Mathematical Statistics
Lab 2

Exercise 1

Let's consider traffic accidents and lets define some events:

- death – the driver of a vechicle died because of the accident.
- restraint - driver of a vechicle involved in an accident had the safety belt fastened

For a randomly selected traffic accident the following probabilities hold for the drivers of the car (according to a ten years old research study about traffic in USA):

$$P(restraint) = 0.89$$
$$P(death|restraint) = 0.007$$
$$P(death|norestraint) = 0.04$$

Find the following probability:

$$P(restraint|death) = \ldots$$

Exercise 2

Suppose a random variable X can take the values -1, 0, 1 with corresponding probabilities such as 1/4, 1/2, 1/4. Create the distribution table for random variable X and compute

a) $P(X \leq -1)$,

b) $P(X < 0)$,

c) $F_X(0)$,

d) $F_x(0.5)$,

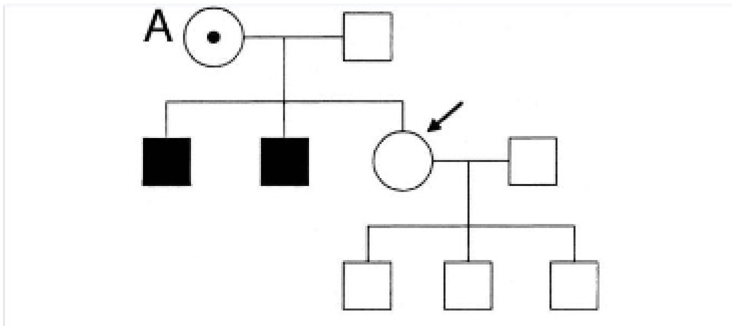
e) EX.

Exercise 3

A worried individual, Jane, comes to a medical genetics centre to seek counselling. She thinks she might be a carrier of a mutation witch causes a serious disease on male offspring (if the mutation is inherited by a male child). There are some real reasons to be worried. Although Jane's parents seem to be healthy, her two brothers suffered from this disease. The disease is known to be a recessive X-chromosome based disease (the mutation causing the disease is situated in the X-chromosome). An example of this type of disease is haemophilia.

1. Given the information – what is the probability the person is a carrier of the bad mutation?

2. We talk with Jane about the weather and her family and discover that she has already 3 healthy male offspring. If one takes into account this additional information – what is the probability for her to be the carrier of the disease (given the information we have)?



3. There exists also a test to test for the mutations causing the disease. But this gene test tests only a set of common mutations that might cause the disease – because it is easier to discover the link between the mutation and the disease for common mutations. Approximately 80% of the disease cases are caused by common mutations (known to science) and in 20% cases the cause of a disease is a rare or novel mutation (unknown causes). It is also known, that specificity of the test is 100%.

   A medical geneticist, instead of asking questions about the persons' children (how are they doing?), proceeds with a gene test. The test yields a negative result. What is the probability of Jane being a carrier given the genetics test she took yielded a negative test result?

Exercise 4

A coin is tossed 10 times. One counts the number of heads out from the 10 tosses – the outcome is denoted by Y. What is the distribution of Y?

$$Y \sim \ldots\ldots$$

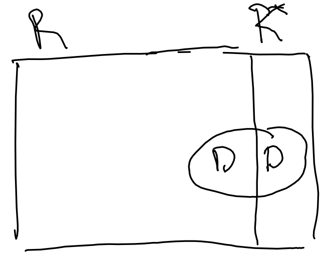
Calculate the probability

$$P(Y = 5) = \ldots\ldots$$

Exercise 5

You buy a Euro Jackpot lottery ticket. In this lottery the goal is to match 5 correct numbers out of 50, plus another 2 supplementary numbers out of another 10. The ̈jackpotön 10th September 2021 will be 48 million € and the ticket costs 2€. What are your winnings on average?

$\boxed{1}$



$P(\text{restraint}) = 0{,}89$

$P(\text{death} \mid \text{restraint}) = 0{,}007$

$P(\text{death} \mid \text{norestraint}) = 0{,}04$

$P(\text{restraint} \mid \text{death}) = ?$

$$P(\text{restraint} \mid \text{deat}) = \frac{P(\text{restraint}, \text{death})}{P(\text{death})}$$

$$P(\text{death} \mid \text{restraint}) = \frac{P(\text{death}, \text{restraint})}{P(\text{restraint})}$$

By total probability theorem:

$$P(\text{death}) = \underbrace{P(R)\,P(D \mid R)}_{P(R,D)} + \underbrace{P(R^*)\,P(D \mid R^*)}_{P(R^*,D)}$$

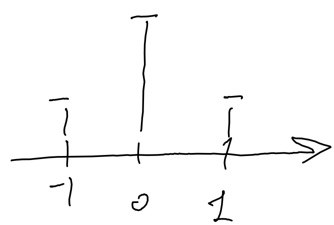
By Bayes' Theorem

$$P(R|D) = \frac{P(R) \cdot P(D|R)}{P(R)P(D|R) + P(R^*)P(D|R^*)} =$$

$$= \frac{0{,}89 \cdot 0{,}007}{0{,}89 \cdot 0{,}007 + 0{,}11 \cdot 0{,}04} = 0{,}586$$

$\boxed{2}$

$$P_X(x) = \begin{cases} 1/4 & \text{if } x = -1 \\ 1/2 & \text{if } x = 0 \\ 1/4 & \text{if } x = 1 \\ 0, & \text{otherwise} \end{cases}$$



a) $P(X \leq -1) = 1/4$

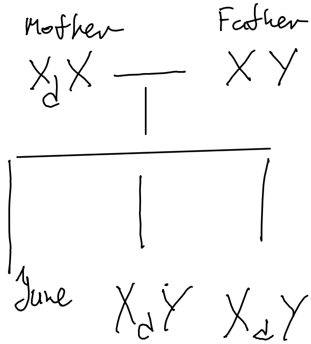
b) $P(X < 0) = 1/4$

c) $F_X(0) = 3/4$

d) $F_X(0{,}5) = 3/4$

e) $E[X] = 0$

$\boxed{3}$ X_d -chromosome

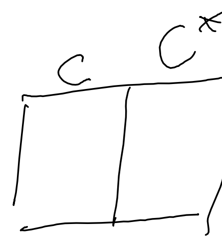


! parents are healthy =>
- father has X chrom. without mutation
- Mother has X chr. without mutation.

1) $P(\{\text{Jane has } X_dX\}) = 0{,}5$

2) H = „Jane has 3 healthy offspring"

C = „Jane has desea

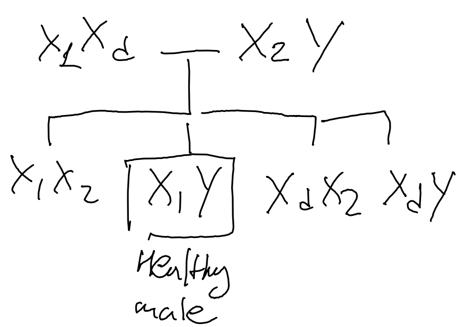


$$P(C|H) = \frac{P(C,H)}{P(H)}$$

$$P(H_1|C) = 0{,}25$$

$$P(H|C) = P(H_1|C)^3$$

$$P(C,H) = P(H|C) \cdot P(C)$$



$$P(C|H) = \frac{P(H|C)\,P(C)}{P(H|C)\,P(C) + P(H|\bar{C})\,P(\bar{C})} =$$

$$= \frac{\left(\frac{1}{4}\right)^2 \cdot \cancel{\frac{1}{2}}}{\left(\frac{1}{4}\right)^3 \cancel{\frac{1}{2}} + \left(\frac{1}{2}\right)^3 \cdot \cancel{\frac{1}{2}}} = \frac{\left(\frac{1}{4}\right)^{\cancel{3}\,2}}{\cancel{\frac{1}{4}}\left(\left(\frac{1}{4}\right)^2 + \frac{1}{2}\right)} = \frac{\left(\frac{1}{2}\right)^{\cancel{4}\,3}}{\cancel{\frac{1}{2}}\left(\left(\frac{1}{2}\right)^3 + 1\right)} =$$

$$= \frac{\frac{1}{8}}{\frac{9}{8}} = \frac{\cancel{8}}{\cancel{8} \cdot 9} = \frac{1}{9} = 0{,}11$$

c) G = „Known disease mutations"

T^+ and T^- = „test pos / negative"

C^+ = „Jane carrier"

$$\text{specificity} = \frac{\text{True negative}}{\text{True Negative + False Positive}} = 100\%$$

$$P(T^+|C^+) = 80\%$$

$$P(T^-|C^+) = 20\%$$

$$P(C^+) = 0{,}5$$

$$P(T^-|C^-) = 1$$

from specificity

$$P(C^+|T^-) = \frac{P(T^-|C^+) \cdot P(C^+)}{P(T^-|C^+)P(C^+) + P(T^-|C^-)P(C^-)}$$

$\boxed{4}$

Y – number of heads out of 10 tosses

$$Y \sim \text{Binomial}(\overset{p=}{0{,}5}, \overset{n=}{10})$$

$$P(Y=5) = \binom{10}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 = \frac{\cancel{10}^{2}\cdot 9\cdot \cancel{8}^{4}\cdot 7\cdot \cancel{6}^{2}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1}\cdot\left(\frac{1}{2}\right)^{10}$$

$$= \frac{\cancel{2}\cdot\cancel{9}^{4,5}\cdot\cancel{7}^{3,5}\cdot\cancel{2}}{2^{\cancel{10}\,\cancel{8}\,6}} = \frac{4{,}5\cdot 3{,}5}{2^6} \approx 0{,}246$$

$\boxed{5}$

Uniform probability distribution

$$\binom{50}{5} = \frac{50!}{5!(50-5)!} = \frac{\cancel{50}^{25}\cdot 49\cdot \cancel{48}^{12}\cdot 47\cdot 46}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} =$$

$$\binom{10}{2} =$$