Eigen vector

eigenvetors

Ax parallel to x

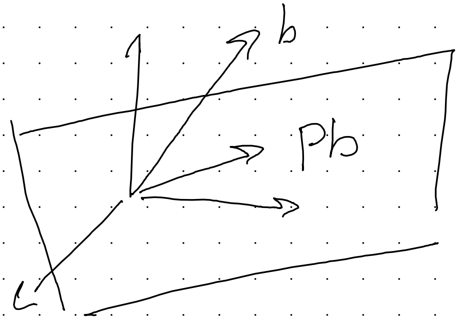
$$Ax = \lambda x$$

eigenvalue

if A is singular, $\lambda = 0$ is eigen value

Let's look at projection m.

What are x's and λ's for projection matrix



Pb is not eigen vector

Any x in the plane would be eigen vector $\lambda = 1$

Any $x \perp$ plane: $Px = 0$ $\lambda = 0$

Let's consider premertation m.

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad x = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \lambda = 1$$

$$x = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \lambda = -1$$

Fact:

sum of λs $= a_{11} + a_{22} + \dots a_{nn}$

How to solve:

$$Ax = \lambda x$$

$$\underbrace{(A - \lambda I)}_{\text{singular}} x = 0$$

$\Rightarrow$ $\det(A - \lambda I) = 0$

Find n λs

Example:

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 - 1 =$$

$$= \lambda^2 - \underset{\substack{\uparrow \\ \text{trace}}}{6}\lambda + \underset{\substack{\uparrow \\ \text{det of } A}}{8}$$

$\lambda_1 = 4$ $\lambda_2 = 2$

$$A - 4I = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \qquad x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A - 2I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Observation if i add $3I$ to matrix its evalues increa but evectors constand if $Ax = \lambda x$

$$(A+3I)x = \lambda x + 3x = (\lambda+3)x$$

Not so great

if $Ax = \lambda x$, B has eigenvalues $\alpha_1, \ldots$

$$\cancel{(A+B)x = (\lambda+\alpha)x}$$

we have no gurantee x is eigenvector for B

Example:
rotation matrix

$$Q = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

rotates $90°$

$\text{trace} = 0 + 0 = \lambda_1 + \lambda_2$

$\det = 1 = \lambda_1 \lambda_2$

$$\det(Q - \lambda I) = \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0$$

$\lambda_1 = i \qquad \lambda_2 = -i$

Example

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 1 \\ 0 & 3-\lambda \end{vmatrix} = (3-\lambda)^2$$

$\lambda_1 = 3 \qquad \lambda_2 = 3$

$$(A - \lambda I)x = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x = 0 \qquad x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

no second independent eigenvector x_2

Recitation

Given invertable m.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 1 & 4 \end{bmatrix}$$

Find eigenvalues and evectors of A^2 and $A^{-1} - I$

$$A^2 = \begin{bmatrix} 1 & 4 & 9 \\ 0 & 1 & 4 \\ 0 & 1 & 16 \end{bmatrix}$$

$$(A^2 - \lambda I)x = 0$$

$$\det(A^2 - \lambda I) = \begin{vmatrix} 1-\lambda & 4 & 9 \\ 0 & 1-\lambda & 4 \\ 0 & 1 & 16-\lambda \end{vmatrix} =$$

$$= (1-\lambda)\left((1-\lambda)(16-\lambda) - 4\right) = 0$$

$\lambda = 1$ $\qquad (1-\lambda)(16-\lambda) - 4 = 0$

$$16 - 17\lambda + \lambda^2 - 4 = 0$$

$\lambda^2 - 17\lambda + 12 = 0$

$D = 17^2 - 4 \cdot 12 = 241$

$\lambda_1 = \dfrac{+17 + \sqrt{241}}{2} \approx 16,262$

$\lambda_2 \approx 0,738$

$\boxed{\lambda_1 = 1}$

$$\begin{bmatrix} 0 & 4 & 9 \\ 0 & 0 & 4 \\ 0 & 1 & 15 \end{bmatrix} x = 0$$

$(A^2 - \lambda I)$

$$\begin{bmatrix} 0 & \boxed{4} & 9 \\ 0 & 0 & \boxed{4} \\ 0 & 0 & 0 \end{bmatrix} x$$

$$X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

9

$\boxed{\lambda_2 = 16,262}$

$$\begin{bmatrix} -15,262 & 4 & 9 \\ 0 & -15,262 & 4 \\ 0 & 1 & 0,262 \end{bmatrix} x = 0$$

smarter solution of the problem d

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 1 & 4 \end{bmatrix} \qquad \det(A - \lambda I) =$$

$$= \begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 1-\lambda & -2 \\ 0 & 1 & 4-\lambda \end{vmatrix} = (1-\lambda)\left((1-\lambda)(4-\lambda)+2\right) =$$

$$= (1-\lambda)\left(\lambda^2 - 5\lambda + 6\right) = (1-\lambda)(\lambda-3)(\lambda-2)$$

$\boxed{\lambda_1 = 1}$ $\qquad$ $\boxed{\lambda_2 = 3}$ $\qquad$ $\lambda_3 = 2$

$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix} x = 0, \qquad \begin{bmatrix} -2 & 2 & 3 \\ 0 & -2 & -2 \\ 0 & 1 & 1 \end{bmatrix} x = 0 \qquad \begin{bmatrix} -1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 1 & 2 \end{bmatrix} x = 0$$

$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} x = 0 \qquad \begin{bmatrix} -2 & 2 & 3 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix} x = 0$$

$$x_1 = \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} \qquad x_2 = \begin{bmatrix} 0{,}5 \\ -1 \\ 1 \end{bmatrix} \qquad x_3 = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

Case when A^2

$Ax = \lambda x$

$AAx = A(\lambda x) = \lambda A x = \lambda^2 x$

hence for A^2 evectors are same and evalues are squared.

$\lambda_1 = 1$ $\quad \lambda_2 = 9$ $\quad \lambda_3 = 4$

Case $A^{-1} - I$

$Ax = \lambda x$

$Ix = A^{-1} \lambda x$

$A^{-1} x = \frac{1}{\lambda} x$

i assume for inverse evalues have power -1.

if we shift A^{-1} by I

$\lambda_1 = \lambda_1^{-1} - 1 = 0$

$\lambda_2 = \frac{1}{3} - 1 = -\frac{2}{3}$

$\lambda_3 = \frac{1}{2} - 1 = -\frac{1}{2}$

* evalues of invertable matrix are allways non zero