Karlsruhe Institute of Technology

Efficient Long-Haul Truck Driver Routing

Master Thesis of

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Statement of Authorship

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Karlsruhe, May 21, 2022

Abstract

A short summary of what is going on here.

Deutsche Zusammenfassung

Kurze Inhaltsangabe auf deutsch.

Contents

1. Introduction

This chapter should contain

1. A short description of the thesis topic and its background.
2. An overview of related work in this field.
3. Contributions of the thesis.
4. Outline of the thesis.

2. Preliminaries

This chapter should provide the foundations of the thesis.

3. Problem and Definitions

The long-haul truck driver routing problem is an extension to the common shortest path problem (SPP) which is defined as follows. Let $G = (V, E, \omega)$ be a graph where V is the set of nodes, E is the set of edges (u, v) with $u, v \in V$, and ω is the weight function $\omega : E \to \mathbb{R}_{\geq 0}$ which assigns each edge a nonnegative weight or length. Given a start node $s \in V$ and a target node $t \in V$, the SPP searches a shortest path p from s to t, i.e., a path $p = \langle s = v_0, v_1, ..., t = v_k, \rangle$ with $(v_i, v_{i+1}) \in E$ and minimal $len(p) = \sum_{i=0}^{k-1} \omega((v_i, v_{i+1}))$.

We introduce a set $P \subseteq V$ of parking nodes and a set R of driving time constraints r_i. Each driving time constraint is defined by a maximum allowed driving time $r_{i,d}$ and a pause time $r_{i,p}$. Thereby, the driving time constraints define a relation $r_i \leq r_{i+1}$ with $r_i \leq r_j \implies r_{i,d} \leq r_{j,d} \wedge r_{i,p} \leq r_{j,p} \forall i, j$. In words, a longer driving time restriction must have a longer or equal driving and pause time and there must be no restriction r_i with a longer driving time limit, but shorter pause time than another restriction r_j. Before exceeding a driving time of $r_{i,d}$, the driver must stop and pause for a time of at least $r_{i,p}$. Afterwards, the driver is allowed to drive for a maximum time of $r_{i,d}$ again without stopping. Stops can only take place at nodes $v \in P$.

A path p from s to t now includes not only the sequence of visited nodes $p = \langle s = v_0, v_1, ..., t = v_k, \rangle$, but also a pause time $\rho : p \to \{0, r_{i,d}\}$ at a node i. It is $\rho(v_i) = 0$ $\forall v_i \notin P$.

Definition 3.1 (Driving Time)**.** *The driving time $len_{dt}(p)$ of a path $p = \langle s = v_0, v_1, ..., t = v_k, \rangle$ is defined analogously to the length of a path in the ordinary SPP. It is $len_{dt}(p) = \sum_{i=0}^{k-1} \omega((v_i, v_{i+1}))$.*

Definition 3.2 (Travel Time)**.** *The travel time $len_{tt}(p)$ of a path $p = \langle s = v_0, v_1, ..., t = v_k, \rangle$ is defined as the sum of the driving time of the path and the total accumulated pause time $len_{tt}(p) = len_{dt}(p) + \sum_{i=0}^{k} \rho(v_i)$.*

Definition 3.3 (Path Compliance)**.** *A valid path $p = \langle s = v_0, v_1, ..., t = v_k, \rangle$ must comply with the driving time restrictions R. The path p complies with R if it complies with all $r_l \in R$.*

Let v_i be the starting node s or any node on the path with $\rho(v_i) \geq r_{l,p}$ and let v_j be the target node t or any node on the path with $\rho(v_j) \geq r_{l,p}$ and $i < j$. Then let q be the subpath of p from v_i to v_j. A path complies with driving time restriction r_l if $len_{dt}(q) < r_{d,l}$ for all possible subpaths q or there is a node v_m on q with $\rho(v_m) \geq r_{l,p}$ and $i < m < j$.

The long-haul truck driver routing problem now can be defined as follows.

Long-Haul Truck Driver Routing
Input: A graph $G = (V, E, \omega)$, a set of parking nodes $P \subseteq V$, a set of driving time constraints R, and start and target nodes $s, t \in V$
Problem: Find the path p from s to t in G which minimizes travel time $len_{tt}(p)$ and complies with the driving time restrictions R.

In many practical applications, the number of different driving time constraints is limited to only one or two constraints, i.e., $|R| = 1$ or $|R| = 2$. Therefore, we will often only consider one of these special cases.

4. Algorithm

We introduce a labeling algorithm which solves the shortest path problem with driving time constraints. todo

4.1. Dijkstra's Algorithm with One Driving Time Constraint

A driving time constraint is a rule which defines a maximum allowed non-stop driving time t_d and a pause time t_p. Before the driving time limit t_d is exceeded, the driver must park at designated parking for a minimum time period of t_p before continuing.

The base algorithm with one driving time constraint extends a Dijkstra search with pruning rules to comply with the constraint. It uses distance labels which it propagates between the nodes. The search operates on a graph $G = (V, E, \omega)$ with the available parking nodes defined as a subset $P \subseteq V$. The search can decide to *park* at a node v if $v \in P$.

Each node v holds a set $L(v)$ of *labels*. Each label at a node v holds two distances d_0 and d_1 and a link to the previous label. The chain of linked labels represents a unique *path* from s to v. A path is characterized by the sequence of visited nodes v_i and a subset of all $v_i \in P$ to describe the parking nodes on the path which were used for parking. The distance d_0 describes the distance on the path from the start node s and d_1 since the last pause, i.e., the distance from the last node $v_i \in P$ which was used for parking.

4.2. A* with Driving Time Constraints

4.2.1. Potential for Driving Time Constraints

Given a target node t, the CH potential $\pi_{t,ch}$ yields a perfect estimate for the distance $d_{direct}(v, t)$ from v to t without regard for driving time restrictions and pauses. A lower bound for the time $d(v, t)$ from v to t with breaks due to the driving time limit can be calculated by taking the minimum necessary amount of breaks on the shortest path into account:

$$\pi'_t(v) = \left\lfloor \frac{d_{direct}(v, t)}{t_d} \right\rfloor * t_p + d_{direct}(v, t)$$

A node potential is called *feasible* if it does not overestimate the distance of any edge in the graph, i.e.

$$len(u, v) - pot(u) + pot(v) \geq 0 \quad \forall (u, v) \in E \tag{4.1}$$

Algorithm 4.1: DIJKSTRA+DTC

Input: Graph $G = (V, E, \omega)$, parking nodes $P \subseteq V$, driving time restriction r, source node $s \in V$

Data: Priority queue Q, per node priority queue $\mathsf{L}(v)$ of labels for all $v \in V$

Output: Distances $\mathsf{d}(v)$ for all $v \in V$, shortest-path tree of s given by $\mathsf{pred}(\cdot)$

`// Initialization`

$\mathsf{Q}.\text{INSERT}(s, (0, 0))$
$\mathsf{L}(s).\text{INSERT}((\perp, \perp), (0, 0))$

`// Main loop`

while Q *is not empty* **do**
    $u \leftarrow \mathsf{Q}.\text{DELETEMIN}()$
    $(d_0, d_1) \leftarrow \mathsf{L}(u).\text{MINKEY}()$
    $l \leftarrow \mathsf{L}(u).\text{DELETEMIN}()$
    if $\mathsf{L}(u)$ *is not empty* **then**
        $k_{dist} \leftarrow \mathsf{L}(u).\text{MINKEY}()$
        $\mathsf{Q}.\text{INSERT}(u,\ k_{dist})$
    forall $(u, v) \in E$ **do**
        if $d_0 + \omega(u, v) < r_d$ **then**
            $D \leftarrow \{(d_0 + \omega(u, v), d_1 + \omega(u, v))\}$
            if $v \in P$ **then**
                $D.\text{INSERT}((d_0 + \omega(u, v) + r_p, 0))$
            forall $x \in D$ **do**
                if x *is not dominated by any label in* $\mathsf{L}(v)$ **then**
                    $\mathsf{L}(v).\text{REMOVEDOMINATED}(x)$
                    $\mathsf{L}(v).\text{INSERT}((l, (u, v)), x)$
                    if $\mathsf{Q}.\text{CONTAINS}(v)$ **then**
                        $\mathsf{Q}.\text{DECREASEKEY}(v, x)$
                    else
                        $\mathsf{Q}.\text{INSERT}(v, x)$

Following example of a query using the graph in Fig. 4.1 shows that π'_t is not feasible. With a driving time limit of 6 and a pause time of 1, the potential here will yield a value $\pi_t(s) = 8$ since the potential includes the minimum required pause time for a path from s to t. Consequently, with $\pi_t(v) = 5$ and $len(s, v) = 2$, $len(s, v) - \pi_t(s) + \pi_t(v) = -1$.

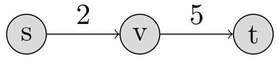

Figure 4.1.: A graph with the potential to break the potential.

A variant of the potential accounts for the distance $d(p, v)$ with p being the last parking node that was used for a pause to calculate the minimum required pause time on the *v-tpath*. Since the potential now uses information from a label l with $l \in L(v)$, it no longer is a node potential but also depends on the chosen label at v.

Algorithm 4.2: A*+DTC

Input: Graph $G = (V, E, \omega)$, parking nodes $P \subseteq V$, driving time restriction r, potential $\mathsf{pot}()$, source node $s \in V$
Data: Priority queue Q, per node priority queue $\mathsf{L}(v)$ of labels for all $v \in V$
Output: Distances for all $v \in V$, tree of allowed shortest paths according to the restriction r from s, given by l_{pred}

```
// Initialization
Q.insert(s, (0,0))
L(s).insert((⊥,⊥), pot((0,0)))
// Main loop
while Q is not empty do
    u ← Q.deleteMin()
    (d_0, d_1) ← L(u).minKey()
    l ← L(u).deleteMin()
    if L(u) is not empty then
        k_dist ← L(u).minKey()
        Q.insert(u, k_dist)
    forall (u,v) ∈ E do
        if d_0 + ω(u,v) < r_d then
            D ← {(d_0 + ω(u,v), d_1 + ω(u,v))}
            if v ∈ P then
                D.insert((d_0 + ω(u,v) + r_p, 0))
            forall x ∈ D do
                if x is not dominated by any label in L(v) then
                    L(v).removeDominated(x)
                    L(v).insert((l, (u,v)), x)
                    if Q.contains(v) then
                        Q.decreaseKey(v, x)
                    else
                        Q.insert(v, x)
```

$$\begin{aligned}\pi_t(l,v) &= \left\lfloor \frac{d_{direct}(p,v)+d_{direct}(v,t)}{t_d} \right\rfloor * t_p + d_{direct}(v,t)\\ &= \left\lfloor \frac{d_1(l)+d_{direct}(v,t)}{t_d} \right\rfloor * t_p + d_{direct}(v,t)\end{aligned}$$

Since the potential π_t now uses label information it no longer is a node potential and the feasibility definition as defined in inequality 4.1 can no longer be applied. We still want to use potential and label information to calculate lower bound estimates for the length of paths.

Lemma 4.1. *Let $p = \langle s = v_0, v_1, ..., t = v_k, \rangle$ be a path with labels l_i at nodes v_i. Then $d_0(l_{i-1}) + \pi_t(l_{i-1}, v_{i-1}) \leq d_0(l_i) + \pi_t(l_i, v_i)$.*

The lower bound estimate for the length of the entire path to which a label belongs can only increase when propagating labels to a next node.

Proof. Given a Graph $G = (V, E)$ with a set of parking nodes $P \subseteq V$, let $p = \langle s = v_0, v_1, ..., t = v_k, \rangle$ be a path in G with labels l_i at nodes v_i. Let $p, q \in P \cup \{s\}$ the last parking node which was used by label l_{i-1} and l_i or s, if no parking node was used.

$$\begin{aligned}d_0(l_{i-1}) + \pi_t(l_{i-1}, v_{i-1}) &= d_0(l_{i-1}) + \left\lfloor \frac{d_1(l_{i-1})+d_{direct}(v_{i-1},t)}{t_d} \right\rfloor * t_p + d_{direct}(v_{i-1},t)\\ &= d_0(l_{i-1}) + \left\lfloor \frac{d_{direct}(p,v_{i-1})+d_{direct}(v_{i-1},t)}{t_d} \right\rfloor * t_p + d_{direct}(v_{i-1},t)\\ &= d(s,p) + d_{direct}(p,v_{i-1})\\ &\quad + \underbrace{\left\lfloor \frac{d_{direct}(p,v_{i-1})+d_{direct}(v_{i-1},t)}{t_d} \right\rfloor * t_p}_{\text{minimum required pause time on p-t subpath}} + d_{direct}(v_{i-1},t)\end{aligned} \tag{4.2}$$

Case 1: $p = q$

$$\begin{aligned}d_{direct}(p,v_{i-1}) + d_{direct}(v_{i-1},t) &= d_{direct}(p,v_{i-1}) + len(v_{i-1},v_i) + d_{direct}(v_i,t)\\ &= d_{direct}(q,v_{i-1}) + len(v_{i-1},v_i) + d_{direct}(v_i,t)\\ &= d_{direct}(q,v_i) + d_{direct}(v_i,t)\end{aligned} \tag{4.3}$$

With equations 4.2 follows

$$\begin{aligned}d_0(l_{i-1}) + \pi_t(l_{i-1}, v_{i-1}) &= d(s,p) + d_{direct}(p,v_{i-1}) + d_{direct}(v_{i-1},t)\\ &\quad + \left\lfloor \frac{d_{direct}(p,v_{i-1})+d_{direct}(v_{i-1},t)}{t_d} \right\rfloor * t_p\\ &= d(s,q) + d_{direct}(q,v_i) + d_{direct}(v_i,t)\\ &\quad + \left\lfloor \frac{d_{direct}(q,v_{i1})+d_{direct}(v_i,t)}{t_d} \right\rfloor * t_p\\ &= d_0(l_i) + \pi_t(l_i,v_i)\end{aligned} \tag{4.4}$$

Case 2: $p \neq q$. In this case, $q = v_i$ and $d(p, v_i) = d(p, q) = d_{direct}(p, v_i) + t_p =$. With 4.2 follows

$$\begin{aligned}
d_0(l_{i-1}) + \pi_t(l_{i-1}, v_{i-1}) &= d(s,p) + d_{direct}(p, v_{i-1}) + d_{direct}(v_{i-1}, t) \\
&\quad + \left\lfloor \frac{d_{direct}(p, v_{i-1}) + d_{direct}(v_{i-1}, t)}{t_d} \right\rfloor * t_p \\
&= d(s,p) + d_{direct}(p, v_i) + d_{direct}(v_i, t) \\
&\quad + \left\lfloor \frac{d_{direct}(p, v_i) + d_{direct}(v_i, t)}{t_d} \right\rfloor * t_p \\
&\leq d(s,p) + d(p,q) - t_p + d_{direct}(v_i, t) \\
&\quad + \left\lfloor \frac{d_{direct}(v_i, t)}{t_d} \right\rfloor * t_p + t_p \qquad (4.5) \\
&= d(s,q) + 0 + d_{direct}(v_i, t) \\
&\quad + \left\lfloor \frac{0 + d_{direct}(v_i, t)}{t_d} \right\rfloor * t_p \\
&= d(s,q) + d_{direct}(q, v_i) + d_{direct}(v_i, t) \\
&\quad + \left\lfloor \frac{d_{direct}(q, v_i) + d_{direct}(v_i, t)}{t_d} \right\rfloor * t_p \\
&= d_0(l_i) + \pi_t(l_{i1}, v_i)
\end{aligned}$$

□

Lemma 4.2. *The potential $\pi_t(l, v)$ of a label l at a node v is a lower bound for the distance including pauses from v to t.*

Proof. Let $p = \langle s = v_0, v_1, ..., t = v_k, \rangle$ be a path with labels l_i at nodes v_i. With $d_0(l_{i-1}) + \pi_t(l_{i-1}, v_{i-1}) \geq d_0(l_i) + \pi_t(l_i, v_i)$ for all edges on p, the total length $len(p)$ of the path must follow $\pi_t(l_i, v_i) \leq len(p) + \pi_t(l_k, t) \Leftrightarrow l(p) \geq \pi_t(l_i, v_i) - \pi_t(l_k, t)$. Since $\pi_t(l_k, t) = 0$, $l(p) \geq \pi_t(l_i, v_i)$ holds. □

Theorem 4.3. *The search can be stopped when the first label at t is removed from the queue.*

Proof. When a label l at t is removed from the queue during a s-t query, all remaining label m of a node v in the queue fulfill $d_0(t) + \pi_t(l, t) \leq d_0(v) + \pi_t(m, v)$. Assume that $d_0(t)$ is not the shortest distance from s to t. Then, a shorter path $p = \langle s = v_0, v_1, ..., t = v_k, \rangle$ exists which uses at least one unsettled label $m \in L(v_i)$. Since l was already removed from the queue, $d_0(t) = d_0(t) + \pi_t(l, t) \leq d_0(v) + \pi_t(m, v) \leq l(p)$ which contradicts the assumption that p yields a shorter s-t distance than $d_0(t)$. □

4.2.2. Multiple Driving Time Constraints

4.3. Core Contraction Hierarchy Variant

4.3.1. Building the Contraction Hierarchy

4.4. Combining A* and Core Contraction Hierarchy

Algorithm 4.3: CORE-CH WITH DRIVING TIME CONSTRAINTS

Input: Graph $G = (V, E, \omega)$, parking nodes $P \subseteq V$, driving time restriction r, potential pot(), source node $s \in V$

Data: Priority queue Q, per node priority queue L(v) of labels for all $v \in V$

Output: Distances for all $v \in V$, tree of allowed shortest paths according to the restriction r from s, given by l_{pred}

```
// Initialization
Q.insert(s, (0,0))
L(s).insert((⊥, ⊥), pot((0,0)))

// Main loop
while Q is not empty do
    u ← Q.deleteMin()
    (d_0, d_1) ← L(u).minKey()
    l ← L(u).deleteMin()

    if L(u) is not empty then
        k_dist ← L(u).minKey()
        Q.insert(u, k_dist)

    forall (u, v) ∈ E do
        if d_0 + ω(u, v) < r_d then
            D ← {(d_0 + ω(u, v), d_1 + ω(u, v))}
            if v ∈ P then
                D.insert((d_0 + ω(u, v) + r_p, 0))

            forall x ∈ D do
                if x is not dominated by any label in L(v) then
                    L(v).removeDominated(x)
                    L(v).insert((l, (u, v)), x)
                    if Q.contains(v) then
                        Q.decreaseKey(v, x)
                    else
                        Q.insert(v, x)
```

5. Evaluation

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6. Conclusion

Summary and outlook.

Bibliography

Appendix

A. Appendix Section 1

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Figure A.1.: A figure