

PS 3.2

118

constituted a similar one
for recitation

24

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$C(A) = c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$N(A) = s_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + s_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Ch 3.2 Worked examples

3.2 A

$$S_1 = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} -2 \\ 0 \\ -6 \\ 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$X = x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 0 \\ -6 \\ 1 \end{bmatrix}$$

23

$$C(A) = c_1 \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

$$N(A) = s \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

R

37

$$-y_1 + y_3 - y_4 = 0$$

$$y_1 - y_2 - y_5 = 0$$

$$y_2 - y_3 - y_6 = 0$$

$$+y_4 + y_5 + y_6 = 0$$

$$\begin{bmatrix} -1 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left[\begin{array}{cccccc} \boxed{-1} & 0 & 1 & 0 & 1 & 1 \\ 0 & \boxed{-1} & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & \boxed{-1} & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad \left[\begin{array}{cccccc} \boxed{1} & 0 & -1 & 0 & -1 & -1 \\ 0 & \boxed{+1} & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & \boxed{+1} & +1 & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$N = \begin{bmatrix} -F \\ I \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$-F$ mit free
not f.
free
 I not f.
free
free

$$y = y_3 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + y_5 \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} + y_6 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

36

$$\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ 0 & 0 & 0 & 0 & 0 \\ 1 & & & & \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} A \\ B \end{bmatrix}$$

$$N(C) = N(A) \cap N(B)$$

$$1 \ 0 \ 0$$

PS 3.3

19

$$\text{rank}(AB) \leq \text{rank}(A)$$

$$AB = I$$

$$\text{rank}(I) = n$$

$$\text{rank}(AB) = \text{rank}(I) = n \Rightarrow \text{rank}(A) = n$$

25

$$A = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & 2 & 6 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

27

a)

$$R = \begin{matrix} \underbrace{\quad}^r & \underbrace{\quad}^{n-r} \\ \begin{matrix} r \\ n-r \end{matrix} \left[\begin{array}{cc} I & R \\ 0 & 0 \end{array} \right] \end{matrix}$$

b)

$$RB = I$$

$$R = [I \ F]$$

$$\underbrace{\begin{bmatrix} I & F \end{bmatrix}}_R \begin{bmatrix} \quad \\ \quad \end{bmatrix}_B = \begin{bmatrix} \square \\ \square \\ \quad \end{bmatrix}_{I_r}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 3 \end{bmatrix}}_R \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$c) \quad \begin{matrix} r & m \\ \left[\begin{array}{cc} I & 0 \\ 0 & 0 \end{array} \right] \end{matrix} \begin{matrix} r \\ n \\ \left[\begin{array}{cc} I & 0 \\ 0 & 0 \end{array} \right] \end{matrix} \begin{matrix} r \\ R \end{matrix} = I_{r \times r}$$

$$d) \quad R^T = \begin{matrix} r \\ \left[\begin{array}{cc} I & F \\ 0 & 0 \end{array} \right]^T \end{matrix} \begin{matrix} r & m-r \\ \left\{ \begin{array}{cc} I & 0 \\ F^T & 0 \end{array} \right\} \end{matrix}$$

$$e) \quad R^T R \begin{matrix} n \\ \left[\begin{array}{cc} I & 0 \\ F^T & 0 \end{array} \right] \end{matrix} \begin{matrix} n \\ \left[\begin{array}{cc} I & F \\ 0 & 0 \end{array} \right] \end{matrix} = \begin{matrix} r & n-r \\ \begin{array}{cc} I & F \\ F^T & F^T F \end{array} \end{matrix}$$

[28] weird

PS 3.4

13

a) When $r=n$ $X=X_p$ or \nexists

b) $AX=b$

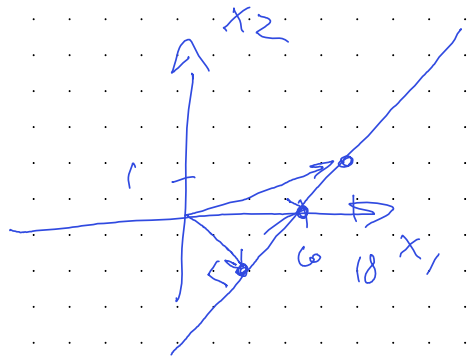
c)

$$\begin{bmatrix} 1 & 2 \\ 5 & 10 \end{bmatrix} x = \begin{bmatrix} 10 \\ 50 \end{bmatrix}$$

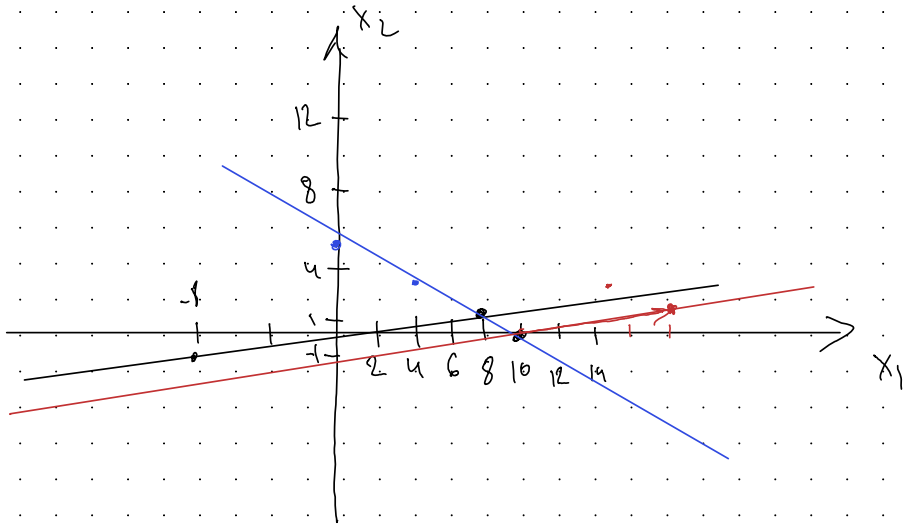
$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} x = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$x_2 = 0 \quad x_1 = 10$$

$$x = \begin{bmatrix} 10 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$



$$x = \begin{bmatrix} 10 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$



28

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 4 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 0 & 4 & 8 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$X = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

35 Done in Matlab

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & & \\ 0 & -1 & 2 & & \\ 0 & 0 & & \ddots & \\ & & & & 2 & -1 \\ & & & & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_9 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ \vdots \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & & \\ 0 & -1 & 2 & & \\ 0 & 0 & & \ddots & \\ & & & & 2 & -1 \\ & & & & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_9 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ \vdots \\ 10 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 2 & -1 & 0 & 0 & 0 & 10 \\ 0 & 1.5 & -1 & & & 15 \\ 0 & -1 & 2 & & & \end{array} \right]$$

30