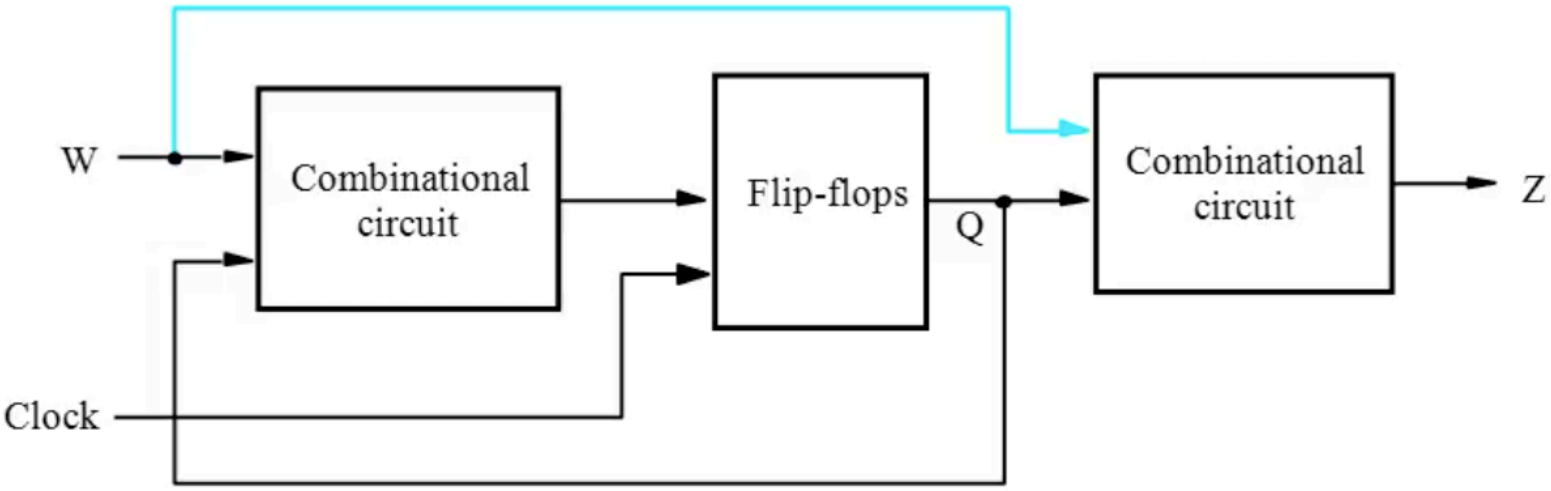
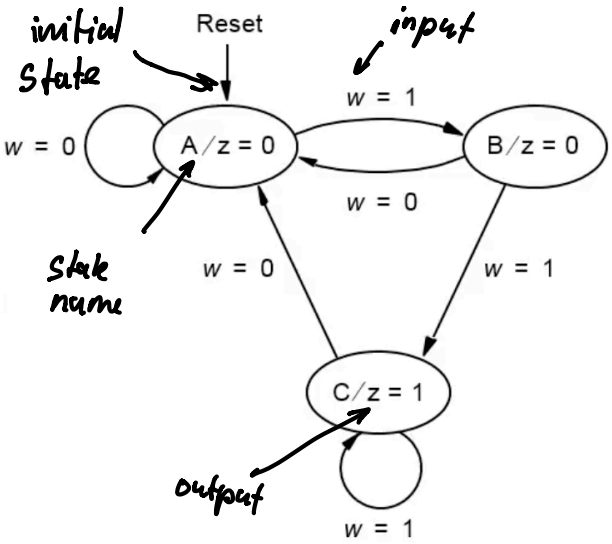
Finite state machine





1. Obtain the user specification of the circuit.
2. Create a state diagram.
3. Create a state table from the state diagram.
4. Perform state assignment.
5. Choose the type of flip-flops.
6. Design the next-state and output logic expressions.
7. Implement the circuit.

The change in state depends on:

- Current state (Q)
- Input (w)

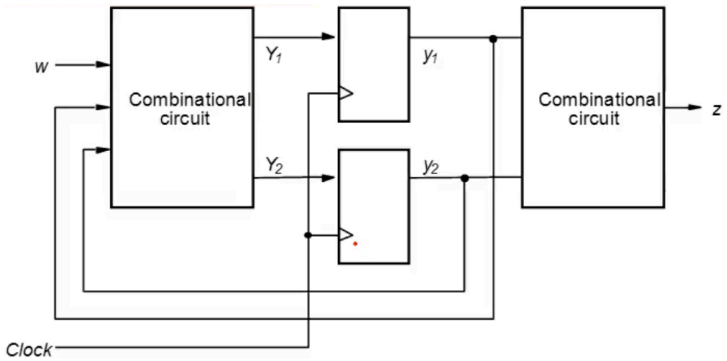
Output (z) depends on:

- Q (in Moore FSM)
- Q and w (in Mealy FSM)

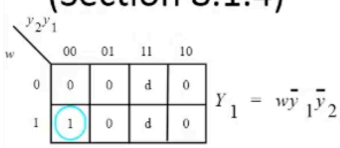
Present state	Next state		Output
	$w = 0$	$w = 1$	z
A	A	B	0
B	A	C	0
C	A	C	1

2^n states can be represented with n Flip-Flops

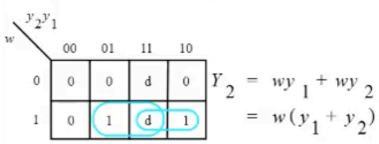
Present state	Next state		Output
	$w = 0$	$w = 1$	z
y_2y_1	Y_2Y_1	Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	10	1
11	*dd*	*dd*	*d*



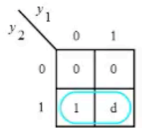
Logic Expressions (Section 8.1.4)



$$Y_1 = w\bar{y}_1\bar{y}_2$$

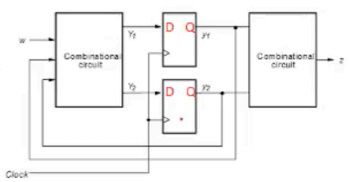


$$Y_2 = wy_1 + wy_2 = w(y_1 + y_2)$$

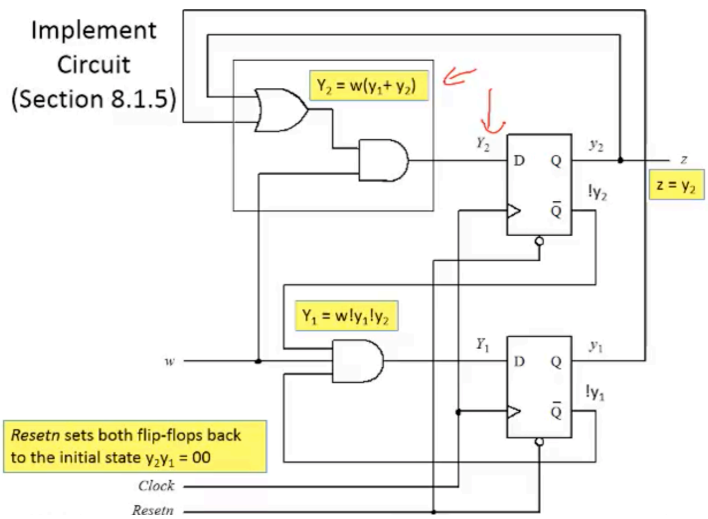


$$z = y_2$$

Present state	Next state		Output
	$w = 0$	$w = 1$	z
y_2y_1	Y_2Y_1	Y_2Y_1	
A 00	00	01	0
B 01	00	10	0
C 10	00	10	1
11	*dd*	*dd*	*d*

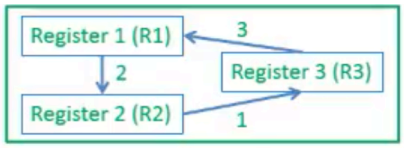


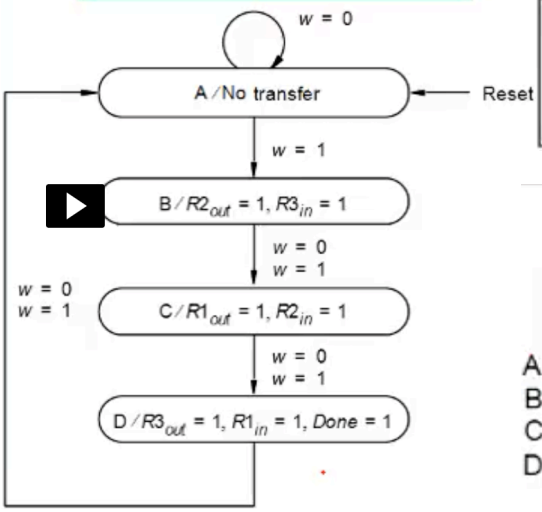
Implement Circuit (Section 8.1.5)



Swap registers:

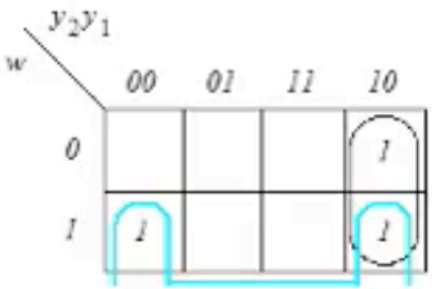
gn

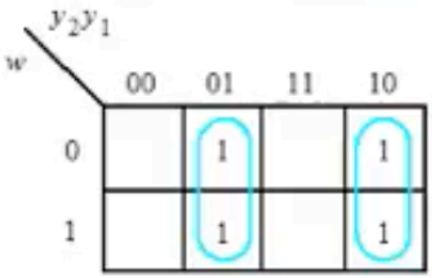




Present state	Next state		Outputs						
	$w = 0$	$w = 1$	$R1_{out}$	$R1_{in}$	$R2_{out}$	$R2_{in}$	$R3_{out}$	$R3_{in}$	$Done$
A	A	B	0	0	0	0	0	0	0
B	C	C	0	0	1	0	0	1	0
C	D	D	1	0	0	1	0	0	0
D	A	A	0	1	0	0	1	0	1

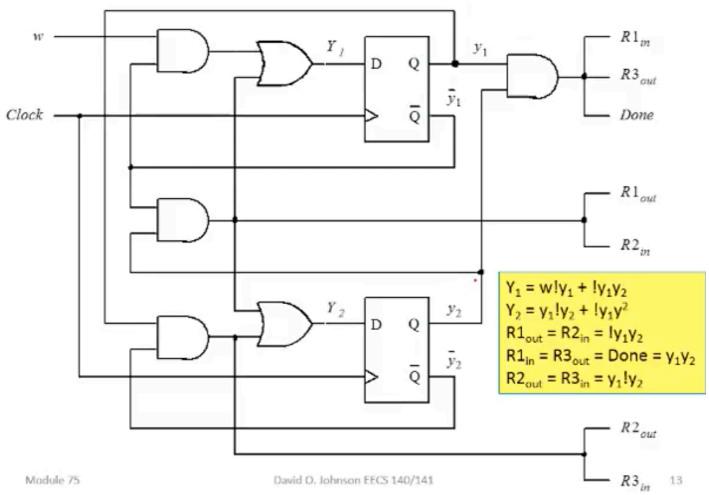
	Present state	Next state		Outputs						
		$w = 0$	$w = 1$							
	y_2y_1	Y_2Y_1	Y_2Y_1	$R1_{out}$	$R1_{in}$	$R2_{out}$	$R2_{in}$	$R3_{out}$	$R3_{in}$	$Done$
A	00	00	01	0	0	0	0	0	0	0
B	01	10	10	0	0	1	0	0	1	0
C	10	11	11	1	0	0	1	0	0	0
D	11	00	00	0	1	0	0	1	0	1





$$Y_1 = w\bar{y}_1 + \bar{y}_1 y_2$$

$$Y_2 = y_1\bar{y}_2 + \bar{y}_1 y_2$$



$R1_{out} = R2_{In} = !y_1y_2$

$R1_{in} = R3_{out} = Done = y_1y_2$

$R2_{out} = R3_{in} = y_1!y_2$

Hot encoding: (faster, but more expensive)

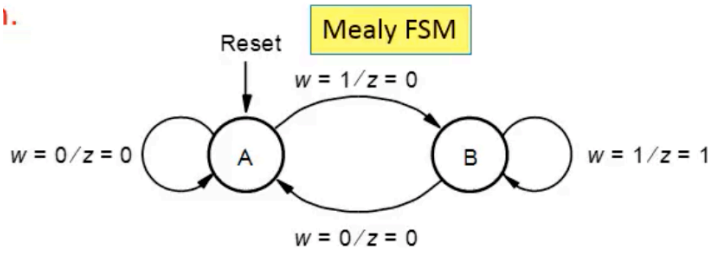
	Present state	Next state $w=0$	Next state $w=1$	Outputs $R1_{out}$	$R1_{in}$	$R2_{out}$	$R2_{in}$	$R3_{out}$	$R3_{in}$	$Done$
	y_2y_1	Y_2Y_1	Y_2Y_1							
A	00	00	01	0	0	0	0	0	0	0
B	01	10	10	0	0	1	0	0	1	0
C	10	11	11	1	0	0	1	0	0	0
D	11	00	00	0	1	0	0	1	0	1

$Y_1 = w!y_1 + !y_1y_2$
$Y_2 = y_1!y_2 + !y_1y^2$
$R1_{out} = R2_{in} = !y_1y_2$
$R1_{in} = R3_{out} = Done = y_1y_2$
$R2_{out} = R3_{in} = y_1!y_2$

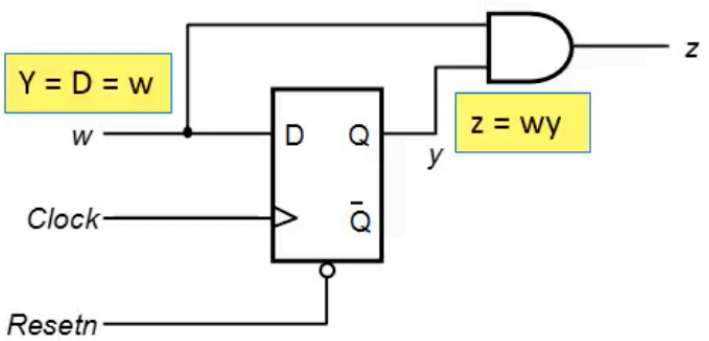
	Present state	Nextstate $w=0$	Nextstate $w=1$	Outputs $R1_{out}$	$R1_{in}$	$R2_{out}$	$R2_{in}$	$R3_{out}$	$R3_{in}$	$Done$
	$y_4y_3y_2y_1$	$Y_4Y_3Y_2Y_1$	$Y_4Y_3Y_2Y_1$							
A	0 001	0001	0010	0	0	0	0	0	0	0
B	0 010	0100	0100	0	0	1	0	0	1	0
C	0 100	1000	1000	1	0	0	1	0	0	0
D	1 000	0001	0001	0	1	0	0	1	0	1

$Y_1 = !wy_1 + y_4$
$Y_2 = wy_1$
$Y_3 = y_2$
$Y_4 = y_3$
$R1_{out} = R2_{in} = y_3$
$R1_{in} = R3_{out} = Done = y_4$
$R2_{out} = R3_{in} = y_2$

1.



Present state	Next state $w=0$	Next state $w=1$	Output z $w=0$	Output z $w=1$
A	A	B	0	0
B	A	B	0	1



	Present state	Next state $w=0$	Next state $w=1$	Output $w=0$	Output $w=1$
	y	Y	Y	z	z
A	0	0	1	0	0
B	1	0	1	0	1

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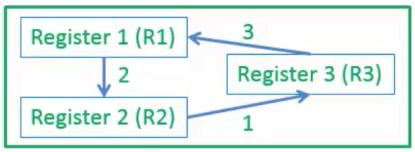
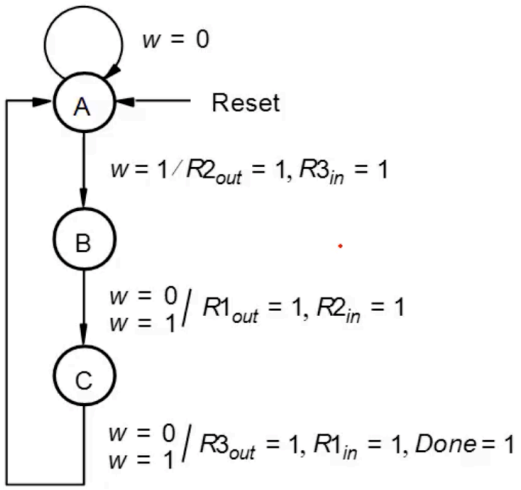




	Next State	Outputs (w=0)						
	w=0	$R1_{out}$	$R1_{in}$	$R2_{out}$	$R2_{in}$	$R3_{out}$	$R3_{in}$	Done
A	A	0	0	0	0	0	0	0
B	C	1	0	0	1	0	0	0
C	A	0	1	0	0	1	0	1

	Next State	Outputs (w=1)						
	w=1	$R1_{out}$	$R1_{in}$	$R2_{out}$	$R2_{in}$	$R3_{out}$	$R3_{in}$	Done
A	B	0	0	1	0	0	1	0
B	C	1	0	0	1	0	0	0
C	A	0	1	0	0	1	0	1

	Pres State	Next State	Outputs						
			w=0						
	y_2y_1	Y_2Y_1	$R1_{out}$	$R1_{in}$	$R2_{out}$	$R2_{in}$	$R3_{out}$	$R3_{in}$	Done
A	00	00	0	0	0	0	0	0	0
B	01	10	1	0	0	1	0	0	0
C	10	00	0	1	0	0	1	0	1
	11	dd	d	d	d	d	d	d	d

	Pres State	Next State	Outputs						
			w=1						
	y_2y_1	Y_2Y_1	$R1_{out}$	$R1_{in}$	$R2_{out}$	$R2_{in}$	$R3_{out}$	$R3_{in}$	Done
A	00	01	0	0	1	0	0	1	0
B	01	10	1	0	0	1	0	0	0
C	10	00	0	1	0	0	1	0	1
	11	dd	d	d	d	d	d	d	d

	y_2y_1			
	00	01	11	10
w 0	0	0	d	0
w 1	1	0	d	0

$Y_1 = w!y_1!y_2$

$R2_{out} = R3_{in} = w!y_1!y_2$

	y_2y_1			
	00	01	11	10
w 0	0	0	d	0
w 1	0	0	d	0

$Y_2 = y_1$

$R1_{out} = R2_{in} = y_1$

	y_2y_1			
	00	01	11	10
w 0	0	0	d	1
w 1	0	0	d	1

$R1_{in} = R3_{out} = Done = y_2$

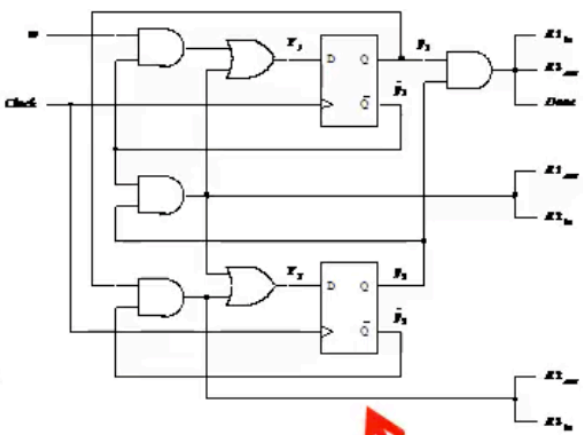
$$Y_1 = w!y_1!y_2$$

$$R2_{out} = R3_{in} = w!y_1!y_2$$

$$Y_2 = y_1$$

$$R1_{out} = R2_{in} = y_1$$

$$R1_{in} = R3_{out} = Done = y_2$$



	Mealy	Moore
Y_1	1 gate + 3 inputs = 4	6 gates + 12 inputs
Y_2	0 gates + 0 inputs = 0	
$R2_{out} = R3_{in}$	Share with Y_1 = 0	
$R1_{out} = R2_{in}$	Share with Y_2 = 0	
$R1_{in} = R3_{out} = Done$	0 gates + 0 inputs = 0	
Total Cost	4	18