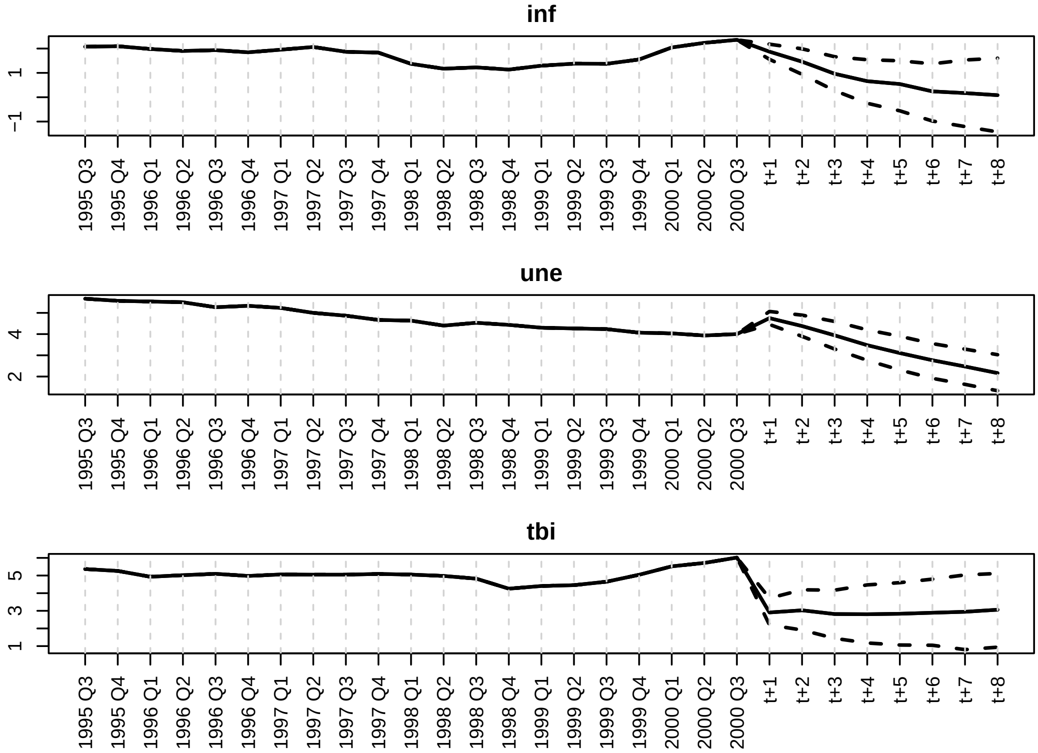
Sign IRFs



Cholesky IRFs



Predictions



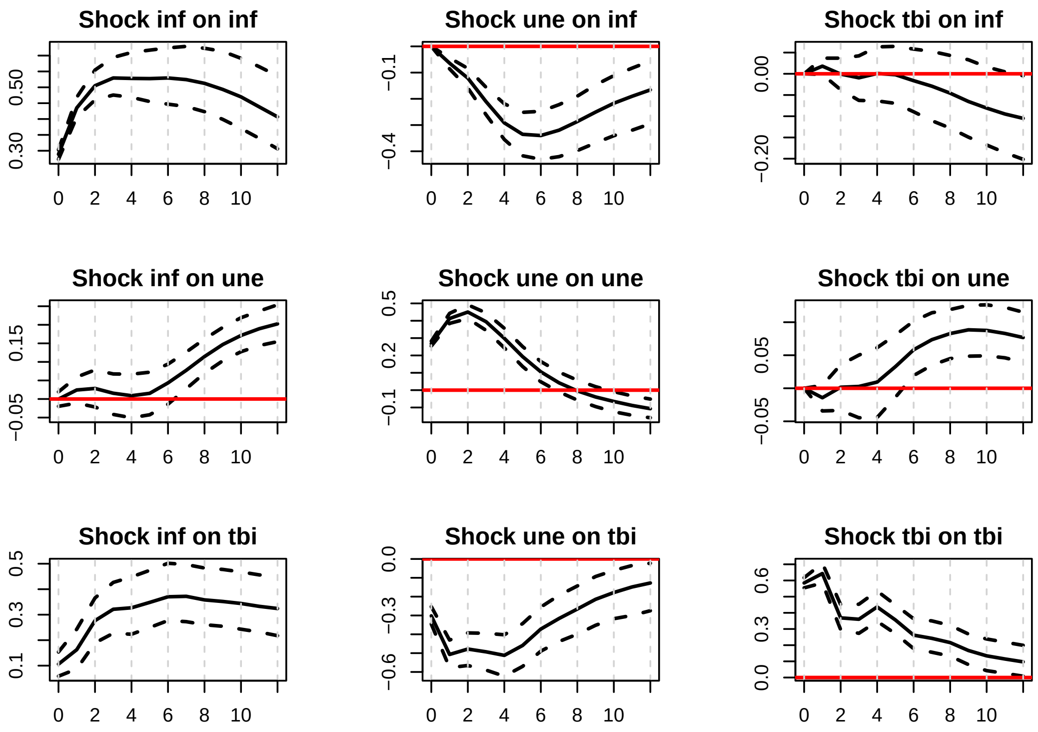
Coefficients

Coefficient means and standard deviations for the values $\lambda_1 = 0.1$ and $\lambda_2 = 100$ are as follows:

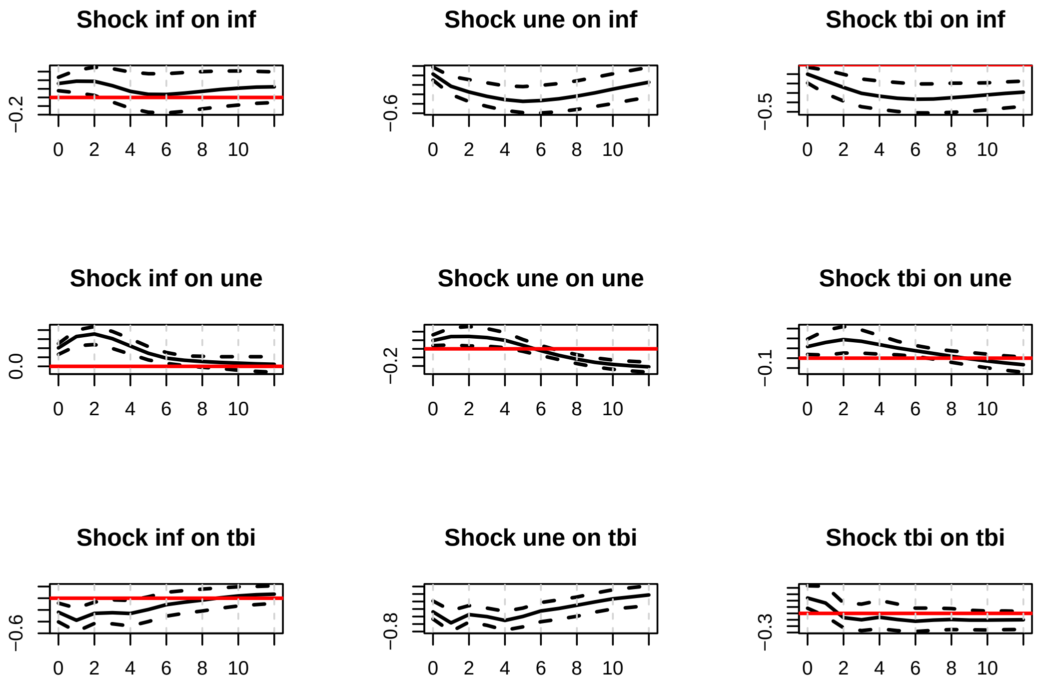
	inf	une	tbi
inf, 1	1.1835 (0.0406)	0.1634 (0.08)	0.0068 (0.192)
une, 1	-0.1703 (0.0945)	1.1092 (0.0458)	-0.3072 (0.2006)
tbi, 1	0.0384 (0.0372)	-0.0518 (0.0313)	0.9487 (0.0478)
inf, 2	-0.0961 (0.0419)	-0.1722 (0.1382)	0.3344 (0.3247)
une, 2	0.1064 (0.162)	-0.1384 (0.0429)	0.4981 (0.3448)
tbi, 2	-0.0596 (0.0524)	0.0139 (0.0443)	-0.0837 (0.0416)
inf, 3	-0.0551 (0.0287)	-0.0598 (0.1319)	-0.2478 (0.3235)
une, 3	-0.2261 (0.1573)	-0.066 (0.0283)	-0.6519 (0.313)
tbi, 3	0.0121 (0.0527)	0.0618 (0.0436)	0.0212 (0.0293)
inf, 4	-0.0244 (0.0208)	0.1042 (0.0738)	0.0043 (0.1863)
une, 4	0.2239 (0.0841)	-0.0215 (0.0211)	0.4959 (0.1631)
tbi, 4	0.0033 (0.0352)	0.0122 (0.0296)	0.0043 (0.0226)
const	0.3953 (0.1079)	0.3577 (0.1077)	0.0558 (0.2643)

Variant 2: $\lambda_1 = 100$, $\lambda_2 = 0.1$

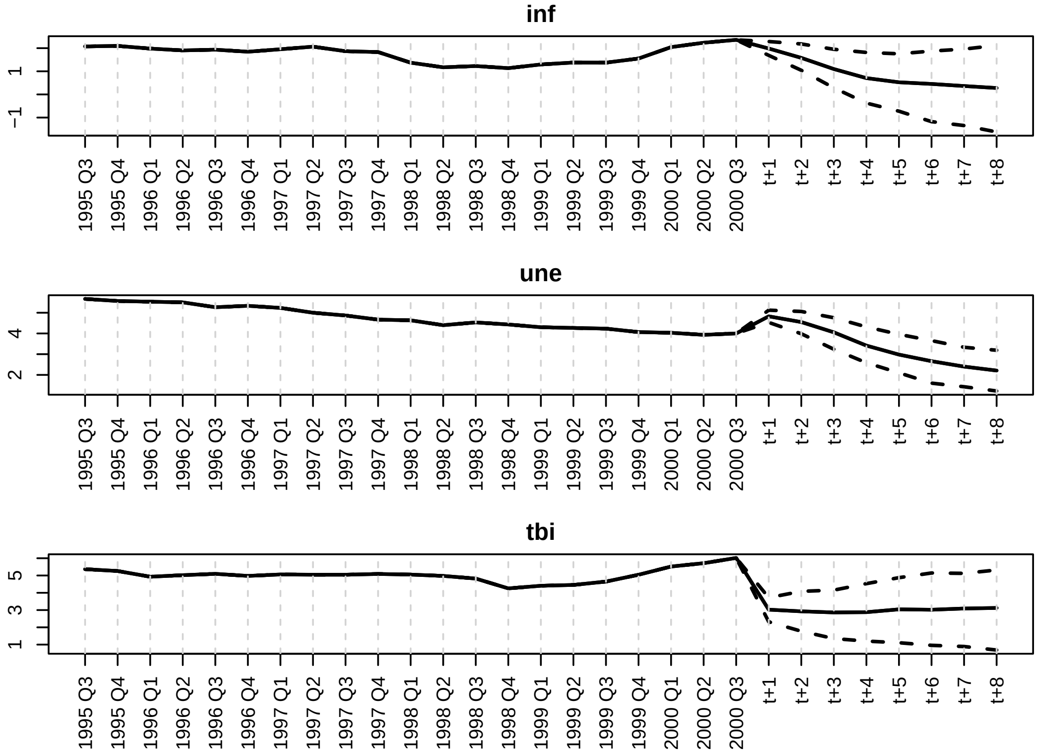
Sign IRFs



Cholesky IRFs



Predictions



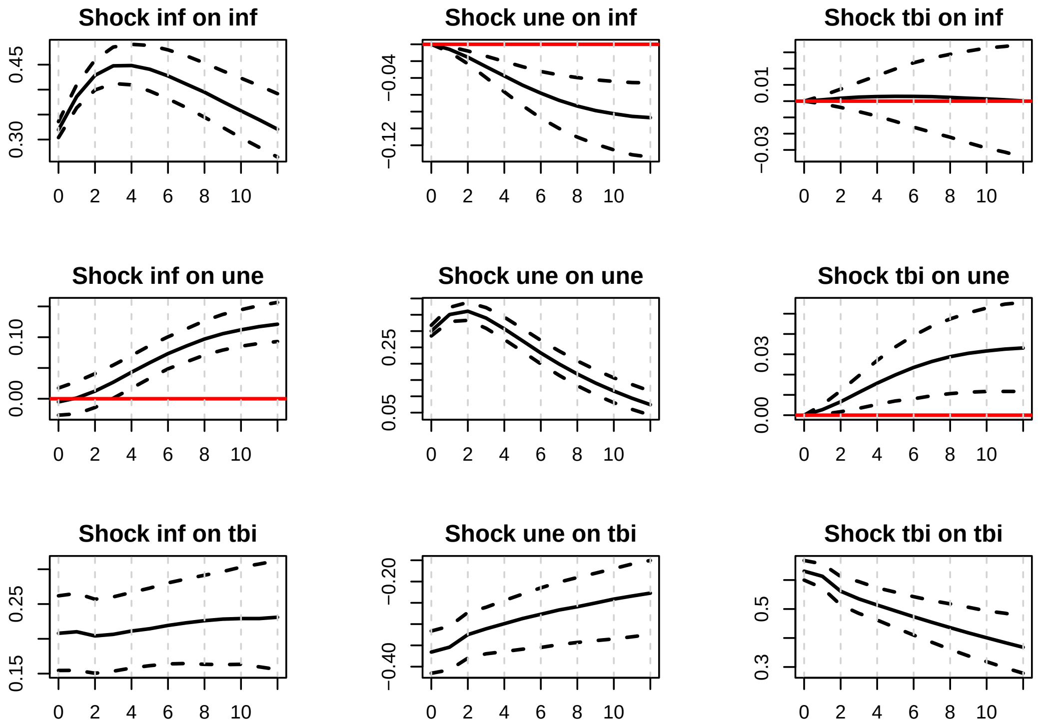
Coefficients

Coefficient means and standard deviations for the values $\lambda_1 = 100$ and $\lambda_2 = 0.1$ are as follows:

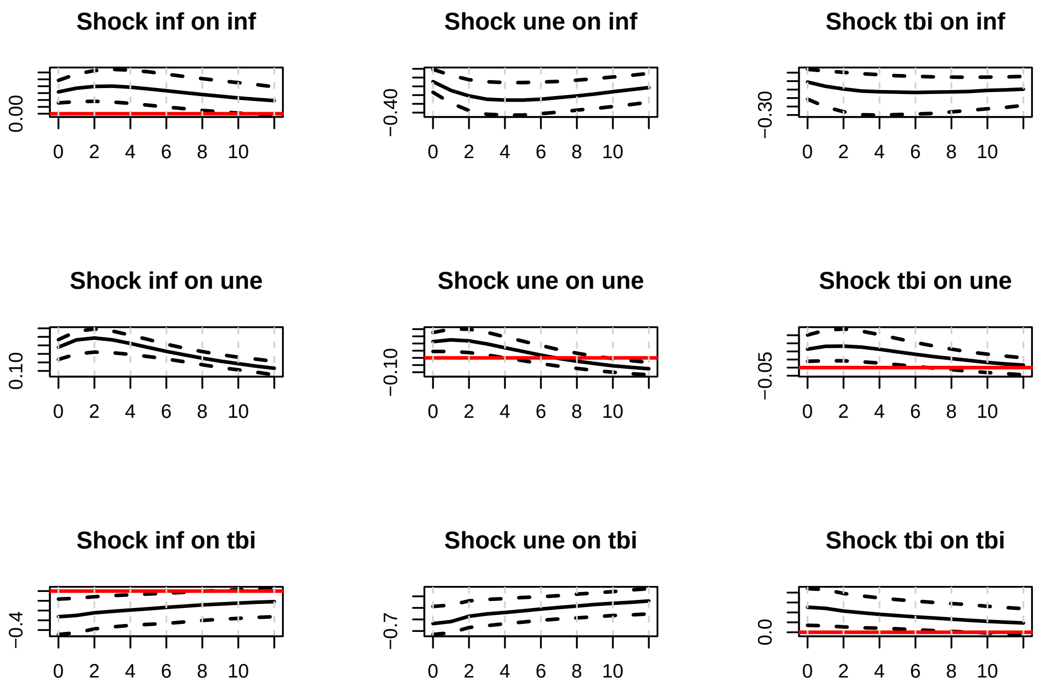
	inf	une	tbi
inf, 1	1.5011 (0.0764)	0.0954 (0.0677)	0.1701 (0.1724)
une, 1	-0.2067 (0.0872)	1.5152 (0.0803)	-0.6461 (0.1999)
tbi, 1	0.0304 (0.0342)	-0.0256 (0.0326)	1.1007 (0.0838)
inf, 2	-0.4847 (0.134)	-0.1869 (0.1198)	0.3483 (0.3152)
une, 2	0.1926 (0.1507)	-0.6062 (0.1428)	0.661 (0.3684)
tbi, 2	-0.0844 (0.0482)	0.0669 (0.0455)	-0.6016 (0.1181)
inf, 3	-0.0682 (0.1315)	0.0773 (0.123)	-0.6176 (0.3153)
une, 3	-0.1901 (0.146)	-0.1435 (0.1392)	-0.1227 (0.3658)
tbi, 3	0.0764 (0.0507)	-0.0667 (0.0458)	0.59 (0.1173)
inf, 4	0.0548 (0.0766)	0.0429 (0.0714)	0.2011 (0.1743)
une, 4	0.153 (0.0802)	0.1414 (0.0726)	0.1177 (0.1905)
tbi, 4	-0.0249 (0.0362)	0.0519 (0.0339)	-0.1844 (0.0847)
const	0.3069 (0.1031)	0.2949 (0.0966)	0.1108 (0.2431)

Variant 3: $\lambda_1 = 0.1,\ \lambda_2 = 0.1$

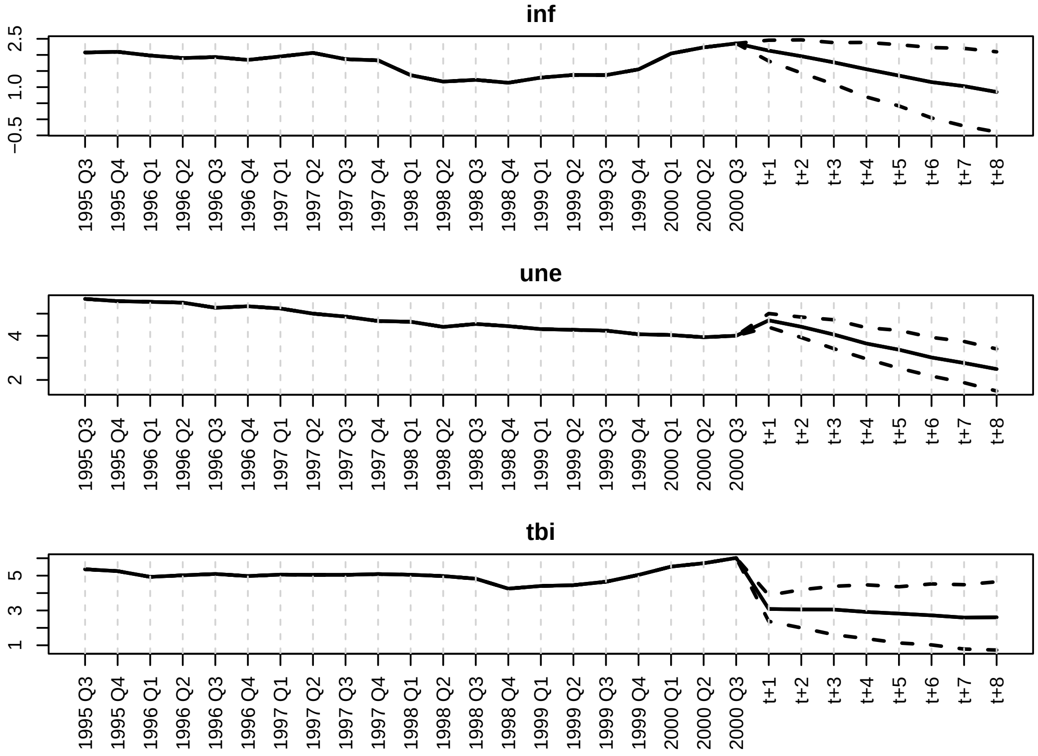
Sign IRFs



Cholesky IRFs



Predictions



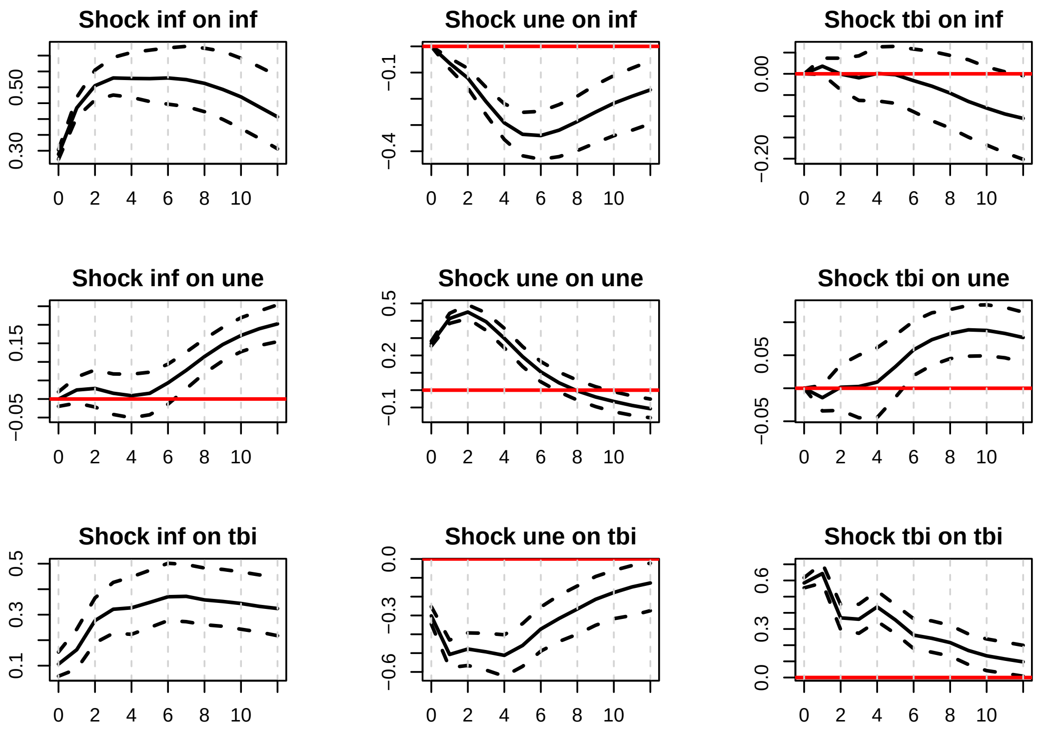
Coefficients

Coefficient means and standard deviations for the values $\lambda_1 = 0.1$ and $\lambda_2 = 0.1$ are as follows:

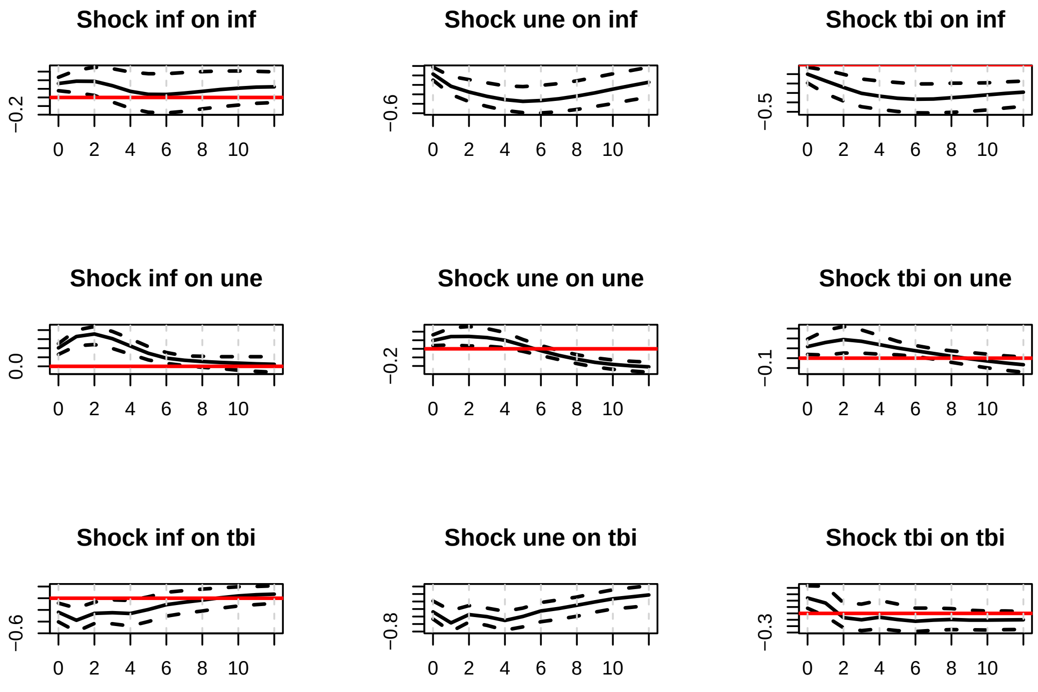
	inf	une	tbi
inf, 1	1.2083 (0.0398)	0.0201 (0.0074)	0.0246 (0.0187)
une, 1	-0.0189 (0.0098)	1.1708 (0.0391)	-0.0032 (0.0213)
tbi, 1	0.0013 (0.0041)	0.0044 (0.0038)	0.9706 (0.0416)
inf, 2	-0.1217 (0.0419)	0.0043 (0.0044)	0.0046 (0.0108)
une, 2	-0.0035 (0.0052)	-0.1604 (0.0417)	-1e-04 (0.0119)
tbi, 2	0 (0.0021)	0.0015 (0.002)	-0.0526 (0.0403)
inf, 3	-0.0707 (0.0274)	0.0018 (0.003)	0.0011 (0.0077)
une, 3	-9e-04 (0.0034)	-0.0775 (0.0271)	3e-04 (0.0083)
tbi, 3	-2e-04 (0.0015)	7e-04 (0.0013)	0.0358 (0.0288)
inf, 4	-0.0404 (0.0207)	0.001 (0.0022)	6e-04 (0.0058)
une, 4	0 (0.0027)	-0.0105 (0.0194)	0.001 (0.006)
tbi, 4	-2e-04 (0.0011)	4e-04 (9e-04)	0.0028 (0.0213)
const	0.2241 (0.0679)	0.3182 (0.0834)	0.1472 (0.1619)

Variant 4: $\lambda_1 = 100$, $\lambda_2 = 100$

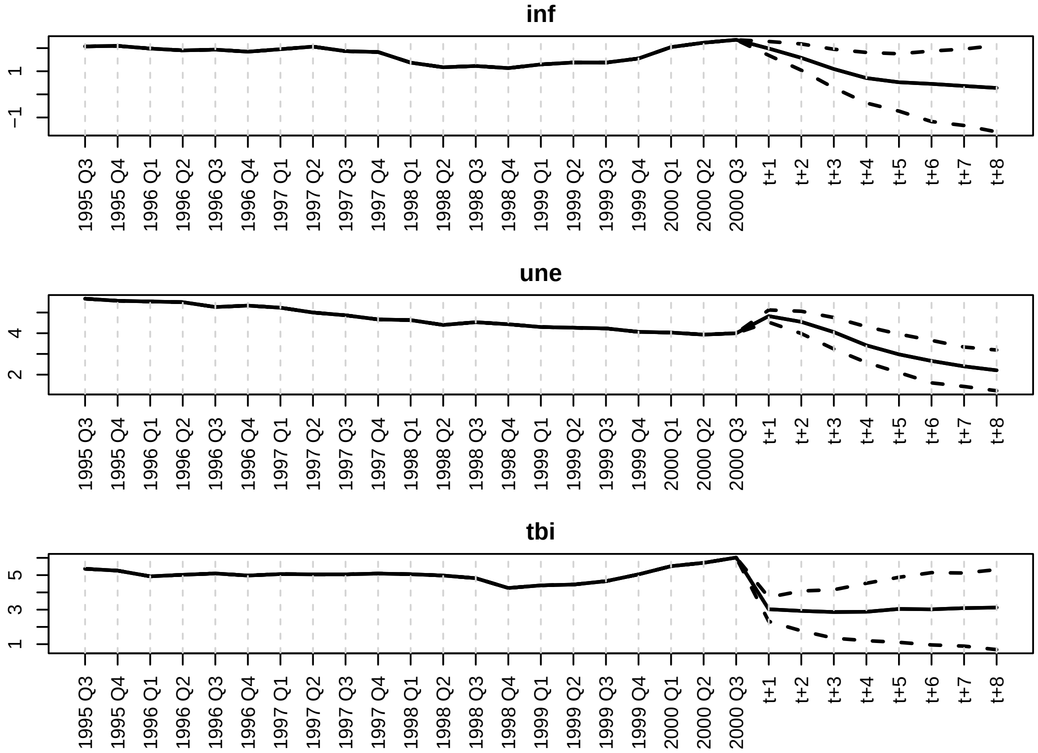
Sign IRFs



Cholesky IRFs



Predictions



Coefficients

Coefficient means and standard deviations for the values $\lambda_1 = 100$ and $\lambda_2 = 100$ are as follows:

	inf	une	tbi
inf, 1	1.5011 (0.0764)	0.0955 (0.0677)	0.1698 (0.1725)
une, 1	-0.2068 (0.0872)	1.5153 (0.0803)	-0.6465 (0.2)
tbi, 1	0.0304 (0.0342)	-0.0256 (0.0326)	1.1008 (0.0838)
inf, 2	-0.4846 (0.134)	-0.1872 (0.1199)	0.3495 (0.3155)
une, 2	0.1931 (0.1509)	-0.6062 (0.1429)	0.6619 (0.3688)
tbi, 2	-0.0844 (0.0483)	0.067 (0.0456)	-0.6018 (0.1182)
inf, 3	-0.0683 (0.1315)	0.0778 (0.1232)	-0.6192 (0.3158)
une, 3	-0.1907 (0.1462)	-0.1435 (0.1393)	-0.1234 (0.3664)
tbi, 3	0.0765 (0.0508)	-0.0668 (0.0458)	0.5902 (0.1174)
inf, 4	0.0548 (0.0766)	0.0427 (0.0715)	0.2019 (0.1745)
une, 4	0.1532 (0.0803)	0.1415 (0.0726)	0.1179 (0.1907)
tbi, 4	-0.025 (0.0362)	0.052 (0.0339)	-0.1846 (0.0847)
const	0.3069 (0.1031)	0.2949 (0.0966)	0.1109 (0.2431)

Discussion

In the above, we estimated the VAR model with 4 lags (unchanged from the provided code) using different combinations of values for λ_1 and λ_2 (where $\lambda_3 = 100$ as usual):

- The "default" values, $\lambda_1 = 0.1$ and $\lambda_2 = 0.5$,
- $\lambda_1 = 0.1$ and $\lambda_2 = 100$,
- $\lambda_1 = 100$ and $\lambda_2 = 0.1$,
- $\lambda_1 = 0.1$ and $\lambda_2 = 0.1$, and
- $\lambda_1 = 100$ and $\lambda_2 = 100$.

To see what these different values do in theory, we revisit the setup of the Minnesota prior for the variance:

$$\underline{\boldsymbol{V}} = \mathrm{Var}(\boldsymbol{A}) = \begin{cases} \left(\frac{\lambda_1}{k}\right)^2 & \text{for } i = j \text{ and the } k\text{-th lag} \\ \left(\frac{\sigma_i^2}{\sigma_j^2}\right)\left(\frac{\lambda_1 \lambda_2}{k}\right)^2 & \text{for } i \neq j \text{ and the } k\text{-th lag} \\ \lambda_3 \sigma_i^2 & \text{for the deterministic part of the model} \end{cases}$$

We can see that λ_1 governs shrinkage for own-variable lags, while λ_2 governs shrinkage for cross-variable lags, where in both cases, a higher value of the respective hyperparameter means less shrinkage.

That is, we chose four combinations of a small value (0.1) imposing considerable shrinkage and a very high value (100) imposing very little shrinkage.

With regard to making a decision regarding which hyperparameters to use, it makes sense to consider the literature background as well as the empirical data at hand and make an informed decision on how much shrinkage to impose.

Coefficients

In theory, we should obtain coefficient values for the first-lag own-variable coefficients that are much closer to one when setting $\lambda_1 = 0.1$ than when setting it to 100. We observe that this is the case. In both cases where we set the stricter lambda value, we get values between 0.9 ans 1.2, while we observe parameter values higher than 1.5 with the lax prior setting. Similarly, we can see that cross-variable autoregressive parameters are more spread out from their prior mean of 0 when we set $\lambda_2 = 100$. Note that the figure in parentheses in the coefficient tables denotes the standard deviation.

Impulse Response Functions

What we observe regarding the impulse response functions is that generally, stricter lambda values (imposing more shrinkage) lead to more "lingering" behavior, i.e. effects that are greater in magnitude and return to zero in a slower fashion. This is because the means that we specified in our prior translate to random walk behavior, and setting lower prior variance means that the prior "pushes" the posterior more firmly towards random walk behavior. Indeed, comparing specifically Variant 3, where both lambda values are chosen to be very low, with Variant 4, where both are high, we can see that while generally, impulse response behavior is similar, the IRFs computed with the lower lambda values resemble more of a random walk behavior than those computed with high lambda values.

Predictions

The directions of the predictions seem to be quite similar regardless of the lambda values chosen. The only differences we do observe are regarding the magnitude of changes in the predicted variables and how confident the predictions are. Inflation is expected to fall in all specifications, as is unemployment, and all variants predict a drop in the interest rate with a subsequent constant development. Specifications with high lambda values translate into wider confidence bounds around the predictions.

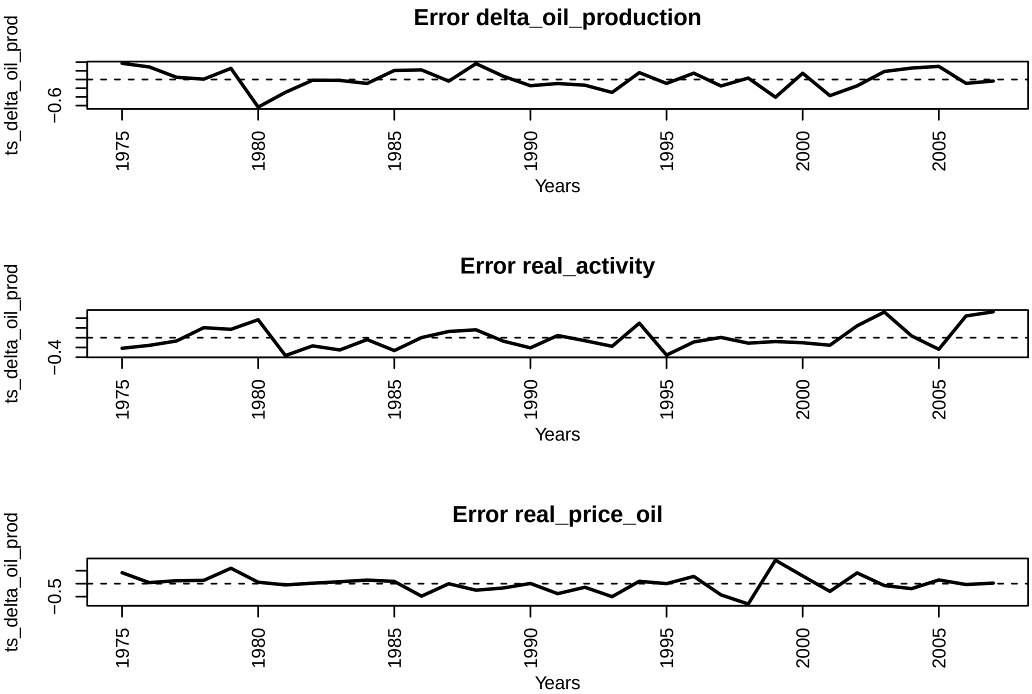
Exercise 2 – Replicating Kilian (2009)

Exercise 2.1 – Estimating the Bayesian VAR

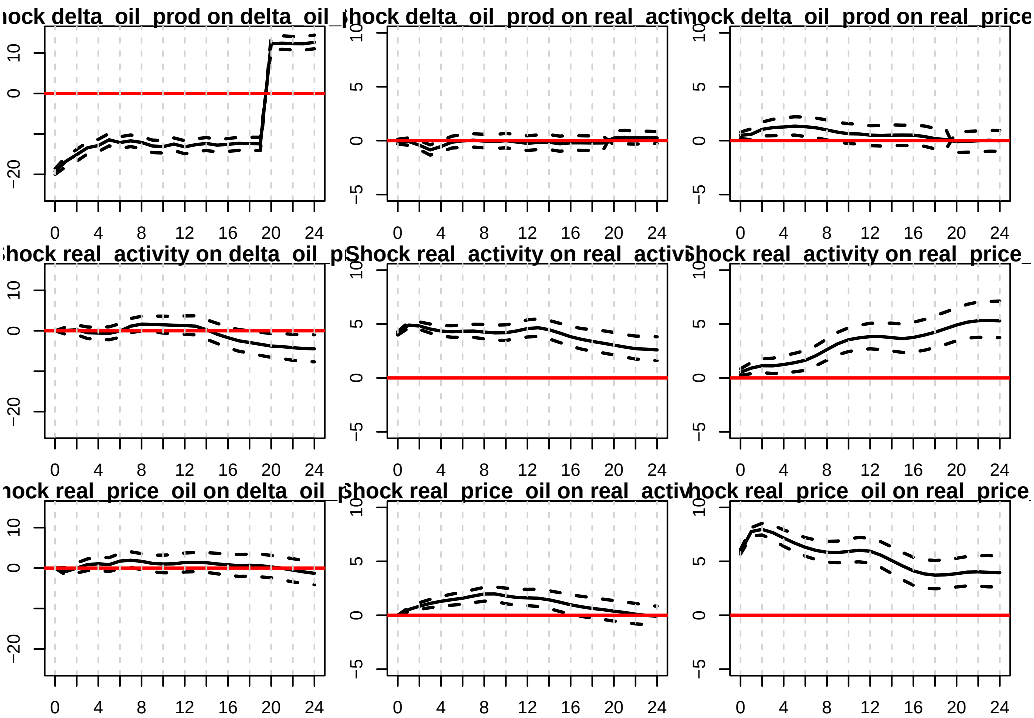
We solved this question by using the code provided for the Bayesian VAR. We were supposed to adjust for the aggressive shrinkage of the Minessota Prior. We did so by adjusting λ_1 and λ_2 by multiplying it by 3 (since there are three months in a quarter). The adapted code can be found in the Rmd file on GitHub.

Exercise 2.2 – Replication of Figures 2 and 3

First, we replicate Figure 2:



Second, we replicate Figure 3:

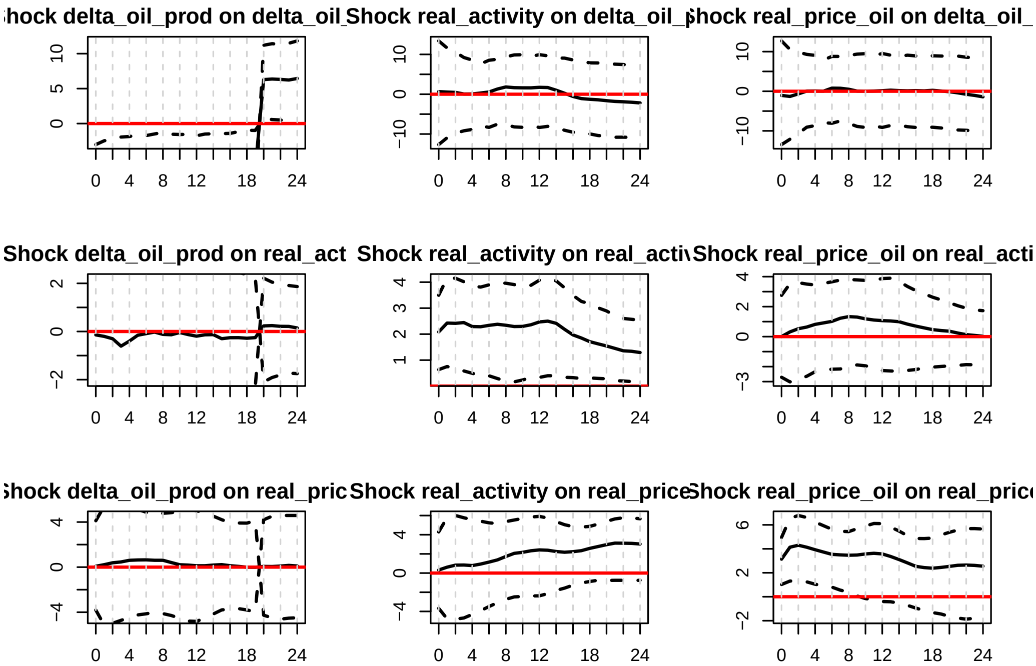


Exercise 2.3 – Sign Restrictions, Recreating Figure 3

We try to set the sign restriction such that it makes sense from both an economic point of view and an econometric point of view. To make sense from an econometric point of view, the sign restrictions need to be set in such a way that the columns are not linearly dependent. If they were, we could not identify shocks. Economically we decided on the following sign restrictions:

- An oil price shock: lowers oil production/ lowers real economic activity/ increases the real price of oil
- An aggregate demand shock: increases oil production/ increases economic activity/ increases the real price of oil
- An oil market specific demand shock: increases oil production/ lowers economic activity / increases the real price of oil

We replicate figure 3 using these sign restrictions:



Discussion

The IRFs are looking quite similar in both cases. What comes to immediate attention is that the variance around the IRFs is much higher in the case where we used sign restrictions. Additionally, we see a "jump" in both approaches in the IRF of an Oil Production shock on Oil Production that is not present in the original figure from Kilian (2009). We suspect that this is due to a coding error which we were not able to identify in time before the submission deadline.

Briefly discuss potential shortcomings of both identification schemes

Recursive ordering: in a Bayesian VAR has several drawbacks. Firstly, it assumes a predetermined causal ordering of variables, which may not accurately capture the true underlying relationships. This can lead to misspecification and biased inference if the assumed order is incorrect. Secondly, recursive ordering cannot capture feedback effects and endogenous relationships among variables, limiting its ability to capture complex dynamic interactions. Thirdly, the choice of the initial ordering can heavily influence the results, introducing subjectivity and potential instability in the analysis. Fourthly, recursive ordering may not be suitable for systems with multiple variables influencing each other simultaneously, as it oversimplifies the interdependencies and may miss important interactions. Lastly, changes in the ordering can have a significant impact on the estimated parameters, making the results sensitive to the chosen ordering and raising concerns about the robustness of the findings.

Sign restrictions: in a Bayesian VAR suffer from several limitations. Firstly, they rely heavily on subjective assumptions made by the researcher in specifying the sign restrictions, which can introduce bias and uncertainty into the analysis. Secondly, sign restrictions may not uniquely identify the shocks in the model, leading to identification problems and potential ambiguity in interpreting the results. Thirdly, the sign restrictions approach assumes a linear relationship between variables.

Exercise 2.4 – Other Variables

We use the unemployment rate.

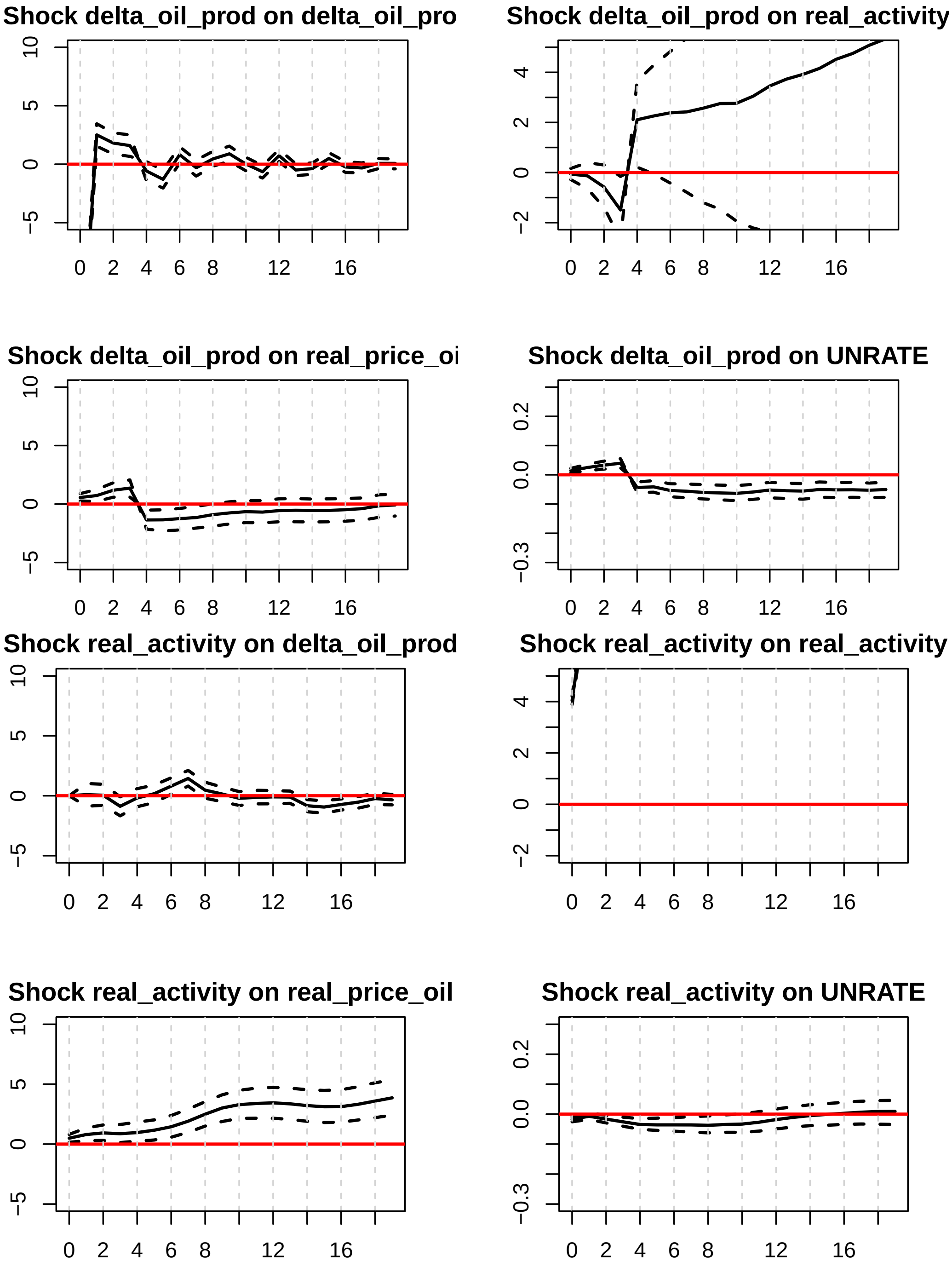
First, we use a Dickey-Fuller Test to see whether the time series is stationary:

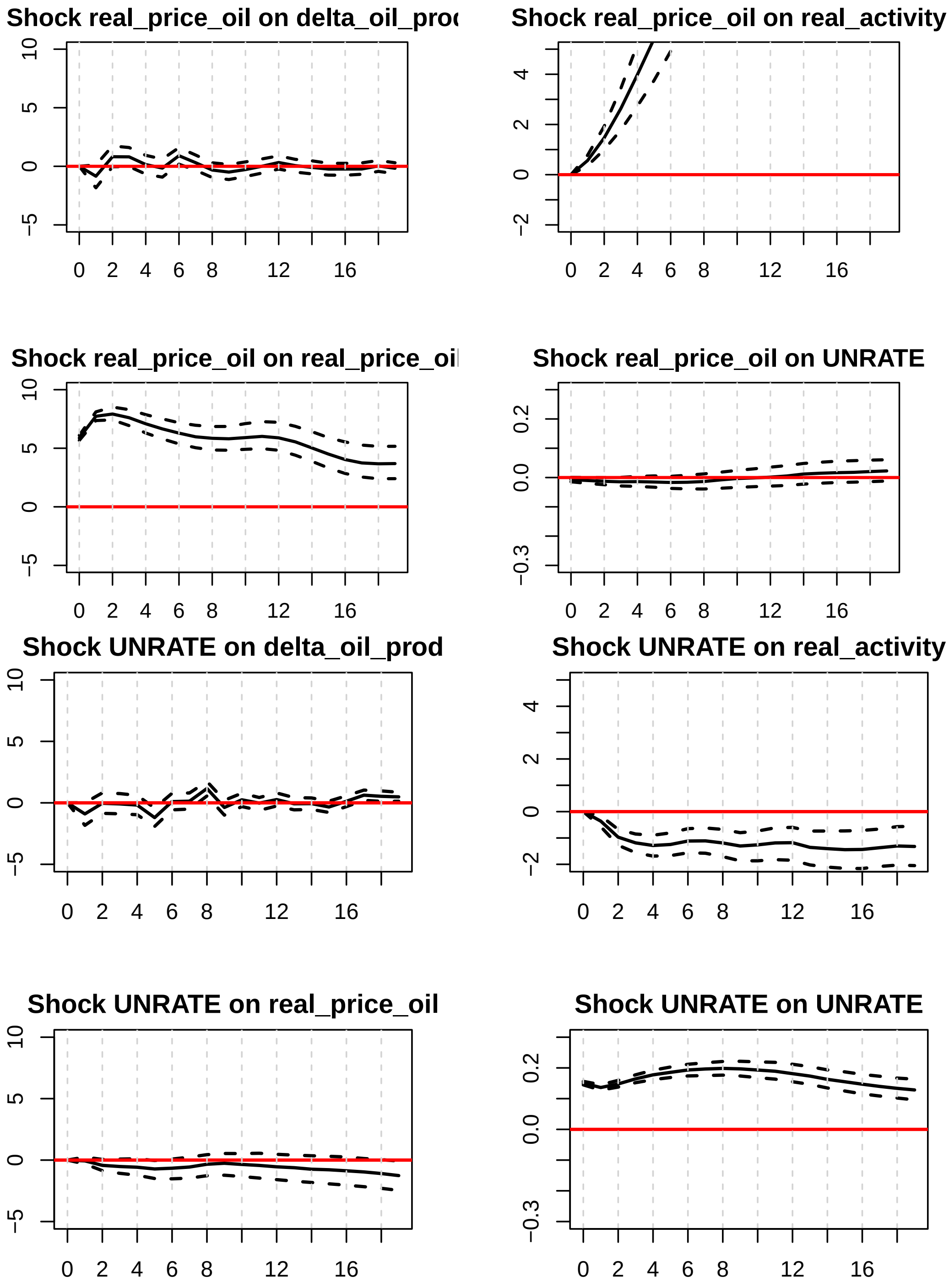
```
adf.test(UNRATE$UNRATE)
```

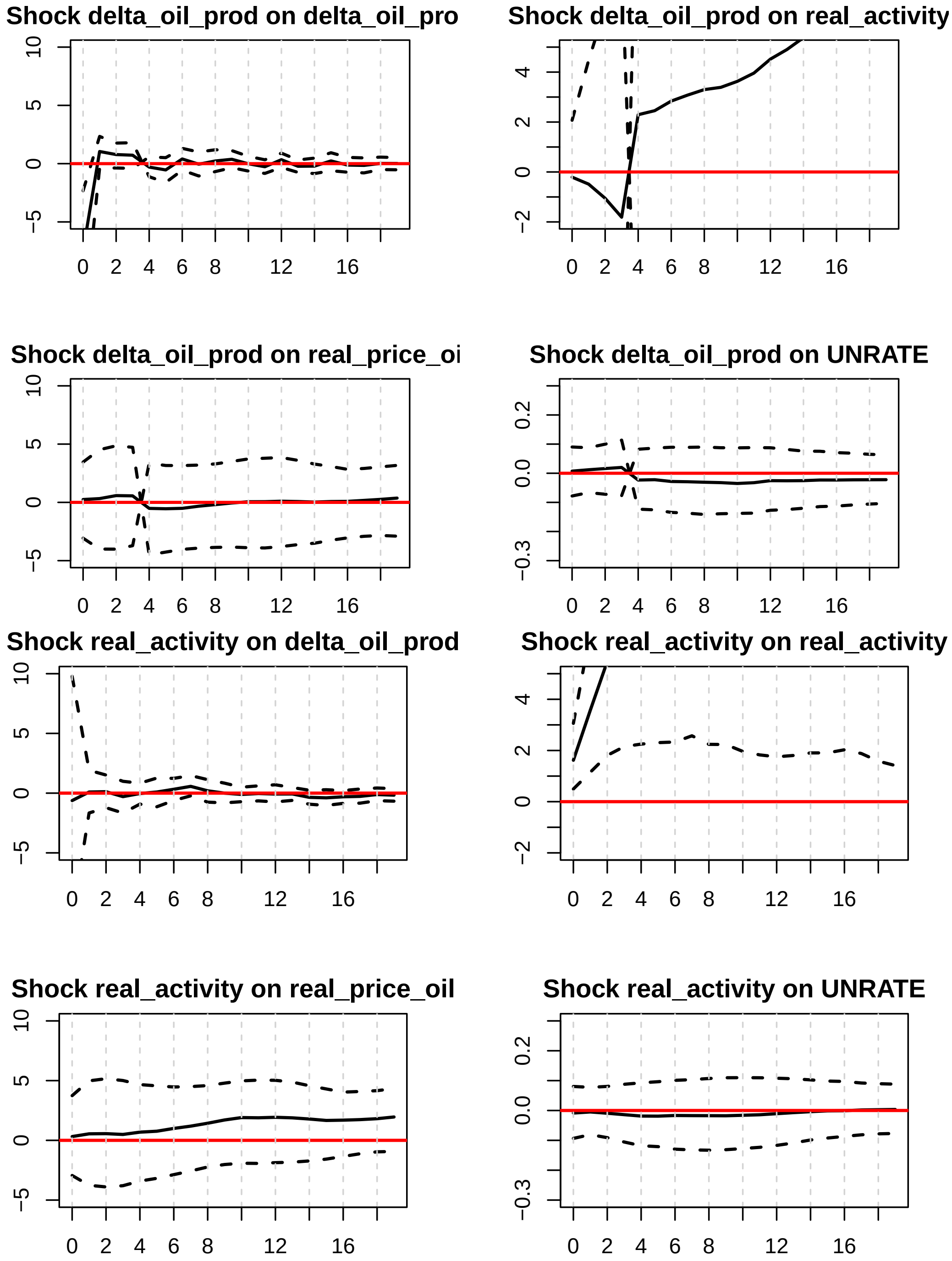
```
##
##  Augmented Dickey-Fuller Test
##
## data:  UNRATE$UNRATE
## Dickey-Fuller = -3.8742, Lag order = 9, p-value = 0.01529
## alternative hypothesis: stationary
```

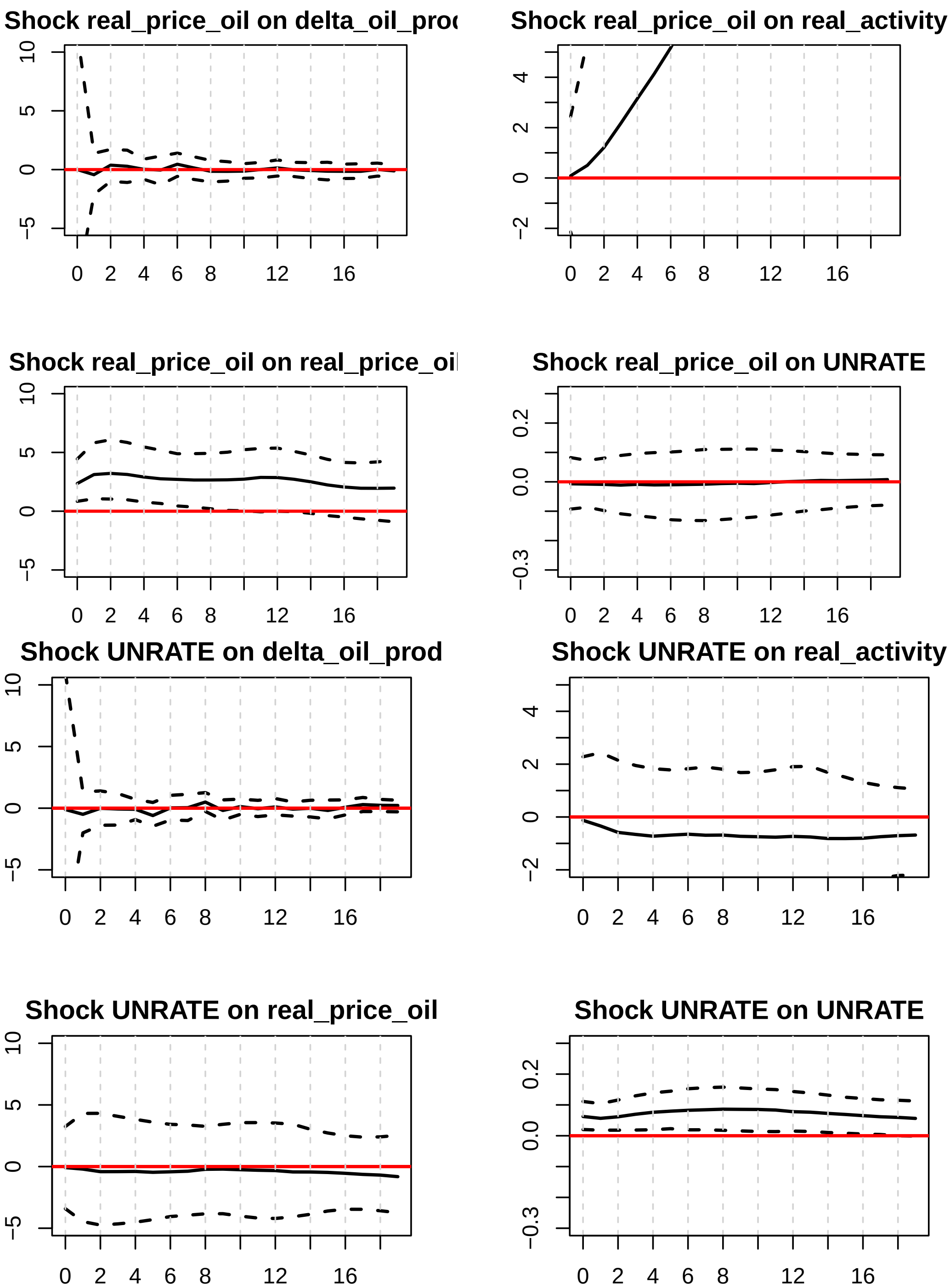
We reject the null hypothesis that the time series is stationary. We now add the unemployment rate to the dataset and adjust for the time periods.We also rewrite the code from (a) and estimate the model again.

Now, we can estimate figure 3 of Kilian (2009) with the added UNRATE first with recursive ordering and afterwards with sign restrictions:


Shock delta_oil_prod on delta_oil_pro
Shock delta_oil_prod on real_activity
Shock delta_oil_prod on real_price_oi
Shock delta_oil_prod on UNRATE
Shock real_activity on delta_oil_prod
Shock real_activity on real_activity
Shock real_activity on real_price_oil
Shock real_activity on UNRATE


Shock real_price_oil on delta_oil_proc
Shock real_price_oil on real_activity
Shock real_price_oil on real_price_oil
Shock real_price_oil on UNRATE
Shock UNRATE on delta_oil_prod
Shock UNRATE on real_activity
Shock UNRATE on real_price_oil
Shock UNRATE on UNRATE


Shock delta_oil_prod on delta_oil_pro
Shock delta_oil_prod on real_activity
Shock delta_oil_prod on real_price_oi
Shock delta_oil_prod on UNRATE
Shock real_activity on delta_oil_prod
Shock real_activity on real_activity
Shock real_activity on real_price_oil
Shock real_activity on UNRATE
0 2 4 6 8 12 16



Discussion

From the impulse response functions (IRFs), it is evident that a decrease in oil production leads to an increase in US unemployment, whereas an increase in real activity results in a decrease in unemployment. However, the impact of the real price of oil on unemployment remains uncertain. On the other hand, a positive shock to US unemployment has an ambiguous effect on oil production, while it significantly reduces real activity and slightly lowers the price of oil.