4 Subspaces

Column space $\quad$ $C(A)$ in R^M

Nullspace $\quad$ $N(A)$ in R^n

Rowspace $\quad$ $C(A^T)$ in R^n

Nullspace of A^T $\quad$ $N(A^T) =$ left null space of A $\quad$ in R^M

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} & & \\ & & \end{bmatrix}$$

$$C(A) \neq C(R)$$

Same row space

Basis for row space is first
r rows of R

$$N(A^T)$$

$$A^T y = 0$$

$$(A^T y)^T = 0^T$$

$$y^T A^{TT} = 0 \qquad \text{left nullspace}$$

$$[A \quad I] \rightarrow [R_{m \times n} \quad E_{m \times m}]$$

$$EA = R$$

Recitation

$$B = \begin{bmatrix} 1 & & \\ 2 & 1 & \\ -1 & 0 & 1 \end{bmatrix} \overset{L}{} \begin{bmatrix} 5 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \overset{U}{} = \begin{bmatrix} 5 & 0 & 3 \\ 10 & 1 & 7 \\ -5 & 0 & -3 \end{bmatrix}$$

dim $C(B) = 2$

$$C(B) = \left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

dim $N(B) = 1$

$$N(B) = x_3 \begin{bmatrix} -\frac{3}{5} \\ -1 \\ 1 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 5 & 10 & -5 \\ 0 & 1 & 0 \\ 3 & 7 & -3 \end{bmatrix} = \begin{bmatrix} 5 & ⑩ & -5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\dim \; C(B^T) = 2$

$$C(B^T) = \left\{ \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$$

$$N(B^T) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} x_3$$