# **Mostly Pointless Spatial Econometrics**

(Gibbons & Overman, 2012)
Spatial Economics Seminar Presentation

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# **Spatial Econometric Models And Their Issues**

**The Experimentalist Paradigm And Spatial Econometrics** 

### The Spatial Durbin Model

The Spatial Durbin Model nests all other models. We can express the reduced form by recursive substitution:

$$y_i = \rho \mathbf{w}_i' \mathbf{y} + \mathbf{x}_i' \boldsymbol{\beta} + \mathbf{w}_i' \mathbf{X} \boldsymbol{\gamma} + u_i,$$

$$u_i = \lambda \mathbf{w}_i' \mathbf{u} + v_i$$
(1)

$$y_{i} = \rho \mathbf{w}_{i}'(\rho \mathbf{W} \mathbf{y} + \mathbf{X} \boldsymbol{\beta} + \mathbf{W} \mathbf{X} \boldsymbol{\gamma} + \mathbf{u}) + \mathbf{x}_{i}' \boldsymbol{\beta} + \mathbf{w}_{i}' \mathbf{X} \boldsymbol{\gamma} + u_{i}$$

$$= \rho^{2} \mathbf{w}_{i}' \mathbf{W} \mathbf{y} + \rho \mathbf{w}_{i}' \mathbf{X} \boldsymbol{\beta} + \rho \mathbf{w}_{i}' \mathbf{X} \boldsymbol{\gamma} + \rho \mathbf{w}_{i}' \mathbf{u} + \mathbf{x}_{i}' \boldsymbol{\beta} + \mathbf{w}_{i}' \mathbf{X} \boldsymbol{\gamma} + u_{i}$$

$$= \rho^{2} \mathbf{w}_{i}' \mathbf{W} \mathbf{y} + \mathbf{x}_{i}' \boldsymbol{\beta} + \rho \mathbf{w}_{i}' (\mathbf{X} \boldsymbol{\beta} + \boldsymbol{\gamma}) + \rho \mathbf{w}_{i}' \mathbf{W} \mathbf{X} \boldsymbol{\gamma} + \mathbf{v}_{i}$$

$$= \dots$$

$$= \rho^{n} (\mathbf{w})_{i}' \mathbf{W}^{n-1} \mathbf{y} + \mathbf{x}_{i}' \boldsymbol{\beta} + \mathbf{w}_{i}' \mathbf{X} (\rho \boldsymbol{\beta} + \boldsymbol{\gamma})$$

$$+ \rho \mathbf{w}_{i}' \mathbf{W} \mathbf{X} (\rho \boldsymbol{\beta} + \boldsymbol{\gamma}) + \rho^{2} \mathbf{w}_{i}' \mathbf{W}^{2} \mathbf{X} (\rho \boldsymbol{\beta} + \boldsymbol{\gamma}) + \dots + \mathbf{v}_{i},$$

$$(2)$$

Under standard regularity conditions:  $\lim_{n\to\infty} \rho^n(\mathbf{W}')^{n-1}\mathbf{W}^{n-1} = 0$ 

#### So what does this tell us?

Spatial interaction in  $y_i$ , spatial externalities through  $x_i$ , or spatial dependence in the error term leads to different econometric specifications

**However**, all of these models have the same reduced form, namely:

$$y_i = \mathbf{x}_i' \beta + \mathbf{w}_i' \mathbf{X} \pi_1 + \mathbf{w}_i' \mathbf{W} \mathbf{X} \pi_2 + \mathbf{w}_i' \mathbf{W}^2 \mathbf{X} \pi_3 + \dots + \mathbf{v}_i$$
 (3)

Spatial Economics uses neighbors characteristics ( $\mathbf{w}_i'\mathbf{X}$ ,  $\mathbf{w}_i'\mathbf{W}\mathbf{X}$ , ...) under the assumption that these are exogenous to instrument for  $\mathbf{w}_i'\mathbf{y}$ 

# The Reflection Problem in the Context of Spatial Econometrics

Remember the **Linear-in-Means** model of neighborhood effects? (Manski, 1993) No?

$$\mathbf{y}_{i} = \rho_{1} \mathbf{E}[\mathbf{y}_{i}|a] + \mathbf{x}_{i}'\beta + \mathbf{E}[\mathbf{x}_{i}'|a]\gamma + \mathbf{v}_{i}, \tag{4}$$

Solving for the reduced form by taking the expectation of (4) and rearranging yields:

$$y_i = \mathbf{x}_i'\beta + \mathbf{E}[\mathbf{x}_i'|a]\frac{(\beta\rho_1 + \gamma)}{(1 - \rho_1)} + \frac{\rho_1}{1 - \rho_1}\mathbf{E}[\mathbf{v}_i|a] + \mathbf{v}_i$$
(5)

No chance that we can distinguish the endogenous ( $\rho_1$ ) from the exogenous peer effects ( $\gamma$ )!

# The Difference Between the Peer Effects Literature and Spatial Econometrics

Spatial Durbin Model again:

$$\mathbf{y}_i = \rho_1 \mathbf{w}_i' \mathbf{y} + \mathbf{x}_i' \beta + \mathbf{w}_i' \mathbf{X} \gamma + u_i$$
 (6)

According to Gibbons and Overman (2012) the spatial econometrics literature is disregarding the identification issues as shown in Manski (1993). Why? In spatial econometrics (6) is assumed to be the **true** data generating process.

Identification of parameters works because:

- The structure of the spatial weights matrix **W** is said to be known
- Exogeneity of X and w', WX, ...

$$y_i = \mathbf{x}_i'\beta + \mathbf{w}_i'\mathbf{X}(\beta\rho_1 + \gamma) + \rho_1\mathbf{w}_i'\mathbf{W}\mathbf{X}(\beta\rho_1 + \gamma) + \rho_1^2\mathbf{w}_i'\mathbf{W}^2\mathbf{X}(\beta\rho_1 + \gamma) + \dots + \varepsilon_i$$
(7)

# The Difference Between the Peer Effects Literature and Spatial Econometrics

Gibbons and Overman (2012) note that there are serious problems in relying on the spatial lags of X to identify the parameters.

Identification breaks down in most spatial econometric models because:

- the exact structure of **W** is not known! (exclusion restriction is not fulfilled)
- Weak Instruments because of high correlation between spatial lags w'<sub>i</sub>X, w'<sub>i</sub>WX, w'<sub>i</sub>W<sup>2</sup>X, .....

**Spatial Econometric Models And Their Issues** 

# The Experimentalist Paradigm And Spatial Econometrics

## Standard Spatial Models Are Plagued! (By Identification Problems)

"[Identification] problems are so fundamental that they must sit at centre stage of applied work"

"[...] we argue that spatial research would be best served by turning away from the application of generic spatial models"

"[Any] alternative approach also has to solve the identification problems that plague spatial economic analysis"<sup>1</sup> Consider a **standard SD specification**:

$$y_i = \rho \mathbf{w}_i' \mathbf{y} + \mathbf{x}_i' \boldsymbol{\beta} + \mathbf{w}_i' \mathbf{X} \boldsymbol{\gamma} + u_i$$
 (8)

- No way to induce exogenous change in w'<sub>i</sub>y without changing w'<sub>i</sub>X (or w'<sub>i</sub>u)
- But can this even be the true DGP? Our decisions probably rely on expectations of y and not y itself
- These reflection issues transfer to differenced specifications and are not solved by randomization<sup>2</sup>

<sup>&</sup>lt;sup>1</sup>Gibbons and Overman (2012, p. 181)

<sup>&</sup>lt;sup>2</sup>Assuming we do not know the true functional form and spatial weights.

#### Two Alternatives Considered

#### **Instrumental Variables Estimation**

- w'<sub>i</sub>X provides instruments for w'<sub>i</sub>y in a correctly specified SAR model
- Exclusion restriction: Has to be met, is often neglected
- How sensible is it to assume that  $\mathbf{w}_{i}^{\prime}\mathbf{X}$  affects  $y_{i}$  only through  $\mathbf{w}_{i}^{\prime}\mathbf{y}$ ?
- Sometimes institutional arrangements provide plausibly exogenous variation
- Another opportunity: w'<sub>i</sub>y represents expectations

#### **Reduced Form SLX Models**

- Composite reduced form parameter describes the influence of neighbors' X or y
- It doesn't distinguish, but the information is useful anyway
- The SAR model's identification problem is absent,
- But: Exogeneity of x<sub>i</sub> and w<sub>i</sub>X is still not credible
- How to proceed?

#### How to Proceed With the SLX Model—And What Problems Remain

#### "Natural Experiments"

- Intuition: A change in w<sub>i</sub> over time allows identification
- Example: The German Reunification

#### Standard IV / Differencing

 Use IV for parameters of interest (this time of the SLX model)

#### **Spatial Differencing**

 Removing unobserved spatial components by differencing observations with their neighbors

- Doesn't all of this lack generalizability outside the experiment sub-group?
   It might, but at least estimates are plausibly causal
- Are places simply too spatially unique to be treated with experimentalist techniques?
   No, a counterfactual only should be comparable along the explanatory dimensions

"[Any] empirical research that aims to find out if x causes y needs to find a source of exogenous variation in x!"<sup>3</sup>

**Problems of These Approaches** 

<sup>&</sup>lt;sup>3</sup>Gibbons and Overman (2012, p. 187)

# Thank you!

# **Appendix: Spatial Econometric Models**

Starting point:

$$y_i = \mathbf{x}_i' \boldsymbol{\beta} + u_i \tag{9}$$

To incorporate spatial dependence we know the

SAR model:

$$y_i = \rho \mathbf{w}_i' \mathbf{y} + \mathbf{x_i} \boldsymbol{\beta} + \mathbf{u_i}$$
 (10)

**SLX** model:

$$\mathbf{y}_i = \mathbf{x}_i' \boldsymbol{\beta} + \mathbf{w}_i' \mathbf{X} \boldsymbol{\gamma} + u_i$$
 (11)

SE model:

$$y_i = \mathbf{x}_i'\boldsymbol{\beta} + u_i, \tag{12}$$

where 
$$u_i = \rho \mathbf{w}_i' \mathbf{u} + \mathbf{v}_i$$
 (13)

#### References I

Gibbons, S., & Overman, H. G. (2012). Mostly Pointless Spatial Econometrics? *Journal of Regional Science*, 52(2), 172–191. https://doi.org/10.1111/j.1467-9787.2012.00760.x

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