Mathematical Statistics
Lab 1

1. Let us take a casting of a single (ordinary six-sided) die (*täring*) as a random event. How many different elementary events are there? How many events can we construct based on these elementary events?

2. What are the chances that at the end of this lab session a female student will leave the classroom first?

3. What is the probability that the sum of two dice is odd?

4. A random point is chosen within a unit square ($S = 1$). What is the probability that this point is below the function line $y = \sin(\pi x)$?

5. Four cards are drawn from a 52-card deck without replacement. Find the probability that the fourth card is from clubs' suit if it is known, that 2 first cards were clubs.

6. Tennis player Bob fails the first serve with 20% probability. If the first serve is successful, then Bob will win the point with 60% probability. Bob fails the second serve with 5% probability, but if the second serve is successful, he will win the point with 20% probability. What is the probability that Bob wins a point?

7. We cast a dice two times. What is the probability that on both throws we got a 6, if...
   - ... we know that at least once we got a 6?
   - ... we know that the first result was 6?

8. In an exam there are 3 examiners. Student Alice wants to take the exam and she knows that with her current knowledge she would pass the exam with a "more gracious" examiner with probability 0.8; with two not so generous examiners with probability 0.4 and 0.4. She does not know who is in which room and the room has to be chosen randomly. What is the probability of Alice passing the exam? What is the probability that after passing the exam that she got the more gracious examiner?

9. How many persons do we need so that with 50% probability we can say that two persons have birthdays on the same day of the year? Assume that all days are equally likely and that there are 365 days in a year.

10. A chessboard is 80x80cm and we throw a coin (with a diameter of 2cm). The central point of the coin will always land somewhere in the 64 squared chess board. What is the probability that the coin will land completely inside a white square?

11. Monty Hall problem.

DONE

$\boxed{1}$ 6 elementary events

$\Omega = \{1, 2, \dots, 6\}$

How many events we can construct based on those elementary events?

$$2^6 - \underset{\substack{\text{elementary}\\\text{events}}}{6}$$

DONE

$\boxed{2}$ Probability that at the end of lab session a female student will leave the classroom first?

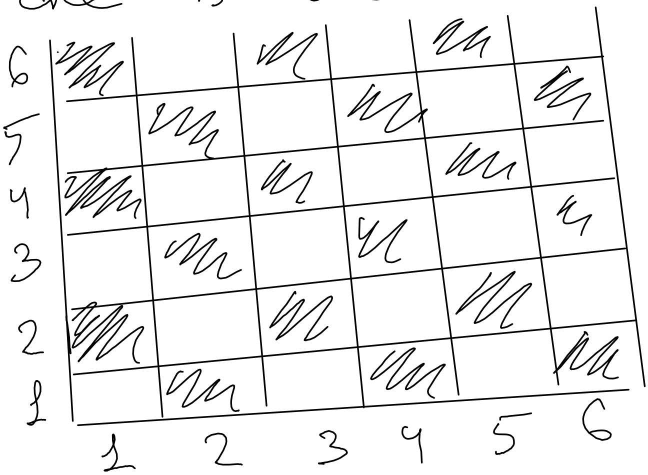
N_f

N_m

$$P\left(\left\{\substack{\text{female}\\\text{leave}\\\text{first}}\right\}\right) = \frac{N_f}{N_f + N_m}$$

DONE

$\boxed{3}$ Prob. that the sum of two dice is odd?

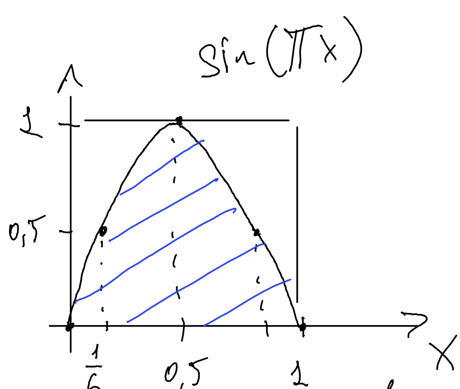


$$P\left(\left\{\begin{matrix}\text{sum of 2}\\ \text{is odd}\end{matrix}\right\}\right) = \frac{1}{2}$$

DONE

$\boxed{4}$

Unit square $(S = 1)$



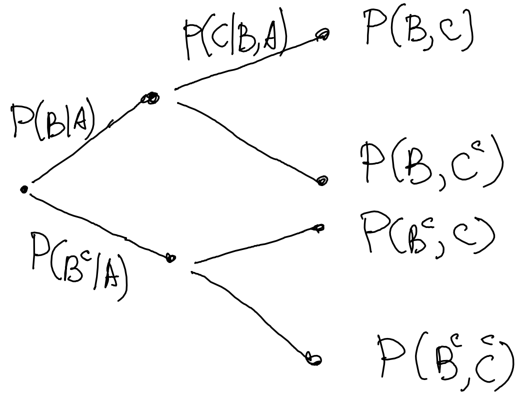
$$P\left(\left\{\begin{matrix}\text{random point is}\\ \text{below function}\\ \sin \pi x\end{matrix}\right\}\right) = \int_0^1 \sin(\pi x)\,dx \Big/ S$$

5 DONE

A = „First 2 card are clubs"

B = „3rd drawn card is club"

C = „4th is club"



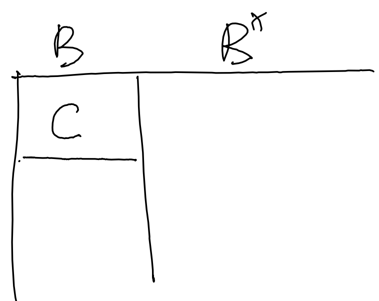
$P(C|A)$ - ?

$$P(B|A) = \frac{11}{50}$$

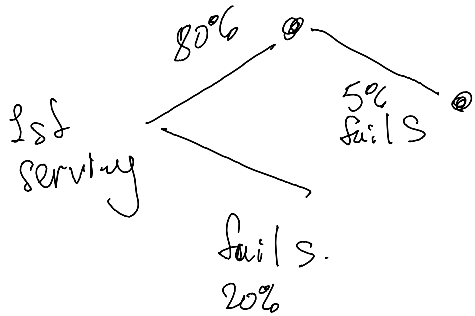
$$P(C|A) = P(B,C|A) + P(B^c,C|A) =$$

$$= P(B|A)\cdot P(C|B,A) + P(B^c|A)\cdot P(C|B^c,A) =$$

$$= \frac{11}{50}\cdot\frac{10}{49} + \frac{39}{50}\cdot\frac{11}{49} = \frac{11\cdot\cancel{49}}{50\cdot\cancel{49}} = \frac{11}{50}$$



$\boxed{6}$



A - first serving successful

B - win the point