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In[349]:= NotebookDirectory[]
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Out[349]= /Users/maxfrankel/Desktop/Jun Ye/Daily Files/20241224/
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ABCD matrix evolution inside a confocal cavity

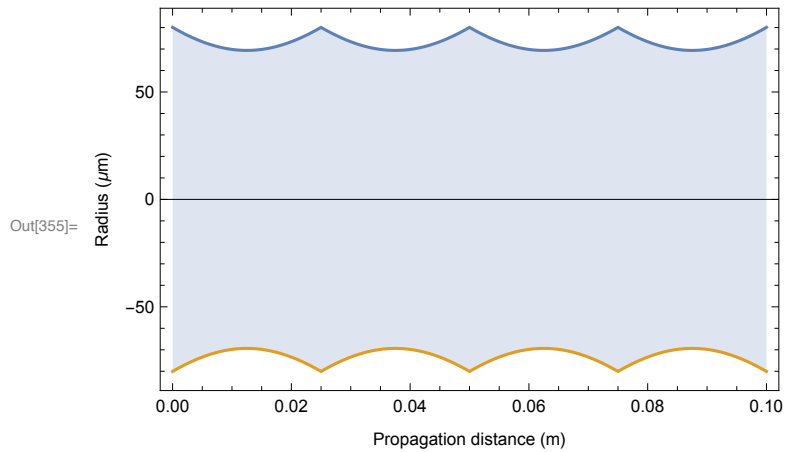
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In[350]:= ABCD[z_] := Which[
  z < L,  $\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}$ ,
  z == L,  $\begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ ,
  L < z < 2 L,  $\begin{pmatrix} 1 & z-L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ ,
  z == 2 L,  $\begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ ,
  2 L < z < 3 L,  $\begin{pmatrix} 1 & z-2L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ ,
  z == 3 L,  $\begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ ,
  3 L < z < 4 L,
   $\begin{pmatrix} 1 & z-3L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ ,
  z == 4 L,  $\begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot$ 
   $\begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ 
];

(* incident beam parameters *)
wi = 0.00008008615870885669; (* initial waist in m *)
λ = 698 * 10-9; (* wavelength in m *)
Ri = -0.05; (* initial wavefront radius in m *)

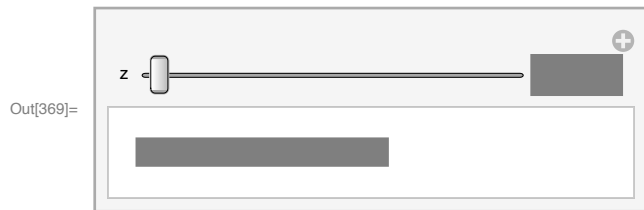
(* cavity parameters *)
Rlens = 50 * 10-3; (* lens curvature in m *)
L = 25 * 10-3; (* cavity length in m *)

(* evolution of waist and radius of curvature inside cavity *)
qi =  $\frac{1}{\frac{1}{Ri} - i \frac{\lambda}{\pi wi^2}}$ ;
qf[z_] :=  $\frac{qi * ABCD[z][[1, 1]] + ABCD[z][[1, 2]]}{qi * ABCD[z][[2, 1]] + ABCD[z][[2, 2]]}$ 
w[z_] :=  $\sqrt{\frac{\lambda}{-\pi \text{Im}[\frac{1}{qf[z]}]}}$  // N
R[z_] := 1 / (N[Re[1/qf[z]]])
plt = Plot[{w[z] * 106, -w[z] * 106}, {z, 0, 4 L}, Filling → {1 → {2}},
  Frame → True, FrameLabel → {"Propagation distance (m)", "Radius (μm)"}]
```



In[356]:=
$$w02[z_] := \frac{w[z]}{\text{Sqrt}\left[1 + \left(\frac{\pi w[z]^2/\lambda}{R[z]}\right)^2\right]} \quad (* \text{ waist size } *)$$

In[369]:= **Manipulate**[N[w02[z]], {z, 0, 4 L}] (* smallest waist within section of cavity *)
Manipulate[w[z], {z, 0, 4 L}] (* beam radius *)
Manipulate[R[z], {z, 0, 4 L}] (* wavefront radius *)



Calculation of stable cavity mode

In[360]:= (* round-trip ABCD matrix *)

ABCDrt = $\begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot$
 $\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -2/Rlens & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix};$
 ABCDrt // MatrixForm

Out[361]//MatrixForm=

$$\begin{pmatrix} -1 & -\frac{1}{40} \\ 40 & 0 \end{pmatrix}$$

In[362]:= (* find solution q that remains unchanged under round-trip *)

qsols = NSolve[q == $\frac{ABCDrt[[1, 1]] * q + ABCDrt[[1, 2]]}{ABCDrt[[2, 1]] * q + ABCDrt[[2, 2]]}$, q]

Out[362]= {{q → -0.0125 - 0.0216506 i}, {q → -0.0125 + 0.0216506 i}}

In[363]:= wq[q_] := $\sqrt{\frac{\lambda}{-\pi \operatorname{Im}\left[\frac{1}{q}\right]}}$ // N

Rq[q_] := 1 / (N[Re[1 / q]])

In[365]:= wq[q /. qsols[[1]]] (* not feasible, imaginary waist *)

Rq[q /. qsols[[1]]]

wq[q /. qsols[[2]]]

Rq[q /. qsols[[2]]]

Out[365]= 0. + 0.0000800862 i

Out[366]= -0.05

Out[367]= 0.0000800862

Out[368]= -0.05