Mathematical Statistics
Lab 1

Exercise 1

Common midwife toad (*Alytes obstetricans*) lives in half of the ponds in some area, $P(\text{toad}) = 1/2$. If it is a sunny spring day (soil temperature is 14°C) then an observer can spot the toad with probability 0.6 – if the toad actually lives in a given pond. Look at the graphs below (Tanadini & Schmidt, 2011):

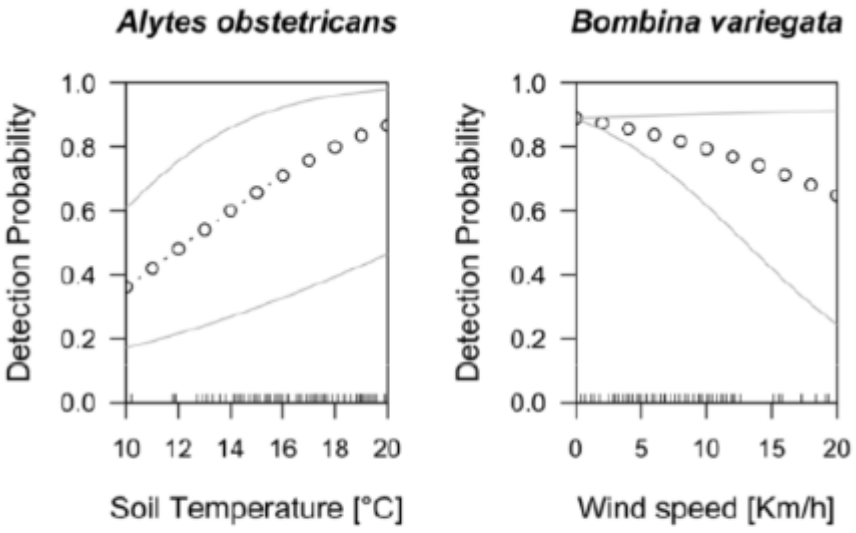


Figure 1. The relationship between meteorological variables and detection probabilities in two anurans. Thin gray lines are 95% confidence intervals. Small ticks inside the box indicate observed soil temperatures and wind speeds, respectively.
doi:10.1371/journal.pone.0028244.g001

If an observer visits a randomly selected pond in this area, what is the probability to observe a toad there?

Exercise 2

Observer visits ponds in a particular area and sees toads in 20% of the ponds in this particular area, $P(\text{sees a toad}) = 0.2$. The soil temperature was 14°C during the survey trip. We assume, that the observer does not see ghosts (if (s)he sees a toad, then a toad is actually there). What is the probability a toad actually lives in a randomly selected pond in this area?

$$P(\text{toad}) = ?$$

Exercise 3

Similar weather (soil temperature 14°C) lasts for several days in spring. Two observers visit separately (independently) ponds in a given area. Find the probability that at least one of them will spot a toad in a pond where a toad actually lives.

Exercise 4

Instead of experienced observers, schoolchildren went for a nature survey. Half of the ponds in this particular area are inhabited with the toads. Unfortunately schoolchildren can sometimes mix up two different species of toads – so sometimes a child may report seeing a common midwife toad in a pond where actually no toads live (with probability 0.1 a child thinks seeing a toad in a pond where actually no toads live). We assume that the soil temperature is still 14°C. What is the probability a child reporting a toad in a randomly selected pond in this particular area?

Exercise 5

Children report seeing toads in 15% of ponds. What is the proportion of ponds actually habited with common midwife toad?

Exercise 6

A supermarket sells light-bulbs from three different companies (A, B, C). Light-bulbs made by company A last at least a year with probability 0.9; company B light-bulbs work a year (or longer) with a probability 0.8 and company C light-bulbs go on working for at least a year with probability 0.4. On the shelf – space dedicated to light-bulbs company A products take up 50%, company B and C products both have 25% of the shelf-space.

What is the probability for me to get a light-bulb that will work at least a year from this supermarket if I just go there and pick a bulb randomly (I do not have any preliminary knowledge about the reliabilities of the light-bulbs).

Hanna

12

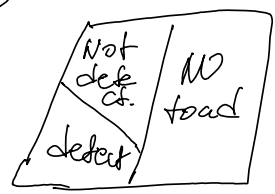
Events:

A = {detect toad at 14°C}

B = {toad lives in pond}

$P(B) = 0,5$

$P(A|B) = 0,6$



$P(A \cap B) = P(B) \cdot P(A|B)$

$= 0,3$

~~A and B are independent events~~

events are dependent

12

$P(A \cap B) = 0,2 \qquad P(A|B) = 0,6$

$P(B) = ?$

$$P(B) = \frac{P(A \cap B)}{P(A|B)} = \frac{0,2}{0,6} = \frac{1}{3}$$

3

$P(A|B) = 0,6$ two observers visit independently pond where toad lives

C – at least 1 observer sees toad

$P(C^*) = P(A^*|B) \cdot P(A^*|B)$

* instructor used $P(A \cap B) = 0,2$ from ex2

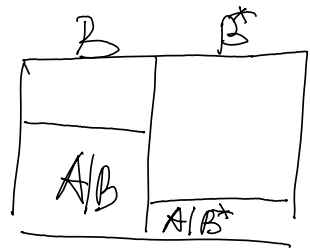
$P(C) = 1 - P(C^*)$

$P(C) = 1 - 0,4^2 = 0,84$

4

B – toad lives in the pond

by total probability theorem



$P(A) = P(B)P(A|B) +$

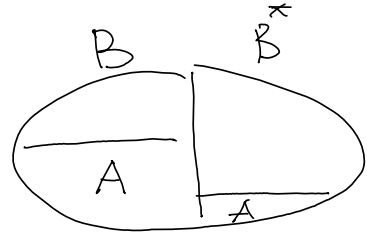
$+ P(B^*)P(A|B^*) = \frac{1}{2} \cdot 0,6 + \frac{1}{2} \cdot 0,1 = 0,35$

5

A - seeing toads

$P(A) = 0,15$

$P(B) - ?$



$P(A|B) = 0,6$

$P(A|B^*) = 0,1$

$$P(B|A) = \frac{P(B)\,P(A|B)}{P(A \cap B)} = \frac{P(B)P(A|B) + P(B^*)P(A|B^*)}{}$$

$$P(A) = P(B)\,P(A|B) + (1-P(B))\,P(A|B^*)$$

$$P(A) = P(B)\left(P(A|B) - P(A|B^*)\right) + P(A|B^*)$$

$$P(B) = \frac{P(A) - P(A|B^*)}{P(A|B) - P(A|B^*)} =$$

$$= \frac{0,15 - 0,1}{0,6 - 0,1} = \frac{0,05}{0,5} = \frac{1}{10}$$

6

$H_1 = \{$ light bulb from company A $\}$

$H_2 = \{$ -11- B $\}$

$H_3 = \{$ -11- C $\}$

$P(H_1) = 0,5$

$P(H_2) = P(H_3) = 0,25$

$A_1 = \{$ light bulb from company A lasts for a year $\}$

$A_2 = \{$ -11- B $\}$ $D = \{$ bulb lasts for a year $\}$

$A_3 = \{$ -11- C $\}$

$P(D|H_1) = 0,9$ $P(D|H_2) = 0,8$ $P(D|H_3) = 0,4$

$$P(D) = P(H_1) \cdot P(D|H_1) + P(H_2) P(D|H_2) + P(H_3) P(D|H_3)$$

$$= 0.5 \cdot 0,9 + 0,25 \cdot 0,8 + 0,25 \cdot 0,4 =$$

$$= 0,45 + 0,2 + 0,1 = 0,75$$