# **Mostly Pointless Spatial Econometrics**

(Gibbons & Overman, 2012)
Spatial Economics Seminar Presentation

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# **Spatial Econometric Models And Their Issues**

**The Experimentalist Paradigm And Spatial Econometrics** 

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$$y_{i} = \rho \mathbf{w}_{i}'(\rho \mathbf{W} \mathbf{y} + \mathbf{X} \boldsymbol{\beta} + \mathbf{W} \mathbf{X} \boldsymbol{\gamma} + \mathbf{u}) + \mathbf{x}_{i}' \boldsymbol{\beta} + \mathbf{w}_{i}' \mathbf{X} \boldsymbol{\gamma} + u_{i}$$

$$= \rho^{2} \mathbf{w}_{i}' \mathbf{W} \mathbf{y} + \rho \mathbf{w}_{i}' \mathbf{X} \boldsymbol{\beta} + \rho \mathbf{w}_{i}' \mathbf{X} \boldsymbol{\gamma} + \rho \mathbf{w}_{i}' \mathbf{u} + \mathbf{x}_{i}' \boldsymbol{\beta} + \mathbf{w}_{i}' \mathbf{X} \boldsymbol{\gamma} + u_{i}$$

$$= \rho^{2} \mathbf{w}_{i}' \mathbf{W} \mathbf{y} + \mathbf{x}_{i}' \boldsymbol{\beta} + \rho \mathbf{w}_{i}' (\mathbf{X} \boldsymbol{\beta} + \boldsymbol{\gamma}) + \rho \mathbf{w}_{i}' \mathbf{W} \mathbf{X} \boldsymbol{\gamma} + \mathbf{v}_{i}$$

$$= \dots$$

$$= \rho^{n} (\mathbf{w})_{i}' \mathbf{W}^{n-1} \mathbf{y} + \mathbf{x}_{i}' \boldsymbol{\beta} + \mathbf{w}_{i}' \mathbf{X} (\rho \boldsymbol{\beta} + \boldsymbol{\gamma})$$

$$+ \rho \mathbf{w}_{i}' \mathbf{W} \mathbf{X} (\rho \boldsymbol{\beta} + \boldsymbol{\gamma}) + \rho^{2} \mathbf{w}_{i}' \mathbf{W}^{2} \mathbf{X} (\rho \boldsymbol{\beta} + \boldsymbol{\gamma}) + \dots + \mathbf{v}_{i},$$

$$(2)$$

Under standard regularity conditions:  $\lim_{n\to\infty} |\rho^n(\mathbf{W}')^{n-1}\mathbf{W}^{n-1}| = 0$ 

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**However**, all of these models have the same reduced form, namely:

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Spatial Economics uses neighbors characteristics ( $\mathbf{w}_i'\mathbf{X}$ ,  $\mathbf{w}_i'\mathbf{W}\mathbf{X}$ , ...) under the assumption that these are exogenous to instrument for  $\mathbf{w}_i'\mathbf{y}$ 

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$$\mathbf{y}_{i} = \rho_{1} \mathbf{E}[\mathbf{y}_{i}|a] + \mathbf{x}_{i}'\beta + \mathbf{E}[\mathbf{x}_{i}'|a]\gamma + \mathbf{v}_{i}, \tag{4}$$

Solving for the reduced form by taking the expectation of (4) and rearranging yields:

$$y_i = \mathbf{x}_i'\beta + \mathbf{E}[\mathbf{x}_i'|a]\frac{(\beta\rho_1 + \gamma)}{(1 - \rho_1)} + \frac{\rho_1}{1 - \rho_1}\mathbf{E}[\mathbf{v}_i|a] + \mathbf{v}_i$$
(5)

No chance that we can distinguish the endogenous ( $\rho_1$ ) from the exogenous peer effects ( $\gamma$ )!

Spatial Durbin Model again:

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Identification breaks down in most spatial econometric models because:

- the exact structure of **W** is not known! (exclusion restriction is not fulfilled)
- Weak Instruments because of high correlation between spatial lags w'<sub>i</sub>X, w'<sub>i</sub>WX, w'<sub>i</sub>W<sup>2</sup>X, .....

**Spatial Econometric Models And Their Issues** 

# The Experimentalist Paradigm And Spatial Econometrics

<sup>&</sup>lt;sup>1</sup>Gibbons and Overman (2012, p. 181)

<sup>&</sup>lt;sup>2</sup>Assuming we do not know the true functional form and spatial weights.

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- These reflection issues transfer to differenced specifications and are not solved by randomization<sup>2</sup>

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- How to proceed?

<sup>&</sup>lt;sup>3</sup>Gibbons and Overman (2012, p. 187)

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"[Any] empirical research that aims to find out if x causes y needs to find a source of exogenous variation in x!"<sup>3</sup>

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# Thank you!

# **Appendix: Spatial Econometric Models**

Starting point:

$$y_i = \mathbf{x}_i'\beta + u_i \tag{9}$$

To incorporate spatial dependence we know the

**SAR** model:

$$y_i = \rho \mathbf{w}_i' \mathbf{y} + \mathbf{x_i} \boldsymbol{\beta} + \mathbf{u_i}$$
 (10)

**SLX** model:

$$\mathbf{y}_i = \mathbf{x}_i' \boldsymbol{\beta} + \mathbf{w}_i' \mathbf{X} \boldsymbol{\gamma} + u_i$$
 (11)

SE model:

$$y_i = \mathbf{x}_i' \boldsymbol{\beta} + u_i, \tag{12}$$

where 
$$u_i = \rho \mathbf{w}_i' \mathbf{u} + \mathbf{v}_i$$
 (13)

### References I

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Manski, C. F. (1993).Identification of Endogenous Social Effects: The Reflection Problem. The Review of Economic Studies, 60(3), 531. https://doi.org/10.2307/2298123