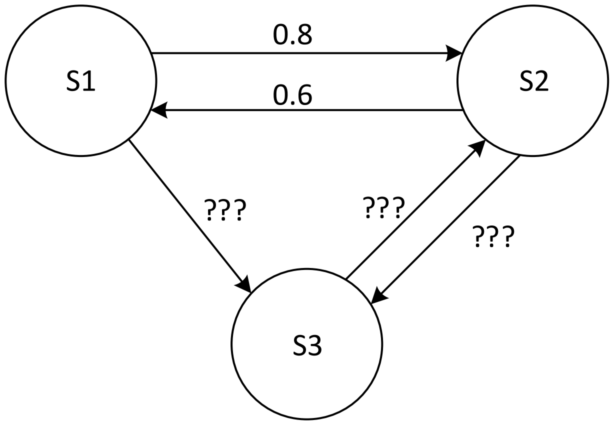
Problem hmm

A particular Markov process is given by the following state transitions.



a) Establish the transition matrix P and replace the acronym « ??? » by the correct values. (1 pt)

Note : if you are blocked here, continue with $P = \begin{pmatrix} 0 & 0.2 & 0.8 \\ 1 & 0 & 0 \\ 0.4 & 0.4 & 0.2 \end{pmatrix}$, but a penalty of -1pt will be considered in problem 1b !

b) Find the stationary distribution of P. (3 pts)

The Markov process is now hidden. Three observations O_1, O_2 and O_3 are visible with the following emission matrix : $O = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0.1 & 0.2 & 0.7 \end{pmatrix}$

The initial state probabilities are S_1: 30%, S_2: 20% and S_3: 50%. We note the following observation sequence : $Y_t = \{O_1; O_3; O_1\}$.

c) Give the total probability P_{tot} of this sequence. (3 pts)
Note : the details of the computations are required. Direct results will be considered as wrong.

d) Give the most likely hidden sequence using the Viterbi-algorithm. (3 pts)
Note : the details of the computations are required. Direct results will be considered as wrong.

Correction

a) We have $P = \begin{pmatrix} 0 & 0.8 & 0.2 \\ 0.6 & 0 & 0.4 \\ 0 & 1 & 0 \end{pmatrix}$

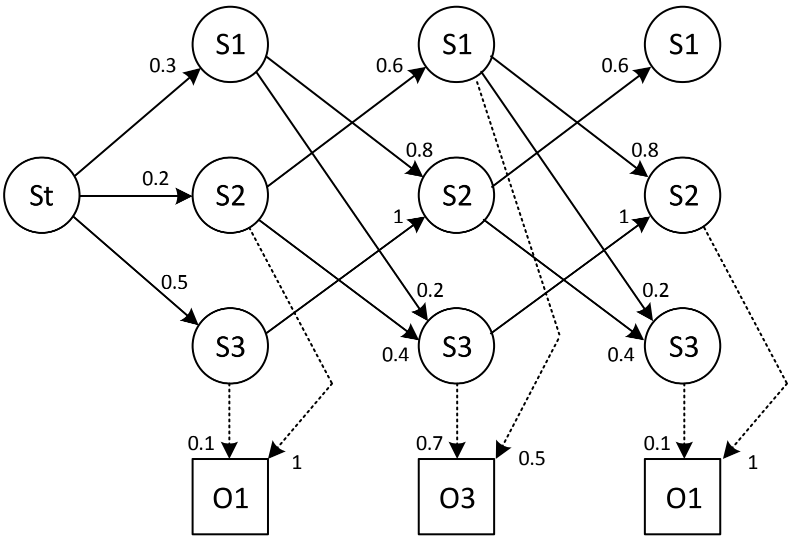
b) When $t \to \infty$, we have $\pi_\infty = \pi_\infty \cdot P$:

$$\begin{cases} (x \quad y \quad z) = (x \quad y \quad z) \cdot \begin{pmatrix} 0 & 0.8 & 0.2 \\ 0.6 & 0 & 0.4 \\ 0 & 1 & 0 \end{pmatrix} \\ x + y + z = 1 \end{cases}$$

We develop further :

$$\begin{cases} 10x = 6y \\ 10y = 8x + 10z \\ 10z = 2x + 4y \\ x + y + z = 1 \end{cases} \Rightarrow \begin{cases} y = 5x/3 \\ 50x/3 = 8x + 10z \ \Rightarrow z = 26x/30 = 13x/15 \\ 10z = 2x + 20x/3 \ \Rightarrow z = 26x/30 = 13x/15 \\ x + 5x/3 + 13x/15 = 1 \Rightarrow \ x = 15/53 \end{cases} \Rightarrow \begin{cases} x = 15/53 \\ y = 25/53 \\ z = 13/53 \end{cases}$$

HMM part



α-table

	t=1, o=O1	t=2, o=O3	t=3, o=O1
S1	0	0.2·0.6·0.5=0.06	0
S2	0.2·1=0.2	0	0.06·0.8·1+0.056·1·1=0.104
S3	0.5·0.1=0.05	0.2·0.4·0.7=0.056	0.06·0.2·0.1=0.0012

c) $P_{tot} = 0.104 + 0.0012 = 0.1052$

β-table

	t=1, o=O1	t=2, o=O3	t=3, o=O1
S1	0.2·1·0.7=0.14	0.8·1·1+0.2·1·0.1=0.82	1
S2	0.6·0.82·0.5+0.4·1·0.7=0.526	0.4·1·0.1=0.04	1
S3	0	1·1·1=1	1

γ-table

	t=1, o=O1	t=2, o=O3	t=3, o=O1
S1	0	0.0492	0
S2	0.1052	0	0.104
S3	0	0.056	0.0012
Sum	0.1052	0.1052	0.1052

Viterbi

	t=1, o=O1	t=2, o=O3	t=3, o=O1	
S1	0	max(0, **0.2·0.6·0.5**, 0) = 0.06	max(0, 0, 0) = 0	
S2	0.2·1=0.2	max(0, 0, 0) = 0	max(0.06·0.8·1, 0, **0.056·1·1**) = 0.056	<- final max
S3	0.5·0.1=0.05	max(0·0.2·0.7, **0.2·0.4·0.7**, 0) = 0.056	max(**0.06·0.2·0.1**, 0·0.4·0.1, 0) = 0.0012	

d) Conclusion : $X_{viterbi}$ = {S2, S3, S2}