Characterizing a Cold Gas Thruster System

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Abstract

A cold gas thruster (CGT) system was designed with pre-existing nozzle theory in mind. This paper deals with characterizing the system and provides an analysis of the force production for it. It was found that the CGT performed similarly to the predicted theory, but the data collected was not sufficient to characterize the system as was previously expected. This required the use of a more general analysis scheme.

1 Introduction

A cold gas thruster (CGT) is a system that uses expanding gas to generate a force. This force is typically used in reaction control systems (RCSs) to stabilize space craft or simply change their attitude. This paper is primarily concerned with reaction control systems to be developed for high altitude balloons (HABs.) These HABs experience intense and sporadic winds. Winds which make data collection for certain sensors difficult. There are several ways in which a RCS can achieve stabilization, but the method of choice here is the CGT.
There are several components important to the CGT RCS. Here, there will only be a brief discussion on two of these components such that the analysis is not lacking information.
The first consideration to make is the type of gas to be used. The primary question here is, *what makes one gas better than another?* One parameter that tries to answer this question is the *specific impulse* (I_{sp}). This is a value specific to a gas. Experimentally, it is measured by integrating a force (F) versus time (t) plot generated by a CGT using that gas. That will give the total impulse, this is divided by the change in weight of the gas through that time period (τ). In other words:

$$I_{sp} = \frac{\int_{t=0}^{\tau} F(t)dt}{mg} \tag{1}$$

where m is mass and g is acceleration due to gravity. This is an excellent start to creating a standard for comparing gases, but there is much more that should be considered. Factors such as safety, availability, cost, energy storage density, and so on all contribute to the choice of gas. Additionally, each one of these factors has a different weight per say depending on the scenario in which they are being applied. After consideration, the choice of gas for this system is CO_2.

The other component is the nozzle. These are used to set the mass flow rate for the system and to accelerate the propellant to supersonic speeds. They consist of a converging section that leads into a throat of minimum radius and a diverging section. Nozzles that have been made to achieve maximum thrust obey the following characteristics. In the converging section, the gas is accelerated; once the throat is reached the gas is traveling at the speed of sound. Then, as it proceeds into the diverging section, it continues to accelerate until it reaches the exit plane of the nozzle. Ideally, the nozzle is designed such that the pressure of the gas exiting the nozzle is equal to the ambient pressure.
The thrust produced by a nozzle can be written in terms of the variables listed below. First, though, it is given by equation 2 in [1].

- Chamber Temperature (T_c)
- Exit Temperature (T_e)
- Chamber Pressure (P_c)
- Exit Area of Nozzle (A_e)
- Throat Area of Nozzle (A_t)

The derivation will not be shown here, but reference [1] discusses it in detail. The text derives equation 2 from the laws of thermodynamics and some general assumptions.

$$C_F = \sqrt{\frac{2\gamma^2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right)} + \frac{(P_e - P_a)}{P_c}\frac{A_e}{A_t} \tag{2}$$

Where C_F is defined by:

$$C_F = \frac{F}{A_t P_c} \tag{3}$$

It can be seen C_F, ergo the thrust, is at maximum when $P_a = P_e = 0$. It is also shown that $\frac{P_e}{P_c}$ is given by equation 4.

$$\frac{P_e}{P_c} = \left(1 + \frac{(\gamma - 1)}{2}M^2\right)^{\frac{1}{\gamma-1}} \tag{4}$$

Additionally, we can define the mach number as:

$$M^2 = \frac{2}{\gamma - 1}\left(\frac{T_c}{T_e} - 1\right) \tag{5}$$

allowing us to make a substitution for $\frac{P_e}{P_c}$ into equation 2. With this substitution, we will eliminate P_e. The motivation for this is that measuring the exit plane pressure is more difficult than measuring the exit plane temperature. After the subtitutions, we retrieve equation 6.

$$F = A_t P_c \left(\sqrt{\frac{2\gamma^2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(1 - \frac{T_c}{T_e}\right)} + \left(\frac{T_c}{T_e}\right)^{-\frac{\gamma}{\gamma-1}}\frac{A_e}{A_t}\right) \tag{6}$$

This equation was developed given some assumptions and in different systems some of the assumptions may have a more or less dominant effect. It can also be seen that since the thrust is highly dependent on the external pressure, the nozzle should be designed to maximize the thrust in the pressure it will spend the most time in. Because of this an experiment was developed to determine the reliability of equation 6.
In measuring the thrust at several different area ratios and comparing it to the predicted thrust, the hope is to create an "effective" proportionality constant. This would then allow the nozzle design process to be maximized for pressures that cannot be tested on Earth's surface.

2 The Data and Analysis

3 Results of Analysis

4 Conclusion

5 Nomenclature

References

[1] Norman Harry Langton. *Rocket propulsion.* Space research and technology: v. 2. American Elsevier Pub. Co., 1970.