Characterizing a Cold Gas Thruster System

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Abstract

A cold gas thruster (CGT) system was designed with pre-existing nozzle theory in mind. This paper deals with characterizing the system and provides an analysis of the force production for it. It was found that the CGT performed similarly to the predicted theory, but the data collected was not sufficient to characterize the system as was previously expected. This required the use of a more general analysis scheme.

1 Introduction

A cold gas thruster (CGT) is a system that uses expanding gas to generate a force. This force is typically used in reaction control systems (RCSs) to stabilize space craft or simply change their attitude. This paper is primarily concerned with reaction control systems to be developed for high altitude balloon payloads (HABPs.) These HABPs experience intense and sporadic winds. Winds which make data collection for certain sensors difficult. There are several ways in which a RCS can achieve stabilization, but the method of choice here is the CGT.

There are several components important to the CGT RCS. Here, there will only be a brief discussion on two of these components such that the analysis is not lacking information.

The first consideration to make is the type of gas to be used. The primary question here is, *what makes one gas better than another?* One parameter that tries to answer this question is the *specific impulse* (I_{sp}). This is a value specific to a gas. Experimentally, it is measured by integrating a force (F) versus time (t) plot generated by a CGT using that gas. That will give the total impulse, this is divided by the change in weight of the gas through that time period (τ). In other words:

$$I_{sp} = \frac{\sum_{t=0}^{\tau} F(t)t}{\Delta mg} \tag{1}$$

where Δm is mass and g is acceleration due to gravity. More precisely, specific impulse is defined by equation 2.

$$I_{sp} = \frac{F}{dm/dt} \tag{2}$$

You may notice the descrepency in units between equations 1 and 2. Often times the I_{sp} is given with units of s, but in reality it is defined with units of Ns/kg.

This is an excellent start to creating a standard for comparing gases, but there is much more that should be considered. Factors such as safety, availability, cost, energy storage density, and so on all contribute to the choice of gas. Additionally, each one of these factors has a different weight per say depending on the scenario in which they are being applied. After consideration, the choice of gas for this system is CO_2.

The other component is the nozzle. These are used to set the mass flow rate for the system and to accelerate the propellant to supersonic speeds. They consist of a converging section that leads into a throat of minimum radius and a diverging section. Nozzles that have been made to achieve maximum thrust obey the following characteristics. In the converging section, the gas is accelerated; once the throat is reached the gas is traveling at the speed of sound. Then, as it proceeds into the diverging section, it continues to accelerate until it reaches the exit plane of the nozzle. Ideally, the nozzle is designed such that the pressure of the gas exiting the nozzle is equal to the ambient pressure.

The thrust produced by a nozzle can be written in terms of the variables listed below. First, though, it is given by equation 3 in reference [1].

- Chamber Temperature (T_c)
- Exit Temperature (T_e)
- Chamber Pressure (P_c)
- Exit Area of Nozzle (A_e)
- Throat Area of Nozzle (A_t)

The derivation will not be shown here, but reference [1] discusses it in detail. The text derives equation 3 from the laws of thermodynamics and some general assumptions.

$$C_F = \sqrt{\frac{2\gamma^2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \frac{(P_e - P_a)}{P_c}\frac{A_e}{A_t} \tag{3}$$

Where C_F is defined by:

$$C_F = \frac{F}{A_t P_c} \tag{4}$$

It is also helpful to define the nozzle's expansion ratio (ϵ), given by equation 5.

$$\epsilon = \frac{A_e}{A_t} \tag{5}$$

It can be seen C_F, ergo the thrust, is at maximum when $P_a = P_e = 0$. It is also shown that $\frac{P_e}{P_c}$ is given by equation 6.

$$\frac{P_e}{P_c} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{1}{\gamma-1}} \tag{6}$$

Additionally, we can define the mach number as:

$$M^2 = \frac{2}{\gamma - 1}\left(\frac{T_c}{T_e} - 1\right) \tag{7}$$

allowing us to make a substitution for $\frac{P_e}{P_c}$ into equation 3. With this substitution, we will eliminate P_e. The motivation for this is that measuring the exit plane pressure is more difficult than measuring the exit plane temperature. After the subtitutions, we retrieve equation 8.

$$F = A_t P_c \left(\sqrt{\frac{2\gamma^2}{\gamma - 1} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \left(1 - \frac{T_c}{T_e} \right)} + \left(\frac{T_c}{T_e} \right)^{-\frac{\gamma}{\gamma-1}} \epsilon \right) \tag{8}$$

This equation was developed given some assumptions and in different systems some of the assumptions may have a more or less dominant effect. It can also be seen that since the thrust is highly dependent on the external pressure, the nozzle should be designed to maximize the thrust in the pressure it will spend the most time in. Because of this an experiment was developed to determine the reliability of equation 8.
In measuring the thrust at several different area ratios and comparing it to the predicted thrust, the hope is to create an "effective" proportionality constant. This would then allow the nozzle design process to be maximized for pressures that cannot be tested on Earth's surface.

2 Data Collection and Analysis

2.1 Data Collection

The data taken was for the variables in equation 8. There were a total of 10 nozzle expansion ratios used with 2 trials each. The expansion ratio kept A_t constant and varied A_e. There were 4 over expanded, 4 underexpanded, 1 optimum, and 1 with no expansion. The naming scheme goes something like ERT, where E stands for either U (Underexpanded), N (None), or O. O stands for either optimum or overexpanded. R is the ratio number (1-4) except in the case that there is no third character. In this case the O stands for optimum, which there is only one of. T stands for trial number which can be either 1 or 2. So for example, U22 is underexpanded, ratio number 2, trial number 2. O1 is optimum ratio, trial number 1. These are explicitly defined in the nomenclature section 4. This naming scheme will be upated in future experiments because it was slightly problematic in the analysis of the data.

Data Sensors

There should be some light discussion on the sensors used for data collection. There was a single pressure transducer located near the inlet of the nozzle, a thermocouple located on the nozzle, a thermocouple located inside of the adaptor that the CO_2 connected to the rest of the plumbing, and a load cell setup to measure the thrust produced from the CGT. For sake of simplicity I will refer to these as "sensors" preceeded by the variable they measured. So, respectively, P_c sensor, T_e sensor, T_c sensor, and F sensor. Additionally, γ is dependent on temperature, but only changes slighty in the temperature differences noticed here. In later models, γ will be set to be a function of temperature for greater accuracy. For now it is not necessary because the force has a small dependence on γ compared to the temperature . So, for the sake of this project

γ is a constant. The data acquisition system was running python on a RaspberryPi 3 and since all of these sensors are analog devices 2 different analog-to-digital converters (ADCs) was used to interface with the Pi. The P_c, T_e, and T_c sensors interfaced with a 10-bit ADC and the F sensor interfaced with a 24-bit ADC. In future experiments, a 24-bit ADC will be used for all data acquisition because of the limited resolution with the 10-bit ADC. This is especially evident with the thermocouple data. All of the python scripts for data collection are available at my GitHub repository.

The code began the experiment after all sensors were calibrated and functioning properly. Data collection ran until the force production was below a certain threshold for a predefined number of points. This would also need to be updated because often times the force sensor would lose its zero position so defining the threshold value was not so easy. A better method would be using the change in force to determine when the experiment was over. This would work well because the force is transient until the CO_2 was all spent. After it was over the mass of the CO_2 canister was measured. All of this data was written to two seperate .txt files. One included the calibration values for the sensors and changes in mass, the other included P_c, T_c, T_e, and F. The previous was not recorded in a useful format and later work had to be done to resolve this. In the future this should be updated. The latter was simply comma delimited and worked fine for reading into python for analysis. The data files can be found at my GitHub repository as well.

2.2 Data Analysis

To begin the analysis, simple plots were made to look at the sensor data over time. Immediately, it was found that the temperature data is quite poor! The resolution of the ADC is evident in the discrete steps that the temperature takes. Figure 1 shows an example of the temperature data.

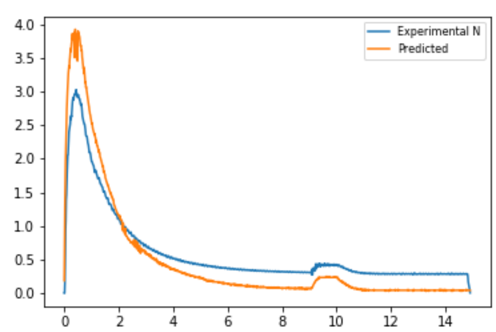

Figure 1: Example plot of the temperature over time for a trial

3 Results of Analysis

4 Conclusion

Nomenclature

γ	Ratio of specific heats
A_e	Exit area of nozzle
A_t	Throat area of nozzle
C_F	Thrust coefficient
F	Force or thrust
g	Acceleration due to gravity
I_{sp}	Specific impulse
M	Mach number
m	Mass
$N1$	No expansion, trial 1
$N2$	No expansion, trial 2
$O1$	Optimum area ratio, trial 1
$O11$	Overexpanded area ratio 1, trial 1
$O12$	Overexpanded area ratio 1, trial 2
$O2$	Optimum area ratio, trial 2
$O21$	Overexpanded area ratio 2, trial 1
$O22$	Overexpanded area ratio 2, trial 2
$O31$	Overexpanded area ratio 3, trial 1
$O32$	Overexpanded area ratio 3, trial 2
$O41$	Overexpanded area ratio 4, trial 1
$O42$	Overexpanded area ratio 4, trial 2
P_a	Ambient pressure
P_c	Chamber pressure

P_e Exit pressure

t Time

T_c Chamber temperature

T_e Exit temperature

$U11$ Underexpanded area ratio 1, trial 1

$U12$ Underexpanded area ratio 1, trial 2

$U21$ Underexpanded area ratio 2, trial 1

$U22$ Underexpanded area ratio 2, trial 2

$U31$ Underexpanded area ratio 3, trial 1

$U32$ Underexpanded area ratio 3, trial 2

$U41$ Underexpanded area ratio 4, trial 1

$U42$ Underexpanded area ratio 4, trial 2

References

[1] Norman Harry Langton. *Rocket propulsion.* Space research and technology: v. 2. American Elsevier Pub. Co., 1970.