Statistical Methods for Financial Data
TP2: Project 1 – Asset Management

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Summary

We study the behaviour and robustness of two long-only risk-based portfolio strategies through a comprehensive series of in-sample and out-of-sample experiments. The two portfolio strategies are:

- **Minimum Variance** (*minvol*)
- **Maximum Diversification** (*maxdiv*)

First, we conduct an in-sample sensitivity analysis to assess how portfolio dimension, asset return variances, and correlations affect risk–return trade-offs. Second, we quantify parameter uncertainty by constructing Gaussian and Student-t bootstrap confidence intervals for the mean, volatility, and Sharpe ratio, showing that heavy-tailed assumptions yield wider, more conservative intervals. We then compare three covariance matrix estimators (sample, Ledoit–Wolf shrinkage, and three-factor model) and show that shrinkage substantially reduces estimation noise, leading to smoother risk profiles and more stable portfolios. We also investigate three resampling schemes (iid, block, and parametric Gaussian bootstrap) to evaluate their impact on portfolio performance. Finally, using a rolling-window backtest over 104 week look-back periods, we show that, while *maxdiv* can achieve higher absolute returns, *minvol* delivers consistently lower volatility and superior risk-adjusted returns out of sample. We also test the shrinkage and the resampling approaches in the out-of-sample exercise. Our results highlight the importance of robustification techniques for portfolio stability.

Method	Portfolio	Mean Ret.	Vol.	Sharpe	Cum. Ret.
Baseline (sample covariance)					
Sample covariance	maxdiv	0.0608	0.151	0.404	0.104
	minvol	0.0732	0.131	0.559	0.138
Shrinkage estimators					
Ledoit–Wolf	maxdiv	0.0776	0.149	0.408	0.143
	minvol	0.0791	0.131	0.541	0.151
3-Factor model	maxdiv	0.0824	0.147	0.437	0.154
	minvol	0.0812	0.131	0.524	0.156
Resampling schemes					
Plugin (no resampling)	maxdiv	0.0608	0.151	0.403	0.104
	minvol	0.0732	0.131	0.559	0.138
IID bootstrap	maxdiv	0.0589	0.144	0.408	0.102
	minvol	0.0708	0.131	0.541	0.133
Block bootstrap	maxdiv	0.0593	0.146	0.407	0.102
	minvol	0.0714	0.131	0.544	0.134
Gaussian bootstrap	maxdiv	0.0650	0.149	0.437	0.114
	minvol	0.0686	0.131	0.524	0.128

Table 1: Key out-of-sample performance across methods.

Contents

1 Data

This project considers the first 50 stocks (alphabetical order) in the FTSE 100 universe. We download data from January 2012 to December 2015 from the package `qrmdata` and compute discrete weekly returns. Due to data issues, we remove two firms (DLG.L and III.L), and then add two firms based on alphabetical order.

2 In-sample Sensitivity Analysis

2.1 Impact of the Dimension Given the Covariance Structure

This subsection presents a comparative analysis of the in-sample risk-return profiles of two types of risk-based portfolios: maxdiv and minvol. In **Figure 1**, each subplot corresponds to one of the portfolio construction strategies, with the left panel showing results for maxdiv and the right panel for minvol. The x-axis measures the in-sample volatility, while the y-axis captures the mean return obtained in-sample. Each dot in the plot represents a simulated portfolio (`num_subsets = 100`), and the portfolios are differentiated by the number of assets they include: 5 (red), 10 (green), and 25 (blue).

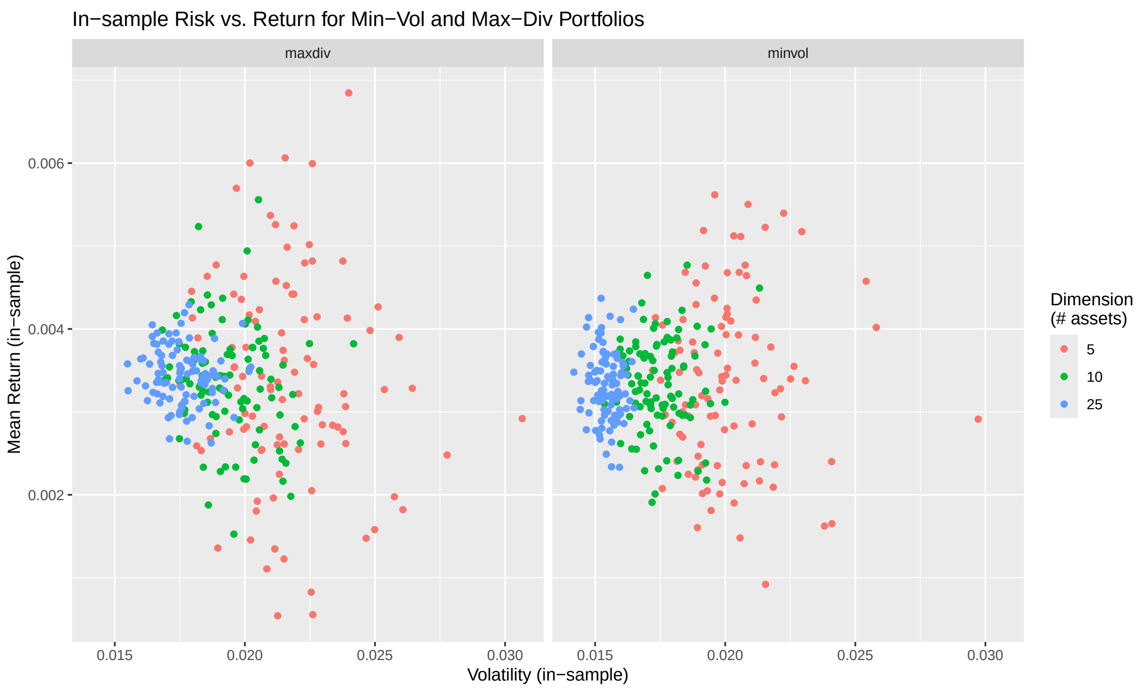


Figure 1: In-sample Risk vs. Return for Min-Vol and Max-Div Portfolios.

In the maxdiv panel, we observe a broader dispersion in both risk and return. Portfolios with a larger number of assets (particularly those with 25 assets, shown in blue) tend to cluster toward the lower end of the volatility scale, suggesting that diversification indeed lowers portfolio risk in this strategy. Moreover, these portfolios achieve relatively stable levels of mean return, forming a concentrated cluster. In contrast, portfolios with only 5 or 10 assets display higher variance in both return and volatility, indicating greater sensitivity to the specific composition of the assets chosen.

In the minvol panel, volatility is slightly more compressed, particularly for larger portfolios. Most portfolios, regardless of their dimension, achieve a lower volatility than the maxdiv portfolios. This reflects the core objective of the minvol strategy, that is to minimize risk regardless of return.

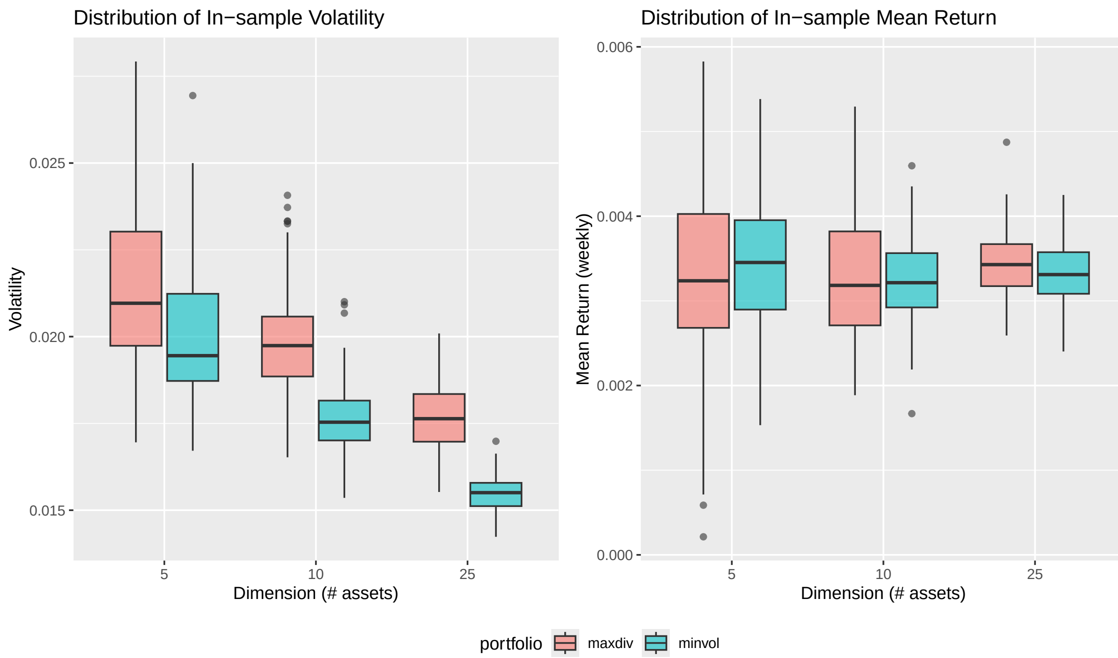


Figure 2: Distribution of in-sample mean return and in-sample volatility.

Comparing across the two strategies, in **Figure 2** we see that maxdiv portfolios offer greater return potential but at the cost of higher risk, particularly when the portfolio dimension is small. As the number of assets increases, both risk and return distributions become more concentrated. In contrast, minvol portfolios deliver lower risk. The visual evidence strongly suggests that dimension (number of assets) plays a critical role in stabilizing the performance of both strategies.

2.2 Impact of Variances Given Dimension and Correlation Structure

We now analyze the impact of variances, given dimension and correlation structure. The set of plots in **Figure 3** illustrate how increasing asset-level variances affects portfolio volatility. This is operationalized by scaling the asset variances uniformly via a "scale factor," applied equally across all assets in the universe. The correlation structure is fixed and hence portfolio weights remain unchanged, given that the scalar multiplies all asset variances.

When every asset variance is multiplied by a common scalar $c \geq 1$ while the correlation matrix is held fixed, the entire covariance matrix scales homothetically,

$$\widetilde{\Sigma} = c\,\Sigma, \qquad \widetilde{\mu} = \mu.$$

Because $(c\Sigma)^{-1} = c^{-1}\Sigma^{-1}$, the weights of both the global minvol and maxdiv portfolios are invariant to this transformation. Portfolio variance, however, grows linearly with c, so that

$$\widetilde{\sigma}_p^2 = w^\top \widetilde{\Sigma}\, w = c\, w^\top \Sigma w = c\, \sigma_p^2,$$

implying $\widetilde{\sigma}_p = \sqrt{c}\,\sigma_p$. **Figure 3** confirms this monotonic, deterministic rise in volatility, the spread across simulations widens more for sparse portfolios, reflecting their diminished capacity to diversify idiosyncratic shocks as all variances rise. **Figure 4** shows that mean returns remain constant across scale factors, as expected. Only the variances are scaled, not the return process.

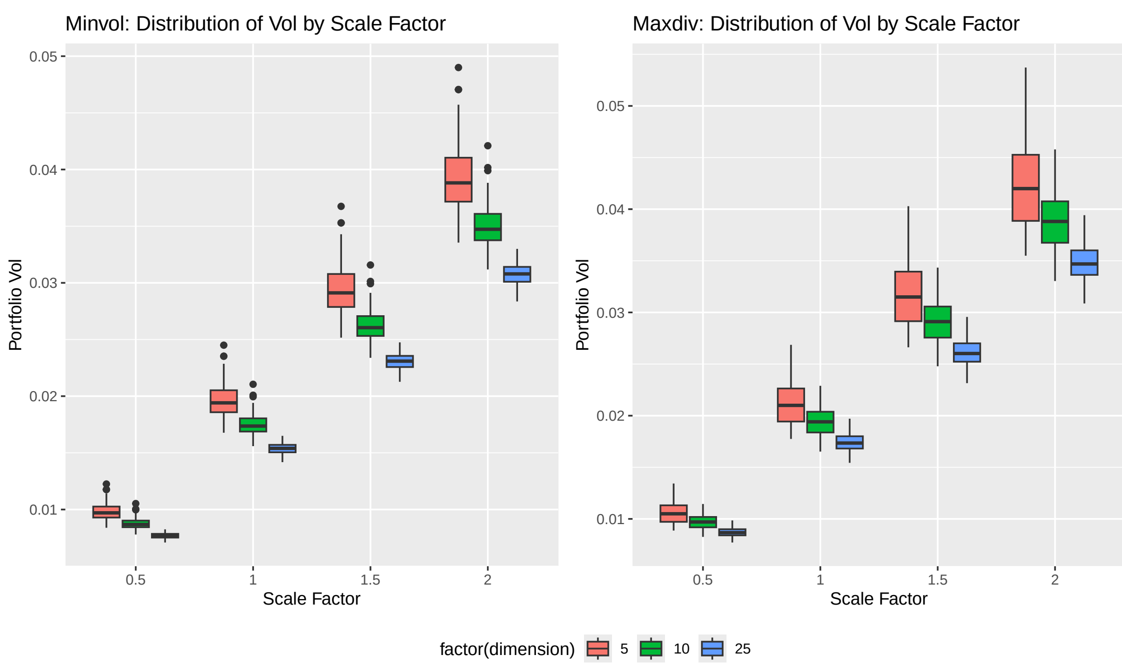


Figure 3: Volatility scale factors

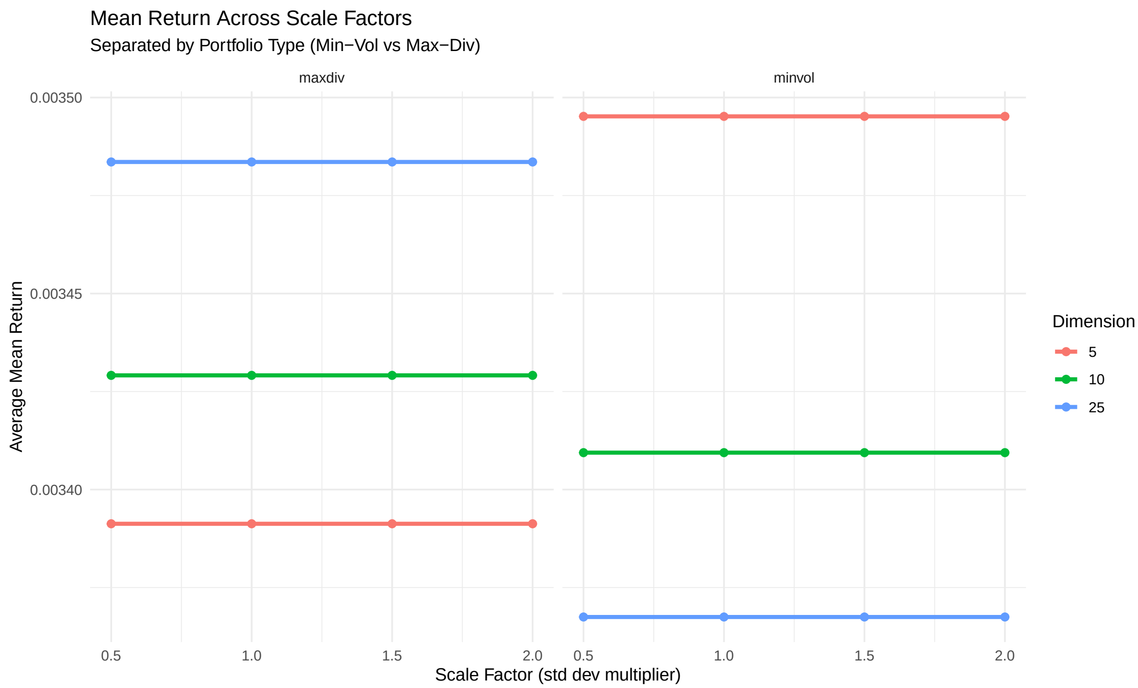


Figure 4: Mean Return Across Scale Factors.

2.3 Impact of Correlation Given Dimension and Variances

Next, **Figure 5** illustrates how the volatility of the minvol portfolio evolves with increasing equicorrelation levels (ρ) for different portfolio sizes. Each panel represents a different portfolio dimension (d = 5, 10, 25 assets), and the bold line highlights the average volatility across multiple random subsets. As expected, portfolio volatility increases monotonically with the level of correlation. This reflects that higher correlation reduces diversification benefits: when assets become more correlated, the ability to cancel out individual risks declines, leading to a rise in total portfolio risk. Importantly, portfolios with more assets (d = 25) achieve lower volatility compared to portfolios with a fewer assets. However, as the equicorrelation increases, the portfolio volatility among all portfolio dimensions becomes closer.

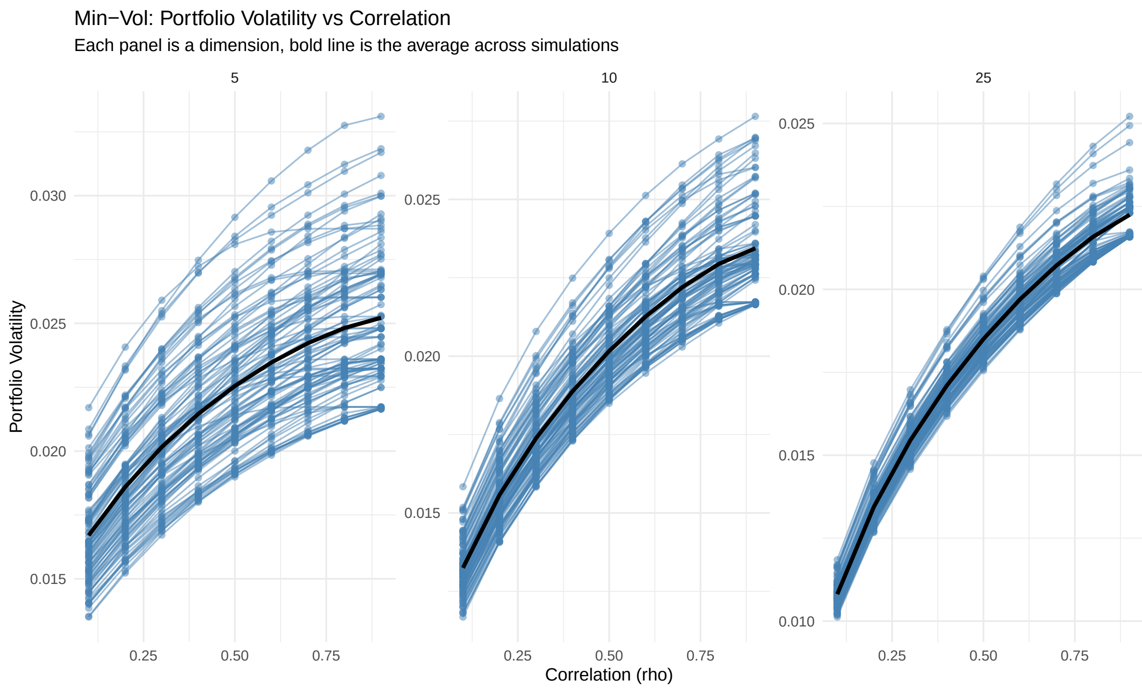


Figure 5: MinVol: Volatility vs. Correlation.

The second figure (**Figure 6**) presents the same structure but focuses on the maxdiv portfolio. The pattern is qualitatively similar, portfolio volatility increases with correlation, and larger portfolios again benefit from lower volatility levels. However, compared to minvol, the maxdiv portfolios tend to exhibit slightly higher average volatilities. This is consistent with the objective of the maxdiv strategy, which does not minimize volatility directly, but instead maximizes the ratio of weighted average asset volatility to overall portfolio volatility.

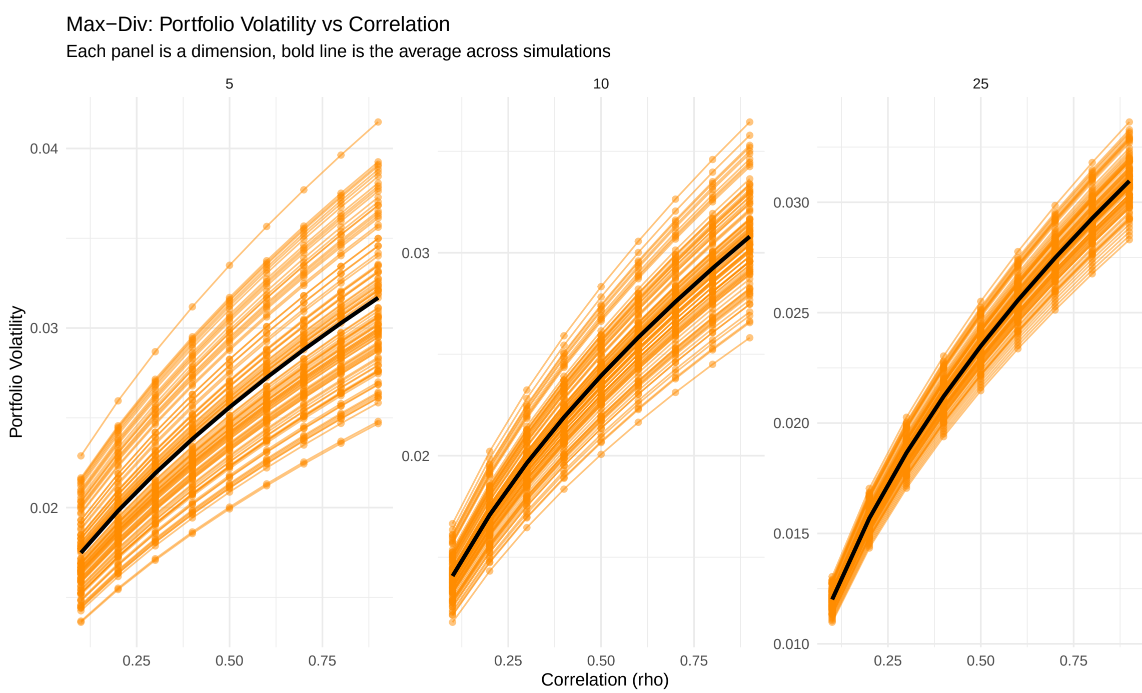


Figure 6: MaxDiv: Volatility vs. Correlation.

Figure 7 shifts focus to the mean return of the portfolios as a function of correlation. For maxdiv portfolios, the mean return remains unchanged across all levels of correlation. This behavior aligns with theory: under the assumption of equicorrelation, it's possible to prove that the maxdiv weights are invariant to ρ. The result is based on Choueifaty & Coignard (2008) and the fact that

$$w^{\star} \propto \Sigma^{-1}\boldsymbol{\sigma}$$

with $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)^{\top}$. In contrast, the minvol portfolios show a mild but consistent decline in mean return as correlation increases, especially in smaller dimensions, with exception to the case the number of assets is 25 and larger values of equicorrelation.

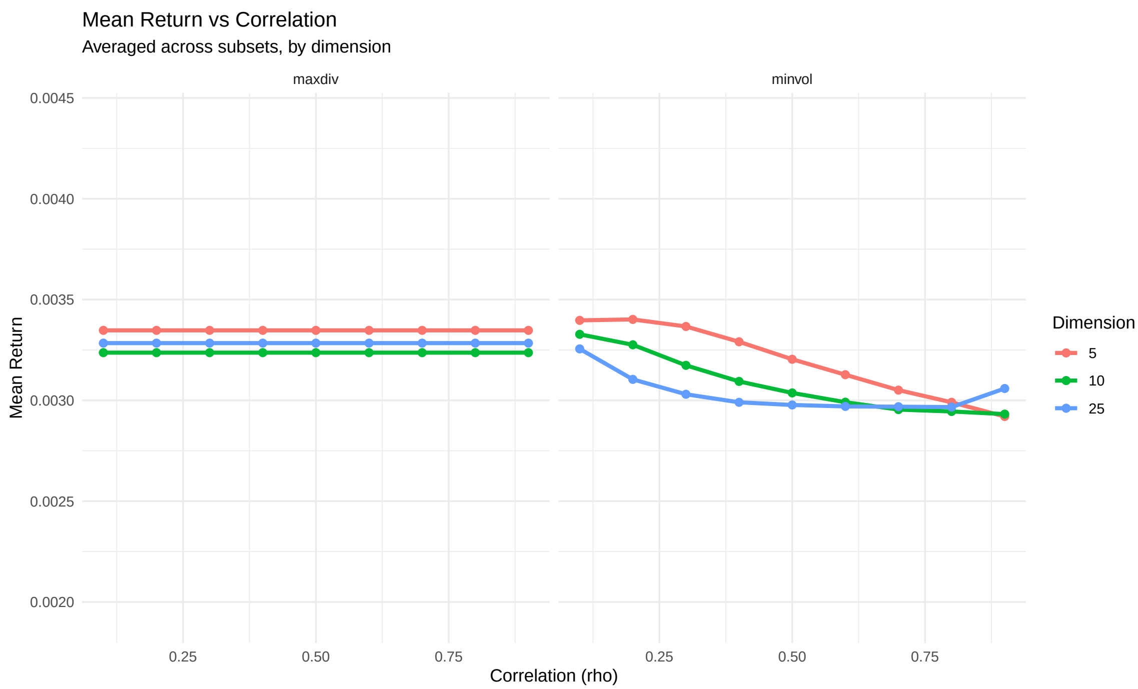


Figure 7: Mean Return vs. Correlation for MinVol and MaxDiv Portfolios.

3 In-sample Uncertainty

3.1 Gaussian Model

This section first investigates the impact of parameter uncertainty in the in-sample performance using a Gaussian model calibrated on the data. Table 2 presents 95% confidence intervals computed via Gaussian bootstrap simulations (B=1000) for two portfolio strategies: minvol and maxdiv. **Figures 8, 9, 10** present visual evidence based on the simulations.

Starting with the mean, the maxdiv portfolio has a 95% CI ranging from approximately 0.127% to 0.575% per week, with a median of 0.361%. In contrast, the minvol portfolio shows a CI of 0.134% to 0.542%, with a median of 0.337%. Although the minvol return is lower on average, the intervals overlap somewhat, suggesting that the difference in returns may not be statistically significant depending on context and risk preferences.

Regarding volatility, the maxdiv portfolio is associated with a CI between 1.32% and 1.67%, and a median of 1.50%, whereas the minvol portfolio shows a CI between 1.20% and 1.46%, with a median of 1.33%. This aligns with the core objective of the minvol strategy.

The Sharpe ratio provides further insight into risk-adjusted performance. For maxdiv, it falls between 0.087 and 0.386 (median 0.241). Minvol displays a CI of 0.099 to 0.416 (median 0.251), suggesting that while maxdiv may yield higher raw returns, minvol offers marginally better consistency in risk-adjusted performance.

Portfolio	Metric	2.5%	50%	97.5%
maxdiv	mean	0.00127	0.00361	0.00575
maxdiv	sharpe	0.08690	0.24100	0.38600
maxdiv	vol	0.01320	0.01500	0.01670
minvol	mean	0.00134	0.00337	0.00542
minvol	sharpe	0.09910	0.25100	0.41600
minvol	vol	0.01200	0.01330	0.01460

Table 2: Bootstrap quantiles of in-sample performance metrics

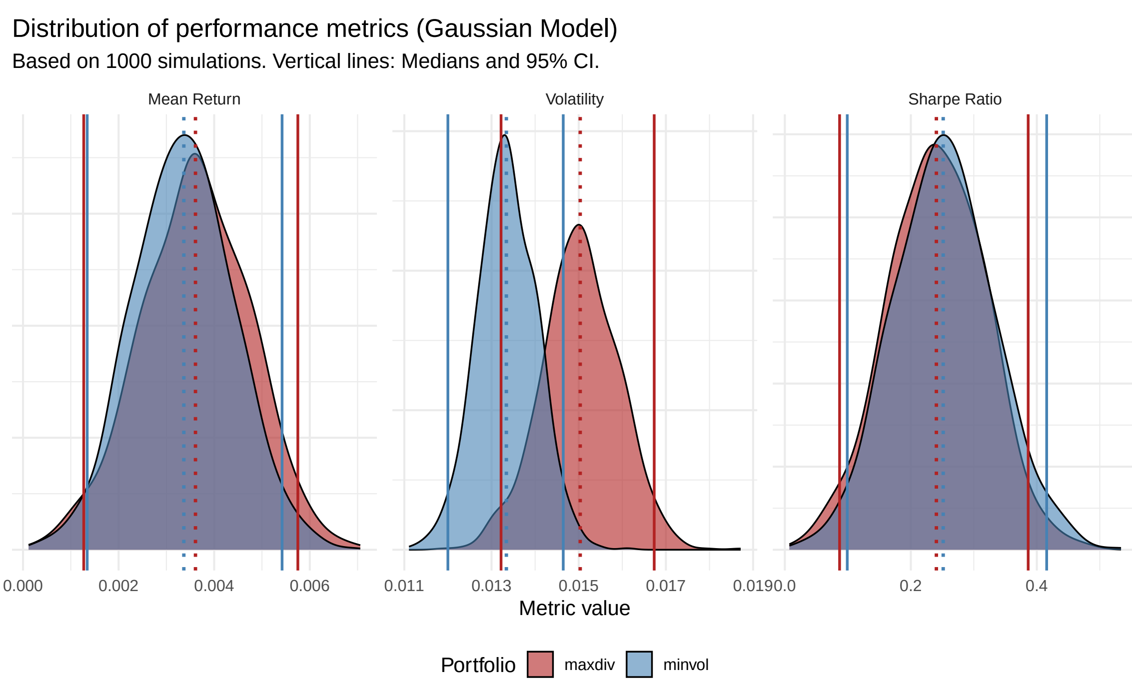


Figure 8: Gaussian Density Plots and Confidence Intervals

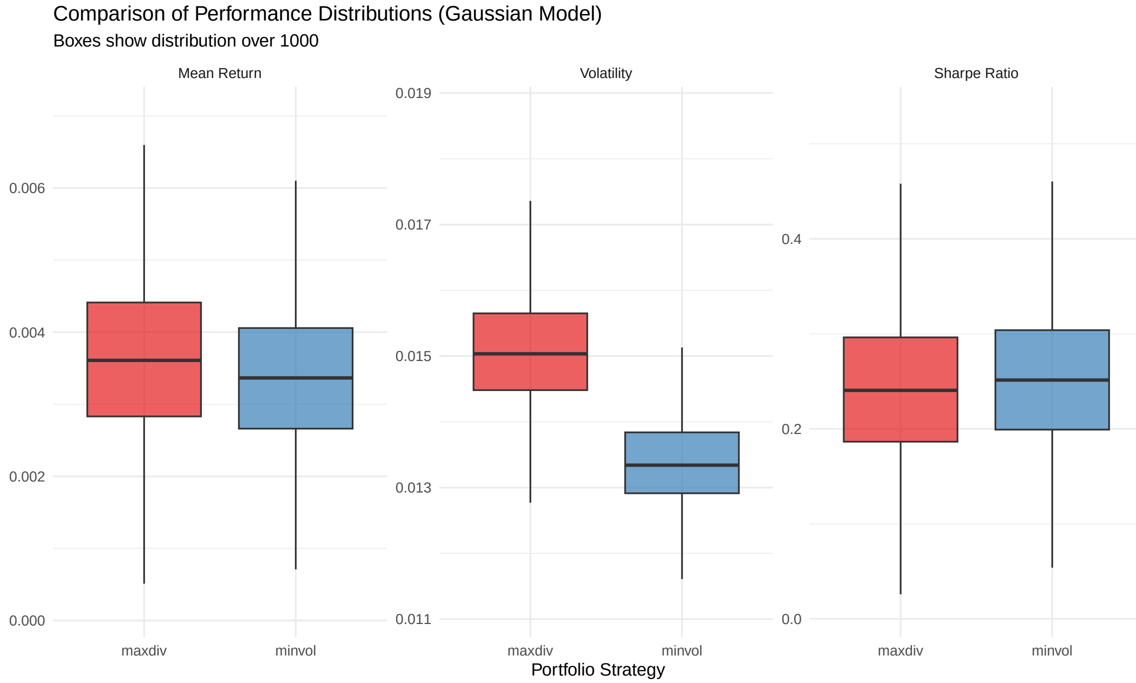


Figure 9: Performance Distributions.

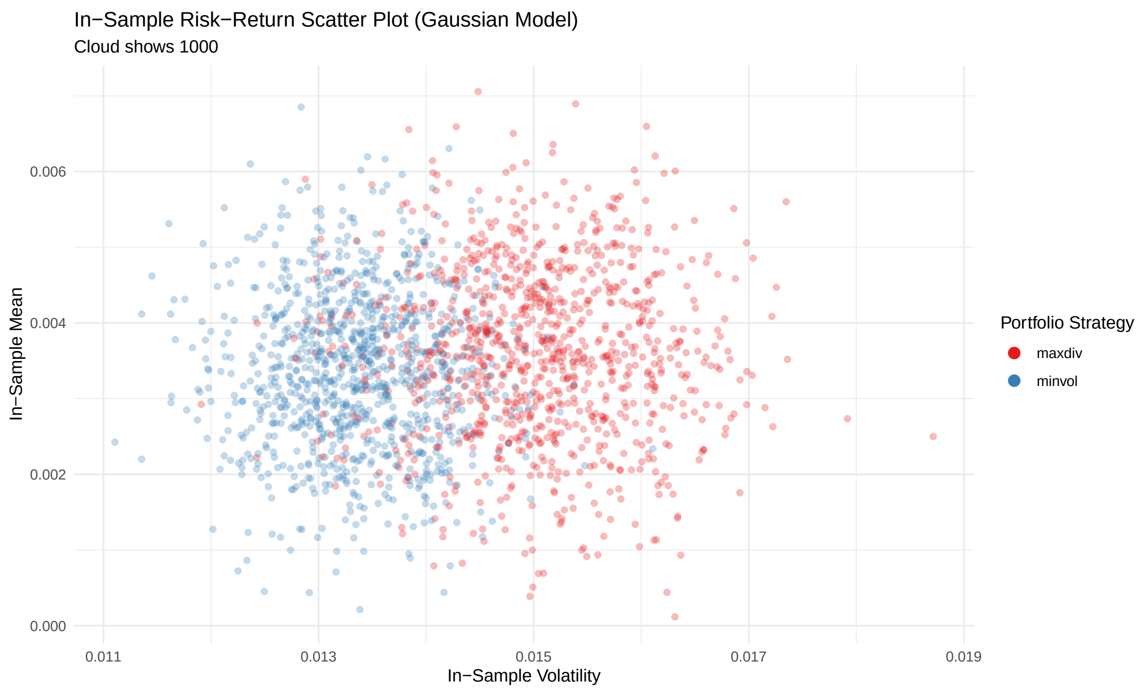


Figure 10: Risk-Return Scatter.

Overall, these Gaussian bootstrap-based confidence intervals suggest that both portfolios deliver positive returns and Sharpe ratios, but with differing trade-offs. The maxdiv strategy tends to achieve higher average returns at the cost of higher and more variable risk, while minvol offers more stable, lower-risk performance with comparable risk-adjusted returns.

3.2 Student-t Model

This subsection considers the same approach as in subsection 2.1 but with a Student-t model. Table 3 displays the 95% bootstrap confidence intervals for the mean, volatility, and Sharpe ratio under the Student-t model, across two portfolios: maxdiv and minvol. Starting with the mean, the maxdiv portfolio shows a 95% confidence interval ranging from approximately 0.040% to 0.533% per week, with a median of 0.300%. For the minvol strategy, the interval is tighter and lower, from 0.031% to 0.487%, with a median of 0.268%. These results suggest that, even under the Student-t model, the maxdiv strategy is expected to yield slightly higher returns on average, although the distributions are relatively close, as we can see in **Figure 11**.

Turning to volatility and risk-adjusted performance, The maxdiv portfolio exhibits a higher volatility profile, with a 95% confidence interval ranging from 1.45% to 1.94%, and a median of 1.67%, while the minvol strategy shows lower volatility, between 1.33% and 1.74%, with a median of 1.52%. These estimates confirm that minvol fulfills its core objective of risk reduction, and notably, the volatility intervals under the Student-t distribution remain quite stable and closely aligned with those observed under the Gaussian model. In terms of Sharpe ratio, maxdiv spans a wider interval, from 0.024 to 0.332, with a median of 0.179, while minvol is in the range of 0.195 to 0.336, with a median of 0.175.

Portfolio	Metric	2.5%	50%	97.5%
maxdiv	mean	0.00040	0.00300	0.00534
maxdiv	sharpe	0.02360	0.17900	0.332
maxdiv	vol	0.01450	0.01670	0.0194
minvol	mean	0.00031	0.00268	0.00487
minvol	sharpe	0.01950	0.17500	0.336
minvol	vol	0.01330	0.01520	0.0174

Table 3: Bootstrap quantiles of out-of-sample performance metrics (updated)

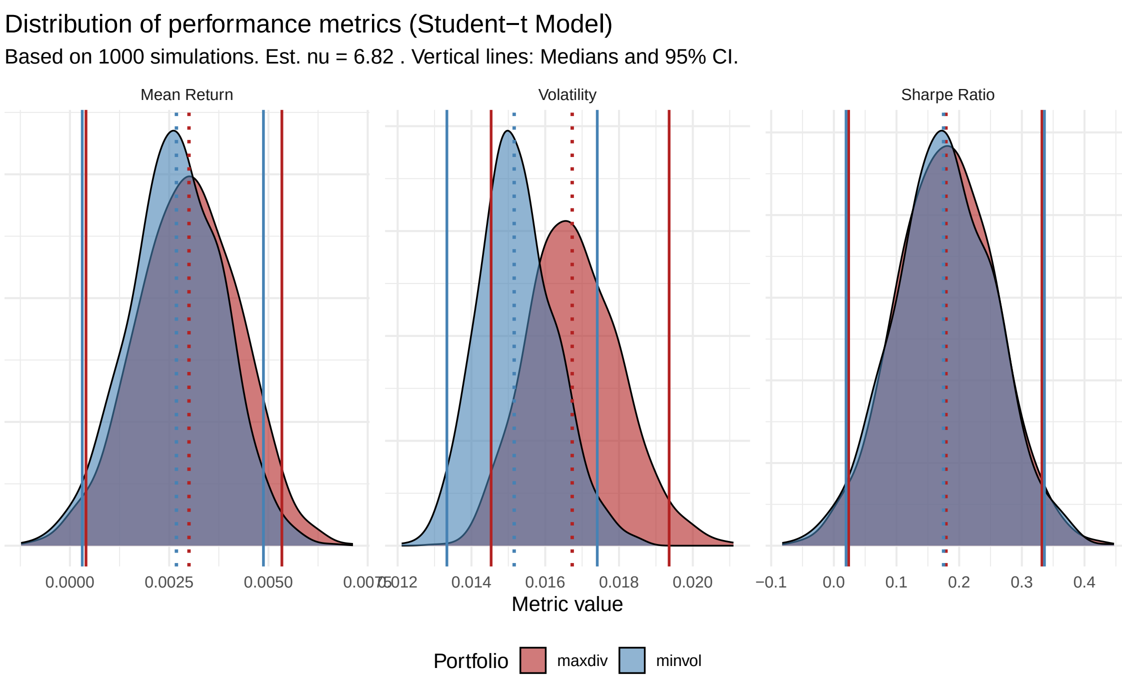


Figure 11: Student-t Density Plot and Confidence Intervals

Tables 4 and 5 present additional results for the Student-t and Gaussian fitted models, respectively. Comparing the Gaussian and Student-t bootstrap results reveals several important insights about how distributional assumptions affect perceived performance. The Student-t model consistently yields wider and more conservative confidence inter-

Portfolio	Mean	SE (Mean)	Vol	SE (Vol)	Sharpe	SE (Sharpe)	# Sims
maxdiv	0.00299	0.00127	0.0168	0.00124	0.1790	0.0775	1000
minvol	0.00267	0.00117	0.0152	0.00103	0.1760	0.0790	1000

Table 4: Student-t fitted-model summary

Portfolio	Mean	SE (Mean)	Vol	SE (Vol)	Sharpe	SE (Sharpe)	# Sims
maxdiv	0.00361	0.00113	0.0150	0.000876	0.241	0.0772	1000
minvol	0.00336	0.00103	0.0134	0.000677	0.253	0.0790	1000

Table 5: Gaussian fitted-model summary

vals. The comparison between the two models shows how distributional assumptions materially affect performance evaluation. Across all metrics, the Student-t model delivers more conservative estimates, especially for mean and volatility. This reflects the Student-t's ability to capture heavy tails, thereby accounting for rare but impactful extreme events. In contrast, the Gaussian model, by assuming thin tails, understates uncertainty and compresses confidence interval.

3.3 In-sample Shrinkage Approaches

We now analyse the impact of robustification techniques. In particular, we compare the results considering the sample covariance matrix against two shrinkage estimators from the package `RiskPortfolios`:

- Ledoit–Wolf estimator (see Ledoit and Wolf (2003))
- Factor analysis with a total of 3 factors (see Harman (1976))

First, to provide a visualization about what these alternative estimators are doing, we plot three heatmaps, one for each of the covariance-matrix estimators. These heatmaps consider the entire sample period and all 50 assets. The sample covariance matrix heatmaps reveals a highly detailed and noisy structure, where the clustering patterns appear dense and full of minor variations. This reflects the fact that the sample covariance estimator uses only historical data without any form of regularization. While the heatmap retains all the empirical relationships, it also retains much of the noise, which can lead to unstable portfolio optimization results.

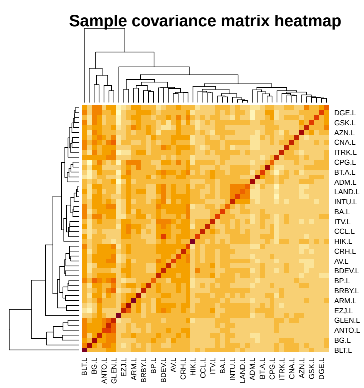
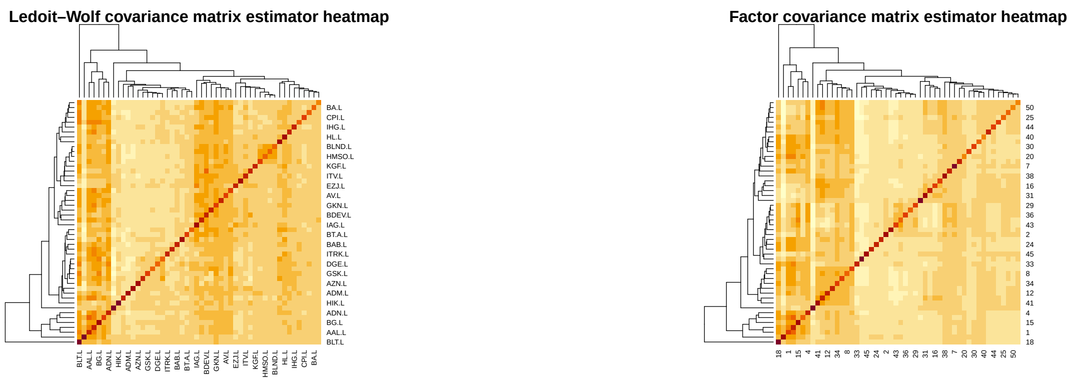


Figure 12: Sample covariance heatmap.

The Ledoit–Wolf covariance matrix heatmap in **Figure 13**, by contrast, exhibits more structured block patterns. This estimator applies shrinkage toward a structured target, which reduces estimation error and enhances numerical stability. The heatmap visually reflects this. The noise is visibly reduced, and asset clusters emerge. We expect that this result in a more stable and reliable covariance estimate, especially valuable in high-dimensional settings.

Figure 13 also shows the factor covariance matrix heatmap. It shows an even more pronounced level of shrinkage and structure. Instead of estimating pairwise covariances directly, this approach models asset covariances via exposures to a small number of underlying factors. This structure implies that much of the correlation is being captured through common latent components, and idiosyncratic relationships are downweighted.



(a) Ledoit–Wolf covariance heatmap

(b) Factor-model covariance heatmap

Figure 13: Covariance heatmaps using shrinkage estimators: (a) Ledoit–Wolf, (b) factor model

We now investigate the in-sample impact of different covariance matrix estimators on the performance of optimized portfolios. Using bootstrap resampling (B = 1000), we evaluate the two portfolio strategies, minimum volatility and maximum diversification.

Figure 14 and **15** present the dispersion under the different covariance matrix estimators for minvol and maxdiv portfolios. **Figure 15** shows that shrinkage reduces volatility dispersion for both portfolios. The sample covariance produces wider spreads and more extreme outliers. In contrast, both Ledoit–Wolf and the factor model result in tighter and lower volatility estimates, implying more stable risk profiles. This is expected, shrinkage reduces estimation error by pulling extreme covariance entries toward a structured prior, leading to greater robustness in the optimized weights.

Importantly, for maxdiv, the LW estimator produces the lowest volatility on average, while the factor model stabilizes but with a slightly higher level. This may be due to the factor model ignoring some idiosyncratic variation. For minvol, the pattern is similar: sample covariance has more noise, LW delivers lower average volatility.

In terms of means, the picture changes. **Figure 14** shows that the sample estimator tends to produce higher median for minvol and a similar level for maxdiv. This suggests that optimizing with the raw sample covariance matrix may lead to more aggressive portfolios, reflecting both upside potential and downside risk due to overfitting noise in the sample. In contrast, Ledoit–Wolf and factor-based portfolios yield more conservative return profiles, with tighter distributions centered around slightly lower means.

Our results results reaffirm that shrinkage estimators improve stability, reducing the risk of overfitting and leading to more reliable portfolio construction, especially when asset dimensionality is high or the sample size is limited. We expect that these results should be even more pronounced when we consider out-of-sample results.

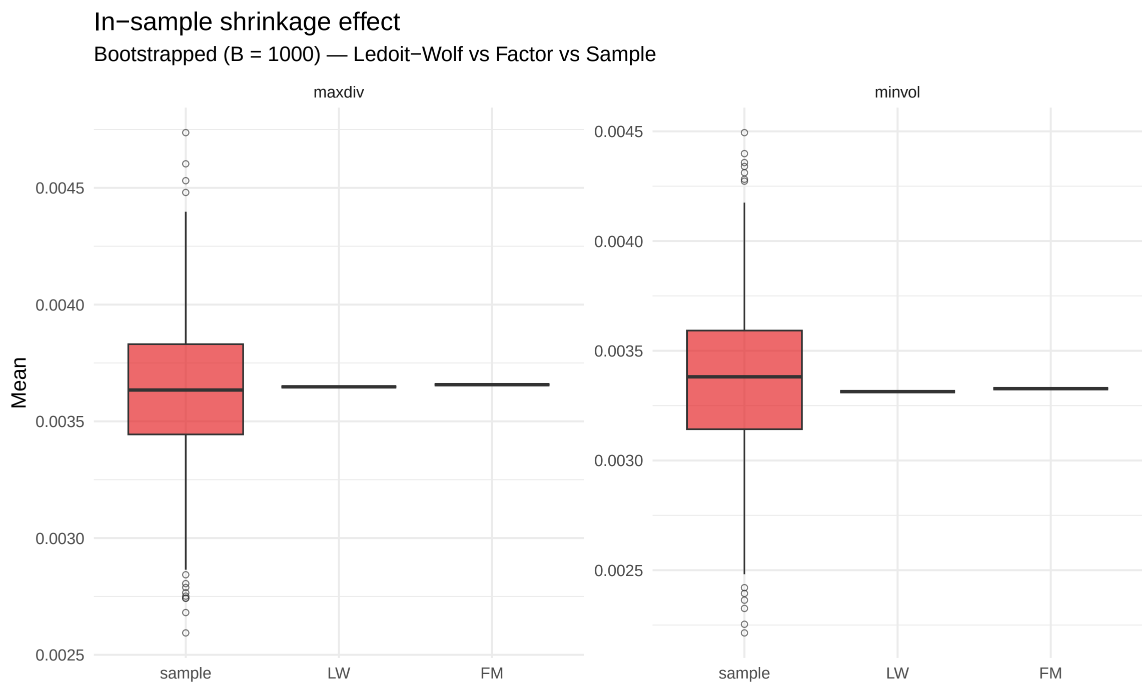


Figure 14: Mean dispersion under different covariance estimators for portfolios minvol and maxdiv.

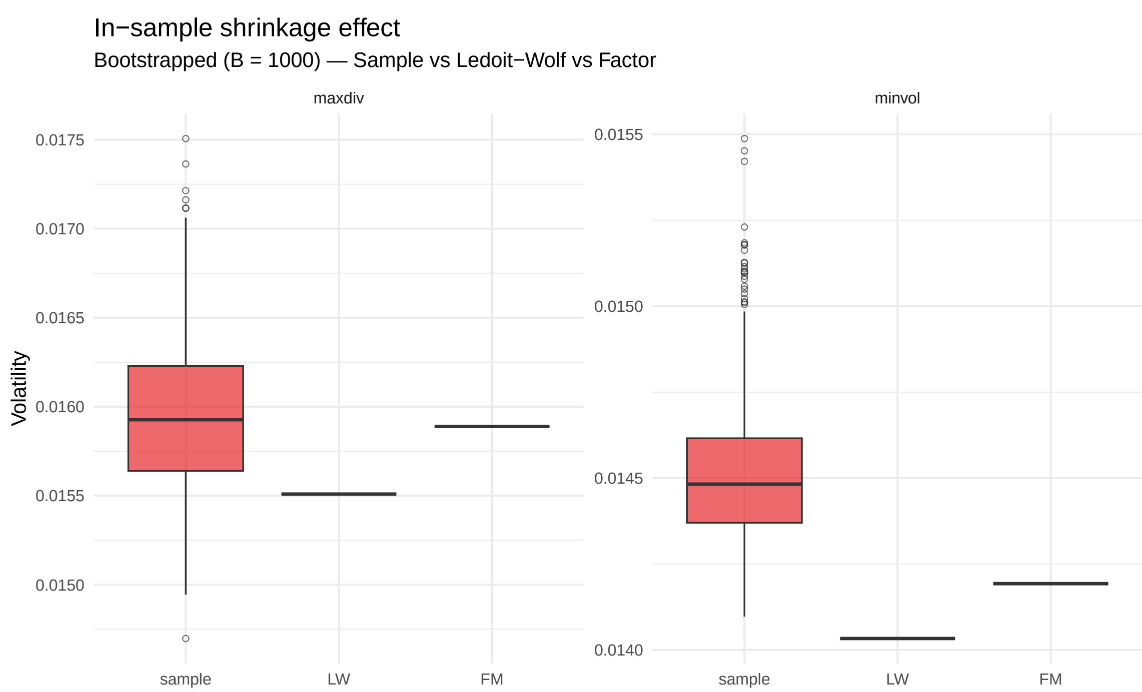


Figure 15: Volatility dispersion under different covariance estimators for portfolios minvol and maxdiv.

Covariance shrinkage (in-sample medians)				
Estimator	Portfolio	Mean Ret. [%]	Vol. [%]	Sharpe
Sample cov.	maxdiv	0.363	1.59	0.228
	minvol	0.337	1.45	0.232
Ledoit–Wolf	maxdiv	0.365	1.55	0.235
	minvol	0.331	1.40	0.236
3-Factor	maxdiv	0.366	1.59	0.230
model	minvol	0.333	1.42	0.234

Table 6: In-sample results: parameter uncertainty and shrinkage effects

3.4 Comparing Different Resampling Approaches

In this analysis, we explore how different resampling strategies impact the construction and stability of optimized portfolios when using the sample covariance matrix estimator. We consider three resampling schemes:

- **IID bootstrap:** resamples rows independently with replacement.
- **Block bootstrap:** resamples consecutive blocks of returns to preserve dependence (Politis and Romano, 1994), implemented via the `tseries` package with the stationarity option.
- **Gaussian resampling:** generates synthetic returns from a multivariate normal distribution calibrated to the data based on the sample mean and covariance matrix.

The plots for the dispersion of mean, volatility and Sharpe ratio (**Figure 16**) summarise how results change according to each resampling scheme for each portfolio strategy. When the two strategies are evaluated under identical resampling schemes, the maximum diversification portfolio consistently posts the higher mean return but also exhibits the larger volatility. Those offsetting movements leave the Sharpe ratio distribution almost unchanged relative to the minvol portfolio, with the exception to the iid bootstrap for the minvol strategy. The medians of the two boxes are virtually coincident under every specification.

The dispersion of Sharpe ratios, however, differs across resampling techniques. Under iid bootstrapping the Sharpe distribution is widest. Resampling individual weeks does not account for serial dependence, producing some resamples with unrealistically low aggregate risk and others with inflated risk, and the optimiser reacts by generating a broad range of outcomes. The Gaussian parametric resampler imposes a multivariate-normal structure on every draw, the absence of fat tails removes the most extreme scenarios. The inter-quartile range is relatively close to the iid case. The stationary block bootstrap yields the tightest Sharpe distribution of all, retaining volatility clustering in each pseudo-sample penalises portfolios that are overly sensitive to transient covariance patterns, leading to a narrower, more reliable band of risk-adjusted returns than those produced by iid or Gaussian resampling.

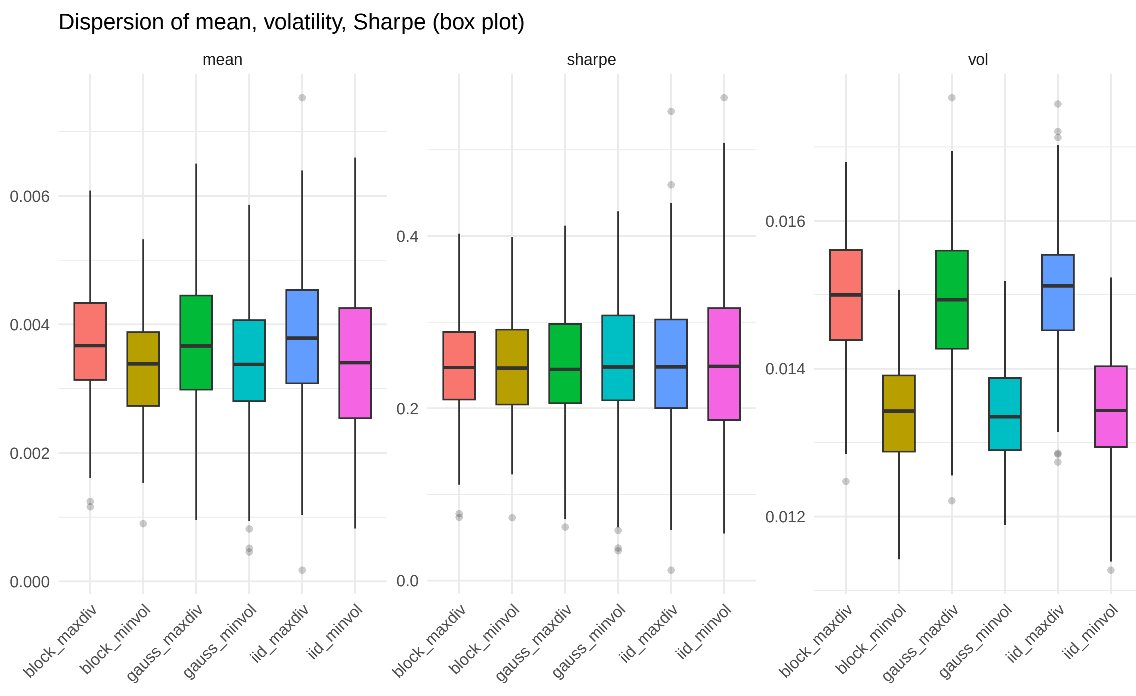


Figure 16: Dispersion of mean, volatility, and Sharpe ratio under each resampling scheme for each portfolio strategy.

4 Out-of-sample Impact

4.1 Out-of-sample Performance Analysis

In this section we select the first 25 equities in the data set in alphabetical order, and estimate our model on a 104-week look-back window ($\approx$ 2 years). For each week we:

(i) compute the plain sample covariance of those 104 observations;

(ii) form two long-only efficient portfolios: the minimum-variance portfolio and the maximum-diversification portfolio; and

(iii) apply the resulting weights to the return of the next week.

All subsequent weeks repeat this roll-forward procedure, so every out-of-sample (OOS) return is generated with information that was available at the time, without data leakage. Table 7 presents a summary of the OOS performance for each strategy. In this set-up, the minvar strategy delivers the smoothest out-of-sample rolling-window volatility and the highest Sharpe ratio, whereas the maxdiv strategy earns a lower average return and exhibits higher volatility. **Figure 17** shows the cumulative return of the portfolios and **Figure 18** shows the OOS rolling 52-week volatility.

Portfolio	Ann. Mean Ret.	Ann. Vol.	Sharpe	Cum. Ret.
maxdiv	0.0608	0.151	0.404	0.104
minvol	0.0732	0.131	0.559	0.138

Table 7: Out-of-sample performance summary.

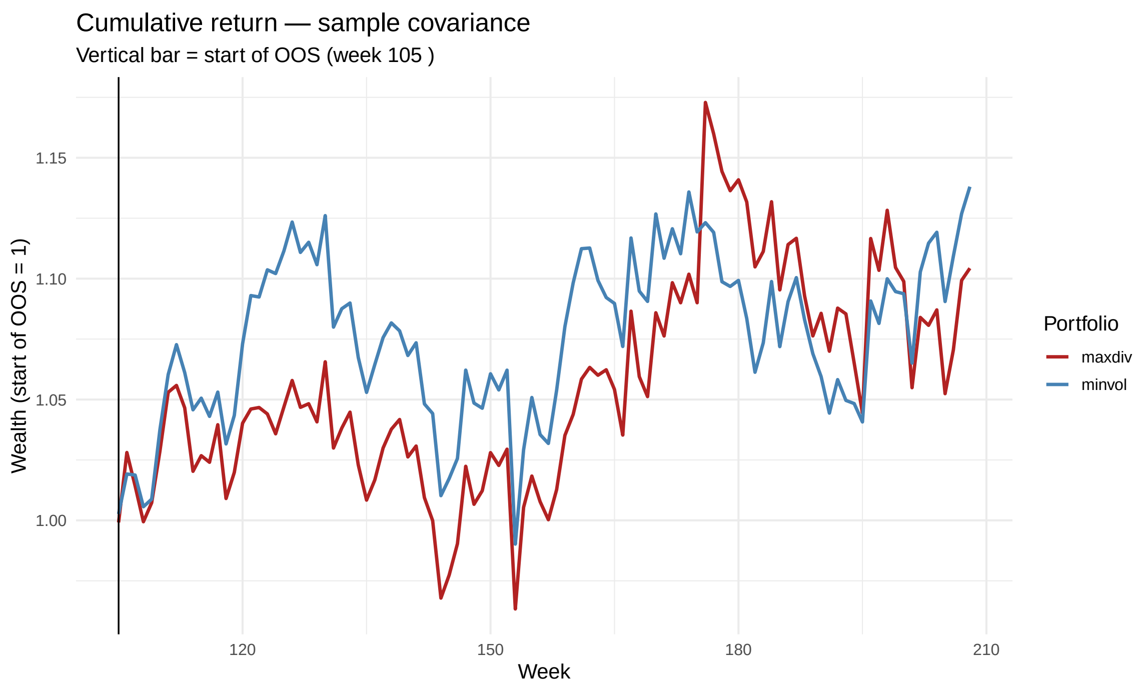


Figure 17: Cumulative return for minvol and maxdiv.

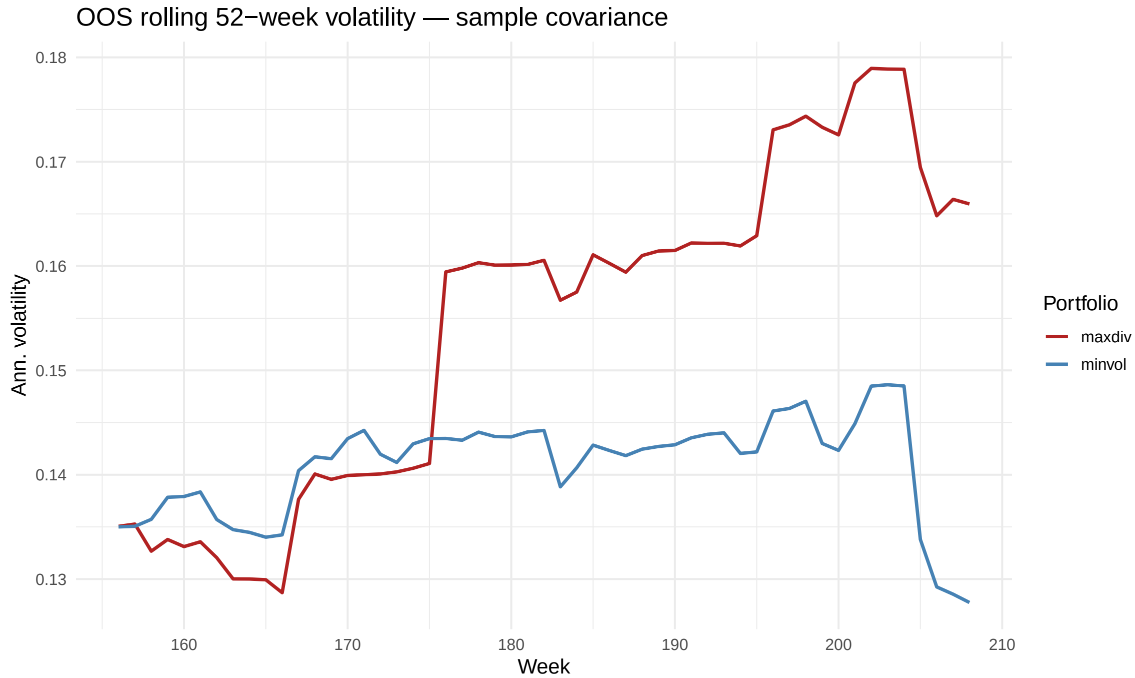


Figure 18: OOS rolling 52-week volatility.

4.2 Out-of-sample Performance and Shrinkage Approaches

Keeping the same rolling window and weekly re-balancing schedule, we now replace the noisy sample covariance with two shrinkage estimators:

(i) Ledoit–Wolf linear shrinkage toward the identity matrix

(ii) A 3-factor model whose covariance is shrunk toward the factor block

These are the same estimators we considered in Section 2. Both estimators deal with estimation errors, lifting the smallest eigenvalues and lowering the condition number. Out-of-sample this translates into visibly smoother return trajectories and lower volatility for both portfolios.

The statistics in table 8 and the cumulative return plots in **Figure 19** show that both portfolios benefit from the shrinkage approaches. For the maxdiv portfolio, both shrinkage estimators outperform the sample covariance in all performance metrics. The mean return increases from 6.08% (sample) to 7.76% (Ledoit–Wolf) and 8.24% (3-factor), while volatility remains largely unchanged. The Sharpe ratio improves from 0.0560 under the sample estimator to 0.0725 with Ledoit-Wolf and 0.0776 with the 3-factor model. The cumulative return also rises, from 10.4% for the sample to 14.3% and 15.4% for the Ledoit-Wolf and 3-factor estimators, respectively.

A similar pattern is observed for the minvol portfolio. The sample covariance yields a mean return of 7.32% and a Sharpe ratio of 0.0776. Both shrinkage methods again improve these figures, with the 3-factor model achieving the highest mean return (8.12%) and Sharpe ratio (0.0857), along with a cumulative return of 15.6%, compared to 13.8% under the sample.

Figure 20 presents the OOS rolling 52-week volatility, for the maxdiv strategy the sample covariance estimator generate a volatility rolling window that is higher than the other, while the result is less pronounced for minvol strategy. Overall, both shrinkage estimators deliver superior out-of-sample performance relative to the sample covariance, with the 3-factor approach offering the best overall results for both portfolios in terms of return, risk-adjusted return, and cumulative return.

Cov. Est.	Portfolio	Ann. Mean Ret.	Ann. Vol.	Sharpe	Cum. Ret.
sample	maxdiv	0.0608	0.151	0.404	0.104
	minvol	0.0732	0.131	0.559	0.138
Ledoit–Wolf	maxdiv	0.0776	0.149	0.408	0.143
	minvol	0.0791	0.131	0.541	0.151
3-factor	maxdiv	0.0824	0.147	0.437	0.154
	minvol	0.0812	0.131	0.524	0.156

Table 8: Out-of-sample performance by covariance estimator and portfolio.

Figure 19: Cumulative Return - Shrinkage

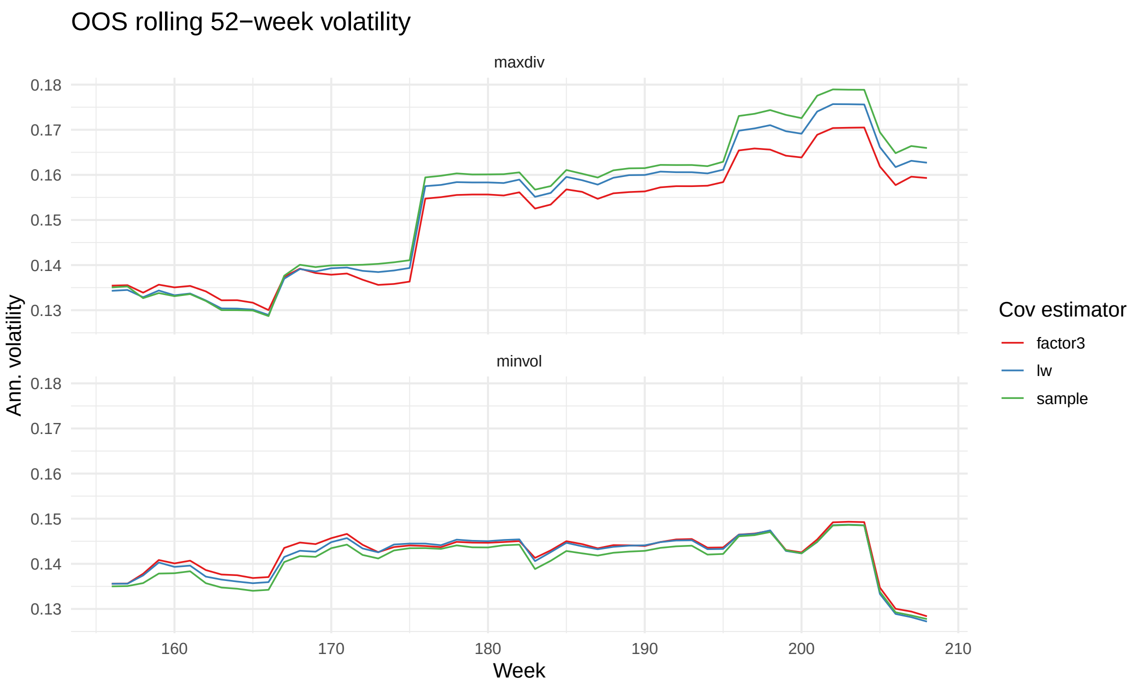


Figure 20: OOS rolling 52-week volatility based on different shrinkage approches.

4.3 Out-of-sample Performance and Resampling Approaches

In this subsection, instead of taming estimation error inside the covariance matrix, we address it at the portfolio construction step. For every weekly window we create 100 pseudo-samples based on iid bootstrap, a stationary block bootstrap, or a Gaussian parametric bootstrap. We optimise each replica, average the resulting weight vectors, and then invest those averaged weights for the next week to compute the out-of-sample statistics and compare the three resampling approaches.

Table 9 present the statistics and **Figure 21** the cumulative returns. The plugin estimator represent the standard sample covariance estimator without any resampling scheme. Our results show that plugin portfolios carry the most estimation noise. For the maxdiv portfolio, performance differences across resampling methods are relatively small. The mean returns range from 5.89% (iid) to 6.50% (Gaussian), with the Gaussian approach yielding the highest return and Sharpe ratio (0.437). Volatility stays narrow across methods, with the exception to the plugin estimator in the maxdiv strategy. **Figure 22** shows that it has higher OOS rolling 52-week volatility compared to the other approaches. Cumulative returns vary modestly, with the Gaussian method again slightly outperforming the rest.

The minvol portfolio displays similar stability across resampling techniques. The plugin method achieves the highest mean return (7.32%) and Sharpe ratio (0.559), followed closely by the block bootstrap and iid bootstrap. The Gaussian method results in slightly lower returns (6.86%) and Sharpe ratio (0.524).

Overall, in our sample, resampling has a relatively minor effect on out-of-sample performance compared to shrinkage approaches analyzed in Section 3.3. While the Gaussian method improves maxdiv results slightly, the plugin estimator remains a good alternative for minvol, suggesting that the benefits of resampling may depend on the portfolio strategy being used.

Resampling	Portfolio	Mean Ret.	Vol.	Sharpe	Cum. Ret.
plugin	maxdiv	0.0608	0.151	0.403	0.104
	minvol	0.0732	0.131	0.559	0.138
iid	maxdiv	0.0589	0.144	0.408	0.102
	minvol	0.0708	0.131	0.541	0.133
block	maxdiv	0.0593	0.146	0.407	0.102
	minvol	0.0714	0.131	0.544	0.134
gauss	maxdiv	0.0650	0.149	0.437	0.114
	minvol	0.0686	0.131	0.524	0.128

Table 9: Out-of-sample performance by resampling method and portfolio.

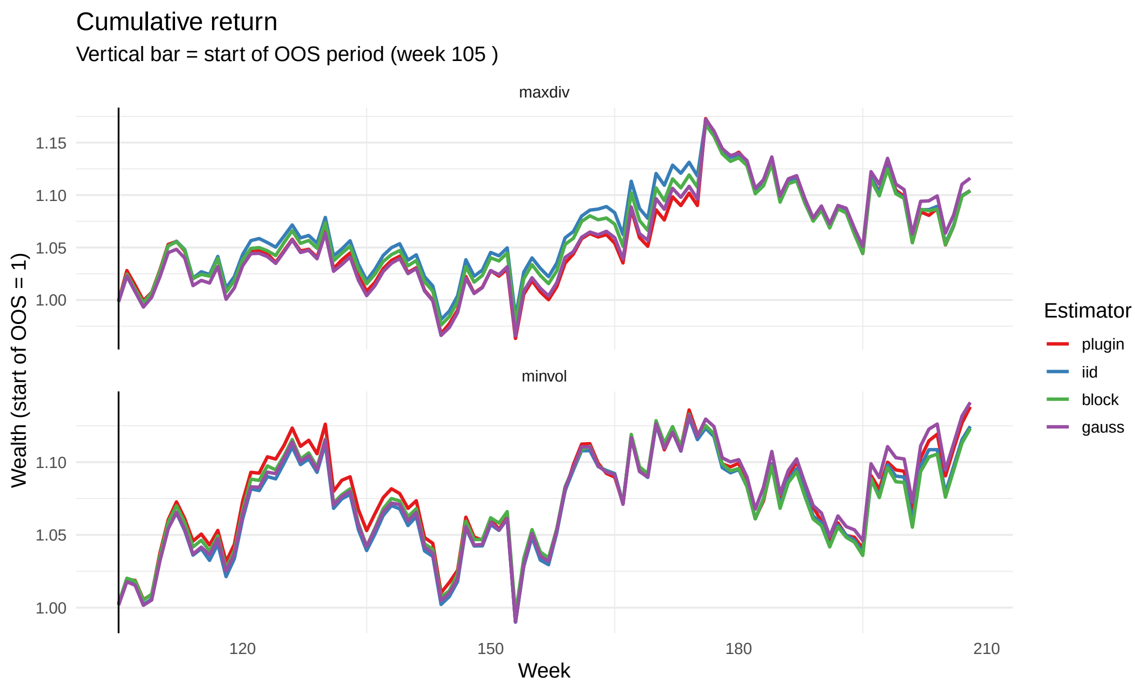


Figure 21: Cumulative return based on resampling for each portfolio strategy.

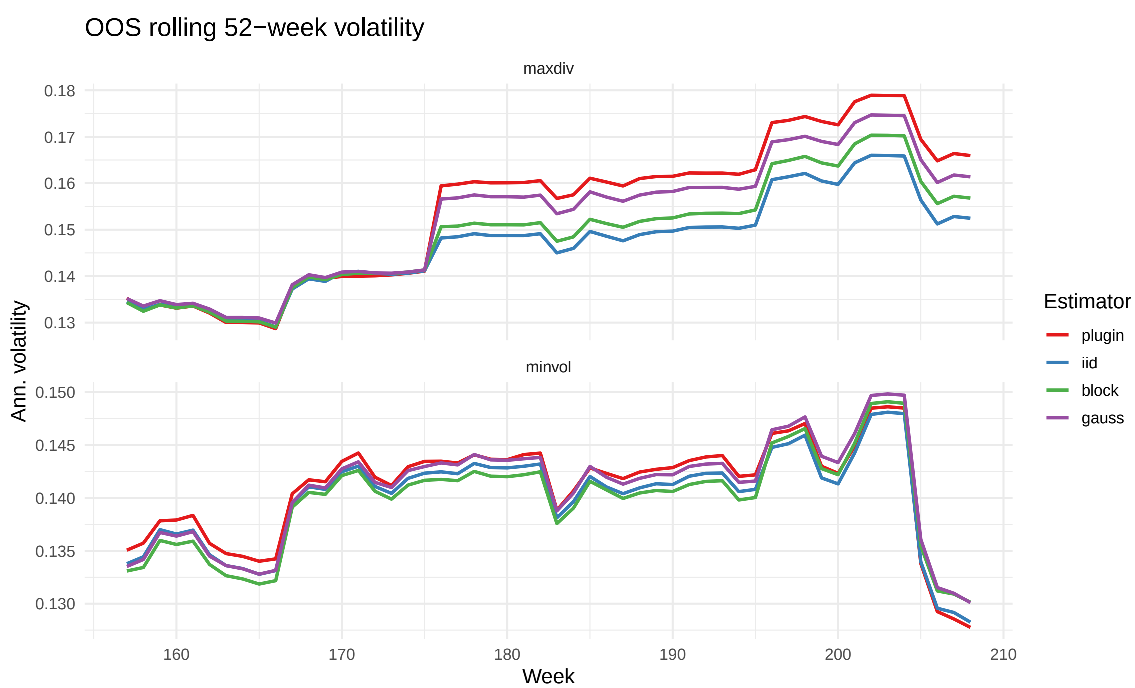


Figure 22: OOS rolling 52-week volatility based on resampling schemes.

5 References

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