

# MATH60633A – TP2

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## Important information

- This assignment is worth 20% of the final grade.
- This work must be done in teams of 3-4 people; fill out the peer evaluation form.
- Send me an e-mail at david.ardias@hec.ca as soon as you have created your team.
- This assignment cannot be done using ChatGPT.
- I will only answer questions asked on the forum.
- Your Rmarkdown or R notebook file must be deposited before the date set for this purpose. Any delay will incur a 5% penalty per hour.
- Clearly document all your R code. It must be executable.
- Put all your files in a single .zip folder and name it `name1_name2.zip`. A single file per team must be submitted.
- The total number of points is 100.

## Context

You work as a quantitative analyst for a large investment bank. You and your team are responsible for challenging the models used by traders and risk managers. You work with R and love reproducible research. All your files are written in Rmarkdown or R notebook.

You can watch the introductory video on Rmarkdown to help you build properly the R file.

## Learning objectives

### Content (scientific rigor, concepts, creativity, clarity) [10 points]

- Choose the right tools.
- Implement the steps correctly.
- Come up with innovative solutions.
- Report results with clarity.

### Form (coding, collaboration, presentation) [20 points]

- Build RStudio project with proper folder structure
- Use Rmarkdown/notebook file to reproduce your results.
- Program with state-of-the-art coding standards.

# Description

Choose one of the projects below:

## Project 1 (Asset management) [70 points]

The asset allocation team recently launched two long-only risk-based portfolios: 1) minimum-variance portfolio and 2) maximum diversification portfolio. The estimations are all implemented in the package **RiskPortfolios**. Before launching the products, they want to investigate the sensitivity of the portfolios to their inputs: the correlations and the variances. The analysts are interested in the impact on the in-sample and out-of-sample performances.

Your task is to set-up a framework to assess and compare the impact of the parameter uncertainty for the portfolios.

### Instructions

#### Data

1. Consider the first 50 stocks (alphabetical order) in the FTSE 100 universe. Download the data and compute the weekly returns for a period of 4 years.

#### In-sample sensitivity analysis [20 points]

1. Display the sensitivity of in-sample reward and in-sample risk of portfolios of size  $d = 5, 10, 25$  for the volatility and the correlations. Try to investigate the following questions:
  - Given the covariance structure, what is the impact of the dimension? The choice of stocks (sub-universe)?
  - For a given dimension and correlation structure, what is the impact of the variances?
  - For a given dimension and variances, what is the impact of the correlation? (assume an equi-correlation matrix).

#### In-sample uncertainty [25 points]

1. Investigate the impact of parameter uncertainty in the in-sample performance (return and risk, Sharpe). Use first a Gaussian model calibrated on the data.
2. Do the same with a Student model. Calibrate the model on data.
3. Test the impact of robustification techniques. In particular, consider shrinkage approaches.
4. Test the portfolio resampling approach (iid bootstrap, block bootstrap and Gaussian parametric resampling).

#### Out-of-sample impact [25 points]

1. Perform an out-of-sample performance analysis by rebalancing at the weekly frequency. Do this for a selected universe of 25 stocks. Compare the two portfolios.
2. Perform the same analysis using shrinkage approaches. Compare the two portfolios.
3. Perform the same analysis using resampling approaches. Compare the two portfolios.

## Project 2 (Risk management) [70 points]

The objective is to implement (part of) the risk management framework for estimating the risk of a book of European call options by taking into account the risk drivers such as underlying and implied volatility.

### Instructions

#### Data

1. Load the database *Market*. Identify the price of the S&P 500, the VIX index, the term structure of interest rates (current and past), and the traded options (calls and puts).

#### Pricing of a portfolio of options [5 points]

1. Assume the following book of European call options: 1x strike  $K = 1600$  maturity 20 days, 1x strike  $K = 1650$  maturity 20 days, 1x strike  $K = 1750$  maturity 40 days and 1x strike  $K = 1800$  maturity 40 days.
2. Find the price of this book given the latest underlying price and the latest implied volatility (take the VIX for all options). Use Black-Scholes formula to price the options. Take the current term structure and linearly interpolate to find the corresponding rates. Use 360 days/year for the term structure and 250 days/year for the time to maturity of the options.

#### One risk driver and univariate Gaussian model [10 points]

1. Compute the daily log-returns of the underlying stock.
2. Assume they are iid normally distributed.
3. Generate 10 000 scenarios for the one-week ahead (five days) underlying price using the normal distribution fitted to the past invariants.
4. Determine the P&L distribution of the book of options, using the simulated underlying values. Assume the implied volatility stays the same. Take interpolated rates for the term structure.
5. Compute the VaR95 and the ES95.

*Note.* Do not forget that you are moving forward one week (five days). The maturity of the options and the rate for the term structure are therefore different than for the initial book.

#### Two risk drivers and bivariate Gaussian model [15 points]

1. Compute the daily log-returns of the underlying stock.
2. Compute the daily log-returns of the VIX.
3. Assume they are invariants normally distributed.
4. Generate 10 000 scenarios for the one-week ahead underlying price and the one week ahead VIX value using the normal distribution fitted to the past risk drivers.
5. Determine the P&L distribution of the book of options, using the simulated values. Take interpolated rates for the term structure.

#### Two risk drivers and copula-marginal model (Student-t and Gaussian copula) [20 points]

1. Proceed as before, but assume now that the first invariant is generated using a Student-t distribution with  $\nu = 10$  degrees of freedom and the second invariant is generated using a Student-t distribution with  $\nu = 5$  degrees of freedom. Assume the normal copula to merge the marginals. Recompute the P&L and the risk figures.

**Volatility surface [20 points]**

1. Fit a volatility surface to the implied volatilities observed on the market (traded call and put options). Minimize the absolute distance between the market implied volatilities and the model implied volatilities. The parametric surface is given by:

$$\sigma(m, \tau) = \alpha_1 + \alpha_2(m - 1)^2 + \alpha_3(m - 1)^3 + \alpha_4\sqrt{\tau},$$

where  $m = K/S$  is the moneyness and  $\tau$  is the time to maturity of the option in years.

2. Re-price the portfolio using the parametric model. Once you have fitted the volatility surface, consider  $\tilde{\sigma}(m, \tau) = \sigma(m, \tau) - (\alpha_1 + \alpha_4) + \text{VIX}_T$ , where  $\text{VIX}_T$  is the VIX at time  $T$ . Use this for the pricing at time  $T$ . For the pricing at  $T + 5$ , consider  $\tilde{\sigma}(m, \tau) = \sigma(m, \tau) - (\alpha_1 + \alpha_4) + \text{VIX}_{T+5}$  where  $\text{VIX}_{T+5}$  is obtained by simulation.