(1) Formula for A^{-1}

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ab-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} C^T \longleftarrow \text{ products of } n-1 \text{ entries}$$

cofactor matrix

↖ Products of n entries

Check

$$A \, C^T = (\det A) \, I$$

$$\begin{bmatrix} a_{11} & \to & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} C_{11} & \cdots & C_{n1} \\ \vdots & & \vdots \\ C_{1n} & & C_{nn} \end{bmatrix} = \begin{bmatrix} \det A & & \bigcirc \\ & \det A & \\ \bigcirc & & \det A \end{bmatrix}$$

zeros because

$$A_S = \begin{bmatrix} a & b \\ a & b \end{bmatrix}$$

screw up

$$\det A_S = ab + b(-a)$$

Suppose i have some non singular
m. and i move 1,1 element
(e.g. add 1 to it). What happens
to inverse m. ?

② Second application $\boxed{\text{Cramer's rule}}$

$Ax = b$

$x = A^{-1} b = \frac{1}{\det A} C^T b$

$X_1 = \frac{\det B_1}{\det A}$ ← multiplication of
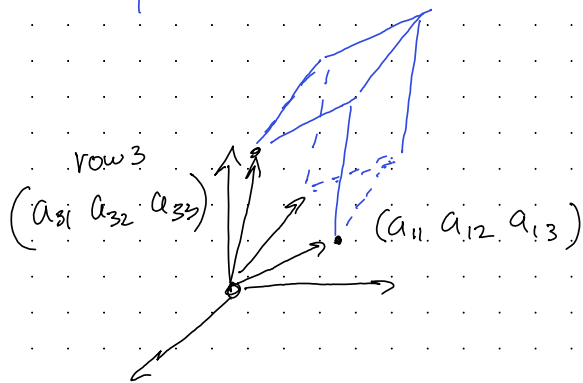b to row of C^T
which produces some
determinant of matrix
B_1

$X_2 = \frac{\det B_2}{\det A}$

$B_1 = \begin{bmatrix} b & \begin{array}{c} n-1 \\ \text{columns} \\ \text{of } A \end{array} \end{bmatrix}$

$B_j = \begin{bmatrix} \text{columns of} \\ A \text{ with} \\ \text{column } j \text{ replaced} \\ \text{with } b \end{bmatrix}$

③ Application

take absolute values if we talk about volume

Claim: $|\det A| =$ volume of box

row 3
$(a_{31} \ a_{32} \ a_{33})$
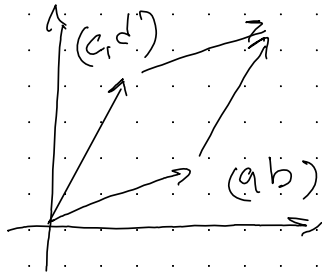
$(a_{11} \ a_{12} \ a_{13})$

Special case $A = I$

Suppos $A = Q$

$$|Q^T Q| = |I|$$

$$\det Q^T \ \det Q = 1$$

$$|Q|^2 = 1$$

If double an edge then volume doubles. $\Rightarrow$ It satisfy prop 3a

(c, d')

$(a b)$

$$\text{area} = \det \begin{bmatrix} A & b \\ c & d \end{bmatrix}$$

Recitation