

## The power function

PF  $h(\theta)$  of a test is the prob-ty of rejecting  $H_0$  when the value  $\theta$  is correct value of the parameter in the parameter space:

$$h(\theta) = P(H_0 \text{ is rejected if } \theta \text{ correct value of the parameter})$$

$$h(\theta) = P(T(X) \in C \mid \theta)$$

So if  $\beta(\theta) = P(\text{fail to reject } H_0 \mid \theta)$   
then  $h(\theta) = 1 - \beta(\theta)$

Properties of the  $h(\theta)$ :

- $0 \leq h(\theta) \leq 1$
- A test is good if  $h(\theta)$  is large  $\forall \theta \in H_1$  (small  $\forall \theta \in H_0$ )
- $\sup_{\theta \in H_0} h(\theta) = \alpha$

## Relationship of $\alpha$ and $\beta$

### Proposition

Suppose an experiment and a sample size are fixed and a test statistic is chosen. Then decreasing the test size of the rejection region to obtain a smaller value of  $\alpha$  results

Using the Normal Approximation