Uebung 11

Multivariate Statistik SoSe 2019 Antje Jahn (FBMN, h_da) Neudert, Baudisch, Kopfmann

Aufgabe 1 a)

```
clust <- read.csv("Ex11.csv")</pre>
library('VIM')
clust <- format(clust, scientific = FALSE)</pre>
nrow(clust)
#> [1] 999
clust[c(1:40)] <- lapply(clust[c(1:40)], as.numeric)</pre>
str(clust)
                   999 obs. of 40 variables:
#> 'data.frame':
#> $ X.0.9619334 : num -0.2925 0.2588 -1.1521 0.1958 0.0301 ...
#> $ X0.4418028 : num -1.139 -0.973 -2.213 0.593 -0.691 ...
#> $ X.0.9750051 : num 0.196 0.588 -0.862 0.283 -0.403 ...
#> $ X1.417504 : num -1.281 -0.8 0.631 0.247 -0.73 ...
#> $ X0.8188148 : num -0.251 -1.82 0.952 1.979 -0.364 ...
#> $ X0.3162937 : num 2.512 -2.059 -1.166 -0.871 1.125 ...
#> $ X.0.02496682: num -0.9222 -0.0648 -0.3916 -0.9897 -1.404 ...
#> $ X.0.063966 : num 0.0595 1.5921 1.0636 -1.0323 -0.8061 ...
#> $ X0.03149702 : num -1.41 -0.173 -0.35 -1.11 -1.238 ...
#> $ X.0.3503106 : num -0.657 -0.121 -1.489 -0.385 0.578 ...
#> $ X.0.7227299 : num -0.116 -0.188 -0.243 1.651 -0.272 ...
#> $ X.0.2819547 : num 0.826 -1.5 -0.433 -1.745 2.177 ...
#> $ X1.337515 : num 0.3464 -1.2287 -0.0388 -0.3789 1.4364 ...
#> $ X0.7019798 : num -0.5695 0.856 -0.0579 -0.6798 -1.0258 ...
#> $ X1.007616 : num -0.132 1.25 -1.398 -2.132 0.298 ...
#> $ X.0.4653828 : num 0.69 -0.898 -0.156 -0.23 -0.556 ...
#> $ X0.6385951 : num -0.909 0.87 -2.736 0.466 0.205 ...
#> $ X0.2867807 : num 1.303 -0.225 0.776 -1.8 -1.192 ...
#> $ X.0.2270782 : num -1.673 0.45 0.614 0.626 0.235 ...
#> $ X.0.2200452 : num -0.5255 0.5514 2.0192 -0.0977 0.671 ...
#> $ X.1.242573 : num 0.798 0.146 1.081 -0.3 0.131 ...
#> $ X.0.1085056 : num -0.69 0.13 -1.08 -0.53 1.07 ...
#> $ X.1.864262 : num 0.9 1.304 -0.243 -2.024 1.231 ...
#> $ X.0.5005122 : num 0.429 -1.662 0.513 -0.511 1.134 ...
#> $ X.1.325008 : num -0.676 -1.63 -0.513 0.046 0.556 ...
#> $ X1.063411 : num -0.5341 -0.0774 2.5517 1.268 -0.3588 ...
#> $ X.0.2963712 : num
                        -1.733 1.306 -2.314 -0.744 1.08 ...
#> $ X.0.1216457 : num -1.603 0.793 -1.276 0.223 -0.206 ...
#> $ X0.08516605 : num -1.08362 1.55947 -1.22927 0.85846 -0.00616 ...
#> $ X0.6241764 : num 0.0334 -0.6885 1.4344 0.2747 0.1643 ...
#> $ X.0.5095915 : num 1.701 -0.615 -0.284 -0.693 1.157 ...
#> $ X.0.2167255 : num 0.00729 0.01 0.19895 -0.84571 0.24177 ...
#> $ X.0.05550597: num 0.0991 0.9458 -0.0918 -0.1775 0.0886 ...
```

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#> $ X.0.4844491 : num 0.564 -0.319 0.35 -0.166 0.183 ...
#> $ X.0.5215811 : num -0.257 -0.118 -0.299 1.483 0.943 ...
#> $ X1.949135 : num -0.582 0.621 1.514 -1.688 -0.21 ...
#> $ X1.324335
                : num -0.1699 -0.0708 0.6712 -0.1414 0.5363 ...
#> $ X0.4681471 : num -0.5423 0.4017 0.0109 0.2008 -1.1852 ...
                 : num 0.3129 -0.0162 -1.0437 -0.6759 -0.4227 ...
   $ X1.0611
#> $ X1.65597
                 : num -1.284 -0.527 1.625 2.221 0.624 ...
```

```
Skalieren der Daten
scaled_data = t(as.matrix(scale(clust)))
#Let us apply kmeans for k=3 clusters
kmm = kmeans(scaled_data,5,nstart = 60,iter.max = 15) #we keep number of iter.max=15 to ensure the algo
#> K-means clustering with 5 clusters of sizes 6, 20, 4, 4, 6
#> Cluster means:
                             3
                                       4
#> 2 -0.37161784 -0.13035899 -0.1191583 -0.27118251 0.09177602 -0.28814260
#> 4 -0.78319049 -0.49090586 -1.0648990 0.51260323 -0.54926363 0.05930680
#> 5  0.52131197 -0.18331527 -0.1358904 -0.83481512  0.04058154 -0.18226590
                   8
                           9
                                    10
                                              11
#> 1 0.2693997 -0.61242370 0.4129149 -0.12586306 -0.11344398 -0.009829228
#> 2 -0.1211400 -0.37586416 -0.3708198 1.62663688 1.72059797 1.664074561
#> 3  0.3126914  0.29568956  0.5338776  0.26831779  -0.70752839  -0.010429615
#> 4 -0.3948767 -0.13357774 -0.2372000 0.27359662 0.68705922 0.277998806
#> 5 0.3908269 0.02740118 0.7588462 0.03165377 0.01290324 0.446714976
         13
                  14
                           15
                                    16
                                            17
#> 2 1.5375534 1.3451297 1.39421046 1.2634387 1.4395779 1.3612312
#> 3  0.7647604  -0.1114630  -0.05920584  0.5384011  0.4240889  0.1055045
#> 4 -0.4819270 -0.2432016 -0.52716355 0.3573333 0.2218558 0.9887323
#> 5  0.8304547 -0.2553001 -0.28482853  0.3531113  0.1755410  0.1823990
          19
                  20
                            21
                                      22
                                               23
#> 1 0.5677292 -0.4456284 0.411038224 -0.58944506 0.80640302 -0.3782759
#> 2 1.4899759 -0.1229951 -0.271069658 -0.09074504 -0.09921251 -0.2118606
#> 3  0.7103414 -0.1542571 -0.003805002  0.49595437 -0.49355629 -0.2892790
#> 4 -0.6681783 1.1072158 0.227614221 0.94218789 -0.06738606 -0.4950885
#>
          25
                   26
                            27
                                       28
                                                29
#> 2 -0.06460533 -0.1320811 -0.36691247 0.025430789 -0.21630879 -0.3471511
#> 4 -0.52013622 -0.4439729 -0.45176058 -0.360283240 -0.57687231 0.5015907
#> 5  0.46540148  0.2379101  0.06653013 -0.006218144  0.24992445  0.0193832
                   32
                             33
                                       34
#> 1 0.43896819 -0.4709800 0.083331800 -0.89495651 0.5262321 0.7807821
#> 2 -0.26477050 -0.3687968 -0.105633410 0.07017628 -0.3253538 -0.2707660
#> 3  0.35846280  0.9947445  0.998880781  -0.58949291  -0.1399450  -0.2028861
#> 4  0.25472249 -0.7221197  0.004067165  0.87426310  0.3446951  0.3958152
```

```
39 40 41 42
#> 37 38
#> 1 0.2445228 0.26612829 -0.48353931 0.4160063 0.05241789 -0.8641055
#> 3 -0.2417528 -0.32482099 0.42163902 0.1685215 0.36940873 0.5768088
#> 4  0.2704832  0.07022468 -0.06312819 -0.8372542  0.02474842 -0.7335332
#> 5 0.5313674 0.16014437 0.90443488 -0.1930511 -0.28692377 -0.2970924
#>
                      45
                              46 47
        43
                44
#> 3 -0.3968998 -0.007011356 -1.0630995 -0.31146595 0.45057031 -0.7733195
#> 4  0.6094372  0.878730844  -0.3116950  0.27882881  -0.09055204  0.7938531
#> 5  0.4642647 -0.114793503  0.5208168 -0.33404405 -0.56532028 -0.3274485
          50 51 52 53
        49
#> 1  0.2519002 -0.47625745 -0.1803616 -0.03397894  0.07006807 0.1521231
#> 2 0.1689789 -0.19732275 -0.1096104 -0.39064545 0.06101166 0.1015477
#> 4  0.6779938  -0.27268341  0.4434038  0.28183570  -0.58060013  0.7697884
#> 5 -0.4747610 -0.03860082 -0.8738160 -0.11553991 -1.15857969 0.4848286
               56
                      57
                              58
                                     59
       55
#> 1 0.2936580 0.83660972 0.96389754 -0.8151255 0.25198474 -0.43293809
#> 2 -0.2237806 -0.19949061 -0.28284603 -0.3253625 -0.02415848 -0.07723587
#> 3 -0.4365878 -1.00325012 -0.22282049 0.5770650 -0.65320219 -1.00676458
#> 4  0.7143900 -0.06877729  0.03029892  0.5050880  1.33557834 -0.73307169
#> 5 0.8711945 0.09202742 -0.24991871 -0.3213844 -0.25011593 -0.06793408
       61
              62
                  63
                         64
                                65
#> 1 -0.2994688 -0.02661648 0.2933676 -0.8687416 0.308321333 0.5603507
#> 4 -0.7439120 -0.52931438 -1.0429400 0.5393907 0.007869601 -0.2978932
#> 5 -0.4378094 -0.20058601 0.3553418 0.4685499 0.300199623 -0.0723357
                68
                        69
                               70
#> 1 0.591758331 0.40415873 -0.24588737 0.40456349 -0.45941591 -0.07123055
#> 2 0.069128344 -0.05795774 -0.20241258 -0.17327019 -0.39225824 -0.27524147
#> 3  0.008976087 -1.32839945 -0.29399634  0.10112407  0.50824908  0.20272775
76
                                     77
        73
               74
                       75
#> 2 -0.4379984 -0.18168741 -0.37088891 -0.4889365 -0.76899089 -0.1778989
#> 4  0.3626320  -0.02847437  -0.29489093  0.9037426  0.18491478  -0.8488177
#> 5 0.7542423 0.68223383 0.04309302 0.4575848 -0.02283312 0.3108973
        79 80
                     81 82 83
#> 1 -0.1338984 -0.4984065 0.51911339 0.408749981 -0.3139950 0.33587365
85
               86
                   87
                          88
                                     89
#> 2 -0.2850831 -0.02402128 -0.3858371 -0.03740294 -0.08604632 -0.4574942
#> 3 0.3181019 -0.49205025 0.6634653 0.08270619 -0.20351583 -0.1439250
#> 4 -0.1639529 -0.85655827 -1.0991932 0.28350738 0.38691686 -0.1134014
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#> 5 -0.1504066 -0.24173598 0.1500079 1.13711974 -0.26136841 0.4605060
   91 92
                  93 94 95 96
#> 1 0.4955692 -0.4213072 -0.4634990 0.08936219 0.37162897 -0.65775772
#> 2 -0.1111301 -0.2107736 -0.2856978 0.22069225 -0.38174396 -0.52005547
#> 4 -0.4460123 -0.3565933 -0.2100048 0.15435173 -0.19036045 0.22589694
99
         97
                98
                              100
                                        101
#> 1 0.82976307 0.2571233 -0.51284064 -0.4281201 0.08908484 -0.05085597
#> 2 -0.38868732 -0.1842695 -0.04823037 -0.2101826 -0.60903738 0.06862975
#> 3 0.40529435 0.9261345 -0.76126211 0.1427432 0.01598073 0.07301713
#> 4 0.06090514 0.1679564 0.33668665 0.1987313 -0.31713962 0.11908894
103
                104
                        105
                                 106
                                         107
#> 1  0.03268454 -0.11937055 -0.02676059 -0.24927610  0.15059039  0.3199368
#> 3 1.34547615 0.60211206 0.23267875 -0.05393762 -0.63221150 -0.9519872
#> 4 -0.34209980 0.57121002 0.56128863 0.32416558 0.46074442 0.1774898
#> 5  0.62912917  0.80418476  0.29707154  0.45139701 -0.05259747  0.6556487
        109
               110 111 112
                                       113
#> 1 0.33426883 0.2592015 0.22660049 0.1795284 -0.32101651 -0.7876200
#> 2 -0.18713798 -0.1828621 -0.41536508 -0.2857062 -0.15815029 -0.6940744
#> 3  0.26624720 -0.4818963 -0.47025473 -0.3792165  0.25472147 -0.5283800
#> 4  0.46605682 -0.0575877  0.07936352 -0.3900128 -0.44369862  0.5067358
#> 5  0.03174276  0.1344284  0.10181235  -0.2478251  -0.03864506  -0.9254685
                               118
                116
                       117
        115
                                        119
#> 2 -0.6548391 -0.28193948 -0.0248672 -0.16917252 -0.46087125 -0.1799984
#> 3 0.1483241 0.94228010 0.2461943 0.39872116 0.05468702 -1.2793512
#> 5  0.7368935  0.72280826  -0.7697384  -0.20577044  -0.38970124  0.1582693
                       123
        121
               122
                               124
                                   125
#> 2 -0.28478555 -0.1280213 -0.6209063 0.14749476 -0.2707339 -0.3693148
#> 3  0.30292271 -0.1560288  0.2000287  0.32729822  0.4387156  0.1027881
#> 5  0.08803798  0.2554742  -0.5178308  -0.28643547  -0.5714683  0.1137160
        127
                128
                        129
                            130
                                        131
#> 2 -0.01486872 -0.04566675 -0.37258822 -0.5944401 0.0184310 -0.12154245
#> 4 -1.07589491 -0.46488600 1.14166708 -0.2481689 0.9262626 -0.17280460
133
                134
                         135
                               136
                                        137
#> 1 0.381552276 0.6330100 0.30126000 -0.2171590 0.1744577 -0.7805140
#> 4 1.641199591 0.6320004 0.10916596 -0.3845967 -0.7995270 -0.1899867
#> 5 0.009754401 0.4819932 0.89140450 0.4213859 -0.4123370 -0.3113772
        139
                140
                       141
                               142 143
#> 1 -0.01086746 -0.25523403 -0.1765432 -0.8543973 0.3045971 -0.6387030
#> 2  0.03507214 -0.59107106  0.1049791 -0.1844108 -0.1877522 -0.1874742
#> 3  0.09309054 -0.45931965 -1.3686985  0.9031501  0.3620551 -0.4161519
```

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#> 4  0.74610899  -0.09379896  -0.5642678  -0.4731945  -0.1859028  -0.6810929
#> 5  0.58590352  0.08246624  0.2591798  -0.1250495  0.9733864  0.3634333
                        147
        145
                146
                                148
                                         149
#> 1 0.1577444 -0.05890027 -0.1452037 0.03267094 0.65830566 0.5704079
#> 2  0.0273816  -0.20573727  -0.4051609  -0.52708075  -0.04983021  -0.1545578
#> 3 1.0048672 -0.15939108 -0.2091676 0.40316442 -0.10935245 -0.8132031
#> 5 -0.2318873 -0.25426588 -1.0078834 -0.53900374 -0.20035935 0.3988125
                                       155
               152
                            154
        151
                        153
                                                 156
#> 1 0.2970520 -0.8772445 0.18926348 -0.66872364 0.04754606 -0.1419597
#> 2 -0.1998927 -0.0248404 -0.33435454 -0.10711989 0.18935530 -0.3908296
#> 4  0.1634228 -0.1315224 -0.52381821  0.07497978 -1.45190328  0.4451738
158
                         159
                                 160
                                         161
          157
#> 1 0.051001559 0.4579355 0.57053811 0.1175550 -0.1962088 0.15791366
#> 2 -0.149268758 -0.4070264 -0.04287047 -0.1365019 -0.1981188 -0.36367445
#> 3 -0.705715128 -0.1213432 -0.05671325 -0.9358367 0.2909226 -0.02494178
#> 5 -0.284328316  0.2598854  0.06729398  0.1153083  0.7159439 -0.44049100
#>
                                          167
        163
                164
                     165
                                 166
                                                  168
#> 1 0.4640424 0.61339821 0.33631649 0.50906346 -0.02988715 0.6917919
#> 3  0.6974189  0.53376175  0.01289270  -0.67337411  0.03596347  -0.2083181
#> 5 -0.2143666  0.29607272 -0.11226322  0.04743149 -0.51400189  0.2658628
        169 170 171 172 173
#> 2 -0.09039814 -0.33426566 -0.69031693 -0.47621529 -0.31287627 -0.17482762
#> 3 1.00738750 -0.01851372 0.41373973 -0.30703778 -0.20341825 -0.08356962
#> 4  0.02904725  0.12550014  -0.18923539  0.02570236  0.01872816  0.28399307
175
                 176
                         177
                                  178
                                          179
                                                    180
#> 2 0.03311640 0.01718515 0.06853988 -0.44442226 -0.06689367 -0.34657703
#> 3 1.02518727 0.09513698 -0.74704077 -0.68471190 -0.77551825 0.12849632
#> 5 -0.27376516 -0.29174453 0.09411501 0.93851945 0.19843762 0.36399061
        181
                 182 183
                                  184 185
#> 1 0.29262780 -0.314645559 0.13251735 0.59204370 -0.3674472 0.4439589
#> 2 -0.20360180 -0.055260829 0.04141384 -0.04641734 -0.3724033 -0.1648925
#> 4 -0.05306831 -0.009699505 -0.05494839 -0.11863998 -0.6723188 -0.1308761
#> 5  0.25994555 -0.479878691  0.55024326 -0.01169700 -0.3764713 -0.1962039
         187
                188
                        189
                                  190
                                          191
                                                  192
#> 1 -0.19458408 -0.3919685 -0.5392328 0.210025585 -0.15642377 -0.1121745
#> 2 -0.46623352 -0.1945033 -0.2765365 -0.437107682 -0.02824591 -0.0137198
#> 3  0.01005885  0.2603170  0.6625878  0.023276500 -1.24224639  0.6568225
#> 4  0.18205663 -1.4367227 -0.7931617 -0.009340915  0.50418100  0.2962404
#> 5 -0.15459726  0.4074572 -0.2700543  0.062493443 -0.05720502 -0.5768854
                     195
#>
        193
                194
                              196
#> 1 -0.42012725 -1.15310695 0.572721202 0.007686597 0.03384184
```

```
#> 3 1.36755593 1.37581409 -0.298102574 -0.422326529 -0.73703229
#> 4 -0.13164775 -0.48136477 -0.186579852 -0.377846946 -0.29811528
#> 5  0.03493856  -0.43660797  0.104683459  -0.016863695  -0.58954363
         198
                199
                         200
                                  201
                                                    203
#> 1 -0.03961779 -0.4600117 -0.09576678 -0.098012810 -0.1945209 0.45427144
#> 2 -0.05257738 -0.2073000 -0.05471169 -0.298138118 -0.4792068 -0.11496417
#> 4 0.17481905 0.4678694 -0.74620114 0.400773257 0.2804912 0.06989101
204
                 205
                          206
                                  207
                                          208
#> 1 0.1191151 0.04870360 0.82670327 0.4946159 -0.2498191 0.79174749
#> 2 -0.1588761 -0.02950543 -0.29732272 -0.1016534 -0.1573904 -0.35680929
#> 3 -0.2002485 -0.59435678 -0.71773896 0.5945637 -0.3591834 -0.25090850
#> 4 1.1189000 -0.19947071 0.35767464 -0.7963723 -0.2264914 -0.45952646
210
                 211
                          212
                                   213
                                            214
#> 1 -0.16636116 -0.09504943 0.09731115 -0.81835864 0.22841629 0.67470185
#> 2 -0.28051747 -0.08381073 -0.10372380 0.11129123 -0.09024021 -0.04638255
#> 3 -0.14316521 -0.68896910 0.05954856 -0.04942479 0.68244976 -0.17353753
#> 4  0.03204052  0.23877756  -0.77338759  -0.60879876  -0.17067634  0.02476100
216
                 217
                         218
                                 219
                                         220
#> 1 0.8054967 0.02464152 0.8158847 0.3803715 0.1182419 -0.07002739
#> 2 -0.2515823 -0.07630523 -0.2811819 -0.4373923 -0.1759834 -0.39606320
#> 3 0.1581896 0.87497295 0.7400291 -0.9960164 -0.7156804 0.30505963
#> 4  0.2221446  0.44570752  0.8641705  -0.5112934  -0.9541388  1.06227736
#> 5 -0.2215647 -0.47148349 -0.6190156 0.3161815 -0.1947727 0.06106580
         222
                 223
                                          226
#>
                          224
                                  225
#> 2 -0.07066302 -0.6015252 -0.20323856 -0.22987130 0.2212300 -0.3419037
#> 3 -0.76860271 -0.4003603 -0.22659122 1.08562603 -0.2782425 -0.2397290
#> 5  0.45953492 -0.6131163  0.09676732 -0.16860680  0.2534536  0.3203354
         228
                 229
                         230
                                 231
                                          232
#> 1 0.06173504 -0.99654239 0.9812396 0.50755148 -0.4950941 -0.677181856
#> 2 -0.06864663 -0.09308773 -0.3247298 -0.04280057 -0.3929308 -0.324002544
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#> 2 -0.04088518 -0.1988928 -0.1105994 -0.24760256 -0.33944795 0.05478381
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#> 1 0.19821541 -0.2575350 0.511767769 -0.1613375 0.8490005 0.4963020
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#> 1 0.7022157 -0.25906953 0.572962910 -0.4662536 -0.62820718 -0.01895617
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#> 4 -0.46175508 -0.602267991 0.20616634 0.78591902 0.15082357 0.01462283
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#>
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#> 4  0.4064875 -0.26476026  0.1121307 -0.6060397  0.04097193  0.7571746
515 516 517 518 519 520
#> 2 1.5559619 1.55896838 0.9514437 1.292185544 1.55367657 1.61736671
#> 3 -0.8179185 1.01707114 0.9075041 0.588472403 0.88142065 0.07829130
#> 4  0.6401386  0.01862849 -1.1016273  0.209676748 -0.31378323  0.09324847
#> 5  0.1432116 -0.56421531 -0.4775998 0.551810970 0.07454074 0.20100697
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                                          525
#> 2 1.5975176 1.5324968 1.4368595 1.14485742 1.60852329 1.4969632
#> 3 0.5923674 -0.4113790 -0.3974979 0.02879990 0.77186286 0.1989546
#> 4 -0.5095456  0.1330409 -0.1350190  0.09448719  0.05082684  0.4668918
#> 5  0.0369825 -0.1566914 -0.6035754 -0.06789964  0.14263415 -0.9849619
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                               530 531
#> 2 1.1416396 1.5170490 1.47116568 1.3995933 1.1143836 1.4742794
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#> 4 -0.4742186 -0.1686239 -0.48356212 0.1659317 -0.1234020 -0.3492770
#> 5 -0.1856317 -0.6540165 0.34711365 -0.1602043 0.1961470 0.3468394
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                                 536
                                           537
                                                     538
#> 1 0.86559317 -0.1575004 -0.01888199 0.9029136 -0.24127670 0.43468781
#> 2 1.74672549 1.5174088 1.46839683 1.5041989 1.65671952 1.45860367
#> 3  0.59149827 -0.1334661 -0.76511816 -0.1468837 -0.63263733 -0.02140968
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#>
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#> 2 1.7832946 1.58668899 1.4969298 1.4424795 1.4690633 1.6410311
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#> 5  0.5886061  0.05077996 -0.4988030 -0.1344771  0.1495193 -0.3000748
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#> 1 -0.7384893 -0.1755618 -0.08937795 -0.1952706 0.009170035 0.31506241
#> 2 1.5515175 1.3050625 1.81634975 1.6864728 1.625985598 1.94061734
#> 3 0.5395583 0.1615067 -0.20169981 -0.2766119 0.176791836 0.45044295
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#> 5 -0.5242727 -0.3442774 0.21067310 -0.2159372 0.747443101 -0.01396852
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                                      555
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#> 3  0.49312298 -0.02852972 -0.4870552 -0.6213101  0.5743329  0.2984626
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                                    560 561
#> 2 1.73024094 1.57494048 1.5913168344 1.75745771 1.52481737 1.40580754
#> 3 0.64687770 -0.34910396 0.0931666517 0.02057299 -1.08568126 0.06008485
#> 5 0.27054341 0.08715501 0.5637357044 -0.04678057 -0.54460236 -0.18654890
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                              566 567
#> 1 0.54965779 -0.47827593 0.8373319 0.45250635 -1.0600953 -0.2131507
```

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#> 2 1.51072908 1.78406743 1.7867569 1.44540937 1.7113705 1.4419683
#> 3 -0.12323700 -0.11755455 0.1213718 0.01761069 -0.1392129 0.1767065
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#> 1  0.2006595  -0.9476748  0.4369781  0.5825721  -0.87012088  0.3426916
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#> 575 576 577 578 579
#> 1  0.2618689 -0.40178125  0.6350788  0.3283936  0.34020312  0.7330385
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#> 3 -0.3899005 -0.20488563 1.4604145 0.1435288 -0.59556496 0.1749258
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                           584 585
#> 2 1.70179035 1.4281791 1.7395391 1.37440276 1.73234008 1.63504939
#> 5  0.49169990 -0.2392124 -0.5597592 0.42695234 0.05166746 -0.08704223
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                588 589 590 591
#> 2 1.40340087 1.89909129 1.5767421 1.5044341 1.38572474 1.7467402
#> 4  0.01821627 -0.14845139  0.1732367 -0.8125931 -0.32982992  0.2576952
#> 5  0.04619608  0.06025608  -0.1727694  -0.1603537  -0.09607106  -0.3927492
                          596 597
        593
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                  595
#> 1 0.44132310 0.1194174 0.4531873 0.1638653 0.59321997 -0.21059849
#> 2 1.45882776 1.4845304 1.5233950 1.5777387 1.29963694 1.48445969
#> 3 0.67856175 0.1359853 -0.3454087 0.2220714 -0.03934745 -0.47885565
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#> 2 1.5823443 -0.24354123 -0.19986019 -0.1975115 0.01489423 -0.1580870
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#> 1 0.1760117 1.1130777 0.28562411 0.15268160 -0.1732629 0.47892041
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#> 3 -0.1585968 -0.7704702 0.33071645 0.04301411 0.1300726 -0.08880430
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** 5 0.2307637 0.8132690 0.36710420 0.42697062 0.3536543 0.46927534
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                                   615
#> 1 0.13539246 0.3986611 -0.02346974 -0.2061695 -0.08922817 -0.49059664
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#> 1 0.5752269 0.42981817 -0.25922601 -0.6245834 0.622114813 -0.3307448
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                            625
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                                             627
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#> 4  0.87841288  0.72336140  0.23651352  0.2538604  -0.4494797  -0.58803042
#> 5 1.36739067 -0.87433932 0.01551737 0.1156511 0.6928576 -0.07965699
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#> 2 -0.11031296 -0.35805827 0.09684187 -0.24698414 0.14492497 -0.5332118
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#> 1 -0.22983272  0.5253708  0.5732194  0.51949860 -0.1343351  0.004305671
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#>
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#> 3 -0.71325369 -0.19693901 -0.03521839 -0.05436589 -0.29214210 0.162226537
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#> 1 0.08658630 0.3803976 -0.001111453 0.51104775 -0.2950923 -0.05458927
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#> 4  0.44307761 -0.6273584 -0.901448861  0.03306812 -0.4019955  0.92256054
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#> 1 0.33622974 -0.1338507 0.5014833 0.36111802 -0.30172664 -0.23181982
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#> 3 0.42100160 0.2867734 -0.7784346 -0.09889806 0.49350348 -0.07594976
#> 5 0.04830504 0.1851324 0.4291430 0.22000322 0.47860513 -0.26035410
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#> 671 672 673 674 675
#> 1 0.03354542 -0.4590974 0.33198050 -0.42848163 0.009364633 0.24216430
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696 697 698 699
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#> 1 0.6733992 0.0002056087 0.267800419 0.05555859 0.04877295 -0.3223088
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#> 5 0.2713987 -0.5399034998 0.065266281 0.51749277 0.32716817 0.1513930
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#> 5 0.4963209 0.76032691 -0.38427786 0.23527791 0.49669584 0.5398775
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                            709
                                     710
                                              711
#> 1 0.08621766 0.60690794 0.228148402 0.1563638 -0.06203594 -0.3337289
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#> 5 -0.25690236 -0.03388069 0.009220486 -0.9959687 0.21231315 0.3383650
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                          715
                                    716
                                            717
#> 1 0.3541947 -0.84861521 -0.07570572 -0.18270293 -0.3494227 0.81568490
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#> 3 -0.7025240 -0.60054073 0.90051561 0.29434212 -0.8635087 -0.47232931
#> 4 -0.5125655 -0.11754422 0.23133304 -0.66637257 1.0155255 0.06261742
#> 5 -0.2060332 -0.76121057 -0.10272078  0.55364598 -0.2407623 -0.02424838
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                                   722
                                           723
#> 1  0.2067350 -0.2204792 -0.3645215  0.54339550 -0.02438388 -0.74684562
#> 2 -0.3116768 -0.2681751 -0.2746189 -0.06196019 -0.11677124 0.03848801
#> 4 0.7775052 0.3550789 0.1279315 -0.77689323 -1.04618006 0.44140501
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#> 5 0.1814499 0.1955472 -0.2408207 0.66006982 -0.77866199 0.02250287
                  726 727 728
                                        729
         725
#> 1 0.4080078 -0.1178629 0.59786457 0.2228159 0.5530162 0.35047523
#> 2 -0.3228195 -0.3745494 0.02705018 -0.2271431 -0.1287634 -0.02936578
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#> 4 -0.5591195 -0.4542465 -0.19271824 -0.3017795 0.2674124 0.31365238
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#> 1 0.5947580 0.5720788 -0.17340199 0.2972158 0.81762118 -0.125557524
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#> 3 0.2947579 0.4840861 -0.21907950 0.4896492 -0.14766780 -0.477305607
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                   738
                            739
                                      740
                                                 741
#> 1 -0.06541808 -0.6739601 -0.39797641 -0.37182339 -0.006726624 0.28575981
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#> 5 -0.15343681 -0.6252137 -0.09732202 0.45804671 -0.623980715 0.32073269
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          743
#> 1  0.12516315  0.21032997  0.3174486  -0.008784443  -0.31504920  -0.11162712
#> 2  0.04289637 -0.23270611 -0.2932020 -0.465098828 -0.16476901  0.04751925
#> 3  0.24108671  0.51036037  -0.2765438  0.002706145  0.96332355  0.46159625
#> 4 -0.79899490 -0.50471703 1.3767690 0.117057644 0.06932312 -0.11901909
749
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                           751
                                    752
                                             753
#> 1 0.2332486 0.5265214 -0.3724518 0.46877457 -0.1307506 0.09236585
#> 2 -0.2248141 -0.2036347  0.1960953 -0.54837525 -0.2473881 -0.05066223
#> 3 -0.6463786  0.5933169  0.1023518  0.03050769  0.3814090 -0.81279309
756
                        757
                                    758
                                             759
         755
#> 2 -0.4164422 -0.1542149 -0.3339746 -0.034104663 -0.1874015 -0.3866756
#> 4  0.5901337 -0.5655443 -0.1784834 -0.761625034 -0.9724554  0.5253020
#> 5 -0.2356788 -0.1281876  0.3914288  0.393100581 -0.2637055  0.1950744
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                               763
                                         764
#> 1 0.513768508 0.234922291 -0.07653395 0.22845824 0.454991737
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#> 4 -0.356317011 -0.436141660 -0.65882871 -0.22716375 0.476256201
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#>
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                   767
                            768
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#> 1 0.45000240 0.2517663 0.09586127 -0.4857805 -0.2005510 0.74937155
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#> 3 0.03056814 0.4754055 -0.20351638 -1.1551389 0.3235668 -0.54346243
#> 5 0.13837141 0.1225086 0.14656546 -0.4429424 0.2268921 -0.36183517
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                 773
                                    775 776
#> 1 0.4215993 -0.4351841 0.06184419 -0.7665268 0.3133624 -0.07344836
#> 2 -0.0575740 -0.1368262 -0.23456369 -0.3483941 -0.2322941 -0.32992470
#> 3 0.3774078 -0.7693286 -0.50140739 0.9144138 0.6384829 0.62102867
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#> 4 -0.1385837 -0.1499774 -0.47236800 -0.4413517 0.8941945 0.42030247
779
                       780
                                781
                                      782
#> 2 -0.08923046 -0.06949418 -0.1893364 0.01323172 -0.1257239 -0.06621611
#> 3 -0.48171580 -0.61314909 0.2274666 0.10569767 -0.2445459 -0.25501468
#> 4 1.03669791 0.55172175 0.6497134 -0.27364456 -0.1824620 0.76456424
#> 5 -0.04577351 -0.20227185 0.1387296 -0.39962358 0.2880411 0.77457722
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                                787
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        784
#> 1 0.7042630 0.490234406 -0.15417123 -0.75762194 0.3074839 0.2071115
#> 4  0.4555658  1.286441873  -0.57616552  -0.30439396  0.3793404  -0.3283243
#> 5 -0.6720685 -0.047216221 0.38381815 -0.02771919 -0.1227310 0.4079164
                791
        790
                        792
                                793
                                       794
#> 1 -0.2733179 -0.05351293 0.02433102 -0.74083224 -0.1922083 -0.71994345
#> 4  0.6420398 -0.69538227 -0.09802471 -0.13055521 -0.3152001 -0.79981358
#> 5 -0.1390884 -0.33186611 0.36589339 0.74787787 -0.6430866 0.47076524
        796
                797
                       798
#>
                                799
                                        800
#> 2  0.01906554 -0.2008655 -0.14746600 -0.21788625 -0.08788291 -0.02340793
#> 3 -0.64750285 -0.5271013 -0.64709692 -0.61529342 1.32356037 -0.65063803
#> 4 0.41791910 0.2002993 0.07564006 2.05008659 0.62022847 -0.19985602
802
           803 804 805 806 807
#> 1 0.4111672 -0.8917421 -0.6103059660 0.5224491 0.05469729 0.3078831
#> 2 -0.3617920 -0.3369655 -0.0004163018 -0.3481966 0.08451213 -0.2743448
#> 3  0.3909614  0.3193829  0.1483275993  0.6780569  0.35365083  0.4347624
#> 5 -0.5253524 -0.6844106 0.2537610718 0.2955570 -0.15957943 -0.8676122
         808
                809
                        810
                               811
                                        812
#> 2 -0.006235752 -0.3073398 -0.29517762 -0.1699731 0.11239790 -0.23711809
#> 3 -0.707885211 -0.2492797 -0.60933866 -0.6147726 -0.41055354 0.96344416
#> 5  0.861854547  0.3966186  0.01299542  -0.6693434  0.05341637  0.05008262
        814
              815
                    816
                          817
                                  818
#> 1 0.1096432 -0.4163902 -0.2505782 -0.5575096 -0.26414160 -0.01259576
#> 2 -0.1097228 -0.2030742 -0.3288650 -0.1852719 -0.23018677 -0.12359768
#> 5 -0.4513159  0.1022995 -0.0378420  0.6608405  0.41456670 -0.46357879
                                      824
                      822
                              823
        820
               821
#> 1  0.5200306  0.2151472  -0.3970393  -0.39985749  -0.23881608  0.07371192
#> 2 -0.1747192 -0.1351410 -0.2638480 -0.27225394 0.05116985 0.09096116
#> 4  0.1240226  0.1622295  1.0908823  0.66855409  0.38972035  0.93271854
#> 5  0.6455757 -0.4147033  0.3692643 -0.06125604  0.19955879  0.16512674
            827 828 829
                                  830
#>
       826
#> 1  0.04900425 -0.3524713  0.5842493  0.1834240 -0.04719248 -0.17000278
#> 2 -0.11598551 -0.3045434 -0.3390843 -0.1282123 -0.48552754 -0.12691506
```

```
#> 4 0.07899116 0.1579410 0.4150498 -1.0704664 0.91448121 0.09486115
#> 5  0.06432228 -0.1295208 -0.2419796 -0.9588546  0.11115675  0.08547814
       832 833 834 835
#> 1 -0.65894500 -0.56466718 -0.3485018 -1.090704e+00 -0.172510944
#> 2 -0.33314617  0.01347359 -0.2420884 -9.235255e-05  0.005421612
#> 3 0.25874745 0.89140036 0.3135121 9.438072e-01 0.272753850
#> 4  0.11602729  0.84739293  -0.3700056  3.268897e-01  -0.138525345
#> 5 -0.06260554 -1.04233690 -0.2577588 -6.106464e-01 -0.734421432
        837
                838
                       839
                                840
#> 2 -0.30751212 -0.002357671 -0.4499222 -0.42660485 0.02551043 -0.2422559
#> 3  0.73366917 -0.113884053  0.0899397  0.05986544 -0.97521206 -0.1754214
#> 4 -0.00981995  0.779034550  0.7190849  0.43096984  0.20966815  0.5004006
843
                844
                       845
                              846 847
#> 1 0.09346708 0.07160959 0.8499797 -0.5802570 0.3431943 -0.009278270
#> 2 -0.08862137 -0.36622634 -0.3334672 0.1180505 -0.4192297 -0.088806143
#> 3  0.91872219  0.87195693  -0.2411419  -0.3494477  0.2649281  0.004440631
#> 4  0.36153819   -0.26570646    0.1057673   -0.1006741   -0.4959904   -0.377359859
#> 5  0.15239122 -0.31092460 -0.5426163  0.5492576  0.6249646 -0.159078365
        849
               850
                      851
                              852
                                     853
#> 1 0.10604588 -0.1296886 0.3681700 -0.364173785 -0.5725783 -1.3354532
#> 2 -0.09474761 -0.3313803 0.1729763 0.418483271 -0.1776955 -0.2526549
#> 4  0.87804096 -0.1878727 -0.8771883 -0.306840792 -0.6953335  0.7443467
#>
       855
              856
                      857
                              858
                                      859
#> 1 0.4123326 0.0414796 -0.16906910 -0.01491473 -1.00378447 -0.04998139
#> 5  0.1675265 -0.6450254  0.17713484 -0.33946009  0.12856302 -0.38430507
       861 862 863 864 865
#> 2 -0.4948565 -0.3579505 -0.002755344 -0.1000324 -0.27694398 0.03525046
#> 3 0.3269846 0.1595729 -0.397572173 -0.1526729 0.14822439 0.45106430
#> 4 0.4005725 0.1125520 0.749127380 0.5008604 -1.12171793 -0.13910812
#> 5 0.6540672 0.1924047 -0.479337138 -0.5146872 -0.01151449 0.45587345
        867
                868
                        869
                             870
                                     871
#> 4  0.626126029  0.03690793  0.25154960  -0.4590444  -0.21133195  -0.08048723
#> 5 -0.743102929 -0.15468317 -0.03367495 0.9872219 0.45737120 0.23738689
        873
                874
                        875
                                876
                                        877
#> 2 0.03897649 -0.07038718 -0.49981753 -0.05106914 0.00593532 -0.06793132
#> 4 -0.14933172  0.56130413  0.43759367  0.38623448 -0.31086788  0.03958836
#> 5  0.71198474  0.56918732  0.09208503  0.44455486  0.19931151  -0.86847439
               880
                   881
                           882
        879
                                       883
#> 1 -0.03916108 -0.24340776 0.1275876 -0.03109459 -0.88908739 -0.43325934
```

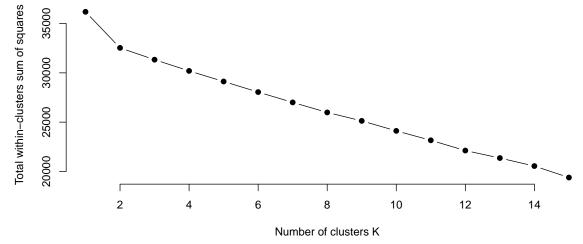
```
#> 2 -0.50894940 -0.60056105 -0.3980324 -0.48852242 -0.36060196 0.08706226
#> 4  0.50869576  0.05321147  0.3743545  0.01603005  -0.35466407  -0.43002404
#> 5 -0.49344274 -0.14061453 -0.5064272 -0.85983119 0.06348581 0.68714568
                 886 887 888
        885
#> 1 0.1781144 -0.31804395 -0.1196628 0.0880550 0.42565603 -0.6723057
#> 2 -0.3820139 -0.24347529 0.2183143 -0.2796119 -0.09181601 -0.5060207
#> 3 0.8517023 0.08273771 1.0042434 0.0885867 0.09213116 -0.5962389
#> 4 -0.9457595 -0.93313609 -0.2558327 -0.5417179 0.08436273 0.4808880
891
                892 893 894
                                         895
#> 1 0.06789178 -0.22696478 0.744027380 0.69106431 -0.2637612 0.381376263
#> 4 -0.12875828 -0.20817937 -0.153610314 -0.62350013 -0.1219857 -0.279500631
898
         897
                         899
                                  900
                                          901
#> 1 -0.37619353 -0.2031186 -0.5478383 0.4726148 -0.3754247 -5.155391e-02
#> 2 -0.06209988 -0.2198317 0.0168379 -0.1872877 -0.2748914 -1.419353e-02
#> 4 -0.62100321  0.5662322 -0.2068409 -0.2812690 -0.6304459 -4.510271e-01
#> 5 -0.56264208 -0.3372480 -0.2022357 -0.3067012 -0.2457104 -3.110955e-05
#>
         903
                  904 905
                                   906
                                            907
#> 1 -0.03851762  0.2861844  0.09994693  0.7561816  0.37038189 -0.1628279
#> 2 -0.24999178 -0.2918445 -0.23570156 -0.0447524 -0.22483243 -0.4847065
#> 4  0.18721121 -0.3038926  0.74872725  0.1813533  0.41095820 -0.3702743
#> 5  0.05673851  0.2596385 -0.70392270 -0.4696924  0.61339249  0.6594252
                  910
         909
                           911 912
                                             913
#> 1 0.02509931 0.31769778 -0.03081071 0.60022939 1.1128917 0.25682801
#> 2 -0.06466259 -0.13073557 -0.42866933 -0.22163836 0.1270801 -0.17090345
#> 3  0.07674486 -0.05937173 -0.06063068  0.07313152 -0.6063177  0.40288928
#> 4 -0.75402286 -0.01253961 0.02543239 -0.14352242 -0.3332395 -0.07713992
#> 5  0.32541359 -0.10079006  0.29543538 -0.21987654  0.1250082  0.75270456
         915
                 916
                      917
#> 1 -1.1786016  0.30585281 -0.5341049 -0.21403790 -0.1704831  0.62774319
#> 2 -0.1115031 -0.08149816 -0.3359158 -0.07038508 -0.2384565 -0.32314244
#> 3 -0.4177336 -1.20729337 0.5518968 -0.56681981 -0.6768762 1.29260645
#> 4  0.8058535 -0.41644672 -0.5578503 -1.07481441 -0.8587315 -0.94533494
#> 5 0.7074596 -0.17590690 0.0802576 0.17475484 -0.5678667 0.04716792
         921
                  922
                          923
                                  924
                                        925
#> 3 -0.29705678 -0.07918715 0.1045943 0.2617422 -0.11245711 -0.3913766
#> 4  0.64586869 -0.05930989 -0.5204091  0.3077301  0.84866841  0.6002481
#> 5 0.31148717 0.43810758 -0.5749079 -0.8501942 -0.25692849 -0.6410251
                          929
                                   930
         927
                 928
                                          931
#> 1 0.15490285 0.3196536 -0.52916218 0.72762371 -1.09595979 -0.3678020
#> 2 -0.21450302 -0.1805928 -0.23057575 -0.25167038 0.02780574 -0.1872114
#> 3 -0.09129663 -0.2262789 -0.31969481 -0.04873302 0.75130060 0.2023166
#> 4 -0.24689772 -0.4340414 0.37161248 -0.24449067 0.04591207 -0.7886644
#> 5 -0.18739461 -0.3107782 0.04012788 -0.31680954 0.17300297 -0.0520401
         933
                934
                           935
                                936
                                            937
```

```
#> 1 0.63446713 0.4533957 -0.26814048 -0.08416369 0.3413637 -0.8615525
#> 2 -0.09000613 -0.4790011 -0.16989772 -0.03780131 -0.2483854 0.1290663
#> 3  0.47492060 -0.2740050 -0.48364044 -0.27242093 -0.2067417 -0.4443152
#> 5  0.09858250 -0.4694131  0.29075234  1.18233416 -0.9281559  0.1878335
        939
                940
                         941
                                  942
                                           943
#> 1 0.4133870 -0.4620677 -0.35859771 0.01334758 0.29335988 0.63161547
#> 2 -0.1515867 -0.2227046 -0.18011008 -0.21777023 0.05833538 -0.01546944
#> 5 0.3756093 -0.3275589 -0.04169907 -0.10426751 0.02307948 -0.64726853
                          947
                                    948
#> 4 -1.1855156 -0.01559563 1.478496995 0.397452361 -0.1783053 0.38157813
951
                 952
                       953
                               954
                                           955
#> 1 -0.03034646 -0.57397316 0.9215768 -0.69966930 -0.07058964 0.45280810
#> 2  0.01540740 -0.27738068 -0.5602273 -0.24046236 -0.05131833 -0.52980757
#> 4  0.21280192  0.06521681  -0.1359811  1.40677852  -0.29236799  0.51700095
#> 5 -0.44509599 -0.37879135 -0.6064055 -0.07824753 -0.05969545 -0.09979373
         957
                  958
                          959
                                   960
                                           961
#> 1 -0.04384551 -0.01564051 -0.72376803 -0.09521153 1.0024562 -0.5036704
#> 2 -0.26531408 -0.17102953 -0.03697572 -0.27363341 -0.2436483 -0.4599659
#> 3  0.12233294  -0.02724637  -0.19582956  -0.20305918  0.3217269  0.2215923
#> 4  0.19456702  -0.43833959  0.63591851  0.49578812  -0.8490087  0.2947275
#>
        963
                 964
                          965
                                   966
                                           967
                                                   968
#> 1  0.2349761 -0.11241934  0.03553782  0.09828602  0.2023683 -0.1645724
#> 2 -0.3605081 -0.23856302 -0.46812316 0.28061348 -0.2555891 -0.4698696
#> 4 1.1604714 -0.06825133 -0.25154210 -0.75390696 -0.5109193 -0.3862390
#> 5 0.0709320 0.09120613 0.22284600 -0.36700155 0.7102059 1.0059299
                        971
                                   972
        969
                 970
                                            973
#> 1 0.2561624 -0.14280886 -0.5185232 0.0791083410 -0.33126645 -0.5315725
#> 2 -0.1698527 -0.03827192 -0.2622173 -0.2149532018 -0.05656158 -0.1150551
#> 3  0.1105636  0.25064843  0.4946791  0.0004459663  0.82161055 -0.7150882
#> 5 -0.6969725 -0.11715858 -0.5977882 0.3234219632 0.16189157 0.4132262
                         977
#>
        975
                976
                                  978
                                            979
#> 1 -0.3527262  0.7951192  0.23564777  0.17931517 -0.283587948 -0.003777557
#> 2 -0.3462707 -0.2572410 -0.34777541 0.08644216 -0.267365188 -0.431048754
#> 3 -0.4296394 -0.3489732 -0.05367794 0.51948617 -0.004130759 0.967823724
#> 5 0.2868030 0.7390336 -0.48256893 -0.24314161 0.739674519 -0.775779960
         981
                 982
                         983
                                   984
                                             985
#> 1 -0.06653708 -0.14259783 -0.2503373 -0.937007985 -0.521694959 0.13818718
#> 2 -0.21156964  0.01731834 -0.3589901 -0.012474619 -0.003492390 -0.21470386
#> 3 0.37461871 0.30936225 0.1321706 0.546442403 -0.235517708 -0.57543986
#> 4  0.53148902  -0.91204050  -0.1061298  0.004624396  -0.439815478  0.08970725
#> 5 0.08876853 0.22209263 -0.8812674 -0.148350368 -0.006195863 -0.61635954
```

```
#> 987 988 989 990 991
#> 1 0.21774822 -0.79287679 0.45399780 -0.7645174 0.190300077 0.45946229
#> 2 -0.14167112 -0.05977998 -0.43172593 -0.1456071 -0.003537708 -0.26031019
#> 3  0.01039599  -0.13231940  0.35084496  0.4139772  0.280944935  -0.63890435
#> 4  0.32870972  0.44429793  -0.32141041  -0.1635785  -0.045666626  -0.05267257
#> 5 -0.86130092  0.27078203  0.06051313  0.3530737  0.384805662  0.13847014
#>
           993
                     994
                               995
                                         996
                                                  997
#> 2 -0.32712730 -0.03168406 -0.2362432 -0.33453575 -0.2623600 -0.30791152
#> 3  0.01770253  -0.78682235  0.1379680  -0.58513481  -0.2231518  0.67548014
#> 4  0.62398628  1.63328452  0.1305629  0.07934175  0.6496092  -0.32123826
#> 5 0.09225582 0.28371015 0.4251850 -0.34694729 -0.1582129 0.20518411
#>
#> 1 0.32975040
#> 2 -0.41400480
#> 3 -0.60957751
#> 4 -0.01658663
#> 5 -0.03068340
#>
#> Clustering vector:
#> X.0.9619334 X0.4418028 X.0.9750051 X1.417504
                                                X0.8188148
#>
           3
                          1
                                       1
#>
    X0.3162937 X.0.02496682
                         X.0.063966 X0.03149702 X.0.3503106
#>
          5
                           5
                                            1
#> X.0.7227299 X.0.2819547
                           X1.337515
                                     X0.7019798
                                                  X1.007616
#>
           3
                                 5
                                            5
                      .3
#>
   X.0.4653828
               X0.6385951
                          X0.2867807 X.0.2270782 X.0.2200452
#>
      1
                           5
                                      1
                                                        1
#>
    X.1.242573 X.0.1085056
                          X.1.864262
                                     X.0.5005122
#>
                                 2
           2
                     2
                                             2
                                                         2
#>
    X1.063411 X.0.2963712 X.0.1216457 X0.08516605
#>
                   2
                                 2
                                        2
  X.0.5095915 X.0.2167255 X.0.05550597 X.0.4844491 X.0.5215811
#>
              2 2 2
#>
     2
#>
     X1.949135
                X1.324335
                          X0.4681471
                                        X1.0611
#>
            2
                       2
                                  2
                                             2
                                                         2
#> Within cluster sum of squares by cluster:
#> [1] 4896.690 13540.079 2934.875 2889.281 4857.595
#> (between_SS / total_SS = 19.5 %)
#>
#> Available components:
#>
#> [1] "cluster"
                  "centers"
                               "totss"
                                            "withinss"
#> [5] "tot.withinss" "betweenss"
                                "size"
                                             "iter"
#> [9] "ifault"
```

Elbow-Method

```
#Elbow Method for finding the optimal number of clusters set.seed(42)
# Compute and plot wss for k=2 to k=15.
k.max <-15
data <- scaled_data
```



Kriterium: Beim Elbow-Kriterium werden die Fehlerquadratsummen in ein Diagramm abgetragen. Zeigt sich im Kurvenverlauf ein Knick ("Ellbogen"), so kann dieser Wert als Entscheidungskriterium für die Clusteranzahl verwendet werden.

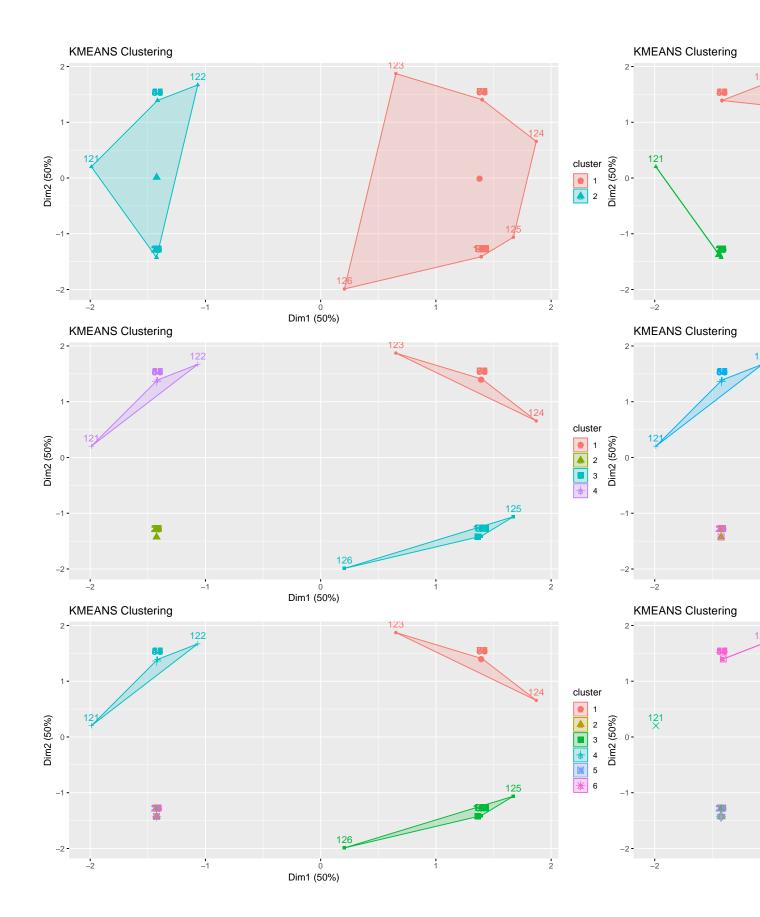
Elbow-

Optimale Anzahl der Cluster K=4.

Aufgabe 2 a)

In diesem Aufgabenteil fu
ehren wir den Kmeans Algorithmus auf den clust Datensatz durch fuer die Werte
 $k=\{2,\ldots,7\}$ und visualisieren anschliessend die Cluster.

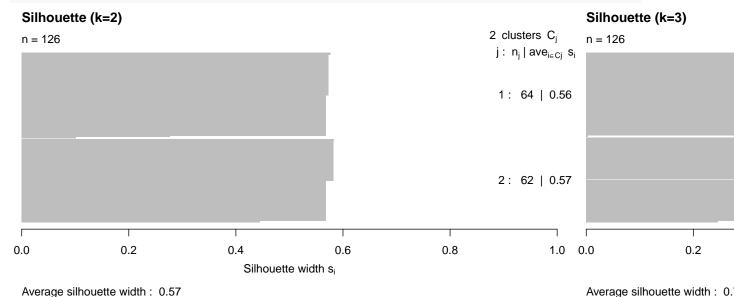
```
load("clust.RData")
packageTest("cluster")
packageTest("factoextra")
data <- clust
colnames(data) <- c("x1", "x2", "x3", "x4")
data <- scale(data)
for(i in 2:7) {
   par(mfrow=c(6,1))
   eclust(data, "kmeans", k=i)
}</pre>
```



b)

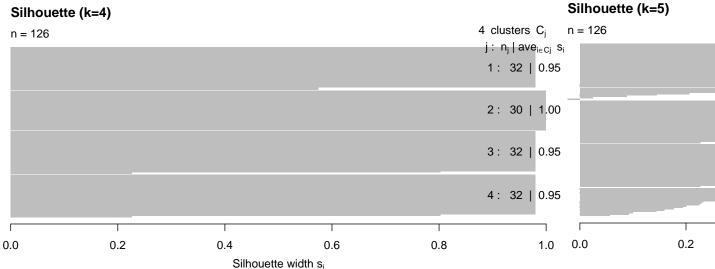
Nun berechnen wir die Silhouetten sowie die Silhouetten-Koeffizienten fuer die Clusterungen aus a) und visualisieren anschliessend die Silhouetten inkl der Koeffizienten.

```
for(i in 2:7){
  km.res <- eclust(data, "kmeans", k=i, graph=FALSE)</pre>
  sil <- silhouette(km.res$cluster, dist(data))</pre>
  plot(sil, main=sprintf("Silhouette (k=%d)",i))
}
```



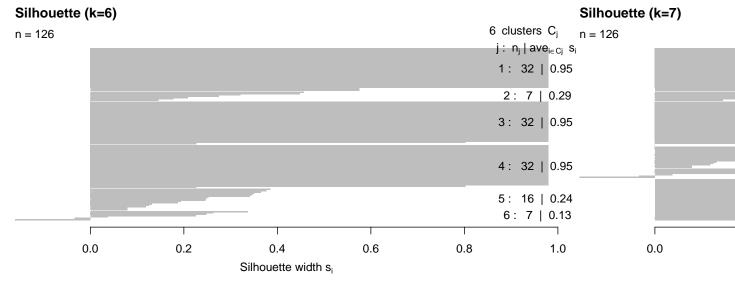
Average silhouette width: 0.57

Silhouette (k=4)



Average silhouette width: 0.96

Average silhouette width: 0.7



Average silhouette width: 0.78

Average silhouette width: 0.7

c)

Die Silhouette eines Datenpunktes entspricht einem Guetekriterium der Cluster. Sie gibt an, wie aehnlich bzw. unaehnlich ein Datenpunk zu seinen Nachbarpunkten im Mittel ist. Dabei werden die Vergleiche zwischen dem Datenpunkt und den Nachbarpunkten innerhalb des eigenen Clusters gemacht sowie zwischen dem Datenpunk und den Nachbarpunkten des nächsten Nachbar-Clusters.

Die Silhouette kann Werte aus dem Intervall [-1,1] annehmen.

- 1 bedeutet dabei, dass der Datenpunkt im Mittel relativ zentral innerhalb des eigenen Clusters liegt und somit weiter entfernt zu den Nachbarklustern ist. Diese Eigenschaft entspricht einem guten Clustering.
- -1 bedeutet, dass der Datenpunkt im Mittel nacher zum Nachbar-Cluster liegt als zum eigenen Cluster.
- 0 wird einem Datenpunkt zugeordnet, wenn ein Cluster ausschliesslich diesen Datenpunkt enthaelt.

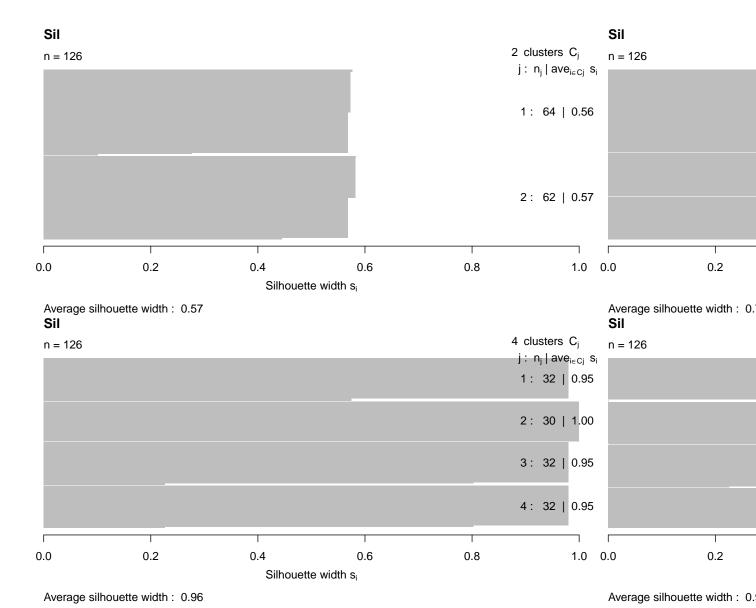
maximimieren wir den Wert der Silhouette, so erhalten wir den Silhouetten-Koeffizienten.

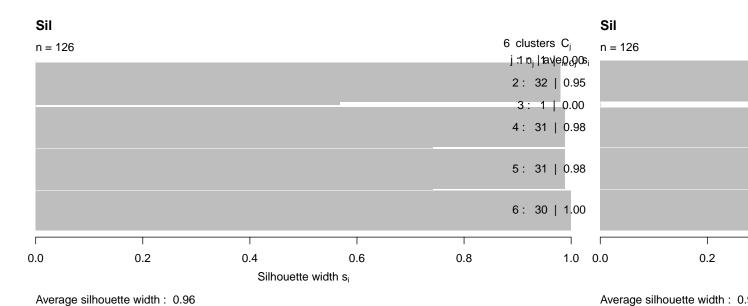
Vergleichen wir die Modelle mit unterschiedlichen Anzahl an Cluster, so waehlen wir das Modell mit k=4, da dieser die hoechste Silhouette besitzt sowie niedrigste Komplexitaet. Alle Cluster sind aehnlich gefuellt. Erhoehen wir die Anzahl der Cluster, so bleibt der Wert der Silhouette gleich, jedoch erhalten wir Silhouetten Werte gleich 0, diese entsprechen einem Cluster von einem Datenpunkt.

d)

Da das Ergebnis des K-Means-Verfahrens sehr stark von dem Initialzustand abhaengt, wiederholen wir im Folgenden den Algorithmus und setzen dabei fuer jeden Wert $k \in \{2, ..., 7\}$ zufaellig einen Startzustand. Diese Option wird mit dem parameter nstart erreicht, den wir z.B. auf 25 setzen, so dass der Algorithmus 25 Mal durchlaeuft mit jeweils zufaelligem Startzustand. Im default betraegt dieser Parameterwert 1.

```
for(i in 2:7) {
   km.res <- eclust(data, "kmeans", k=i, graph=FALSE, nstart=25)
   sil <- silhouette(km.res$cluster, dist(data))
   plot(sil, main="Sil")
}</pre>
```





In unserem Fall erkennen keine signifikanten Unterschiede zu unserer Beobachtung auf Aufgabenteil b).