$\boxed{\text{Section 5.2}}$

$\boxed{16}$

$$|F_4| = \begin{vmatrix} \boxed{1} & -1 & & \\ 1 & 1 & -1 & \\ & 1 & 1 & -1 \\ & & 1 & 1 \end{vmatrix} = 1 \cdot |F_3| - (-1) \cdot \begin{vmatrix} \boxed{1} & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} =$$

$$= 1\,|F_3| - (-1) \cdot (1) \cdot |F_2|$$

Section 5.3

[8] Find cofactors of A

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 6 & -3 & 0 \\ 3 & 1 & -1 \\ -6 & 2 & 1 \end{bmatrix}$$

Find:

$AC^T = I \cdot \det A$

$$\begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 3 & -6 \\ -3 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

If you chang that 4 to 100 why is $\det A$ unchaged?

(1) Claim if 4 changed to 100 $\Rightarrow$ $\det A$ unchanged.

$$A' = \begin{bmatrix} 1 & 1 & 100 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix} \qquad C' = \begin{bmatrix} 6 & -3 & 0 \\ +195 & -95 & -1 \\ -198 & +98 & 1 \end{bmatrix}$$

$\boxed{\text{Section 6.1}}$

a few questions left to think

$\boxed{19}$ Matrix 3×3 has evalues 0, 1, 2

a) Rank of B? if $\lambda = 0$ then B is singular $\Rightarrow$ rank is either 1 or 2 not 0 because the B is Z, which can't have $\lambda = 1$, $\lambda = 2$ (?)

d) $(B^2 + I)^{-1}$

B^2 has λ^2

when shift by I then $\lambda + 1$

if taking inverse then λ^{-1}

b) determinant of $B^T B$

i need to answer if result is singular?

$$|B^T B| = |B^T|\,|B|$$

$\uparrow$ singular $\Rightarrow 0$

$\boxed{29}$ I don't understan solution for C

Find e values

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ $\quad 1+4+6 = \lambda_1 + \lambda_2 + \lambda_3$

$|A| = 1 \cdot 4 \cdot 6 = \lambda_1 \cdot \lambda_2 \cdot \lambda_3$

Upper triangular m. has its evalues along its diagonal $\Rightarrow$

$\lambda_1 = 1 \quad \lambda_2 = 4 \quad \lambda_3 = 6$

$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$ $\quad \lambda_1 + \lambda_2 + \lambda_3 = 0+2+0$

$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = |B| = -6$

$|B - \lambda I| = -\lambda \cdot (2-\lambda) \cdot (-\lambda) + 1\,(2-\lambda) \cdot (-3) = (2-\lambda) \cdot (\lambda^2 - 3) =$

$= (2-\lambda)(\lambda - \sqrt{3})(\lambda + \sqrt{3}) \quad \lambda_1 = 2 \quad \lambda_{2,3} = \pm\sqrt{3}$

$C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$ $\quad$ C is singular $\Rightarrow$

$\lambda_1 = 0$

$|C - \lambda I| = \begin{vmatrix} 2-\lambda & 2 & 2 \\ 2 & 2-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{vmatrix} =$

$\boxed{\text{Section 6.2}}$

$\boxed{6}$ a) ✓

find all evectors:

$$A = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\det |A - \lambda I| = \begin{vmatrix} 4-\lambda & 0 \\ 1 & 2-\lambda \end{vmatrix} = (4-\lambda)(2-\lambda)$$

$\lambda_1 = 4$ $\quad\quad$ $\lambda_2 = 2$

$x_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\quad\quad$ $x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$x_1' = \begin{bmatrix} 2C_1 \\ C_1 \end{bmatrix}$ $\quad\quad$ $x_2' = \begin{bmatrix} 0 \\ C_2 \end{bmatrix}$

$S = \begin{bmatrix} 2C_1 & 0 \\ C_1 & C_2 \end{bmatrix}$ or vice versa columns

b) $(A)^{-1} = (S \Lambda S^{-1})^{-1}$

$A^{-1} = S^{-1} \Lambda^{-1} S$

All evalues turn into $\frac{1}{\lambda}$

evectors stay the same

$\boxed{16}$

Find Λ and S to diagonalize

$$A_1 = \begin{bmatrix} .6 & .9 \\ .4 & .1 \end{bmatrix}$$

* From markov matrix ⟹ the largest evalue is 1

$$\det|A - \lambda I| = \begin{vmatrix} .6-\lambda & .9 \\ .4 & .1-\lambda \end{vmatrix} = (.6-\lambda)(.1-\lambda) - .4 \cdot .9$$

$$= \lambda^2 - .7\lambda - 0{,}3$$

* From trace
$\lambda_1 + \lambda_2 = a_{11} + a_{22}$
$1 + \lambda_2 = 0{,}7$
$\lambda_2 = -0{,}3$

$\lambda_1 = 1$ $\qquad$ $\lambda_2 = -0{,}3$

$$\underbrace{x_1 = \begin{bmatrix} +2{,}25 \\ 1 \end{bmatrix}}_{\text{Steady state}} \quad \begin{bmatrix} -0{,}4 & .9 \\ .4 & -.9 \end{bmatrix} \qquad x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0{,}9 & 0{,}9 \\ 0{,}4 & 0{,}4 \end{bmatrix}$$

all columns of A^k will approach

$k \to \infty$ $\qquad \Lambda^k \to \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$\Lambda = \begin{bmatrix} 1 & \\ & -0{,}3 \end{bmatrix}$$

What's the limit of $S\Lambda^k S^{-1}$?

$$A^k = S\Lambda^k S^{-1} = \underbrace{\begin{bmatrix} x_1 & x_2 \end{bmatrix}}_{S} \Lambda^k S^{-1} =$$

$$= \underbrace{\begin{bmatrix} x_1 & x_2 \end{bmatrix}}_{S} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} S^{-1} = \begin{bmatrix} x_1 & 0 \end{bmatrix} S^{-1} =$$

$$= \frac{1}{3{,}25} \begin{bmatrix} 2{,}25 & 2{,}25 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0{,}6923 & 0{,}6923 \\ 0{,}3077 & 0{,}3077 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 0{,}3077 & 0{,}3077 \\ -0{,}3077 & 0{,}6923 \end{bmatrix} =$$

$$= \frac{1}{3{,}25} \begin{bmatrix} 1 & 1 \\ -1 & 2{,}25 \end{bmatrix}$$

$\boxed{37}$

left eigen vectors

$$A = S \Lambda S^{-1}$$

$$A^T = (S^{-1})^T \Lambda S^T$$

$$A^T y = \lambda y$$

How do we multiply multiply matrices to find this formula for A?

sum of rank-1 matricies

$$A = S \Lambda S^{-1} = \lambda_1 x_1 y_1^T + \dots + \lambda_n x_n y_n^T$$