T-Distribution

$$T = \frac{Z}{\sqrt{\frac{V}{v}}} = \frac{standard\ normal\ distribution}{\sqrt{\frac{X^2\ random\ variable}{degree\ of\ distribution}}} = \frac{\bar{X} - \mu\ (often\ 0)}{\frac{S}{\sqrt{n}}} = \frac{sample\ mean}{sample\ standard\ deviation}$$

$v = n - 1$

$t_{1-\alpha} = - t_{\alpha}$

Symmetric around $t = 0$

$When\ v \to \infty \Rightarrow T \to Z$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Confidence intervals:

$\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

$\bar{x} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$

$\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$

Prediction intervals:

$\bar{x} - z_{\frac{\alpha}{2}} \sigma \sqrt{1 + \frac{1}{n}} < x_0 < \bar{x} + z_{\frac{\alpha}{2}} \sigma \sqrt{1 + \frac{1}{n}}$

$\bar{x} - t_{\frac{\alpha}{2}} s \sqrt{1 + \frac{1}{n}} < x_0 < \bar{x} + t_{\frac{\alpha}{2}} s \sqrt{1 + \frac{1}{n}} \quad v = n - 1$

Hypothesis

H_0: $\mu = 0.25$ # mean

H_1: $\mu = 0.5$

$x > x_c \Rightarrow critical\ region$

	H_0 *is true*	H_1 *is true*
Fail to reject H_0	Correct	Type 2 error
Reject H_0	Type 1 error	Correct

$\alpha = P(type\ 1\ error) = level\ of\ significance = rejected\ H_0,\ but\ it\ was\ true$

$\beta = P(type\ 2\ error);\ 1 - \beta = power\ of\ the\ test = did\ not\ reject\ H_0,\ but\ H_1\ is\ true$

$\alpha = P(x > 8\ if\ H_0\ is\ true) = P(x > 8\ if\ p = 0.25) = \sum_{k=9}^{20} p^k (1 - p)^{20-k} \binom{20}{k} = 0.0409$

$\beta = P(x \le 8\ if\ H_0\ is\ false) = P(x \le 8\ if\ p = 0.5) = \sum_{k=0}^{8} p^k (1 - p)^{20-k} \binom{20}{k} = 0.2517$

$if\ a\ is\ desc,\ b\ is\ asc$
$if\ n\ is\ asc,\ a\ and\ b\ is\ desc$

α - level of significance (divide by 2 for two directional)
$P - value$ - the probability of this or more extreme result (in both directions)

Reject H_0 if $\bar{x}$ is in two sided critical region $(-\ \infty, \mu_0 - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}) \cup (\mu_0 + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}; \infty)$

Reject H_0 if $\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ is in two sided critical region $(-\ \infty, -\ z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}; \infty)$ (or $\sigma \to s,\ s \to t$)

$P(-\ z_{\frac{\alpha}{2}} < Z < z_{\frac{\alpha}{2}}) = P(\mu_0 - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}; \mu_0 + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}) = (1 - \alpha)$?

$P_{value} = 2P(Z <- |\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}|)$ (or $\sigma \to s,\ s \to t$). if $<\ \alpha$, reject H_0

For H_1: $\mu > \mu_0$: reject H_0 if $\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ is in $(z_\alpha; \infty)$ or for $<$, $(-\ \infty, -\ z_\alpha)$ (or $\sigma \to s,\ s \to t$)

$P_{value} = P(Z > \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}})$ or < (or $\sigma \to s,\ s \to t$). if $<\ \alpha$, reject H_0
Can use s instead of σ and t-values with $v = n - 1$ to replace z-values

EXAM

$T = \frac{Z}{\sqrt{\frac{V}{v}}} = \frac{standard\ normal\ distribution}{\sqrt{\frac{X^2\ random\ variable}{degree\ of\ distribution}}} = \frac{\bar{X}-\mu\ (often\ 0)}{\frac{S}{\sqrt{n}}} = \frac{sample\ mean}{sample\ standard\ deviation}$ **(used for n<30)**

$v = n - 1$ $\quad t_{1-\alpha} =- t_\alpha$ $\quad$ Symmetric around $t = 0$ $\quad When\ v \to \infty\ \Rightarrow\ T \to Z$

$Z = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$

$100(1 - \alpha)\%$
Confidence intervals:

$\bar{x} - z_{\frac{a}{2}} \frac{\sigma}{\sqrt{n}} <\ \mu < \bar{x} + z_{\frac{a}{2}} \frac{\sigma}{\sqrt{n}}$

$z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \le\ 10\ within\ hours\ how\ large\ n\ 95\%\ confident$

$\bar{x} - t_{\frac{a}{2}} \frac{s}{\sqrt{n}} <\ \mu < \bar{x} - t_{\frac{a}{2}} \frac{s}{\sqrt{n}}$

$\bar{x} - z_{\frac{a}{2}} \frac{s}{\sqrt{n}} <\ \mu < \bar{x} - z_{\frac{a}{2}} \frac{s}{\sqrt{n}}$?

Prediction intervals:

$\bar{x} - z_{\frac{a}{2}}\sigma\sqrt{1 + \frac{1}{n}} < x_0 < \bar{x} + z_{\frac{a}{2}}\sigma\sqrt{1 + \frac{1}{n}}$

$\bar{x} - t_{\frac{a}{2}}s\sqrt{1 + \frac{1}{n}} < x_0 < \bar{x} + t_{\frac{a}{2}}s\sqrt{1 + \frac{1}{n}} \quad v = n - 1$

Tolerance limits: $x \pm ks$ where k is from the t-factor table ($\gamma\ tollerance, 100(1 - \alpha)\ \%\ of\ all, n$)

$\alpha = P(type\ 1\ error) = level\ of\ significance\ =\ rejected\ H_0,\ but\ it\ was\ true$ **(/use normal** μ**)**

$\beta = P(type\ 2\ error);\ 1 - \beta = power\ of\ the\ test = did\ not\ reject\ H_0,\ but\ H_1\ is\ true$

$\alpha = P(x > 8\ if\ H_0\ is\ true) = P(x > 8\ if\ p = 0.25) = \sum_{k=9}^{20} p^k(1 - p)^{20-k}\binom{20}{k} = 0.0409$

$\beta = P(x \le 8\ if\ H_0\ is\ false) = P(x \le 8\ if\ p = 0.5) = \sum_{k=0}^{8} p^k(1 - p)^{20-k}\binom{20}{k} = 0.2517$

$if\ a\ is\ desc,\ b\ is\ asc \quad if\ n\ is\ asc,\ a\ and\ b\ is\ desc$

α - level of significance (divide by 2 for two directional)

$P - value$ - the probability of this or more extreme result (in both directions)

Reject H_0 if $\bar{x}$ is in two sided critical region $(-\infty, \mu_0 - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}) \cup (\mu_0 + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}; \infty)$

Reject H_0 if $\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ is in two sided critical region $(-\infty, -z_{\frac{\alpha}{2}}) \cup (z_{\frac{\alpha}{2}}; \infty)$ (or $\sigma \to s,\ z \to t$)

- $P(-z_{\frac{\alpha}{2}} < Z < z_{\frac{\alpha}{2}}) = P(\mu_0 - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}; \mu_0 + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}) = (1 - \alpha)$?

$P_{value} = 2P(Z < -|\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}|)$ (or $\sigma \to s,\ z \to t$). if $<\ \alpha$, reject H_0

For $H_1: \mu > \mu_0$: reject H_0 if $\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ is in $(z_\alpha; \infty)$ or for $<$, $(-\infty, -z_\alpha)$ (or $\sigma \to s,\ z \to t$)

$P_{value} = P(Z > \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}})$ or < (or $\sigma \to s,\ z \to t$). if $<\ \alpha$, reject H_0

Can use s instead of σ and t-values with $v = n - 1$ to replace z-values