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Bicycle network design: model and solution algorithm

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Abstract

We propose an optimization framework for urban bicycle network design. The model takes into account interests of the users (who travel along shortest paths) and the planners (available budget). An underlying network composed by street segments suitable to build cycling infrastructure is taken as input. Each network link has construction and user cost, both proportional to the distance. A network link without cycling infrastructure which is part of a path followed by users, has a larger user cost. A multi-commodity network flow mixed-integer mathematical program is proposed and applied to small-sized problem instances to validate the model. The formulation considers the discontinuities of the bicycle network, i.e. the users' paths which include segments without cycling infrastructure. Sensitivity analysis are performed with respect to budget levels and to penalization of user's cost in links without cycling infrastructure. A metaheuristic is proposed to handle large-sized instances. As an additional feature (difficult to formulate in the exact model), the metaheuristic also minimizes the total number of discontinuities by including them into the objective function. The accuracy of the metaheuristic is estimated by comparing with exact results when possible. The methodology is tested using data from the city of Montevideo, Uruguay, including a large-sized underlying street network and origin-destination trips estimated from a household survey. Computational results are obtained with and without minimization of discontinuities, and they are compared with the current bicycle network of the city.

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Keywords: cycling; network design; optimization

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1. Introduction

Cycling has been increasingly identified and promoted as a sustainable transport mode which can contribute to mitigate several non-desirable effects caused by other urban transport modes, like traffic congestion, noise and air pollution (Henao et al., 2015). Both practitioners and researchers have turn their attention to methodologies and technologies for planning and operating bike-oriented transport systems, either to attract new users to this mode or to improve the level of service for those who already use it. As some representative examples, we can mention dedicated bike lanes, traffic signals and specific parking spaces (Bendiks and Degros, 2013). A remarkable initiative which has been implemented in many cities worldwide is bike-sharing (Lin and Yang, 2011), which also poses several challenging management issues.

A key aspect of cycling systems (as of any transportation system) with direct influence over their cost and usability, is the infrastructure. In this context, we understand the infrastructure as the space where people can circulate using bikes. Despite specific regulations, bikes are allowed to circulate using the same infrastructure as cars, that is, the city streets. In some cases, other places banned for cars are allowed for bikes, like park trails or pedestrian streets. Though in principle, city streets are sufficient to ensure connectivity between trip origins and destinations of bike users, it has been recognized that specific infrastructure is needed for cycling. Several types of specific infrastructure for cycling have been implemented, each of them entailing different construction cost and attributes perceived by users, particularly safety and comfort (Bíl et al., 2015; Caulfield et al., 2012; McNeil et al., 2015). Often, the infrastructure is built along urban corridors, either main or secondary streets, connecting places of high demand. Decisions regarding where to build cycling infrastructure usually are taken as part of strategic plans, and therefore long-term ones which may include incremental construction along several years. In this context, the planners are faced to the problem of selecting a set of street sections for building cycling infrastructure over them, in such a way that the result is convenient for the users (people who use bike), and subject to a budgetary constraint. Moreover, as it happens with other transport modes, the users perceive the infrastructure (and services which operate over it, when applies) as a network. This means that users plan and perform their trips by connecting individual elements of the infrastructure/service (street sections, bike lanes, bus lines) in order to reach the destination from the origin. Cycling networks can be considered always connected, since by using the street network, all possible destinations are reachable by bike. However, there has been increasing attention paid to the relevance of connected cycling networks, i.e. cycling infrastructure which allows trips entirely within the bike network. Thus, from both local governments and planning agencies (TFL, 2014) as from the academic field (Buehler and Dill, 2016; Krizek and Roland, 2005) the importance of planning cycling infrastructure as a network has been recently highlighted. In this work, we focus on methodologies for optimal planning of cycling networks.

The literature on this topic is relatively scarce. A considerable amount of grey literature (design standards, reports from civil organizations) has been published (e.g. Sustrans, 2014). Krizek and Roland (2005) study the importance of discontinuities on bike networks, however, they do not propose any method for network planning considering this issue. A discontinuity is defined as a point where bike users are forced to leave the cycling network due to lack of such infrastructure, and therefore they have to share the streets with the cars. Mesbah and Thompson (2011) propose a bilevel optimization model where the upper-level aims to simultaneously maximize the share of bike trips and minimize its impact over car travel time due to reduction of street space. The lower-level represents traffic assignment under user-equilibrium hypothesis, for both bikes and cars. The model is solved by a Genetic Algorithm and tested with an example instance comprising 42 nodes, 142 arcs and 30 origin-destination (OD) pairs. Duthie and Unnikrishnan (2014) propose a mathematical programming model which keeps maximum deviation from the shortest path in users' paths and considers cost in the nodes (a relevant component, either for its desirable features as for its high cost). The model is solved by CPLEX and it is applied to several test instances generated from data related to the downtown Austin region. The corresponding network has 75 nodes, 185 arcs and up to 5625 OD pairs. More recently, Buehler and Dill (2016) review different ways of approaching the design of cycling infrastructure, paying attention to the network concept and its components, i.e., nodes and links. In fact, they conclude that much remains to be done concerning methodologies for planning cycling infrastructure considering the network as a whole.

In this work, we focus on models and algorithms for optimal planning of cycling networks, taking into account interests of the planners and the users. The model takes a network of available street sections and a maximum attainable budget level, and delivers a network (connected or not) which minimizes the distance of bike trips given by an origin-destination matrix. We propose a multi-commodity mixed integer programming formulation based on the classical fixed-charge network design problem (Magnanti and Wong, 1984). In order to solve large-sized instances of the problem, we propose a metaheuristic algorithm based on GRASP (Feo and Resende, 1995); this algorithm also includes a discontinuity minimization feature. The capabilities of the model as well as the accuracy of the heuristic are tested by using small-sized instances of the problem. The model is also applied to a real large-sized case related to the city of Montevideo, Uruguay, for which we compare current solutions against the optimized ones.

The main contributions of this work are: (i) a new formulation for the bicycle network design problem, considering explicitly the existence of discontinuities and different travel cost perceived by the users, depending on whether they travel inside or outside of the network, and (ii) a solution algorithm capable of handling large-sized instances of the problem, which also includes a criterion of discontinuity minimization.

The article is structured as follows. After the introduction given at the present section, both proposed mathematical model and solution algorithm are presented in section 2. Computational experiments and their analysis are reported in section 3. Finally, conclusions and further work are presented in section 4.

2. Mathematical model and metaheuristic solution algorithm

2.1. Model

We formulate the model for bicycle network design as a variant of the fixed-charge multi-commodity network design problem (Magnanti and Wong, 1984). In general terms, the model takes an underlying network of available street sections and an origin-destination matrix of trips between pairs of nodes of the network, and decides where to build cycling infrastructure seeking to minimize the overall travel cost of the users, subject to a budgetary constraint. The distinctive feature of the proposed model is the consideration of discontinuities, for which we include specific parameters and variables. In the following, we present the nomenclature and the mathematical programming formulation.

Nomenclature

G	underlying directed network of available street sections for building cycling infrastructure
N	set of nodes of G, with generic elements i, j and l
n	cardinality of N
A	set of directed arcs of G, with generic element (i,j)
l_{ij}	length of arc (i,j)
L_{tot}	total length of the underlying network, i.e. $\sum_{(i,j)\in A} l_{ij}$
l_{max}	a maximum available length of cycling infrastructure to be built over the entire network
c_{ij}	user cost (e.g. distance or time) of traversing arc (i,j) when cycling infrastructure is built
c'_{ij}	user cost of traversing arc (i,j) when cycling infrastructure is not built, usually $c'_{ij} \geq c_{ij}$
K	set of origin-destination (OD) pairs, with generic element k
$O(k)$	origin node of OD pair k
$D(k)$	destination node of OD pair k
$R(k)$	number of bike trips per time unit within a time horizon, of OD pair k
D_{tot}	total demand of trips in the network, i.e. $\sum_{k\in K} R(k)$
y_{ij}	binary variable which takes value 1 if cycling infrastructure is built over arc (i,j), 0 otherwise
x_{ij}	binary variable which takes value 1 if there is flow of bike users over arc (i,j), even when cycling infrastructure is not built, 0 otherwise
f^k_{ij}	flow of bike users corresponding to OD pair k over arc (i,j) where cycling infrastructure is built
f'^k_{ij}	flow of bike users corresponding to OD pair k over arc (i,j) where cycling infrastructure is not built
F_{in}	percentage of the total demand that flows inside of the bicycle network, i.e. $\sum_{k\in K} \sum_{(i,j)\in A} f^k_{ij} / D_{tot}$
F_{out}	percentage of the total demand that flows outside of the bicycle network, i.e. $\sum_{k\in K} \sum_{(i,j)\in A} f'^k_{ij} / D_{tot}$

The mathematical formulation of the optimization problem is given by expressions (1)-(8) as follows.

$$\min \quad \sum_{k\in K} \sum_{(i,j)\in A} (c_{ij} f^{k}_{ij} + c'_{ij} f'^{k}_{ij}) \qquad (1)$$

$$\text{s.t.} \quad y_{ij} + x_{ij} \leq 1 \qquad \forall (i,j)\in A \qquad (2)$$

$$\sum_{k\in K} f^{k}_{ij} \leq y_{ij} \sum_{k\in K} R_k \qquad \forall (i,j)\in A \qquad (3)$$

$$\sum_{k\in K} f'^{k}_{ij} \leq x_{ij} \sum_{k\in K} R_k \qquad \forall (i,j)\in A \qquad (4)$$

$$\sum_{j\in N} (f^{k}_{ij} + f'^{k}_{ij}) - \sum_{l\in N} (f^{k}_{li} + f'^{k}_{li}) = \begin{cases} R_k & \text{if } i = O(k) \\ -R_k & \text{if } i = D(k) \\ 0 & \text{otherwise} \end{cases} \qquad \forall i\in N, \forall k\in K, \qquad (5)$$

$$\sum_{(i,j)\in A} l_{ij}\, y_{ij} \leq l_{max} \qquad (6)$$

$$f^{k}_{ij} \geq 0, f'^{k}_{ij} \geq 0 \qquad \forall k\in K, \forall (i,j)\in A \qquad (7)$$

$$x_{ij}, y_{ij} \in \{0,1\} \qquad \forall (i,j)\in A \qquad (8)$$

The model is user oriented, since objective function (1) minimizes the overall travel cost of users. We assume that each OD pair with endpoints i and j, will travel along the shortest path from i to j by selecting the best (least costly) combination of arcs which either have or not cycling infrastructure. Thus, depending on the relationship between c_{ij} and c'_{ij}, they may choose using some arcs of the street network which are out of the bicycle network, in order to diminish the overall travel cost. The continuity of flow for each OD pair is ensured by expression (5), a variant of the classical flow conservation constraint which involves flow inside and outside of the bicycle network. We note that additional attributes other than distance and existence of infrastructure (e.g. slope and type of road) influence the path choice of bike users, however, the path length is identified as one the most important ones (Beheshtitabar et al., 2014). Moreover, different bike users may perceive different costs across the set of travel attributes, which leads to consider stochastic assignment models (Gliebe et al., 2009). These features impact on the structure of the model and therefore on its tractability; they could be included in future stages of this research.

Note that the fixed cost of building the infrastructure is not included in the objective function, since it would compete with the interests of the users, thus entailing a multi-objective optimization problem (Ehrgott, 2005). Nevertheless, the building cost is included as a budgetary constraint through expression (6). Because of that, feasible solutions of problem (1)-(8) can exhibit y_{ij} or x_{ij} variables with value equal to 1 even when arc (i,j) is not used by any bike user. This is a consequence of the structure of the model but does not affect its correctness. A simple post-processing can check for arcs with $f^{k}_{ij} = 0$ or $f'^{k}_{ij} = 0$ and set corresponding variables y_{ij} or x_{ij} to 0.

Constraint (2) states that an arc is enabled for cycling, either if cycling infrastructure is built over it ($y_{ij} = 0$) or the users decide to use it even though the infrastructure is not built ($x_{ij} = 0$). Constraints (3) and (4) state that flow of bike users is allowed only over enabled arcs, either through y or x variables. An additional constraint imposing $y_{ij} = y_{ji}$ $\forall (i,j)\in A$ states that each section of the bicycle network is enabled for both forward and backward directions. If we want to allow one-way links, that constraint should be removed.

We should note that this model does not ensure connectivity nor attempts to minimize the number of discontinuities. The latter issue is difficult to formulate explicitly, therefore it will be addressed algorithmically in the solution method.

Formulation (1)-(8) is mixed-integer linear and denotes a high computational complexity problem. Only small instances can be solved to optimality using current state-of-the-art methodologies and technologies. Therefore we address an approximate resolution using a metaheuristic.

2.2. Solution algorithm

In order to solve large instances of the problem, we propose an algorithm based on the GRASP (Greedy Randomized Adaptive Search Procedures) metaheuristic (Feo and Resende, 1995). GRASP performs a greedy randomized construction to obtain an initial solution, which is then improved by applying a local search. Due to the stochastic nature of the construction, several solutions are obtained by running different executions of the routine (GRASP iterations), thus allowing for a wider exploration of the search space. The greedy construction performs by selecting randomly elements from a restricted candidate list (*RCL* for short) to be added to the solution under construction.

Our construction is based on the sub-routine *computePathTrip*, which adds arcs to the solution in order to satisfy the demand given by set K. To do that, the OD pairs are classified into two groups: (1) elements with demand above average, and (2) elements with demand below average. Our *RCL* is the set 1. Once an element (OD pair) is selected, its corresponding origin and destination nodes are connected by one of the m-shortest paths (Yen, 1971) randomly chosen, even when there is not available budget (because we allow flow even if the bicycle network is not built). The demand already satisfied is updated in every step. Note that the general idea is to fix the desirable flows f_{ij} and f'_{ij} (interest of the users), which will determine the values of y_{ij} and x_{ij} (the resulting network).

The local search aims to improve the solution by removing sections from the bicycle network in order to get available budget to improve other sections. To do that, at each step of the search we identify the path with largest total flow over arcs where no bicycle network is built (path 1) and the one with smallest flow over arcs where infrastructure is built (path 2). Then, we try to relocate arcs from path 2 to path 1. Each time a change is performed, the *computePathTrip* sub-routine is called in order to update the corresponding variables.

Each time a neighbor solution is explored in the local search, a control of discontinuities is performed (if it is enabled). This routine counts for each OD pair, the number of times that bike users enter to and leave the bicycle network. This value is added to the objective function with a penalization which is a parameter of the algorithm

3. Computational experiments

We test the proposed methodology by performing different experiments according to the specific aim, namely:

- Performance of both exact and heuristic solving methods.
- Sensitivity analysis, with respect to the budget and the travel cost penalization in discontinuous routes.
- A real case study.

For the exact solving method, we use CPLEX 12.6.3.0. The GRASP heuristic is implemented in Java. Both methods were ran in a Core i7 machine. We use several test instances regarding problem size (see Table 1), including small instances, some others taken from the literature and a real case related to a medium-sized city. Instances 1 to 4 were specifically created by the authors, comprising unitary arc costs and a penalization factor equal to 1.5; this means that bike users perceive a cost overhead of 50% when they traverse an arc which is not part of the bicycle network. The Sioux Falls and Anaheim instances were adapted from (Bar-Gera, 2017), a classical repository of transportation test instances. For the real case, we use data from Montevideo, main city of Uruguay; this is a medium-sized city, comprising about 1.5 million of inhabitants. In order to simplify the construction of the underlying network of available street sections, we consider only the avenues of the city. Origin-destination data of bike trips were obtained from the last available household survey, undertaken in year 2009. The zone system comprises about 150 traffic zones.

Table 1. Main characteristics of the test instances.

Name	Nodes	Arcs	OD Pairs	Arc parameters
1	10	32	11	
2	18	54	14	$l_{ij} = c_{ij} = 1$; $c'_{ij} = 1.5\ c_{ij}\ \forall(i,j)$
3	24	76	7	
4	15	42	12	
Sioux Falls	24	76	528	$l_{ij} = c_{ij} = \{\text{arc length}\}$; $c'_{ij} = 2\ c_{ij}\ \forall(i,j)$
Anaheim	416	1266	1406	
Montevideo	12759	26165	81	$l_{ij} = c_{ij} = \{\text{arc length}\}$; $c'_{ij} = 1.5\ c_{ij}\ \forall(i,j)$

3.1. Performance of solving methods

In this experiment, we execute both exact and heuristic solving methods in order to study:

- The maximum instance size for which the problem can be solved exactly.
- The accuracy of the heuristic method, in the sense of distance to the optimal value.

Table 2 reports percentage gap (difference between best results obtained by both methods, relative to the exact one) and execution times for both methods (in seconds). For the heuristic method we set the number of GRASP iterations equal to 800.

Table 2. Performance of solving methods.

Instance	% gap	T exact	T heuristic
1	0	0.13	5.00
2	0	0.22	83.00
3	0	0.23	30.00
4	5.20	0.22	28.00
Sioux Falls	5.73	13.30	245.00
Anaheim	0.86*	18 h	23 m

We can observe that the exact method is able to solve small-sized instances. For the case of Anaheim (labeled with *), the global optimum was not obtained after a considerable amount of execution time. Therefore, we modified it by reducing the number of nodes, arcs and OD pairs. For that reduced version of the case, the optimal solution was attained after 18 hours. For all cases, the accuracy of the heuristic method is below 6%. Regarding execution time, the heuristic takes more time for solving small cases, but for (the reduced version of) Anaheim, the heuristic takes significantly less time with high accuracy. This is a relevant issue, since this instance exhibits more real characteristics in comparison with the other ones, therefore it validates the application of the heuristic method for large and realistic cases with reasonable accuracy.

3.2. Sensitivity analysis

We test the sensitivity of the model with respect to some relevant parameters, namely the budget level l_{max} and the relationship between the costs of traveling inside (c_{ij}) and outside (c'_{ij}) of the bicycle network.

For the budget level, we perform several runs with the Sioux Fall instance, considering different percentages of the total length of the underlying street network (L_{tot}) where cycling infrastructure can be built. For values of l_{max} from 1% to 30% of L_{tot}, we observe an increase in the percentage of the total demand that flows inside of the bicycle network (F_{in}) from 0% to 91%. This shows the capability of the model to identify a threshold of investment which produces a network of high quality from the point of view of the users. If we increase the budget level from 30% to 90% of L_{tot}, the percentage F_{in} only increases up to 96%; clearly this high level of investment does not produce a significant increase in the quality of the network.

For the second analysis, we consider the ratio $r = c'_{ij} / c_{ij}$ taking values in the set {0.5; 1.0; 1.5; 2.0} and we apply the model to instance number 1, observing the value F_{in} in the solutions obtained. For the first value of the set we observe a percentage of F_{in} equal to 0%, an expectable result since for this setting the users perceive higher cost when they travel inside of the network. This is not a realistic setting but it contributes to validate the model by providing extreme conditions. For $r = 1$, the percentage of F_{in} reaches 14%; in this case, the users do not have incentive to travel inside of the network. For ratios equal to 1.5 and 2.0 we observe percentages of flow inside of the network equal to 89% and 92% respectively. This confirms that as the cost of traveling outside of the network increases, more users will choose traveling inside of the network. Thus, a desirable feature of the model is demonstrated empirically, which is relevant in the context of discontinuous networks.

3.3. Real test case

In this experiment, we aim to study the behavior and capabilities of our proposed methodology when it is applied to a problem instance with real characteristics. It is worth mentioning that a comprehensive case study is not the goal of the study. Table 3 reports results for different values of the budget level l_{max}: the total length of the current bicycle network of the city, half of the total length of the whole street network L_{tot} and the whole length L_{tot}. The heuristic solving method is applied, since the exact approach does not handle this large instance. The algorithm was run either without as with discontinuity penalty. In the first case, formulation (1)-(8) is solved. In the second case, a penalty is included in the objective function, accounting for each time that an OD pair enters to or leaves the bicycle network.

Table 3. Real test case.

Budget	Flow inside of the bicycle network	Flow outside of the bicycle network	Execution time
Without discontinuity penalty			
Current	11%	89%	29:29:57
Half	92%	8%	25:00:33
Whole	100%	0%	08:54:03
With discontinuity penalty			
Current	11%	89%	25:41:45
Half	93%	7%	21:48:58
Whole	100%	0%	08:57:12

We can note that execution times are significantly high, despite they are reduced when the budget is increased. Nevertheless, since this problem arises at strategic planning, execution times are manageable taking into account the potential benefits of the obtained results. By imposing the current budget, we note that only 11% of the demand travels inside of the bicycle network. By imposing the extreme value corresponding to the whole network, the entire demand travels inside of the network, which is an expectable result. If we impose an intermediate budget value (half of the total street network length), we obtain a bicycle network which covers 92% of the bike trips. When we enable the discontinuity penalty, that percentage increases a little. This is an expectable result, since by doing this we are taking care of the interests of bike users. Figure 1 shows the resulting networks for this intermediate budget level, where the blue lines represent the bicycle network, the red lines represent links that are used for bike trips while they are not part of the network and the grey lines represent streets that are unused for cycling. Clearly, the second network seems to be a more connected one, even if it does not necessarily entails lower user total travel cost.

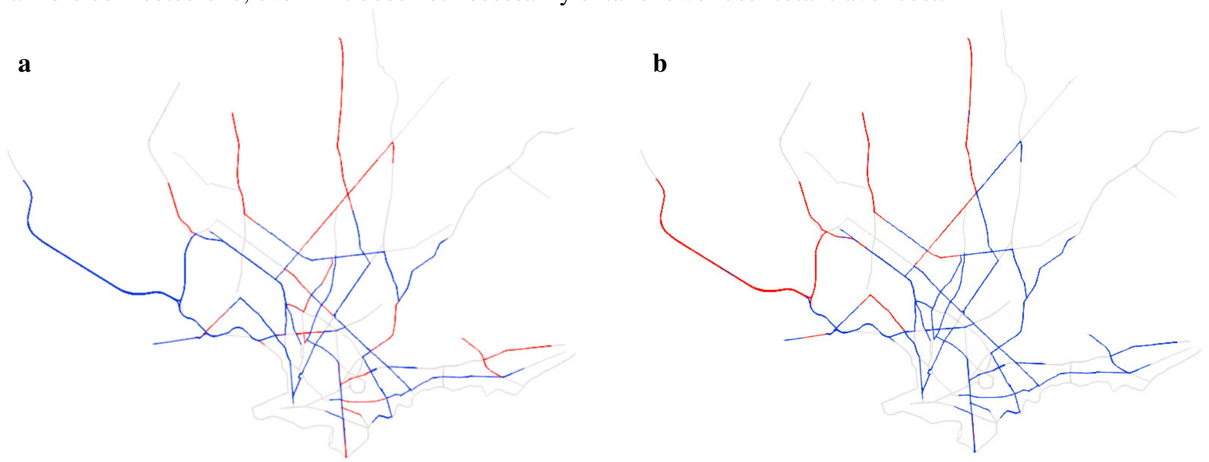

Fig. 1. Results for Montevideo; (a) without discontinuity penalty; (b) with discontinuity penalty.

4. Conclusions and further work

We proposed a new optimization model for bicycle network design, which considers different user travel costs for arcs inside and outside of the network. A heuristic algorithm is proposed to solve large instances of the problem, including a discontinuity minimization feature. The capabilities of both model and algorithm are tested with several instances, including a real one regarding a medium-sized city.

As future work, we identify the need of including information regarding slopes in the user travel cost. Bike users are sensitive to this attribute, especially to positive slopes (Li et al., 2012), therefore it should be considered when designing the network. Also, elastic demand would be a desirable feature for including into the model, since building cycling infrastructure will attract new bike users. The objective function should be reformulated when considering this issue. The accuracy of the heuristic should be assessed by using more test instances. Moreover, the local search would benefit from other neighborhood structures which enable to perform specific improvements in the solution. Finally, the real test case could be improved by using more accurate data. The underlying network could consider other streets, other than only avenues, as candidates for building the cycling network. Also, the origin-destination matrix should be enriched with additional information, other than the one provided by the household survey.

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