

Optical Pumping: Exciting optical and RF transitions simultaneously to perform spectroscopy on the hyperfine splitting of the ground state of Rubidium

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Abstract:

Optical pumping is a common topic investigated in undergraduate and graduate physics labs, as its history is deeply intertwined with the early developments of quantum mechanics and the description of atoms, and the concepts learned when conducting the experiments are still applicable in a wide range of atomic physics experiments today. In a spectroscopy experiment such as this one, one can directly observe phenomena that cannot be explained by a classical description of the atom. In this report, we first discuss the history and theory behind optical pumping, and then go on to show how our optical pumping setup can be used to observe various energy shifts and splittings that emerge in the relativistic mathematical treatment of the electron. We use theoretical models together with experimental data to calculate absorption cross section, the earth's local magnetic field, the Landé g-factor, and verify the Breit-Rabi formula.

I. INTRODUCTION

A. Developing an understanding of atomic structure in the early 20th century: The Zeeman effect and Landé factor

The pace at which atomic physics progressed in the early 20th century is remarkable. The existence of subatomic particles, namely electrons, wasn't accepted until Thomson's experiments with cathode rays in 1897 [1]. Just a year earlier, armed with the newly developed technology of the diffraction grating, Pieter Zeeman, a Dutch physicist who had been investigating the connections magnetism and light, found that the spectral lines of a sodium flame became broadened and split in the presence of an external magnetic field. Zeeman was able to explain the splitting and broadening classically, by considering oscillating electrons as the source of emitted radiation in the sodium flame [2].

Through his investigation of spectroscopic line splittings in different atoms, Zeeman found that the splitting was not the same for all atoms, and incorrectly concluded that the charge/mass ratio varied from electron to electron.

Further confusion came from the discovery by Friedrich Paschen and Ernst Back, in 1913, of two types of Zeeman splitting: one that they dubbed "normal" and one "anomalous" [3]. What they did not know at the time was that the normal Zeeman effect described a singlet system, while the anomalous Zeeman effect came from working with a system that had non-zero spin. Electron spin wasn't discovered until 1925, so physicists spent many years puzzling over the anomalous Zeeman effect.

In an attempt to explain the anomalous Zeeman effect, in 1921, the physicist Alfred Landé created a model based off of empirical results [4]. Part of his model

allowed for the existence of electronic states with half-integer multiples of angular momentum quanta, in order to explain the existence of doublets while still maintaining the $\Delta m = \pm 1, 0$ selection rule. His work involved the assumption that the angular momentum projection is quantized, that angular momentum quanta are vectors, and that atomic angular momenta are combined by vector addition that is also subject to quantization. Although initially looked down upon by other physicists for its lack of theoretical justification, Landé's g-factor turned out to be an important step towards discovering electron spin, interpreting the results of the Stern-Gerlach experiment, and the Pauli exclusion principle [5].

B. Optical pumping and Alfred Kastler

Alfred Kastler's journey with optical pumping is well-outlined in his Nobel lecture and a 1967 article in *Science* [6] [7].

The story begins with a young physicist in the 1920s, setting out into the nascent field of quantum mechanics. Kastler wrote that he became particularly interested in the conservation of angular momentum in the scattering of light from atoms after reading Arnold Sommerfeld's *Atombau und Spektrallinien* in his studies at the Ecole Normale Supérieure. A Polish physicist working under Sommerfeld, Wojciech Rubinowicz, had even shown that the application of conservation of angular momentum led to selection rules for atomic transitions regarding the azimuthal quantum number, corresponding to the polarization of light.

Kastler became interested in work being done to understand conservation of angular momentum in scattering experiments. Excitement came in 1931, when Indian Physicists CV Raman and S Bhagavantam realized that these scattering experiments were experimental evidence of the existence of photon spin [8]. Indian physicist Satyendranath Bose had already predicted the 2-fold quantization of photon polarization in a 1924 paper

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where he re-derived Planck's formula for blackbody radiation purely from statistical mechanics and the assumption of quantization, as he found Einstein's derivation to have "insufficient logical foundation." In that paper, Bose had found that an additional factor of 2 was needed to arrive at the proper number of cells in the phase space volume [Bose].

While at the Ecole Normale Supérieure, for his thesis in 1936, Kastler realized that circularly polarized light could be used to polarize an atomic vapor in successive transitions and selectively excite different magnetic sublevels of the Mercury atom [9].

Following the work of Rabi [10], Lamb, and Retherford [11] in developing nuclear magnetic resonance, Kastler and a former student, Jean Brossel, devised an experiment to measure magnetic resonances between Zeeman sublevels of excited states. The experimental setup involved an optical resonance and an oscillating magnetic field RF resonance, exactly as we do in our optical pumping experiments at Stony Brook today. By this double-resonance method of inducing both optical and RF resonances simultaneously in an atomic vapor of mercury, Kastler showed that one could "pump" atoms and form a concentration either in the positive or negative magnetic sublevels of the ground state, thus polarizing the vapor and creating a non-Boltzmann distribution. He was interested in studying the relaxation of atoms from this non-Boltzmann distribution.

Together with Jean Brossel, Kastler went on to explore a whole range of questions within optical pumping, such as the relaxation times of atomic vapors from this non-Boltzmann distribution, the effects of buffer gases, measurements of the Lande factors, and multi-photon transitions. In 1966, Alfred Kastler was awarded the Nobel Prize "for the discovery and development of optical methods for studying Hertzian resonances in atoms."

C. The importance of optical pumping

Being able to create non-Boltzmann distributions of atoms has broad applications. In 1960, the first optical laser was achieved by T.H. Maiman, which relied on optical pumping to create the population inversion necessary for a laser [12]. Optical pumping has also been used to improve the signal-to-noise in atomic clock experiments [13].

II. THEORY

A. The nonrelativistic atomic model and its energy levels

Rb is an alkali atom with a single valence electron. The closed shells form a spherically symmetric core that shields the nuclear charge. Outside of this core, the electrical potential is approximately identical to that of the

hydrogen atom. Because the valence electron orbital radius is much larger than the radius of this core, the analytic solutions to the hydrogen wavefunction can be used to approximate Rb's valence electron wavefunctions, which have no analytic solution [14]. In order to improve our approximation, we can calculate numerous corrections as perturbations, using perturbation theory.

The hydrogen wavefunctions are the solutions to the Schrödinger wave equation of an electron in the Coulomb potential of a proton [15].

$$i\hbar \frac{\partial}{\partial t} \psi = \hat{H} \psi = \left(\frac{\hbar^2}{2m_e} \nabla^2 - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \right) \psi$$

The Hamiltonian for an alkali atom differs in that the nuclear charge is Ze (for some integer Z equal to the number of protons) instead of e , there are multiple electrons, and there is an additional term representing the interaction between the electrons. When considering the valence electron outside of the spherical core of electrons, we assume that the interaction between electrons counteracts the greater nuclear charge of the alkali atom, yielding the same Hamiltonian as hydrogen.

$$\hat{H} = \sum_i \left(\frac{\hbar^2}{2m_e} \nabla_i^2 - \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_i} \right) + \sum_{i,j} \frac{1}{4\pi\epsilon_0} \frac{e^2}{|r_i - r_j|} \quad (1)$$

The hydrogen wavefunctions consist of radial solutions that depend on the quantum number n multiplied by spherical harmonics that depend on quantum numbers l, m .

$$\psi_{n,L,M} = R_n(r) Y_{L,M}(\theta, \phi)$$

The quantum state of the hydrogen wavefunction can be determined by applying a set of commutative operators describing measurements, \hat{H} , \hat{L}^2 , and \hat{L}_z . When acting on a quantum state $\psi_{n,L,M}$, they return eigenvalues E_n, L, M :

$$\begin{aligned} \hat{H} \psi_{n,L,M} &= E_n \psi_{n,L,M} \\ \hat{L}^2 \psi_{n,L,M} &= L(L+1) \psi_{n,L,M} \\ \hat{L}_z \psi_{n,L,M} &= M \psi_{n,L,M} \end{aligned}$$

where

$$E_n = -\frac{m_e}{2\hbar^2} \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \frac{1}{n^2}$$

In this non-relativistic model, the energy levels are determined solely by n , and the different L, M form degenerate sublevels for a given n .

In this report, we are working with Rubidium. Following the aufbau principle, the core electrons of Rb fill up the orbitals $n = 1, 2, 3, 4$, meaning that, in the ground state, the valence electron is in $n = 5$.

B. Correction to energy levels due the quantum defect

The potential experienced by the Rb valence electron is only identical to that of Hydrogen's electron outside the spherical shell of electrons. Inside this shell, the nuclear charge is no longer shielded, and the potential changes with Ze/r^2 instead of e/r^2 . The valence electron experiences a shift in energy depending on how much its wavefunction overlaps with the core, leading to a shift in energy that is dependent on L . This energy shift is referred to as the quantum defect [14].

For rubidium, the quantum defect causes the normally degenerate 5S and 5P levels to become separated by an energy on the order of 1.6eV [16].

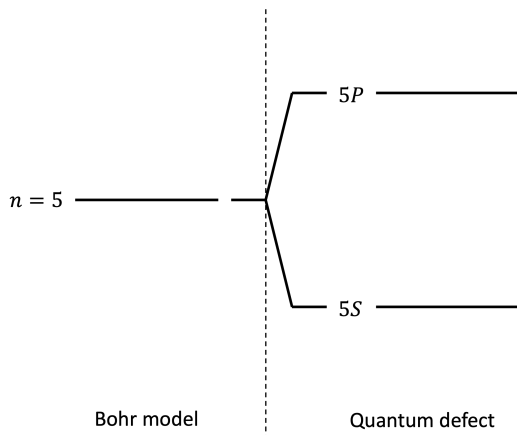


FIG. 1. Diagram showing how the quantum defect lifts the degeneracy of the S and P sublevels within the $n=5$ energy level of the Bohr model. The $L=2,3,4,\dots$ sublevels receive an energy shift as well, but are considered in this experiment and so are not pictured.

C. Relativistic corrections to the hydrogen model energy levels

The Hamiltonian of Eq. 1 is non-relativistic. Thus, the relativistic Hamiltonian for an atom, given by the Dirac formula, gives a better model for our Rb atom [17]. Relativistic treatment of the electron leads to additional terms in the Hamiltonian of Eq. 1, referred to as relativistic corrections.

The purpose of this lab was not to explore all of the different corrections. The corrections of interest were those which lift some degeneracy between quantum states, splitting energy levels, as this creates extra transitions in the absorption spectrum that can be easily observed.

The three relativistic corrections that cause observable splitting are spin-orbit coupling, hyperfine splitting, and the Zeeman effect, all of which deal with the potential energy of a magnetic moment $\vec{\mu}$ in a magnetic field \vec{B} .

$$E = -\vec{\mu} \cdot \vec{B}$$

1. Splitting of energy levels due to spin-orbit coupling

In Rb, spin-orbit coupling creates splitting of the 5P energy level of around 0.03eV, two orders of magnitude smaller than the splitting between the 5S and 5P levels [16].

In the reference frame of an electron, the orbital motion of the nucleus creates a magnetic field. This magnetic field interacts with the electron's intrinsic magnetic moment, μ_e . The Hamiltonian for magnetic moment in an external magnetic field is given by [14]

$$\hat{H}_{SO} = -\vec{\mu}_e \cdot \vec{B} \quad (2)$$

Now, for the orbiting electron, $\vec{B} \propto \vec{L}$, the electron's orbital angular momentum, and $\vec{\mu}_e \propto \vec{S}$, the electron's spin [16]. Therefore,

$$\hat{H}_{SO} \propto \hat{L} \cdot \hat{S}$$

The Hamiltonian composed of both \hat{H}_0 and \hat{H}_{SO} has a term proportional to $\hat{L} \cdot \hat{S}$, and thus \hat{L}_z no longer commutes with the Hamiltonian. Thus a new basis is required to describe the state ψ .

$\hat{J} = \hat{L} + \hat{S}$ does commute with the Hamiltonian, and thus \hat{L}^2 , \hat{S}^2 , and \hat{J}^2 become the new set of operators used to describe a state ψ .

For an electron, $S = 1/2$, and $L = 0, 1, 2, \dots$. For the 5S energy level of Rb, $L = 0$ and so there is no splitting, as $J = S$. However, for the 5P energy level of Rb, $L = 1$, and thus $J = L \pm S$ can take on the values $J = 1/2, 3/2$. These two states have different energy, as seen in Eq. 2, and thus the 5P energy level is split.

A common notation to keep track of L, S , and their relative orientation is shown below:

$$^{2S+1}L_J$$

Using this notation, we can say that 5P splits into $^2P_{1/2}$ and $^2P_{3/2}$.

Because $L = 0$ for 5S, J is simply equal to S , and thus is written as $^2S_{1/2}$.

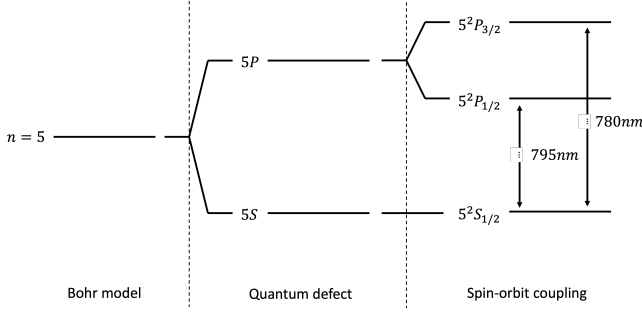


FIG. 2. Diagram showing how spin-orbit coupling splits the 5^2P energy level of Rb into $^2P_{1/2}$ and $^2P_{3/2}$ levels. While the splitting between the S and P levels is on the order of 1eV , the splitting between $^2P_{1/2}$ and $^2P_{3/2}$ due to spin orbit coupling is on the order of 10^{-2}eV .

Only the $5S_{1/2}$ and $5P_{1/2}$ levels are of interest for this report.

2. Hyperfine structure

The hyperfine splitting of each energy level occurs on a scale another few orders of magnitude smaller than the spin-orbit coupling. Rb has a nuclear magnetic moment $\vec{\mu}_I$, which interacts with the total magnetic field created by the electron \vec{B}_e .

$$\hat{H}_{HF} = -\vec{\mu}_I \cdot \vec{B}_e$$

The nucleus's magnetic moment is proportional to the nuclear spin \hat{I} , and the magnetic field of the electron \vec{B}_e is created from the sum of the magnetic moments of the electron's intrinsic magnetic moment, proportional to the sum of its spin \hat{S} and the magnetic moment arising from its orbital motion, proportional to \vec{L} . Thus, $B_e \propto \hat{L} + \hat{S} = \hat{J}$, and therefore,

$$\hat{H}_{HF} \propto \hat{I} \cdot \hat{J}$$

Again, we must choose a new set of operators. In this case, a new operator $\hat{F} = \hat{I} + \hat{J}$ is defined, and the operators \hat{F}^2 , \hat{I}^2 , \hat{J}^2 , and \hat{F}_z and their eigenvalues are used to describe the atom's state ψ .

$$\hat{F}^2\psi = F(F+1)\psi$$

$$\hat{I}^2\psi = I(I+1)\psi$$

$$\hat{J}^2\psi = J(J+1)\psi$$

$$\hat{F}_z\psi = M_F\psi$$

where

$$M_F = -F, -F+1, \dots, F-1, F$$

Thus, each energy level determined by the corrections thus far can take on two values $F = I \pm J$. For each level corresponding to some F , there are $2F+1$ sublevels corresponding to the different possible orientations of \vec{F} with the chosen quantization axis.

The values of I for ^{85}Rb and ^{87}Rb are shown in the table below:

I for ^{85}Rb	I for ^{87}Rb
$5/2$	$3/2$

TABLE I. This table represents the nuclear spin value for each respective isotope of Rb

So far, the corrections to both ^{85}Rb and ^{87}Rb have been equal. Hyperfine splitting is a correction that differs between the two isotopes.

For both the $5S_{1/2}$ and $5P_{1/2}$ levels of ^{85}Rb , the energy levels are split into two values, corresponding to the two values of $F = 5/2 \pm 1/2 = 2, 3$.

For both the $5S_{1/2}$ and $5P_{1/2}$ levels of ^{87}Rb , the energy levels are split into two values, corresponding to the two values of $F = 3/2 \pm 1/2 = 1, 2$.

A diagram of the full set of corrections accounted for in this report for both ^{85}Rb and ^{87}Rb is shown in Fig 3

3. Zeeman splitting

If we now apply an external magnetic field along the chosen quantization axis \hat{z} , there will be additional potential energy from the net dipole moment of the Rb atom interacting with the external field.

$$\hat{H}_{Zeeman} = -\mu_{atom} \cdot B_{ext}$$

However, we can only treat this energy as a perturbation if it is small compared to our Hamiltonian. When the perturbation is much smaller than that which creates hyperfine splitting,

$$\hat{H}_{Zeeman} = g_F \frac{\mu_B B_{ext}}{\hbar} \hat{F}_z$$

where [18]

$$g_F = g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} + g_I \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)} \quad (3)$$

and g_F is known as the Landé g factor.

By first order perturbation theory, this causes an energy shift that affects each magnetic sublevel M_F differently:

$$\Delta E_{|F, M_F\rangle} = \mu_B g_F M_F B_z \quad (4)$$

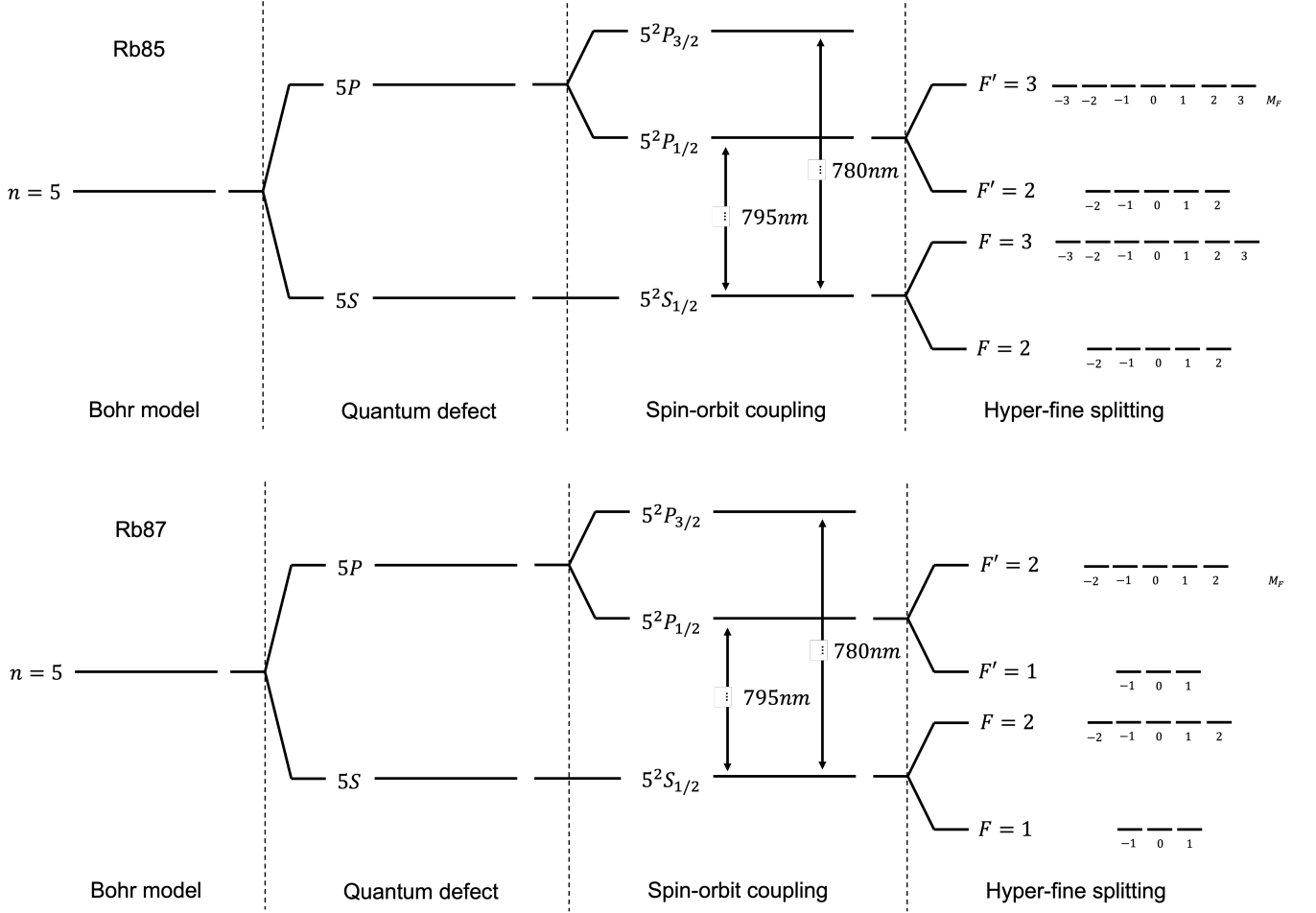


FIG. 3. Diagrams showing the energy corrections to ^{85}Rb and ^{87}Rb due to multiple electrons, spin-orbit coupling, and hyper-fine splitting in order of largest to smallest in magnitude. The two isotopes differ only after the consideration of hyper-fine splitting, where their different nuclear magnetic moments I lead to different possible values for $F = I \pm J$.

This splitting of the M_F sublevels is known as the Zeeman effect. Note that this means that the energy difference between adjacent M_F sublevels is $\mu_B g_F B_z$. These energy differences are what we can actually measure experimentally. By varying B_z and measuring the energy differences, one can calculate g_F and the earth's magnetic field at the site of the experiment.

Eq. 4 is a good approximation for small magnetic fields. There is, in fact, an analytic solution to the energy shifts of the $5^2P_{1/2}$ ground state manifold of Rb, given by the Breit-Rabi formula [18].

$$E_{|J=1/2, m_J, I, m_I\rangle} = -\frac{\Delta E_{hfs}}{2(2I+1)} + g_I \mu_B m B \pm \frac{\Delta E_{hfs}}{2} \left(1 + \frac{4m x}{2I+1} + x^2 \right)^{1/2} \quad (5)$$

where $m = m_I \pm m_J = m_F$,

$$x = \frac{(g_J - g_I) \mu_B B}{\Delta E_{hfs}}$$

and the values of E_{hfs} , g_I , g_J can be found in Steck's Alkali data [18].

For $m = M_F = -F$, the Breit-Rabi formula can be written as

$$E_{|J=1/2, m_J, I, m_I\rangle} = \Delta E_{hfs} \frac{I}{2I+1} - \frac{1}{2} (g_J + 2I g_I) \mu_B B$$

The calculated energy shift according to the Breit-Rabi formula was calculated for $F=3$ in Rb85 and $F=2$ in Rb87, and is shown in Fig. 4 below:

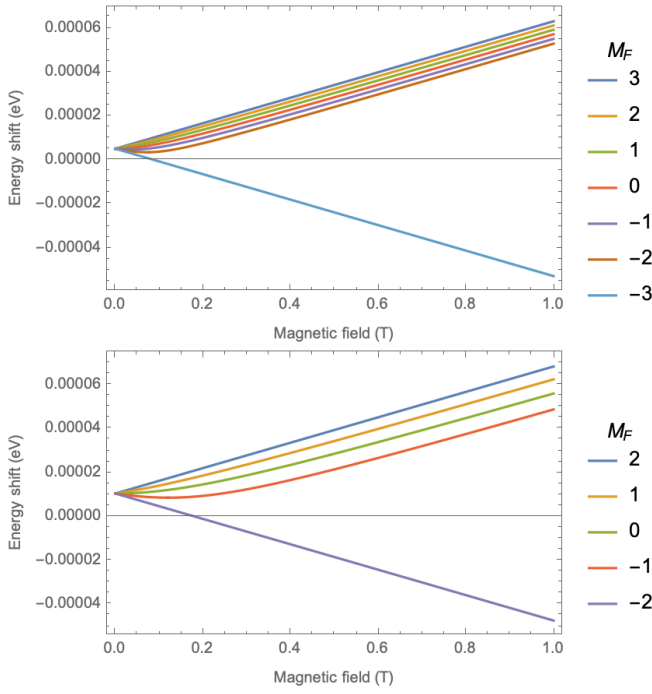


FIG. 4. Calculated energy shift for Rb85 (top) and Rb87 (bottom) according to the Breit-Rabi formula (Eq. 5)

However, in this experiment, the maximum magnetic field achievable was about 0.8mT . Let's focus in on this region in our graphs.

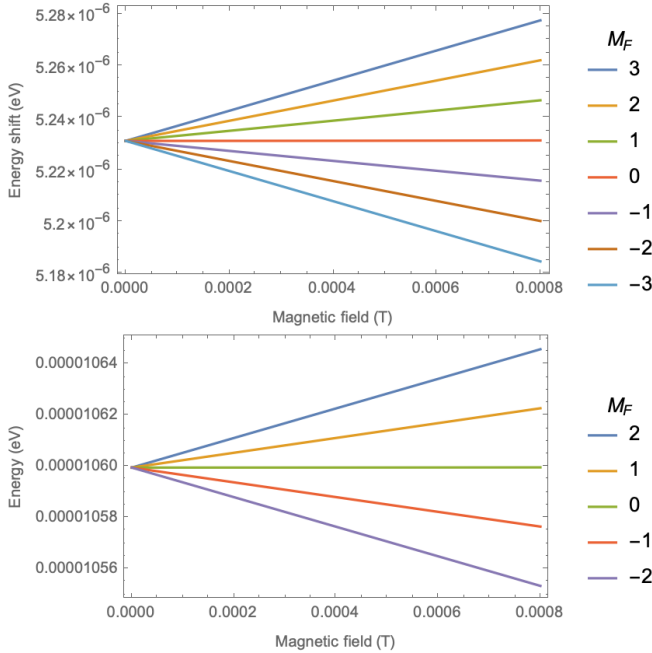


FIG. 5. The graphs from Fig. 4, but over the range of magnetic fields achievable in the experimental setup. Top is Rb85 and bottom is Rb87.

As seen in Fig. 5, the energy difference between ad-

jacent M_F sublevels at our achievable field strengths is very close to equal as magnetic field increases, showing that Eq. 4 is a good approximation.

With spectroscopy, the energy levels themselves can't be measured, but transitions between them can. Transition frequencies (energy/h) between adjacent M_F sublevels were calculated and are shown in Fig. 7.

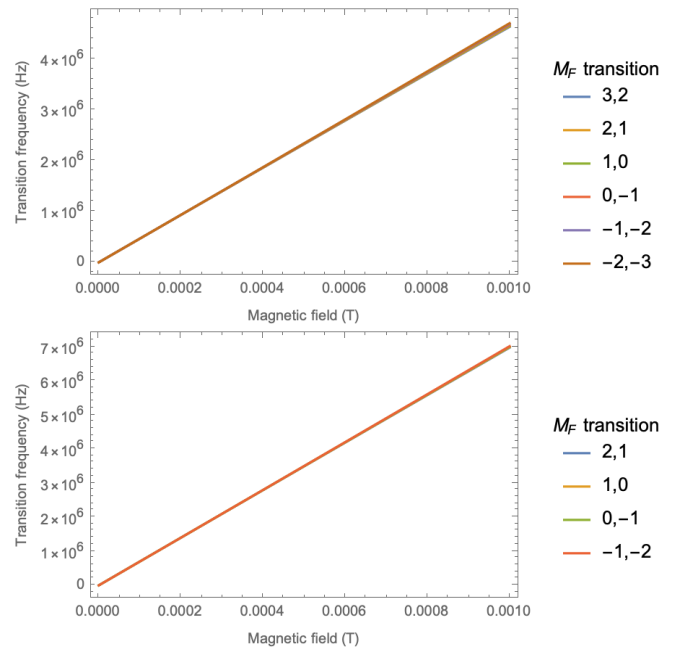


FIG. 6. Theoretical calculation of frequency needed to excite transitions between adjacent M_F sublevels over magnetic field for Rb85 (top) and Rb87 (bottom). There appears to be only one line because the frequencies are very close together.

The transitions are very close in energy, but not equal. The slight difference is only clear when plotting the difference between adjacent transitions.

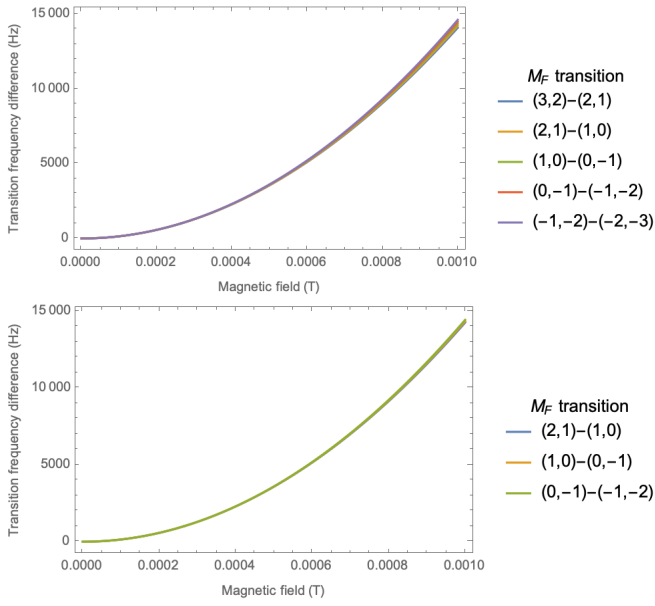


FIG. 7. Theoretical calculation of difference in transition frequencies over magnetic field for Rb85 (top) and Rb87 (bottom). While the transitions are relatively close in frequency, all increasing linearly by about 10MHz per mT as shown in Fig. 7, this graph shows how the transitions split apart, on the order of 10kHz away from their nearest neighbor as the magnetic field increases to 1mT

In this lab, we measured the RF resonances between adjacent M_F sublevels as a function of magnetic field, and compared to theoretical values.

D. Using light to determine differences between atomic energy levels

An atom can transition between two energy levels when stimulated by a photon with energy very close to the difference between two energy levels. For a photon,

$$E_{\text{photon}} = h\nu$$

where h is the Planck constant and ν is the photon frequency. The condition when ν is such that E_{photon} is exactly equal to the energy difference between two levels is called resonance. Atoms can absorb light that is not perfectly resonant with a transition, but the probability of absorption increases the closer the frequency of light is to resonance.

The allowed transitions are determined by the selection rules. For an unpolarized photon, only transitions for which $\Delta M_F = -1, 0, 1$ are allowed. A right-hand circularly polarized photon, denoted as σ_+ , causes transitions for which $\Delta M_F = +1$. For a left-hand circularly polarized σ_- photon, $\Delta M_F = -1$ [19].

By scanning the frequency ν of light illuminating a gas of atoms, the location of resonances corresponding

to different transitions can be observed as dips in the amount of light transmitted through the gas. The rate at which photons are absorbed by a gas of atoms with respect to their detuning $\nu - \nu_0$ from an atomic resonance ν_0 follows the Voigt profile [19],

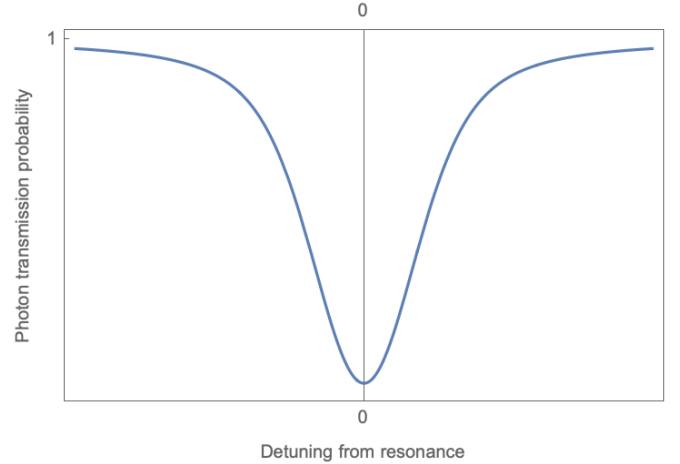


FIG. 8. The Voigt profile models the absorption probability of a photon detuned from an atomic resonance. Even though atoms have discrete energy levels, photons do not need to be perfectly resonant with a transition in order to cause stimulated absorption and emission

where the center of the dip corresponds to resonance with the atomic transition.

E. Optical pumping

Consider the $5^2S_{1/2}$ and $5^2P_{1/2}$ energy levels in ^{87}Rb . For a gas of ^{87}Rb in thermal equilibrium, each energy state will be populated by some fraction of the total number of atoms, following the Boltzmann distribution. In fact, the spacing between hyperfine levels is so small that the thermal energy of a vapor of Rb becomes comparable to the spacing at about 300mK . We are operating at room temperature - around 300K . Thus, even in the presence of an external magnetic field, the populations among the different hyperfine levels is approximately equally distributed in thermal equilibrium.

Suppose 795nm light is shone onto the gas, and that the light has a linewidth great enough that it allows for transitions between any of the sublevels within the $5^2S_{1/2}$ level to and from any sublevels within the $5^2P_{1/2}$ level, but not great enough to create transitions up to any of the sublevels within $5^2P_{3/2}$.

There are selection rules for allowed transitions due to stimulated absorption that depend on the polarization of light. For right-hand circularly polarized light along the \hat{z} direction, denoted σ_+ , the selection rule requires that $\Delta M_F = +1$. The allowed transitions are illustrated in Fig. 9.

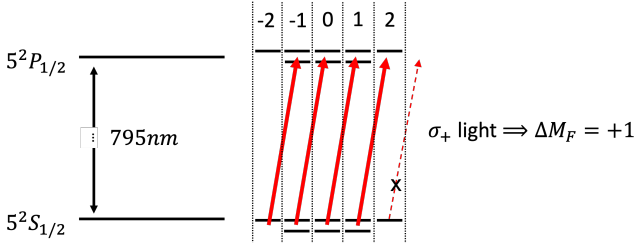


FIG. 9. Diagram of the allowed $\Delta M_F = +1$ transitions from $5^2S_{1/2}$ to $5^2P_{1/2}$ states of ^{87}Rb by stimulated absorption of σ_+ light at 795nm . Clearly, all of the $5^2S_{1/2}$ states can absorb the σ_+ light except for $F = 2, M_F = 2$. A similar diagram can be drawn for $\text{Rb}85$.

Clearly, if a gas of ^{87}Rb atoms is illuminated with σ_+ light at 795nm , atoms in the $F = 2, M_F = 2$ state cannot undergo stimulated absorption.

Once in an $5^2P_{1/2}$ excited state, atoms can transition back down to the ground $5^2S_{1/2}$ states through spontaneous emission. Because spontaneous emission does not have a set polarization, the selection rule requires that $\Delta M_F = +1, 0, -1$ in any transition by spontaneous decay. Only spontaneous decay from $5^2P_{1/2}$ to $5^2S_{1/2}$ is considered, as the transition frequency of spontaneous decay and corresponding lifetime of the $5^2S_{1/2}, F = 2$ state is much smaller than that of any of the $5^2P_{1/2}$ states. An example of the allowed transitions due to spontaneous decay from the $F' = 1, M_F = 1$ is shown in Fig. 10.

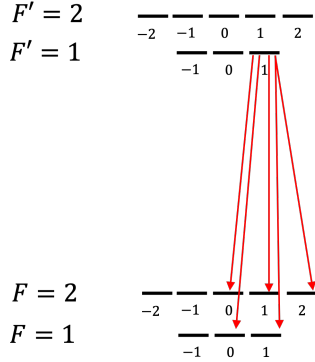


FIG. 10. Diagram of the allowed transitions due to spontaneous emission from the $F' = 1, M_F = 1$ state. Spontaneous emission from any of the $5^2P_{1/2}$ can have $\Delta M_F = 1, 0, -1$.

Once the atom decays back down to a $5^2S_{1/2}$ state, it can absorb σ_+ light again and undergo a $\Delta M_F = +1$ transition again. Combined with the fact that the probabilities of $\Delta M_F = +1, 0, -1$ are equally likely in spontaneous emission, in the presence of σ_+ light, atoms are "pumped" towards the maximum M_F values.

$$\begin{array}{l} F' = 2 \\ F' = 1 \end{array} \quad \begin{array}{ccccc} \hline -2 & -1 & 0 & 1 & 2 \\ \hline & & -1 & 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{l} F = 2 \\ F = 1 \end{array} \quad \begin{array}{ccccc} \hline \bullet & \bullet & \bullet & \bullet & \bullet \\ -2 & -1 & 0 & 1 & 2 \\ \hline \bullet & \bullet & \bullet & \bullet & \bullet \\ -1 & 0 & 1 & & \\ \hline \end{array}$$

FIG. 11. For many atoms and after many cycles of absorption and emission, there will be a buildup of atoms in the ground state with maximum M_F .

Note that the selection rules make it so that, once an atom is in the $F = 2, M_F = 2$ state, it is stuck there, and cannot decay or absorb light. Thus, $F = 2, M_F = 2$ is referred to as a metastable state. Of course, processes such as collisions between atoms and spontaneous emission down to $F = 1, M_F = 1$ do allow atoms in $F = 2, M_F = 2$ to transition out of the metastable state, but if the characteristic times of these processes is much longer than the characteristic time of stimulated absorption of the σ_+ light, a large fraction of the population is pumped away from thermal equilibrium and into the $F = 2, M_F = 2$ state.

F. Absorptive cross section and Beer's Law

The attenuation of light as it passes through an absorptive medium composed of identical two-level systems is described by Beer's law [19] (page 115)

$$I(z, T) = I(0) \exp(-a(T)z) \quad (6)$$

where I is the intensity of light, T is temperature, z is the propagation distance travelled through the medium, and $a(T)$ is the absorption coefficient as a function of temperature T , defined to be

$$a(T) \approx \sigma \frac{g_2}{g_1} n(T) \quad (7)$$

where σ is the absorption cross section, g_1 and g_2 are the degeneracies of the lower and higher energy states involved in the transition, respectively, and $n(T)$ is the density of atoms as a function of temperature. Therefore, in the experimental setup, the attenuation through the Rb cell of length z_{cell} as a function of temperature is

$$I(T) = I(0) \exp(-\sigma n(T) z_{\text{cell}}) \quad (8)$$

Taking the logarithm of both sides of Eq. 8,

$$\log I(T) = \log I(0) - \sigma n(T) z_{cell} \quad (9)$$

As seen in Eq. 9, $\log I(T)$ vs $n(T) z_{cell}$ is a linear relationship with slope $-\sigma$.

The theoretical value of the absorption cross section is [14]

$$\sigma = 3\lambda^2/2\pi \quad (10)$$

III. EXPERIMENTAL SETUP

A. Experimental techniques

1. Measurement of Zeeman shift at low external magnetic field for Rb87

The primary goal of this experiment was to measure the Zeeman shift as a function of external magnetic field. Suppose that a vapor of Rb87 is continuously illuminated with 795nm σ_+ light, causing atoms to become optically pumped to the $F = 2, M_F = 2$ state (for Rb85, it would be the $F = 3, M_F = 3$ state), as shown in Fig. 11. Let's refer to the σ_+ light as the pump light.

Now suppose that a small external field is applied along the \hat{z} direction so that Eq. 4 is a good approximation.

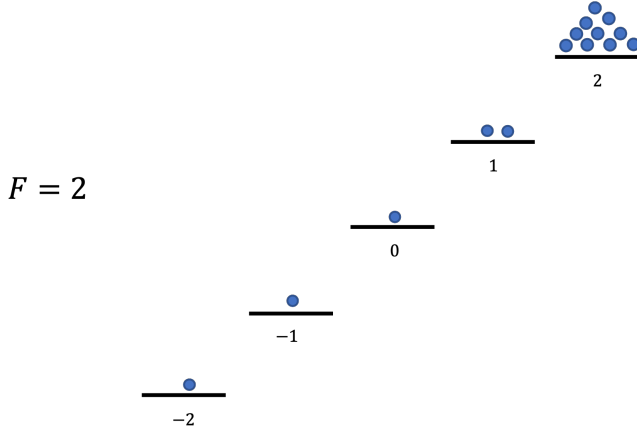


FIG. 12. Zeeman splitting of the M_F sublevels of $F = 2$ for Rb87

If additional electromagnetic radiation is shone in at resonance with the energy difference between the split $M_F = 2$ and $M_F = 1$ states for $F = 2$, atoms in the metastable $F = 2, M_F = 2$ state can undergo stimulated emission and transition with $\Delta M_F = -1$ to the $M_F = 1$ state, in which they can absorb the pump light. If we observe the amount of pump light passing through the vapor, when the RF is on resonance, we will see a dip in

the amount of pump light passing through, as it is being absorbed at a greater rate.

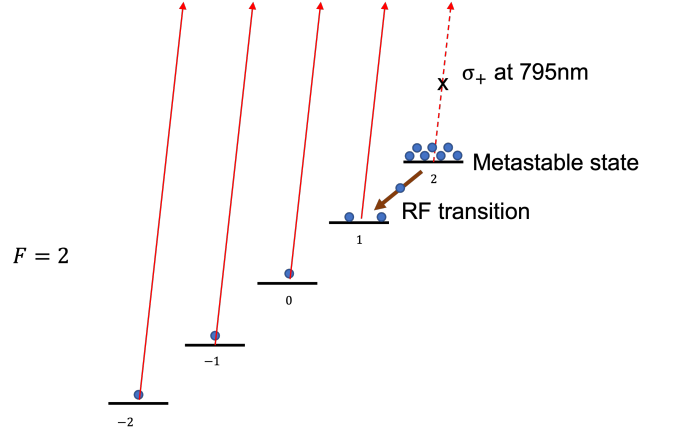


FIG. 13. Atoms in the $F = 2, M_F = 2$ state cannot absorb the 795nm σ_+ light (red arrow) according to the selection rules. However, an RF photon can cause a transition to the $F = 2, M_F = 1$ state (brown arrow), in which the atom can absorb the σ_+ light

For the earth's magnetic field at Stony Brook of $50\mu T$ [20] and $\mu_B \approx 5 * 10^{-5} eV/T$, Eq. 4 implies that the transition between M_F and $M_F - 1$ states are on the order of $2.5 * 10^{-11} eV$, which corresponds to about $600 kHz$

Thus, by applying radio frequency (RF) electromagnetic radiation at varying frequency to the Rb vapor, the Zeeman shift was measured.

By creating an external magnetic field along the same direction as the earth's magnetic field, the Zeeman shift as a function of external magnetic field was measured.

2. Observation of non-linear Zeeman splitting at high external magnetic field

In practice, optical pumping does not pump every single Rb87 atom into the $F = 2, M_F = 2$ state ($F = 3, M_F = 3$ for Rb85). Collisions between atoms can cause state changes, and the rate of spontaneous emission from the $F = 2, M_F = 2$ is low but does not 0. Thus, each M_F sublevel has some nonzero population. Due to pumping, the population increases with increasing M_F .

In low magnetic field, where the splitting between each M_F level is equal, an RF magnetic field resonant with the $M_F = 2, 1$ transition is also resonant with the $M_F = 1, 0$, the $M_F = 0, 1$, and the $M_F = -1, -2$ transitions. As the splitting between M_F levels differs in high magnetic field, there is a substantial difference in the resonant frequency of transitions between adjacent M_F sublevels, making them resolvable in spectroscopic measurement.

But why is there any visible change in transmission for transitions that do not involve the metastable state?

I assume that, due to the long state lifetime of the hyper-fine levels of $5^2S_{1/2}$, the continuous RF light causes Rabi oscillations. Averaged over time, these Rabi oscillations effectively equalize the populations in the two M_F states involved. Pumping is continually working against effects that bring the system back towards thermal equilibrium, and this Rabi oscillation means less pumping into the metastable state where atoms no longer absorb σ_+ light. Less atoms in the metastable state means more atoms absorbing σ_+ light, contributing to the dip in transmission of M_F light.

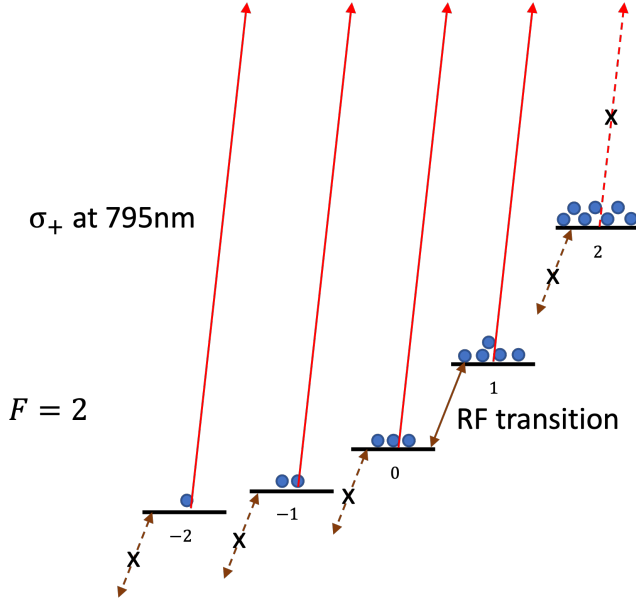


FIG. 14. This diagram is for $\text{Rb}87$, with 795nm σ_+ light in red and RF light in brown. The blue circles represent the population of atoms in each M_F state. In the Paschen-Back effect, the external magnetic field is strong enough that the transitions between adjacent M_F sublevels differ. Thus, a single $\Delta M_F = 1$ transition can be isolated. In this case, the $M_F = 1, 2$ transition is shown to be isolated, and atoms in the $M_F = 0, 1$ states undergo Rabi oscillation between the two states, effectively equalizing their populations and thus working against optical pumping.

B. Components

1. Rb cell and optics

Central to the experimental setup was the Rb cell, a glass cell containing both ^{85}Rb and ^{87}Rb isotopes. This glass cell was heated with a heat gun in order to increase the density of the Rb vapor inside the cell.

The 795nm light, corresponding to the transitions commonly referred to as the D_1 line for Rb, was generated with a Rb lamp. A D_1 filter was used to isolate the 795nm light produced by the lamp. A linear polarizer followed by a quarter waveplate was used to transform

the polarization of the 795nm to σ_+ with respect to the propagation direction, \hat{z} .

The light was focused into the Rb cell by a lens, and the light transmitted through the cell was focused onto a photodiode by another lens. The photodiode's output voltage was measured with an oscilloscope at 60dB gain. The use of 60dB gain was crucial in being able to resolve multiphoton transitions.

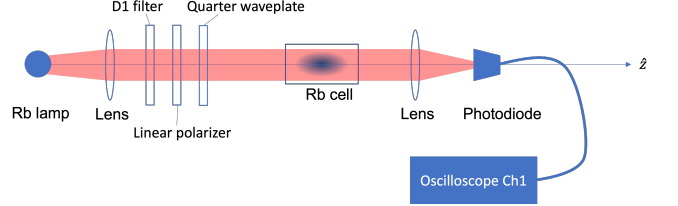


FIG. 15. Optics involved in measuring the transmission of 795nm σ_+ light through the Rb cell

2. Helmholtz coils for oscillating magnetic field

The oscillating RF magnetic field was created by a pair of Helmholtz coils oriented perpendicular to the \hat{z} direction. Thus, the RF photons were unpolarized, and could cause transitions where $\Delta M_F = -1, 0, 1$. Oscillating current was supplied to the coils by the amplified output of a signal generator (SG). The SG was set to sweep linearly through range of frequencies over time around some center frequency. The sweep time was set to 1s , as certain features, such as multi-photon transitions, could not be resolved at lower sweep times. The SG output amplitude was adjusted with each measurement to ensure that the voltage was low enough not to overdrive the amplifier but high enough to observe a dip in absorption.

The SG output a square wave to Ch2 of the scope, which was used as a trigger for making measurements. The SG would sweep from its minimum to maximum value over the period of the trigger signal, with 0s return time between each cycle.

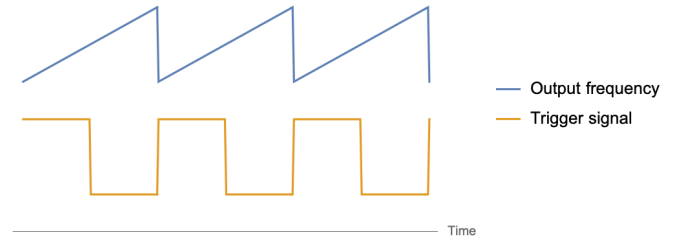


FIG. 16. Diagram showing how the two outputs of the SG are related

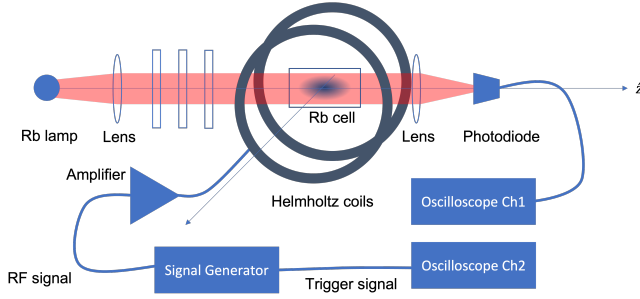


FIG. 17. Diagram showing how the Helmholtz coils are used to create an oscillating magnetic field at RF within the Rb cell.

An example of the photodiode voltage and SG trigger recorded with the oscilloscope in a measurement are shown in Fig. 18:

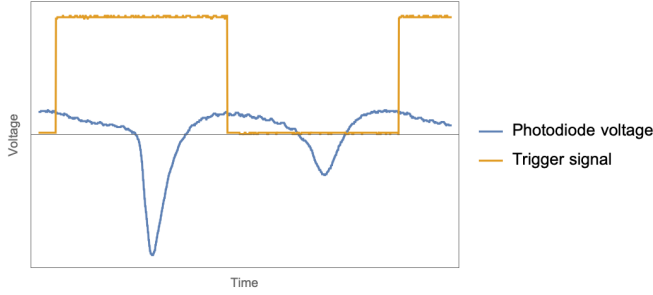


FIG. 18. An example of the signals measured by channel 1 (blue) and channel 2 (yellow) of the oscilloscope for the setup in Fig. 17

In order to locate the frequencies of the dips, the time axis was transformed to frequency by setting lowest frequency in the sweep at the first rising edge of the trigger signal and the highest frequency of the sweep at the second rising edge of the trigger signal. The voltage signals were normalized in the analysis of each measurement.

3. Maxwell coils for constant external magnetic field

In order to create an external magnetic field, three coils in the Maxwell arrangement surrounding the Rb cell were used. The Maxwell coils were oriented along the \hat{z} axis. Because the discussion of Zeeman splitting assumed the external field was along the \hat{z} direction, the whole apparatus was oriented such that \hat{z} pointed along the direction of the earth's magnetic field.

The Maxwell coils were supplied with current from a DC power supply. The current was measured with an ammeter in series.

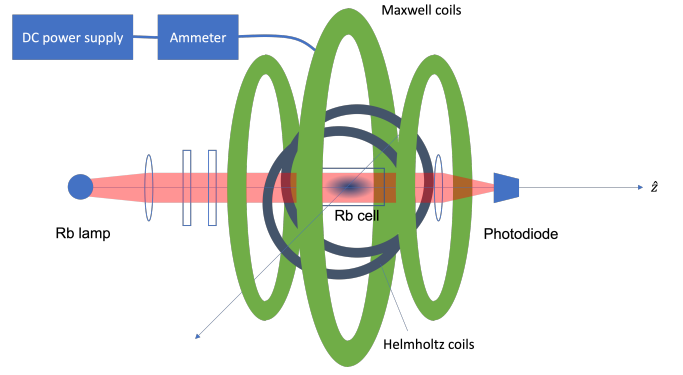


FIG. 19. The Maxwell coils were oriented such that the magnetic field they created was aligned with the \hat{z} direction, parallel to the propagation direction of the σ_+ light and the earth's local magnetic field.

The magnetic field of the Maxwell coils was calculated by approximating each coil as a current loop and applying the Biot Savart law [21]. The magnetic field of a current loop oriented along \hat{z} at a distance of z , a radius of R , and with N turns, as a function of current I , is

$$\vec{B}(I) = N \frac{\mu I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}} \hat{z} \quad (11)$$

where $\mu = 4\pi * 10^{-7} N/A^2$ is the permeability of free space.

The values of z , the diameter, and the number of turns for each of the coils are collected in Tab. II below:

Coil	z location	Diameter	Number of turns
Large	$0 \pm 1cm$	$78.4 \pm 0.1cm$	142 ± 1
Small	$\pm(26.5 \pm 0.1cm)$	$59.1 \pm 0.1cm$	110 ± 1

TABLE II. Specifications of the Maxwell coils used to generate an external magnetic field

By summing together the magnetic fields produced by each coil according to Eq. 11, the magnetic field was calculated for a given current supplied by the DC power supply and measured with the ammeter shown in Fig 19.

Uncertainty in the calculated magnetic field for a single loop was estimated using the following equation:

$$\frac{\delta B}{B} = \sqrt{\left(\frac{\delta N}{N}\right)^2 + \left(\frac{\delta I}{I}\right)^2 + \left(\frac{\delta R}{R}\right)^2 + \left(3z \frac{\delta z}{z}\right)^2}$$

The uncertainty in the total magnetic field was estimated by adding the relative uncertainty of the magnetic field created by each loop in quadrature.

$$\frac{\delta B_{tot}}{B_{tot}} = \sqrt{\sum_{i=1}^3 \left(\frac{\delta B}{B}\right)^2}$$

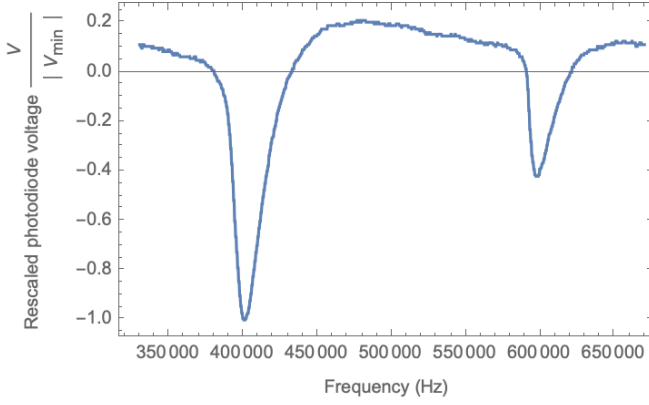


FIG. 20. An example of how the time axis of the data shown in Fig. 18 was rescaled for locating transition frequencies

C. Data analysis

Given a measurement such as that shown in Fig. 18, in order to find the transition frequencies corresponding to the centers of different absorption dips, the time axis of the scope was first transformed into frequencies. As shown in Fig. 16, at the first rising edge of the trigger signal, the SG output is at its lowest frequency, which is set by

$$f_{low} = f_{center} - f_{span}/2$$

where f_{center} and f_{span} are the center and spanning frequencies, set on the SG.

Just before the second rising edge of the trigger signal, the SG output is at its highest frequency,

$$f_{high} = f_{center} + f_{span}/2$$

The period of the sweep and corresponding trigger signal was set on the SG.

In order to convert from time to frequency in the oscilloscope data, the data outside of one period of the trigger signal was dropped and the time axis was rescaled such that the first data point corresponded to f_{low} and the last point corresponded to f_{high} , with a linear sweep in frequency between.

For example, the data shown in Fig. 18 was rescaled for $f_{center} = 500\text{kHz}$ and $f_{span} = 340\text{kHz}$ and is shown in Fig. 20 below:

In order to find the center of the resonance, the frequencies corresponding to the local minima of the data were identified.

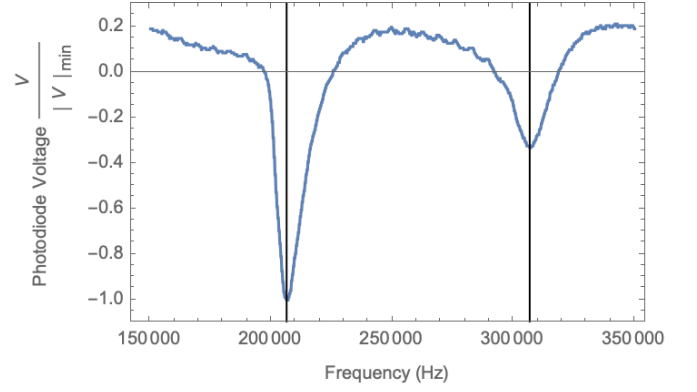


FIG. 21. The black lines indicate the measured local minima of dips in transmission corresponding to transitions

In order to estimate the uncertainty on this measurement, a Gaussian of the form

$$fit(x) = A * \exp\left(\frac{-(x - x_0)^2}{\sigma^2}\right) + offset$$

was fit to the data using Mathematica's Nonlinear-ModelFit, with free parameters A and σ . The parameter x_0 defines the center of the Gaussian, and was taken to be the frequency of the local minimum.

The uncertainty of the measured resonance was taken to be $\sigma/20$. An example of the fit to the first dip shown in Fig. 20 is shown in the figure below:

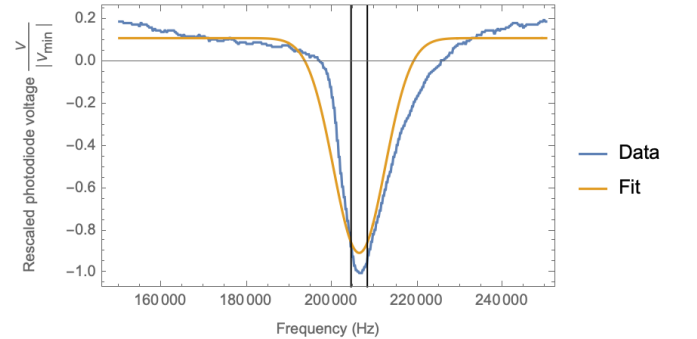


FIG. 22. An example of the Gaussian fitting used to estimate the uncertainty of the transition frequency measurement. The data being analyzed is the first peak shown in Fig. 21.

In calculating the uncertainty on a fitted parameter, the parameter was adjusted until the χ^2 value of the fit increased by 1. The amount that the parameter was adjusted was taken to be the uncertainty.

IV. MEASUREMENTS

A. Rubidium lamp warmup time

When taking measurements, we wanted to ensure that output of the lamp was constant over time. It was found that, after turning on the lamp, it took 400s before the lamp output stabilized.

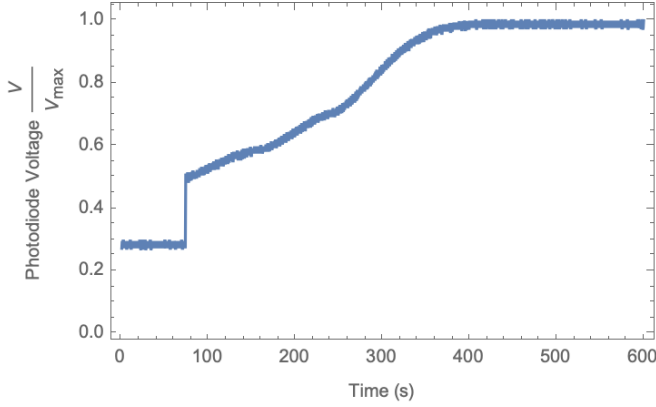


FIG. 23. Measured photodiode voltage over time after the lamp was turned on at $t = 80s$.

B. Measurement of absorptive cross section for Rb

By heating the cell and measuring the photodiode voltage as a function of cell temperature, the relationship between transmitted intensity and cell temperature was measured.

In order to measure the relationship between, transmitted intensity and cell temperature, the lamp was first turned on and warmed up. Then, the heat gun was turned on. Simultaneous measurements of photodiode voltage and cell temperature were made. As the temperature measurements were made using a stopwatch, the uncertainty in time was taken to be 0.25s, the human reaction time.

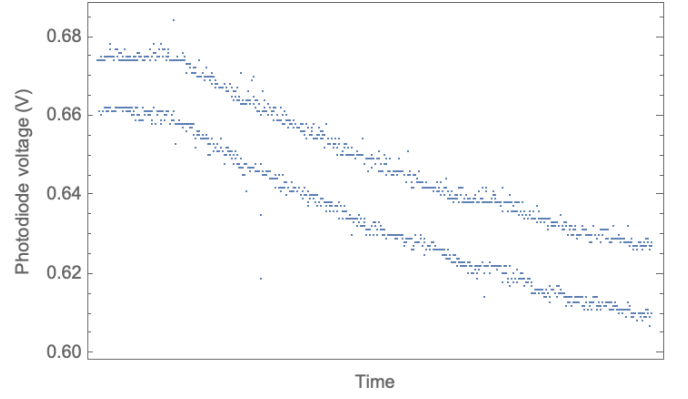


FIG. 24. Photodiode signal as a function of time after the heat gun was turned on, slowly heating the Rb cell from $23^{\circ}C$ to $42^{\circ}C$

As seen in Fig. 24, there was some unfavorable behavior observed in the photodiode voltage signal. It appeared to be splitting into two signals, each with a visually estimated uncertainty of $\pm 0.0025V$. As the general trend of the photodiode voltage and not the exact voltage was of interest, the photodiode signal was averaged, and the uncertainty was taken to be $\pm 0.0025V$.

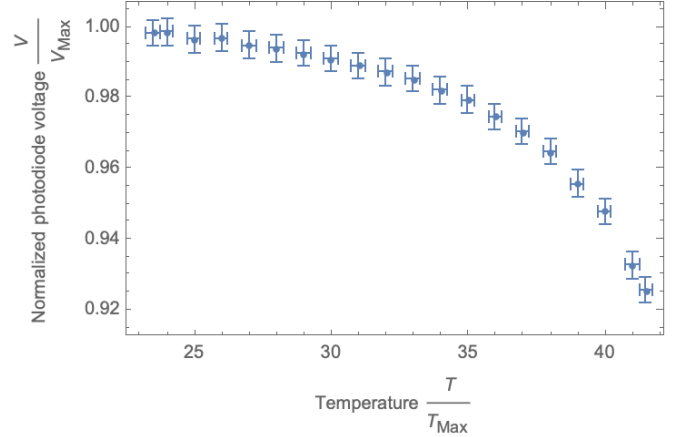


FIG. 25. Graph of normalized photodiode voltage vs Rb cell temperature, showing the relationship between vapor temperature and the transmission of light through the cell. This data was used to calculate the absorptive cross section of the Rb vapor in the cell.

In order to find the absorptive cross section σ from the data shown in Fig. 25, the linear relationship between $\log I(T)$ and σ needed to be found, as discussed in Sec. II F. This required the calculation of $n(T)$ for each temperature corresponding to a data point in Fig 25.

In this experiment, the absorptive medium was composed of Rb vapor in the solid phase. When the Rb cell is heated to temperature T the density of atoms in the gas phase can be found by the ideal gas law

$$n(T) = \frac{P_v(T)}{k_B T} \quad (12)$$

where $P_v(T)$ is the vapor pressure of Rb in the solid phase as a function of temperature and k_B is the Boltzmann constant.

The vapor pressure of Rb in the solid phase can be approximated by [18]

$$P_v = 10^{2.881 + 4.857 - \frac{4215}{T}} * 133.3224 \quad (13)$$

where P_v is vapor pressure in Pa, and T is temperature in Kelvin.

Thus, $n(T)$ was calculated for each data point.

The plot of $\log I(T)$ vs $n(T)z_{cell}$ is in Fig. 26, together with a linear fit. The uncertainty in $\log I$ was taken to be

$$\begin{aligned} \frac{\delta \log I}{\log I} &= \frac{\log(I + \delta I) - \log I}{\log I} \\ &= \frac{\log I \log(1 + \frac{\delta I}{I}) - \log I}{\log I} \\ &= \frac{\log(1 + \frac{\delta I}{I})}{\log I} \\ &\approx \frac{\delta I/I}{\log I} \end{aligned}$$

by expanding around the small variable $\delta I/I$ to lowest order.

The uncertainty in $n(T)$ was taken to be

$$\frac{\delta n(T)}{n(T)} = \sqrt{\left(\frac{\delta P_v}{P_v}\right)^2 + \left(\frac{\delta T}{T}\right)^2}$$

where

$$\delta P_v(T) = P_v(T + \delta T) - P_v(T)$$

was numerically calculated for each data point.

This uncertainty was added in quadrature with $\delta z_{cell}/z_{cell}$, where $z_{cell} = 5.0 \pm 0.1 \text{ cm}$ in the experimental setup.

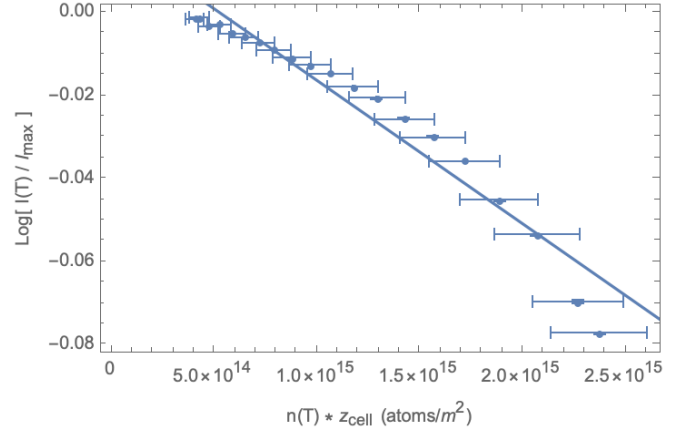


FIG. 26. Graph of the logarithm of the transmission vs the calculated density multiplied by cell length, used to calculate the absorptive cross section of Rb in the cell through Beer's law (Eq. 6)

The absorptive cross section, calculated as the absolute value of the slope of the fit shown in Fig. 26, was found to be

$$\sigma = (2.45 \pm 0.02) * 10^{-17} \text{ m}^2$$

Compared to the theoretical value of $2.9 * 10^{-13} \text{ m}^2$ using Eq. 10, this calculated value was off by a factor of 10^4 .

When initially taking the measurement, it was noticed that the graph of intensity vs temperature was different depending on if measurements were taken when the temperature was increasing or decreasing. This indicates that the thermometer is not a good indication of the true temperature of the cell, and that this method for calculating absorption cross section cannot be trusted.

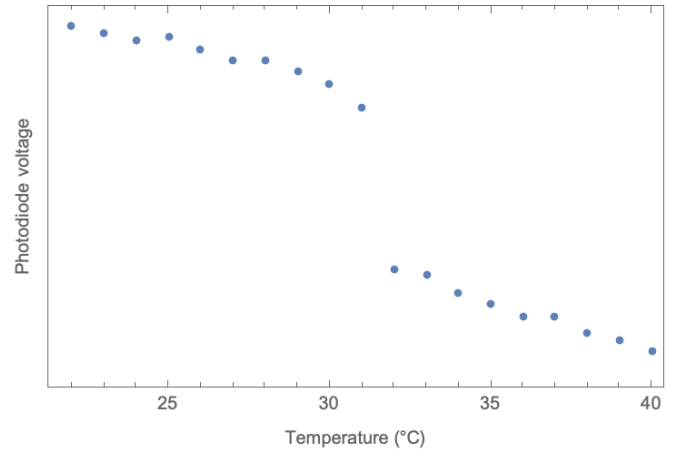


FIG. 27. Plot showing the discrepancy in the relationship between transmission and Rb cell temperature. The first 10 data points were taken as temperature was increasing in the Rb cell from 22°C to 31°C , and the last 9 data points were taken as Rb cell was decreasing in temperature from a maximum of 40°C to 32°C .

C. Power broadening

In order to determine the optimal signal amplitude for the SG producing the RF magnetic field at low external magnetic field, the transmission spectrum was measured for various SG amplitudes, from the minimum of 10mV.

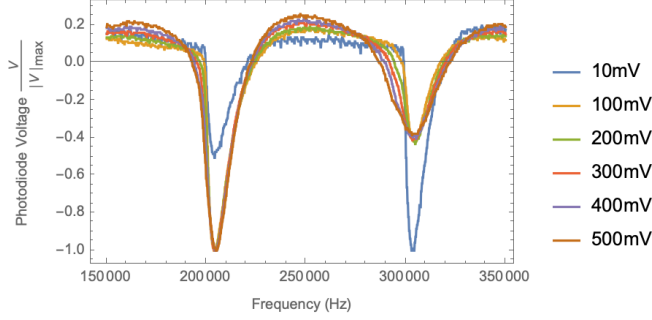


FIG. 28. This graph demonstrates the power broadening associated with our sample at a variety of frequencies and voltages.

Notably, for 10mV, the first dip, corresponding to Rb85, is smaller in magnitude than the second dip corresponding to Rb87. For all of the larger signal amplitudes, Rb85 has a larger dip.

Additionally, above an amplitude of 600mV, the amplifier connected to the SG began triggering its overdrive, ruining the spectrum, as seen in Fig 29 below:

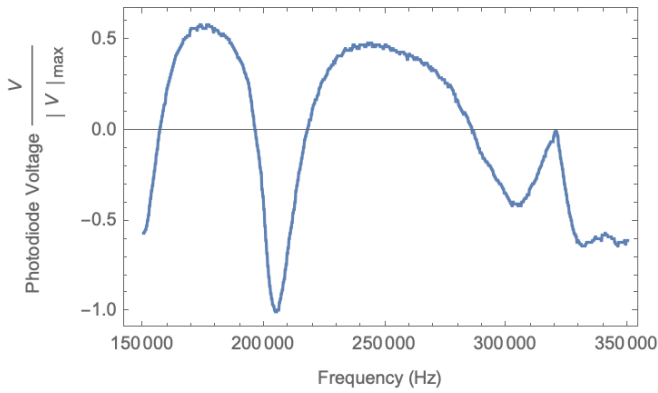


FIG. 29. Resonance spectrum in the presence of amplifier overdrive at a SG amplitude of 600mV, showing that the SG amplitude should not be set to more than 600mV when taking the low field measurements

An amplitude of 250mV was chosen for the SG in the experiment described in the following section as it is in the middle of the range of available amplitudes.

D. Zeeman splitting for small magnetic field

At low magnetic field, where transitions between adjacent M_F sublevels are approximately equal and thus are not resolvable, the transition frequencies as a function of magnetic field are

$$\nu = \frac{\mu_B g_F B_z}{h} \quad (14)$$

Measurements of the frequency corresponding to the Rb85 and Rb87 transitions are shown in Fig. 30:

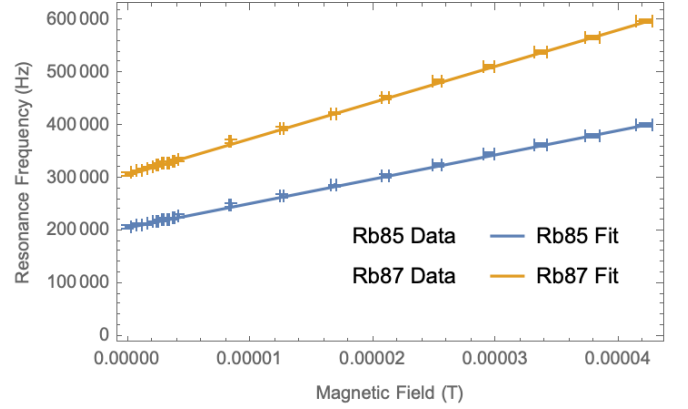


FIG. 30. This graph shows the position of the measured resonance for Rb85 and Rb87 vs external magnetic field

As seen in Eq. 14, the slope of the graphs is $\mu_B g_F B_z / h$. Thus, using the accepted values of μ_B and h together with a calculated estimate for the field generated by the Maxwell coils, B_z , the g_F for Rb85 and Rb87 was calculated.

	Rb85	Rb87
Slope	$4.63 \pm 0.05 \text{ GHz/T}$	$6.91 \pm 0.04 \text{ GHz/T}$

TABLE III. Fitted slope for transition frequency vs external magnetic field at low magnetic field

Isotope	Theoretical g_F	Measured g_F
Rb85	0.333	0.331 ± 0.004
Rb87	0.499	0.493 ± 0.003

TABLE IV. Values of g_F calculated from the slopes in Tab. III

The x-intercept of the linear fit is the offset in magnetic field due to the earth's magnetic field.

Isotope	Calculated earth magnetic field
Rb85	$44.5 \pm 0.3 \mu\text{T}$
Rb87	$44.4 \pm 0.5 \mu\text{T}$

TABLE V. Measured values of the earth's magnetic field, calculated from the x-intercept of the fitted lines in Fig. 30

In future experiments, the magnetic field should be reversed until the splitting is minimized, and the minimum field can be measured so that this calculation can be verified.

E. Paschen-Back effect

At sufficiently external high magnetic fields the Zeeman shift is no longer approximately linear in M_F , and thus the various transitions between adjacent M_F levels become resolvable.

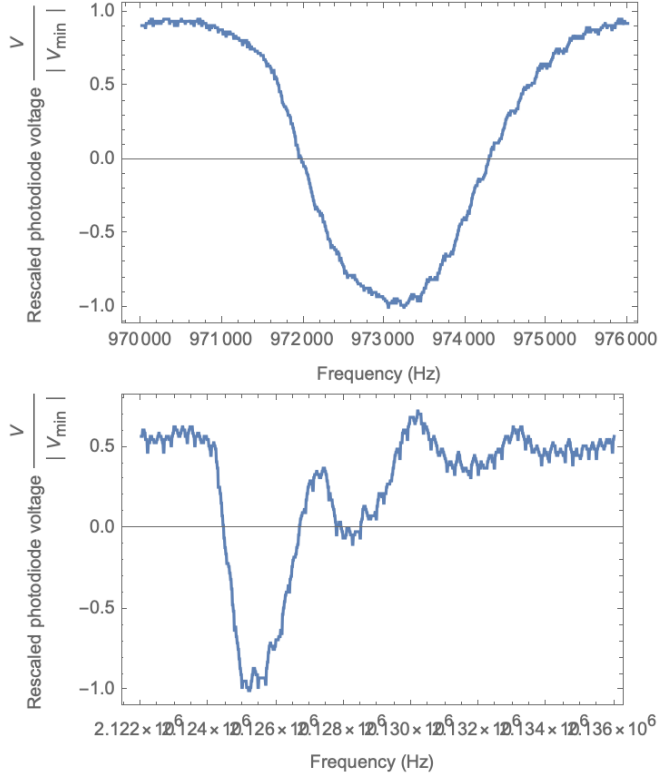


FIG. 31. At $160\mu T$ (top), the different transitions between M_F sublevels could not be resolved. As the magnetic field increased to $420\mu T$ (bottom), the different transitions began to be resolvable.

These energy differences were measured as a function of magnetic field. A plot of the theoretically calculated energy differences vs the measured energy differences is shown in Fig. 32, with the offset due to earth's magnetic field accounted for.

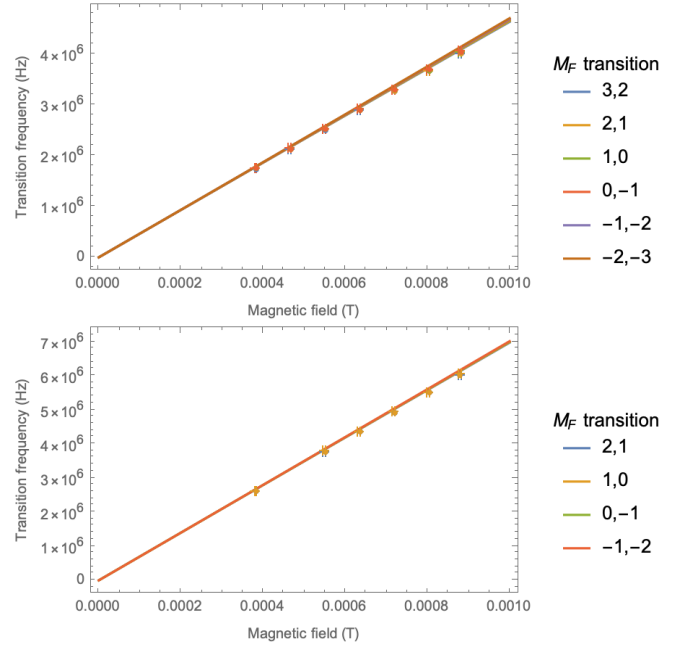


FIG. 32. Comparison between data and theory for transition energy for transitions between adjacent M_F sublevels vs magnetic field for Rb85 (top) and Rb87 (bottom).

Because the transitions were very close in energy, the difference between adjacent transition energies was taken as a function of magnetic field. The comparison between data and theory is shown in Fig. 33 below:

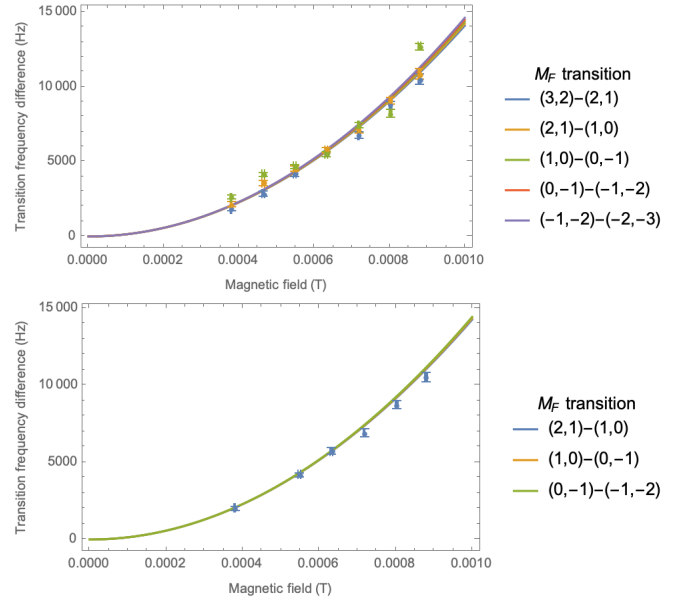


FIG. 33. Measured differences between transition energies compared to theoretical values calculated using the Breit-Rabi formula for Rb85 (top) and Rb87 (bottom)

Fig. 34 shows how the different transitions first became resolvable at about $0.03G$, and the spacing between

transitions increased at an increasing rate as the external magnetic field increased.

F. Multi-photon transitions

At the greatest external magnetic field possible in the setup, the greatest possible amplitude on the SG without overdriving the amplifier, and a sweep time of 1second on the SG, secondary dips were visible in the transmission spectrum, forming directly in between the expected transition frequencies.

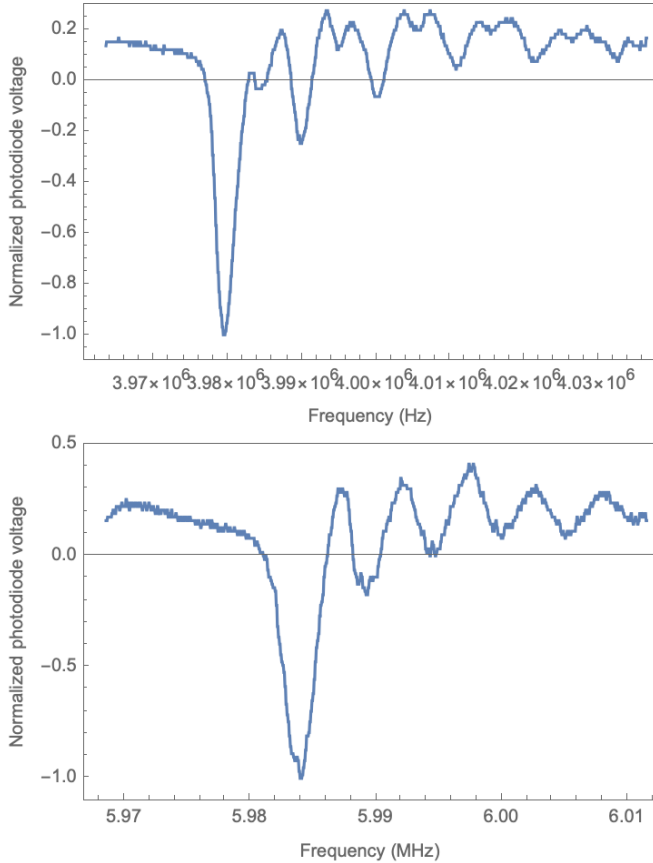


FIG. 35. Plots of measured multiphoton transitions in Rb85 (top) and Rb87 (bottom) at an external field of about 83mT

At sweep times equal to and below 500ms, these dips did not appear.

V. CONCLUSION

A. Measurements

The absorption cross section of Rb was calculated from measured data to be $\sigma = (2.45 \pm 0.02) * 10^{-17} m^2$, 4 orders of magnitude smaller than the theoretically calculated value of $2.9 * 10^{-13} m^2$. This discrepancy requires further investigation. In order to improve this measurement, the temperature should be fixed when taking the measurement.

In the measurement of power broadening at low magnetic field, it was observed that, for low RF field amplitude, the dip in transmission corresponding to ^{85}Rb was less extreme than that of ^{87}Rb , but at high amplitude, this relationship was reversed. It is unclear what causes this behavior.

The measurement of power broadening must be repeated over a range of magnetic fields. There was no clear indication that increased RF field amplitude broadened any features substantially.

Another contributor to the lineshape that should be further investigated is the response time of the Rb vapor to abrupt changes in RF frequency. This effect alters the lineshape at different sweep times. Most of the experiments in this report were done with a sweep time of 500ms. Only for the multiphoton transition measurements was the sweep time increased to 1s, as the transitions did not appear at 500ms.

The calculated earth magnetic field of $44.4 \pm 0.05 \mu T$ was on the same order of magnitude as the expected value of $\approx 51.22 \mu T$ [20], with a percent error of around $\approx 13.05\%$. The discrepancy is likely due to misalignment of the apparatus with the earth's magnetic field. Thus, it is estimated that the lowest possible field that can be generated with the apparatus is $(51.22 - 44.4) \mu T \approx 7 \mu T$. In future experiment, the magnetic field from the Maxwell Coils should be used to cancel out the earth's field as much as possible in order to measure the minimum possible magnetic field.

The measured g_F for Rb85 was 0.331 ± 0.004 , consistent with the theoretical value of 0.333. For Rb87, a g_F of 0.493 ± 0.003 was measured, inconsistent with the theoretical value but within 2%.

At intermediate fields, the Breit-Rabi formula was verified. Multi-photon transitions were also successfully observed.

B. Suggested improvements

One of the largest sources of systematic error in this experiment was the measurement of the photodiode on AC coupling mode. At long sweep times, AC coupling might have significantly distorted the signal. In a future setup, we will set the photodiode to DC coupling and used a power supply to subtract any DC offset to the signal. Additionally, a larger capacitor will be used to filter out 60Hz noise affecting the photodiode signal. Improving the signal-to-noise is vital to improving the setup.

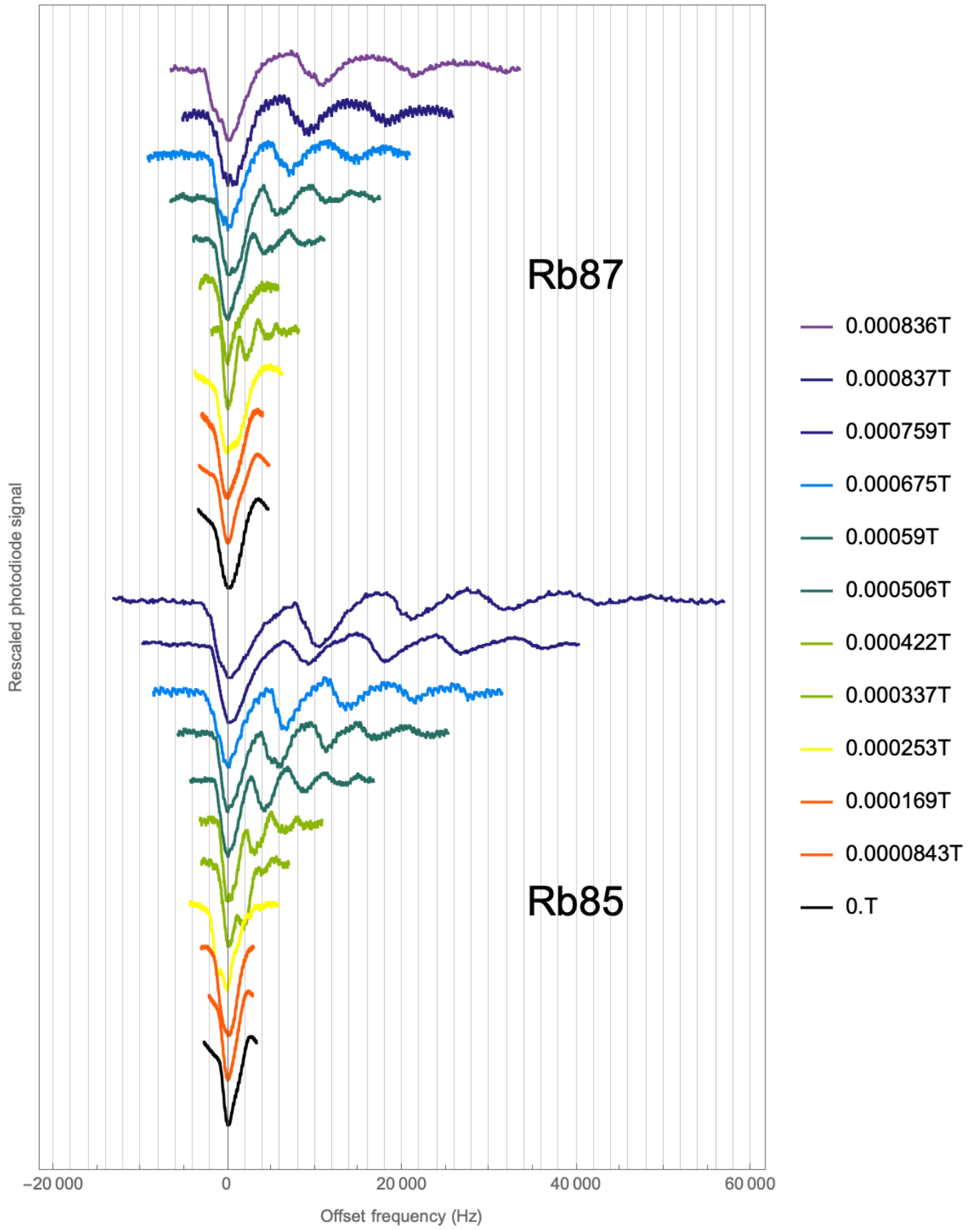


FIG. 34. Plot giving a comparison of the differences between transition frequencies as a function of added external magnetic field

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