

Derive volumetric coefficient of thermal expansion

Research from “Constructal solar chimney configuration” by Koonsrisuk

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Temp deg C	Density kg/m^3	Cp (kJ/kg.K)	K (W/m.K)	Viscosity x10^-6 (m^2/s)	Expansion coeff $\beta$ x10^-3 (1/K)	Prandtl #
-50	1.534	1.005	0.0204	9.55	4.51	0.725
0	1.293	1.005	0.0243	13.30	3.67	0.715
20	1.205	1.005	0.0257	15.11	3.43	0.713
40	1.127	1.005	0.0271	16.97	3.20	0.711
60	1.067	1.009	0.0285	18.90	3.00	0.709
80	1.000	1.009	0.0299	20.94	2.83	0.708
100	0.946	1.009	0.0314	23.06	2.68	0.703
120	0.898	1.013	0.0328	25.23	2.55	0.70
140	0.854	1.013	0.0343	27.55	2.43	0.695
160	0.815	1.017	0.0358	29.85	2.32	0.69
180	0.779	1.022	0.0372	32.29	2.21	0.69

Source: [http://www.engineeringtoolbox.com/air-properties-d\\_156.html](http://www.engineeringtoolbox.com/air-properties-d_156.html)

Units clarification: at 40 deg C,  $\beta = 3.2 \times 10^{-3} \text{ K}^{-1}$

Volumetric coefficient of thermal expansion has units of 1/K

$$\begin{aligned}\beta \Delta T &= \frac{\Delta V}{V} = \frac{\Delta v}{v} \quad \Delta T = T_1 - T_0 \\ \Delta v &= \beta \Delta T v \\ \Delta v &= v_1 - v_0 = \frac{1}{\rho_1} - \frac{1}{\rho_0} \quad v = \frac{1}{\rho}\end{aligned}$$

v is  $v_0$  and  $\rho$  is  $\rho_0$  from inlet, state 0 is the starting point of the expansion. In Koonsrisuk paper they choose average air density for  $\rho$  (between state 1 and 0) and do not specify what temperature  $\beta$  corresponds to. We don't know the density at state 1 yet, so this method seems to be flawed.)

$$\frac{1}{\rho_1} - \frac{1}{\rho_0} = \frac{\beta \Delta T}{\rho_0}$$

$$\frac{1}{\rho_1} = \frac{\beta \Delta T + 1}{\rho_0}$$

$$\frac{\rho_0}{\rho_1} = \beta \Delta T + 1$$

What if want to take the integral approach?

$$\frac{\Delta V}{V} = \int_{T_0}^{T_1} \beta(T) dT$$

if  $\beta(T)$  was constant,  $\int_{T_0}^{T_1} \beta(T) dT = \beta \Delta T$

$$\text{if } \beta(T) = T^2, \int_{T_0}^{T_1} T^2 dT = \left[ \frac{1}{3} T^3 \right]_{T_0}^{T_1} = \frac{1}{3} (T_1^3 - T_0^3)$$

Need a polynomial fit of  $\beta$  data points between  $T_0$  and  $T_1$

Finding pressure drop, as temp increases pressure decreases, pressure gradient causes mass flow in

$$\Delta P = \Delta \rho g H \quad \Delta \rho = \rho_{T_0} - \rho_{T_0 + \Delta T} = \rho_0 - \rho_1$$

$$\Delta \rho = \rho_0 - \frac{\rho_0}{\beta \Delta T + 1} = \rho_0 \left( 1 - \frac{1}{\beta \Delta T + 1} \right) \approx \rho_0 \beta \Delta T$$

(Approximation less than or equal 1% error, for  $\beta \Delta T \leq 10^{-2}$ , this is far less than the % change of the beta value, meaning should get a solution using an integral  $\beta$  approach)

Q: Why can't I use ideal gas law? Instead of  $\beta$ ?

$$\begin{aligned} PV &= nR_u T & n &= \frac{m}{W_M} & P &= \rho \frac{R_u}{W_M} T \\ P &= \rho R_{\text{specific}} T \\ W_{M, \text{air}} &= 28.97 \frac{kg}{kmol} = 0.02897 \frac{kg}{mol} \\ R_u &= 8.3144621 \frac{J}{mol * K} & P &= 101325 Pa & T &= 273.15K \\ J &= \frac{kg * m^2}{s^2} = N * m = Pa * m^3 & Pa &= \frac{kg}{m * s^2} = \frac{N}{m^2} \\ \rho_{air} &= \frac{P W_M}{R_u T} = 1.2925 \frac{kg}{m^3} \end{aligned}$$

$$\text{units check} \quad \frac{\left(\frac{kg}{m * s^2}\right)\left(\frac{kg}{mol}\right)}{\left(\frac{kg * m^2}{mol * K * s^2}\right)(K)} = \frac{kg}{m^3}$$

A: Because both density and pressure are unknown at state 2, but we could calculate pressure from ideal gas law, after using  $\beta$  to calculate density at state 1, rather than calculating  $\Delta P$  using static fluid pressure equation ( $P_{\text{static fluid}} = \rho g H$ ), but this does not take into account the pressure gradient created by having a chimney full of air at temperature  $T_1$

Integral  $\beta$  approach (want to see error compared to just using constant  $\beta$  corresponding to  $T_0$ )

Using  $\beta$  data at 20°C increments from 0°C to 180°C we create a polynomial best fit line of order 3 with matlab function polyfit()

$$p = [-7.4786e - 11 ; 1.0477e - 07 ; -5.4027e - 05 ; 1.2140e - 02]$$

$$\beta(T) = p(1) * T^3 + p(2) * T^2 + p(3) * T + p(4)$$

(note: T is in units of Kelvin)

Our complete integral approach pressure drop equation with no approximations

$$\Delta P = \Delta \rho g H \quad \text{with} \quad \Delta \rho = \rho_0 \left( 1 - \frac{1}{\int_{T_0}^{T_1} \beta(T) dT + 1} \right)$$

Q: When comparing this to constant  $\beta$  approach,  $\Delta \rho = \rho_0 \left( 1 - \frac{1}{\beta \Delta T + 1} \right)$ , what error can we expect given a temperature range  $\Delta T$

A: when comparing  $\int_{T_0}^{T_1} \beta(T) dT$  vs  $\beta \Delta T$  from a starting point of 20 °C we found at a  $\Delta T = 10 K$  introduced an error of 2%, and  $\Delta T = 40 K$  introduced an error of 7%,

Comment: these errors may or may not seem significant as we continue our analysis

Comment: the way we are calculating pressure change is assuming air is acting like a fluid and not a gas? Because don't assume any density change in column of air

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