Proj matrix

$$p = Pb = A(A^TA)^{-1}A^Tb$$

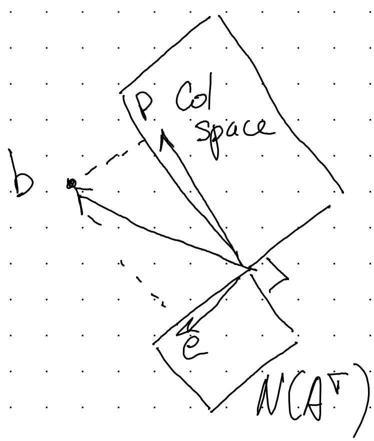
If b in column space $Pb = b$

$$\underbrace{A(A^TA)^{-1}A^TA}_{I}x = Ax = b$$

If $b \perp$ col. space $Pb = 0$

$$A(A^TA)^{-1}\underbrace{A^Tb}_{0}$$

0 becaue b is in $N(A^T)$



Pb $(I-P)b$

$p + e = b$

proj onto $\perp$ space

Least square approximation

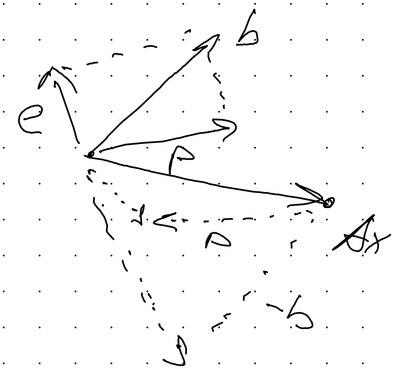
$Ax = b$ has no solution when "too many eq-ns" $(m > n)$

n cols span a small part of m-dimensional space.

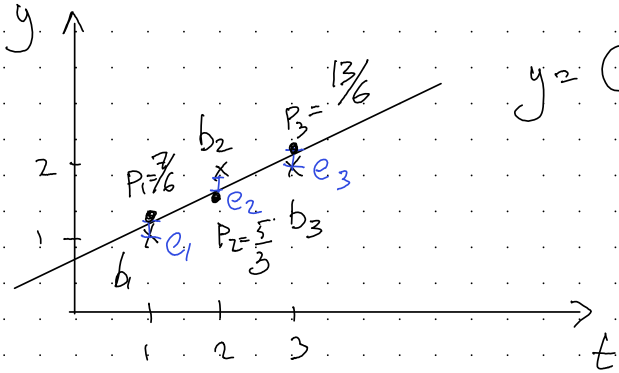
When e is small as possible $\hat{x}$ is a least square solution.

When $Ax = b$ has no solution, multiply $A^T A \hat{x} = A^T b$

$$\|Ax - b\|^2 = \|Ax - p\|^2 + \|e\|$$



Least square



$y = C + Dt$

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} b - p = -1/6 \\ 2/6 \\ -1/6 \end{bmatrix}$$

error vector

$$C + D = 1$$
$$C + 2D = 2$$
$$C + 3D = 2$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

two independent columns, they are bases for col. space, but col. sp. doesn't include b.

Best possible solution, i want to minimize the sum of squares

Minimize

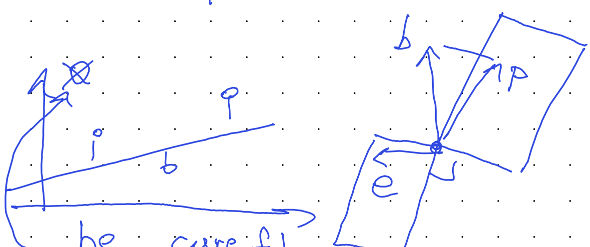
length

$$\|Ax - b\|^2 = \|e\|^2 = e_1^2 + e_2^2 + e_3^2 =$$

Two pictures:

$$= (C+D-1)^2 + (C+2D-2)^2 + (C+3D-2)^2$$

Partial derivative



be careful of outliars

Find $\hat{x} = \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix}$

$$\hat{x} = (A^T A)^{-1} A^T b$$

* Most important equation in stat.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 14 \end{bmatrix}$$

I expect this m. symmetric, invertable

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$$

$A^T A \hat{x} = A^T b$

normal equations

$3C + 6D = 5$

$6C + 14D = 11$

$2D = 1 \qquad D = 1/2$

$C = \frac{2}{3}$

The best line is:

$y = \frac{2}{3} + \frac{1}{2} t$

$b = p + e$

$e_1 = 1/6$

$e_2 = -2/6$

$e_3 = 1/6$

$$\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 7/6 \\ 5/3 \\ 13/6 \end{bmatrix} + \begin{bmatrix} -1/6 \\ 2/6 \\ -1/6 \end{bmatrix}$$

$p \perp e$

Why A^TA is invertable?

Hypotheses: TO prove

If A has independent cols $\Rightarrow$

$\Rightarrow A^TA$ is invertable

Proof:

Suppose $A^TAx=0$ x must be 0

Idea:

$x^TA^TAx=0$

$(Ax)^T(Ax) \Rightarrow Ax=0$

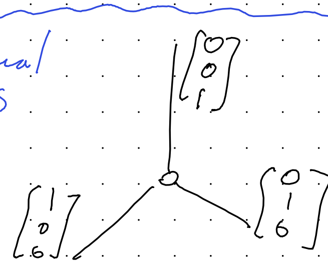
if A has ind. cols then

$x=0$

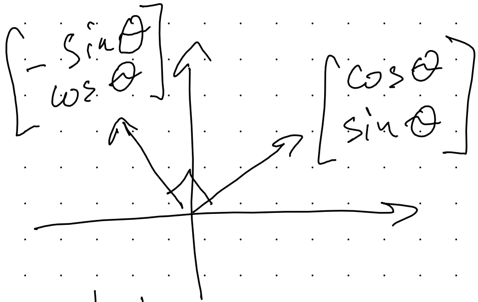
$\square$

Cols def-ly independent if they are perp. unit vectors

orthonormal vectors



if we deeling with orthog-l normal vectors perfect

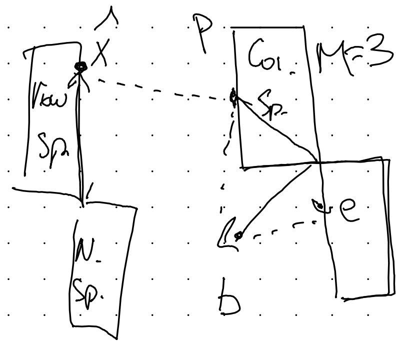


Recitation

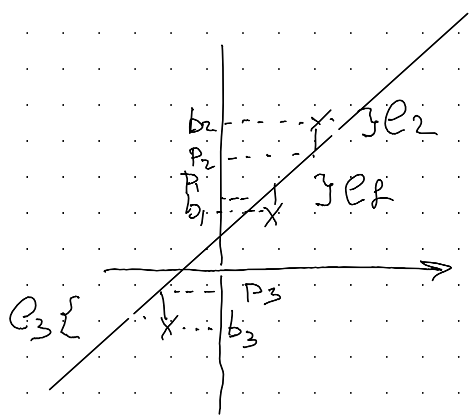
$ax^2 + bx + c$

$c = 0$ because through the origin is given

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 4 & 2 \\ 1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}}_{b}$$



im looking for



$b - p = e$

$A^T(b - p) = 0$

$A^T(b - A\hat{x}) = 0$

$A^T A\hat{x} = A^T b$

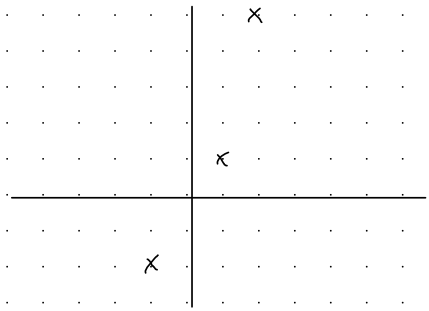
$\hat{x} = (A^T A)^{-1} A^T b$

$A\hat{x} = p$

Recitation

$$y = ax^2 + bx$$

$$a1 + b1 = 1$$
$$a4 + b2 = 5$$
$$a1 + b(-1) = -2$$



$$\underbrace{\begin{bmatrix}1 & 4 & 1\\ 1 & 2 & -1\end{bmatrix}}_{A^T}\underbrace{\begin{bmatrix}1 & 1\\ 4 & 2\\ 1 & -1\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}a\\ b\end{bmatrix}}_{\hat{x}} = \underbrace{\begin{bmatrix}1 & 4 & 1\\ 1 & 2 & -1\end{bmatrix}}_{A^T}\underbrace{\begin{bmatrix}1\\ 5\\ -2\end{bmatrix}}_{b}$$

$$\underbrace{\begin{bmatrix}18 & 8\\ 8 & 6\end{bmatrix}}_{(A^TA)}\underbrace{\begin{bmatrix}a\\ b\end{bmatrix}}_{\hat{x}} = \begin{bmatrix}19\\ 13\end{bmatrix}$$

$$\hat{x} = (A^TA)^{-1} \cdot A^T b$$

$$\hat{x} = \begin{matrix}0{,}2273\\ 1.8636\end{matrix}$$