Mathematical Statistics
Lab 4

1. Body mass index (BMI) is a measure of body fat based on height and weight. Assume that it is normally distributed for men and women in given age groups. The mean BMI for men aged 20-25 is 22 with a variance of 49. Calculate 0.5 quantile and 90th percentile of BMI for young men. What do those values mean?

2. Suppose two rats A and B have been trained to navigate a large maze (labyrinth). Time of run for rat A follows distribution $N(80, 10^2)$ and time of run for rat B follows $N(78, 13^2)$.

   a) On any given day, what is the probability that rat A's run is slower than 90 seconds?

   b) On any given day, what is the probability that rat B's run is quicker than 90 seconds?

   c) What is the probability that rat A runs the maze faster than rat B?

3. A coin is tossed 100 times. What is the probability that the number of heads is less than 45?

4. Let X_i denote the weight of a randomly selected pre-packaged one-kilo bag of carrots. Of course, one-kilo bags of carrots won't weigh exactly one kilo. In fact, history suggests that X_i is normally distributed with a mean of 1.08 kilo and a standard deviation of 0.07 kilo.

   Now, let W denote the weight of randomly selected pre-packaged three-kilo bag of carrots. Three-kilo bags of carrots won't weigh exactly three kilo either. In fact, history suggests that W is normally distributed with a mean of 3.12 kilos and a standard deviation of 0.09 kilo.

   Selecting bags at random, what is the probability that the sum of three one-kilo bags exceeds the weight of one three-kilo bag?

5. Male students' heights are normally distributed with expected value 182 and variance 49.

   a) Find the 95%-prediction interval (0,95-prediction interval) for a randomly sampled male student.

   b) Find the 90%-prediction interval for a randomly sampled male student.

   c) Find the 95%-prediction interval (0,95-prediction interval) for a randomly sampled female student's height – the expected value of height for female students is 168, variance is 36.

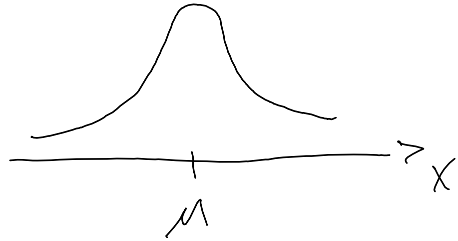
   d) Is your height within the prediction interval?

   e) Find 95% prediction interval for the average of two (randomly selected) female student's heights!

$\boxed{1}$

$\mu = 22$

$\sigma^2 = 49$



$\lambda_{0,5}$ - ?

$\lambda_{0,9}$ - ?

$$P(X > \lambda_{0,5}) = 0,5$$

$$P(X > \lambda_{0,9}) = 0,9$$

$$Y = \frac{X - \mu}{\sigma}$$

$$X = \sigma Y + \mu$$

$$X = 7 \cdot 1,285 + 22 = 30,995$$

$\boxed{2}$

$X_A \sim N(80, 10^2)$

$X_B \sim N(78, 13^2)$

a) rat A slower than 90s

$P(X > 90) = 1 - P(X < 90)$

$P(Y > 1) = 1 - P(Y < 1)$

$$Y = \frac{X - \mu}{\sigma} = \frac{90 - 80}{10} = 1$$

$P(Y < 1) \approx 0,8413$

$P(Y > 1) = 1 - 0,8413$

b) B runs quicker than B

$P(X_B < 90) = P\left(Y_B < \frac{12}{13}\right) \approx 0,8212$

c) Z - rat A runs faster than rat B

$P(Z)$ - ?

$$P(A \leq B) = P(A - B \leq 0)$$

$$A - B = C \sim N(80-78, 10^2 + 13^2) = N(2, 269)$$

$$P\left(Y \leq \frac{0-2}{\sqrt{269}}\right) = P(Y \leq -0{,}122) = 0{,}4562$$

$\boxed{3}$ X - number of heads

$X \sim Bi(100, 0{,}5)$ n is large, $p = 0{,}5$ $\Rightarrow$

* probability is close to 0,5 and n is large we can reuse normal distribution

$$X \sim N(50, 25)$$

$EX = np$ $VX = npq$

$$P(X < 45)$$

$$\Phi(45) = 0{,}1587$$

$\boxed{4}$

$X_i \sim N(1{,}08,\ 0{,}07^2)$

$W \sim N(3{,}12,\ 0{,}09^2)$

$P(X_1 + X_2 + X_3 > W)$

$C \sim X_1 + X_2 + X_3 - W \sim N(1{,}08 \cdot 3 - 3{,}12,\ 3 \cdot 0{,}07^2 + 0{,}09^2) =$

$= N(0{,}12,\ 0{,}0228)$

$P(C > 0) = P\left(Y_c > \frac{0 - 0{,}12}{\sqrt{0{,}0228}}\right) =$

$= P(Y_c > -0{,}79) = 1 - P(Y_c < \quad) =$

$= 1 - 0{,}2148 =$

$\boxed{5}$ X = student height

$$X \sim N(182, 49)$$

$$P(x_{0,025} \leq X \leq x_{0,025}) = 0,95$$

$$P\left(q_{0,025} \leq \frac{X-182}{7} \leq q_{0,975}\right)$$

$$P\left(-1,96 \leq \frac{X-182}{7} \leq 1,96\right)$$

$$-1,96 \cdot 7 + 182 \leq X \leq 7 \cdot 1,96 + 182$$

$$168,28 \leq X \leq 195,72$$

b)

e) $X_i \sim N(168, 36)$

$$\frac{X_1 + X_2}{2} \sim N(168, 18) \qquad V\left[\frac{X_1 + X_2}{2}\right] =$$

$$\frac{1}{4}\left[VX_1 + VX_2\right] =$$

$$= \frac{2 \cdot 36}{4} = 18$$