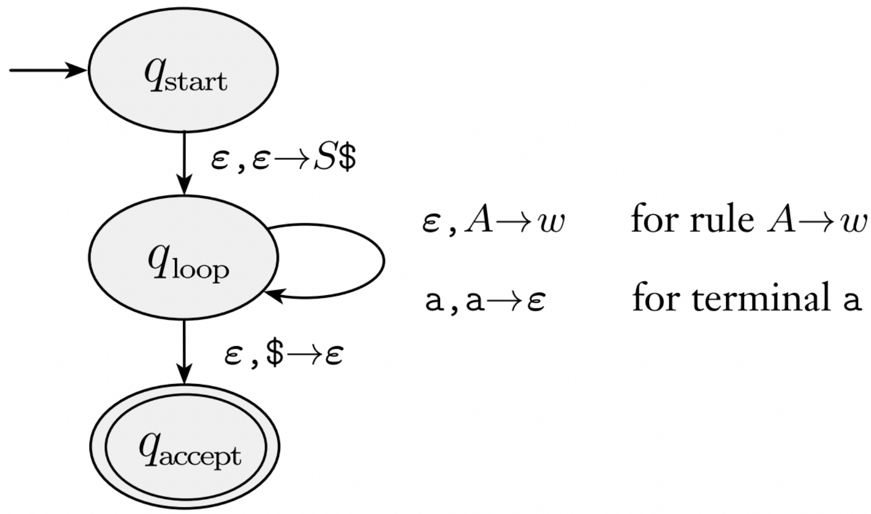
$O(n^x)$ # polynomial complexity



Rice's theorem:
Let P be a language. Given that:
- P is not trivial ($P \neq \emptyset,\ P \neq \Sigma^*$)
- P is only influenced by itself (for two turing machines that have the same language ($Z(M_1) = Z(M_2)$), then $M_1 \in P \Leftrightarrow M_2 \in P$)

Then, P is undecidable

$s \in A,\ |s| \geq p,\ s = xyz \Rightarrow i \geq 0,\ xy^iz \in A,\ |y| > 0,\ |xy| \leq p$ # Pumping lemma

$p \geq 1,\ |s| \geq p,\ s = uvwxy,\ |vx| \geq 1,\ |vwx| \leq p,\ uv^pwx^py \in L\ \ for\ all\ p \geq 0$

$s = uv^nxy^nz$ where n=0+. Show that it can't be pumped

Show that $s = uvxyz = 0^p\#0^{2p}\#0^{3p}$ can not be pumped

$\# \notin v$ and $\# \notin y$ since $s = uv^2xy^2z$ would contain extra #

$if\ \# \notin v\ and\ \# \notin y\ \rightarrow\ u \neq 0^p\ or\ x \neq 0^{2p}\ or\ z \neq 0^{3p}\ \Rightarrow\ |u|:|x|:|z| \neq 1:2:3$

⇒ s can not be pumped

Union:

$S \rightarrow S_1|S_2$

If $S_1 \in CFL$ and $S_2 \in CFL$, then $S_1|S_2 \in CFL$

For a PDA, this can be represented by an ϵ transition from the start state

⇒ set of CFL is closed under a union

$A \leq_p B$ (A mapping reduces to B, A is no harder than B): transform A into B and call B solver

Mapping reduction to prove (not) decidable

To prove undecidability: If $A \leq_{\mathrm{m}} B$ and A is undecidable, then B is undecidable.

To prove non-Turing-recognizability: If $A \leq_{\mathrm{m}} B$ and A is non-Turing-recognizable, then B is non-Turing-recognizable.

To prove NP-completeness: If $A \leq_{\mathrm{P}} B$ and A is NP-complete (and $B \in \mathrm{NP}$), then B is NP-complete.

To prove decidability: If $A \leq_{\mathrm{m}} B$ and B is decidable, then A is decidable.

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq \infty \Rightarrow f(n) = O(g(n))$$

Suppose that we have one of the following cases,

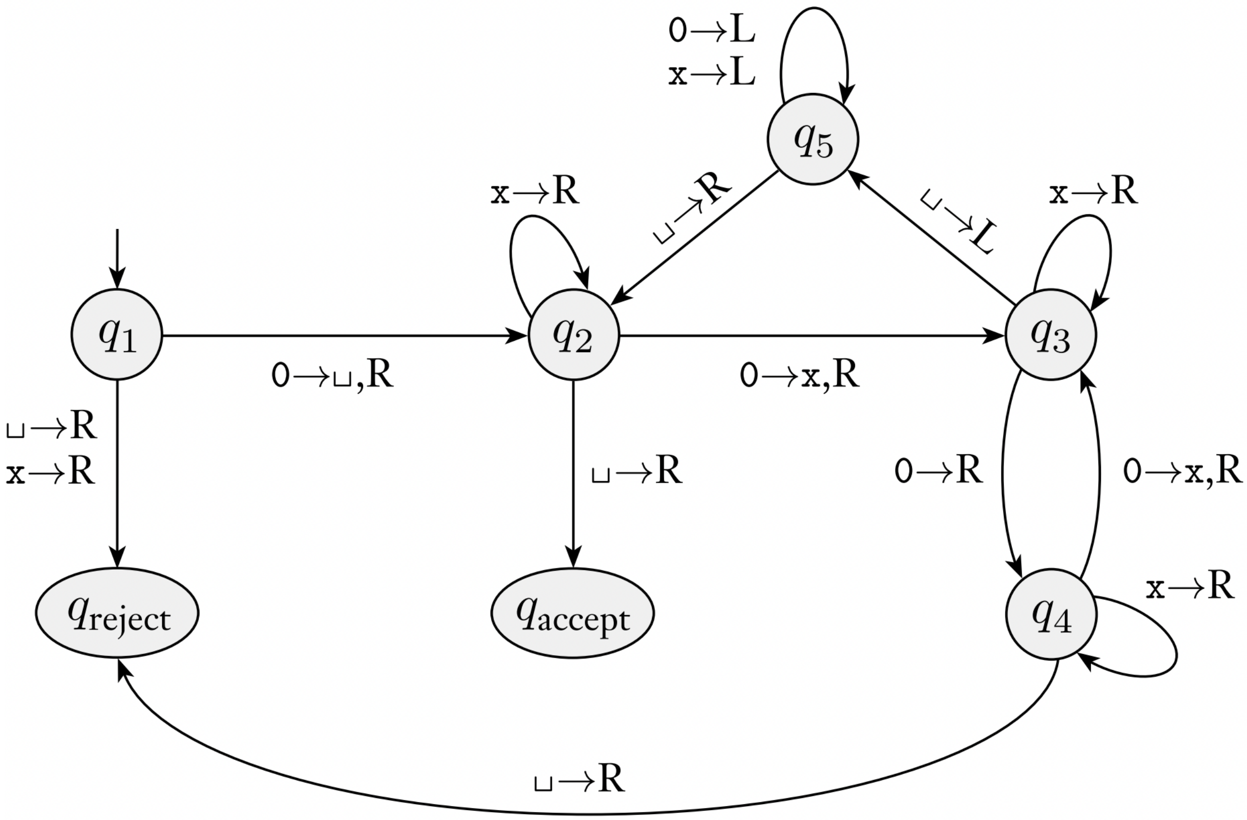
$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0} \qquad \text{OR} \qquad \lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$$

where a can be any real number, infinity or negative infinity. In these cases we have,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Turing description:

1. Mark first unmarked 0. If none, go to 6.
   2. Move the head to the front
   3. Mark first unmarked 1. If none, reject
   4. Move the head to the front
   5. Go to 1
   6. Scan if there is any unmarked 1. If so, reject. Else accept



There exists $L(M') = L_1 \bullet L_2$ with the following description for input $w_1 w_2$

1. Run M_1 on w_1. If M_1 rejected, then reject
2. Run M_2 on w_2. If M_2 rejected, then reject
3. Else, accept

Let T_A and T_B be NP-machines decidable by languages A and B respectively.

Union:

$T_{A \cup B}$ =On input w:

1. Run T_A on w and accept if it accepts
2. Run T_B on w and accept if it accepts
3. Reject

Complexity: $O(T_A + T_B)$