1. Sets

$S = \{1, 2, 3\}$ # sample space
$S = \{height\ of\ students\ in\ the\ classroom\}$
$S = \{x | x \in \mathbb{Z}, 10 \leq x \leq 99\}$
$S = \{a, ab, aab, aaab, ...\}$

A - Event is a subset of the sample set (or full set or empty set ∅)
A' - compliment; all elements in the sample space but not in the event
$A \cap B = A\&B$ # intersection
$A \cup B = A|B$ # union

$(A \cap B)' = A' \cup B'$
$(A \cup B)' = A' \cap B'$

Instead of Venn diagrams for proving these, assign a subset to each interesting Boolean expression and work on those

$|A|$ # size of the set # cardinality of the set
$|A \cup B| = |A| + |B| - |A \cap B|$

Permutations - subsets with order
Combinations - subsets without order
$n_1 * n_2 * ... * n_k = n!$ total outcomes
If need #, can count the opposite case
Can change order of counting
Can consider special cases separately
$\frac{n!}{(n-r)!}$ # permutations of n objects taken r at a time
$\frac{n!}{(n_1!*...*n_k!)}$ # permutations where n1 objects are of kind 1 and are indistinguishable and n2…
$\frac{n!}{r!(n!-r!)} = \binom{n}{r}$ # binomial coefficient **# combinations of n objects r at a time**

2. Probabilities

Independent - don’t influence each other’s probabilities
$P(A \cap B) = P(A) * P(B)$ # product rule. if independent
$P(A \cup B) = P(A) + P(B)$ # sum rule. if independent, else, use Inclusion-Exclusion principle

$P(A) = \frac{|A|}{|B|}$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$, $P(A) > 0$ **can calculate this by hand**

if $P(B|A) = P(B)$ || $P(A \cap B) = P(A)P(B)$, then A and B are independent

If have requirements, intersection combines the requirements

Bayes' rule

$P(A) = P(A \cap B) + P(A \cap B') = P(B)P(A|B) + P(B')P(A|B')$

$P(B_r|A) = \frac{P(B_r | \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)}$

3. Random variables

$f(x) = P(X = x)\ (dis\ only) = F'(x) = F(x) - F(x - 1)$ # probability math/density function

$F(x) = P(X <= x) = \sum_{-\infty}^{\infty} f(x) = \sum_{x}\sum_{y} f(x) = \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x)dxdy$ # cumulative probability/density fn

$F(x) = x\ \ for\ linear\ distribution\ [0, 1]$

$P(a < x < b) = F(b) - F(a) = \sum_{a<x<b} f(x) = \int_{a}^{b} f(x)dx$

$P(x > a) = 1 - F(x)$

$g(x) = \int f(x, y)dy$ (h(y) dx) - marginal distribution **(over range of y) (remember piecewise)**

$f(y|x) = \frac{f(x,y)}{g(x)}$

$f(x|y) = f(x) \leftrightarrow f(y|x) = f(y) \leftrightarrow f(x, y) = g(x)h(y) \leftrightarrow independent$

$if\ f(x|y)\ has\ y,\ then\ not\ independent$

$P(0 < x < 1\,|\,y = 2\,) = \int_{0}^{1} f(x|y)dx$ (substitute for y)

$P(0 < x < 1\,) = \int_{0}^{1} g(h)dx$

$P(X = 1|Y = 1) = f(1|1)$

$\int e^{ax}dx = \frac{e^{ax}}{a} + C$

$\int \frac{1}{x^n}dx = \frac{-1}{(n-1)x^{n-1}} + C$

EXAM

$|A \cup B| = |A| + |B| - |A \cap B|$

Can count opposite case, change order or split cases

$\frac{n!}{(n-r)!}$ # permutations of n objects taken r at a time

$\frac{n!}{(n_1!*...*n_k!)}$ # permutations: n1 objects of kind 1 and identical (and replaced) and n2…

$\frac{n!}{r!(n!-r!)} = \binom{n}{r}$ # binomial coefficient # **combinations of n objects r at a time**

$P(A \cap B) = P(A) * P(B)$ # product rule. if independent

$P(A) = \frac{|A|}{|B|}$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$ **can calculate this by hand**

$P(B|A) = P(B) \Leftrightarrow P(A \cap B) = P(A)P(B)$ ⇔ A and B are independent

If have requirements, intersection combines the requirements

Bayes' rule

$P(A) = P(A \cap B) + P(A \cap B') = P(B)P(A|B) + P(B')P(A|B')$

$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)}$

$P(A) = 1 - P(A')$

$\frac{\binom{1}{1}\binom{8}{2}}{\binom{9}{3}}$ $\frac{total*(total-target)}{total*(left\ after\ 2nd)}$ $\binom{total}{pick}\binom{left}{pick}\binom{rest}{rest}$ $1 - \frac{\binom{4}{3}}{\binom{9}{3}}$ $\#at\ least\ one\ of\ 5\ when\ taking\ 3\ of\ 9$

$f(x) = P(X = x)\ (dis\ only) = F'(x) = F(x) - F(x - 1)$ # probability math/density function

$F(x) = P(X <= x) = \sum_{-\infty}^{\infty} f(x) = \sum_x \sum_y f(x) = \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x)dxdy$ # cumulative probability/density fn

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$f(x|y) = f(x) \leftrightarrow f(x, y) = g(x)h(y) \leftrightarrow independent$

$if\ f(x|y)\ has\ y,\ then\ not\ independent$

$P(0 < x < 1 \mid y = 2) = \int_0^1 f(x|y)dx$ (substitute for y)

$P(0 < x < 1) = \int_0^1 g(h)dx$

$P(X = 1|Y = 1) = f(1|1)$

$\int e^{ax} dx = \frac{e^{ax}}{a} + C$

$\int \frac{1}{x^n} dx = \frac{-1}{(n-1)x^{n-1}} + C$

if

$f(x.y) = \{1/4\ for\ (x, y) = (...), else\ 0\},\ then\ g(x) = \{1/2\ for\ x = 0,\ 1/4\ for\ x \in \{1, -\ 1\},\ 0\ -...\}$