Mathematical Statistics
Week 4 exercises

1. It is known that the top 5% of applicants (as measured by GRE - Graduate Record Exam scores) will receive scholarships. If GRE $\sim N(500, 100^2)$, how high does your GRE score have to be to qualify for a scholarship?

2. It is known that a family annual income follows a normal distribution $N(25000, 10000^2)$. If the poverty level is 10 000 euros, what percentage of the population lives in poverty?

3. A new tax law is expected to benefit "middle income" families, those with incomes between \$20 000 and \$30 000. If family income $\sim N(25000, 10000^2)$, what percentage of the population will benefit from the law?

4. Let us assume that intelligence index IQ is a random variable with the distribution $N(100, 225)$. A student wants to form an "Smartheads Club" which only admits people with the top 1% of IQ scores. What would the student have to set as the cut-off point in the test to allow this to happen?

5. Suppose two rats A and B have been trained to navigate a large maze (labyrinth). Time of run for rat A follows distribution $N(80, 10^2)$ and time of run for rat B follows $N(78, 13^2)$. On any given day what is the probability that rat A runs the maze faster than rat B?

6. A coin is tossed 100 times. What is the probability that the number of heads is less than 45?

7. History suggests that scores on the Math portion of the Standard Achievement Test (SAT) are normally distributed with a mean of 529 and a variance of 5732. History also suggests that scores on the Verbal portion of the SAT are normally distributed with a mean of 474 and a variance of 6368. Select two students at random. Let X denote the first student's Math score, and let Y denote the second student's Verbal score. What is the probability that the first student's Math score is greater than the second student's Verbal score?

8. Let X_i denote the weight of a randomly selected pre-packaged one-kilo bag of carrots. Of course, one-kilo bags of carrots won't weigh exactly one kilo. In fact, history suggests that X_i is normally distributed with a mean of 1.08 kilo and a standard deviation of 0.07 kilo. Now, let W denote the weight of randomly selected pre-packaged three-kilo bag of carrots. Three-kilo bags of carrots won't weigh exactly three kilo either. In fact, history suggests that W is normally distributed with a mean of 3.12 kilos and a standard deviation of 0.09 kilo.

   Selecting bags at random, what is the probability that the sum of three one-kilo bags exceeds the weight of one three-kilo bag?

9. Bayesian statistics - Another approach to probabilities proposed by Thomas Bayes. The main difference would be that data is considered fixed (as opposed as random like in current course - aka the frequentist approach) and the parameters are random. Also, estimates also incorporate prior knowledge.

   Due to the advances in computing power, the Bayesian statistics is gaining popularity and has found applications in many fields.

   A short video that gives a very good overview and explains the difference between Bayesian and frequentist statistics.

$\boxed{1}$ Find a score for top 5 applicants

Rewrite with correct variable naming

$$GRE \sim N(500, 100^2)$$

$$\overline{GRE} = \frac{GRE - 500}{100} \sim N(500, 100^2)$$

$$\lambda_{.95} = 1{,}645$$

$$GRE = 500 + 100 \cdot \overline{GRE} =$$

$$= 500 + 100 \cdot 1{,}645 = 664{,}5$$

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$\boxed{5}$

Let X = r.v. for time of run rafs A

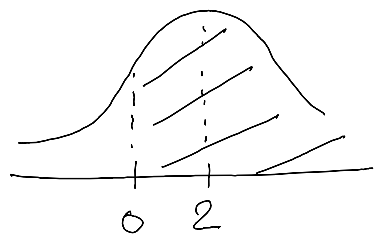
Let Y = -||- rafs B

$X \sim N(80, 10^2)$

$Y \sim N(78, 13^2)$

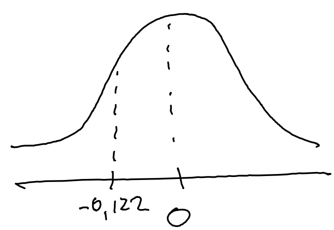
$Z = X - Y \sim N(80-78, 10^2 + 13^2) \sim N(2, \underset{\approx 16.4}{269})$

$P(Z < 0)$ - ?



$Z' = \dfrac{Z - 2}{16.4}$

$Z = Z' \cdot 16{,}4 + 2$



$P\left(Z' < \dfrac{-2}{16{,}4}\right) =$

$= P(Z' < -0{,}122) = \Phi(-0{,}122) =$

$= 1 - \Phi(0{,}122) \approx 1 - 0{,}5478 \approx 0{,}4522$

$\boxed{6}$

$$P = \binom{100}{45} 0{,}5^{45} \cdot 0{,}5^{55}$$

$\mu = np = 50$ $\quad \sigma = \sqrt{Var(npq)} = \sqrt{25} = 5$

$$P = \Phi\left(\frac{44-50}{5}\right) = \Phi(-1{,}2) = 1 - \Phi(1{,}2) =$$

$$= 1 - 0.8849 \approx 0.1151$$

$\boxed{7}$