Def: the space R^n consists of all column vectors v with n components

components of $v \ni \mathbb{R}$

We can add any vectors in R^n, and we can multiply any vector v by any scalar c.

M - vector space of all real value 2by2 m.
F - v.s. of all real functions $f(x)$
Z - v.s that consists of only zero v.

Def: subspace of a vector space is a set of vectors (including 0) that satis-s two requirements: if v and w are vectors in the subspace and c is any scalar, then:

(i) $v+w$ is in the subspace
(ii) cv is in the subspace

A subspace containing v and w must contain all linear comb-s $cv + dw$

Union of 2 subspaces

$P \cup L =$ all vectors in P or L or both

Is not a subspace.

$P \cap L =$ all vectors in both

is subspace

Colum space of A

Def: the CS consists of all lin. comb-s of the columns. The combinations are all possible vectors Ax. They fill the column space $C(A)$.

$$\begin{array}{cc} & n \\ m & \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \end{array}$$

- The system $Ax=b$ is solvable $\Leftrightarrow$ $\Leftrightarrow$ b is in the CS of A
- The CS of A is a subspace of R^m (not R^n).

CS of is subspace of R^4

What's in subspace?

all linear combinations

$Ax=b$ doesn't have solution for every b.

Which b's allow to solve the system?

Can solve $Ax=b$ exactly when b in column space

Are those columns independent?

Pivot columns

Nullspace of A

Nullspace of A = all solutions x to $Ax=0$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \\ b \\ \\ \end{bmatrix}$$ in R^3

NS is line in R^3

Recitation

1) $b = \begin{pmatrix} a \\ b \\ a+b \end{pmatrix}$ b is subspace in $\mathbb{R}^3$

plane

NS

$b = a\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $\quad (1 \;\; 1 \;\; -1)\underbrace{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}}_{\text{b describes a null space}} = 0$

2)

$b_1 b_2 - b_3 = 0$

Let's take $b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ if b spans a subspace then its multiple should be in the same subspace but $b = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ is not.

3) $$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\text{vector is laying on the plane}} + \underbrace{c_1\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}_{\text{span of 2 linearly independent vectors in } \mathbb{R}^3 \text{ is plane}}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ c_1 \\ c_2 \end{bmatrix} = b \quad \Rightarrow \quad \underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}}_{\text{linear span of 2 vectors}} = \begin{pmatrix} b_1 - 1 \\ b_2 \\ b_3 \end{pmatrix}$$

linear combination of 2 and 3 column

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \frac{1}{2} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

hence we can rewrite linear combination

$$b = \left(c_1 + \frac{1}{2}\right) \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \left(c_2 + \frac{1}{2}\right) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

V, subspace is subset of R^3 but it behaves like a vector space itself.

a)

$$c_1 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{\text{is linearly independent vector from vectors } (1,0,-1) \text{ and } (1,0,1)} \neq 0$$

Worked examples

3.1 A

b_1 anb b_2 shuld be in column space of A, but not b_3

3.1 B

- V_2 space has dimention 2, so subspace S is all multiples of one of the bases vectors in V_2. SS is Z.

$u = (1, 2, 1)$ $\quad u \cdot v = 0$

$$v = v_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \quad v_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

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