Permutations P: execute row exchanges

$n!$ - all permutations

$PA = LU$

$P^{-1} = P^T \Rightarrow P^T P = I$

Transpose $(A^T)_{ij} = A_{ji}$

Symmetric matrices $A^T = A$

$R^T R$ ← rectangular is always symmetric

$$\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 11 & 7 \\ 11 & 13 & 11 \\ 7 & 11 & 17 \end{bmatrix}$$

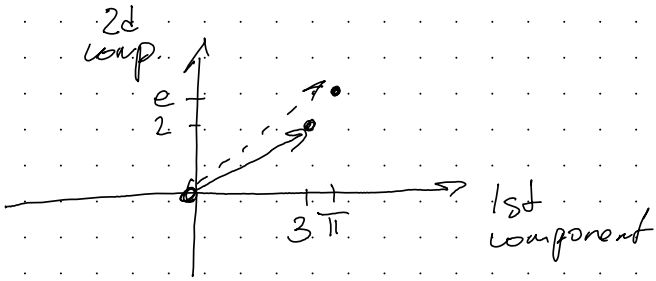
$(R^T R)^T = R^T R^{TT} = R^T R$

Vector spaces

$\mathbb{R}^2$ = all 2d real vectors

($\mathbb{R}$: real numbers)

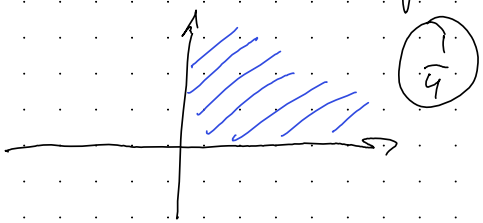
$\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \pi \\ e \end{bmatrix}, \ldots$



$\mathbb{R}^3$ = all 3d (column) vectors (vectors with 3 real components)

$\begin{bmatrix} 3 \\ e \\ 0 \end{bmatrix}$

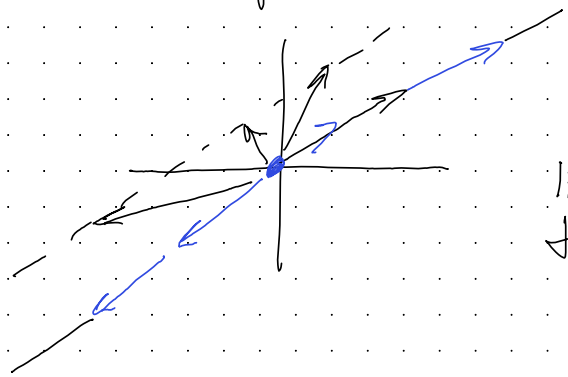
Not a vector space



not closed under multiplication

Example

A vector space inside R^2



subspace of R^2

line in R^2 through 0 vector

Subspaces of R^2

(1) R^2 (2) any line through $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(3) zero

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 4 & 4 \end{bmatrix}$$

columns in R^3

all their linear combinations form a sub space called colum space $C(A)$

Recitation

$$x_1 = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \quad x_2 = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$$

1) V_1, V_2 - line

$V_1 \cap V_2 = \{0\} = Z$

2) $V_3 - \mathbb{R}^2$

$V_1 \cup V_2$ - is two lines?

S - line subspace V_3 e.g. $x_3 = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$

3) $\mathbb{R}^2$