**Derive mass momentum energy material-derivative**

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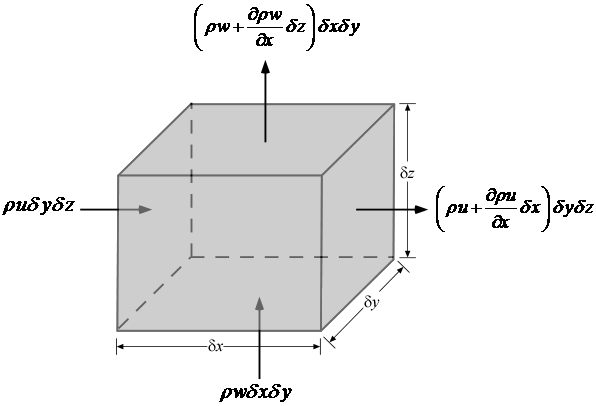
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**Derive continuity equation**

(figure source: <http://pleasemakeanote.blogspot.com/2009/02/derivation-of-continuity-equation-in.html> )



The mass of control volume at time t =

Add up terms

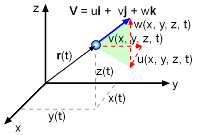
Divide by volume

Form including material derivative D/Dt, non-conservative form (eq3.41 faghri)

**Derive material derivative**

big D is following fluid particle, lagrangian

(figure source: <https://ecourses.ou.edu/cgi-bin/ebook.cgi?doc=&topic=fl&chap_sec=03.4&page=theory> )



Velocity of particle A is a function of location and time

Acceleration is the time rate of change of its velocity, take derivative using chain-rule

Acceleration is the time rate of change of its velocity, take derivative using chain-rule

drop reference to A, because valid for all particles,

General form

**Derive momentum equation**

(following <http://en.wikipedia.org/wiki/Derivation_of_the_Navier–Stokes_equations> )

Reynolds transport theorem, an integral solution relation stating that the sum of the changes of some intensive property (L, does not depend on system size) defined over a control volume (CV) must be equal to what is lost (or gained) through the boundaries of the volume plus what is created/consumed by sources and sinks (Q) inside the control volume.

From RTT we can get generic form of continuity equation

x-direction ()

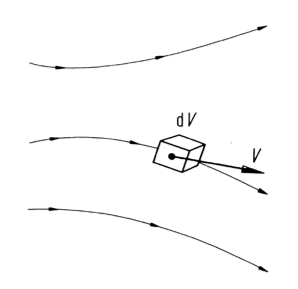
is a source or sink of momentum per volume.

Expanding derivatives

Mass continuity equal to zero

**Derive momentum equation directly from Newton’s 2nd law**

(figure source: http://www.eng.auburn.edu/~tplacek/courses/2610/fluidsreview-1.pdf )



we are using the infinitesimal fluid element approach, where the element moves with the flow field and has a constant mass, but not a constant density as its volume may change.

Newton’s 2nd law states

Since m is a constant property, mass of each element remains constant

for x direction

we are following a moving fluid element therefore time-rate-of-change of u given by material derivative

Conservative form is better for CFD/finite-difference-method/etc, expand material derivative

expand following derivative

expand following derivative

substitute these to get

term in brackets is LHS of continuity equation; hence equal to zero, eq reduces to

pressure, shear, body forces

expand laplace operator

using for effect of viscosity is using incompressible, but is accurate up to 0.3 mach

(source : <http://en.wikipedia.org/wiki/Navier–Stokes_equations> )

This may not make a lot of sense to use incomp viscous term, but then compressible mass equation

need to get from normal stress tensor, why we use that, to the incomp viscosity

(further research: <http://en.wikipedia.org/wiki/Deviatoric_stress_tensor> )

couette flow example, using x-momentum equation

neglect pressure gradient

exact solution

**energy equation**

this is assuming constant heat conductivity across medium

Q: do I use cp or cv? Cv = dU/dT, so probably cv, has to do with when have either constant volume reactor or constant pressure, that enthalpy is only the heat, but internal energy is the heat and pressure energy?

Note: vector identity for divergence of the product of a scalar times a vector

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