$\boxed{1201}$

$X_1 \sim N(9, 15^2)$
$X_2 \sim N(9, 15^2)$

$$\overline{X} = \frac{X_1 + X_2}{2}$$

$$P(\overline{X} = i) \sim N\left(\frac{1}{2}9 + \frac{1}{2}9, \frac{1}{4} \cdot 15^2 + \frac{1}{4} 15^2\right)$$
$$N(9, 15^2/2)$$

$\boxed{1202}$ $X_1, X_2 \sim Bin(1, p)$

$p_1^* = x_1 \qquad p_2^* = \frac{x_1 + x_2}{2}$

a) $p_1^* : X \to \{0, 1\}$

$p_2^* : X \to \{0, \frac{1}{2}, 1\}$

b) $E[p^*] \overset{?}{=} p$
$E[p_1^*] = E[X_1] = np = 1p$
$E[p_2^*] = E\left[\frac{X_1 + X_2}{2}\right] = \frac{1}{2}\left[E[X_1] + E[X_2]\right] = p$

Let's practice finding Expectation of binomial RV $X \sim Bin(n, p)$

$$E[X] = \sum_{k=1}^{n} k \binom{n}{k} \cdot p^{k} (1-p)^{n-k}$$

$$k \binom{n}{k} = \cancel{k} \frac{n(n-1)\cdot \ldots \cdot (n-k+1)}{\cancel{k}(k-1)\cdot(k-2)\ldots 0} = n \binom{n-1}{k-1}$$

change index summation $j = k-1$

$$= n \sum_{i=0}^{n-1} \binom{n-1}{j} p^{j+1} (1-p)^{n-(j+1)} = np \underbrace{\sum_{i=0}^{n-1} \binom{n-1}{j} p^{j} (1-p)^{\overset{(n-1)-j}{n-j-1}}}_{\text{binomial theorem}}$$

$$= np \cancel{\left(p + (1-p)\right)^{\overset{1}{n-1}}} = np$$

binomial theorem

$(x+y)^n = \binom{n}{k} x^k y^{n-k}$

c)

$$Var(p_1^*) = Var(X_1) = npq = p(1-p)$$

$$Var(p_2^*) = Var\left(\frac{X_1 + X_2}{2}\right) = \frac{1}{4}\left[Var X_1 + Var X_2\right] =$$

$$= \frac{npq + npq}{4} = \frac{1}{2} npq$$

p_2^* more efficient than p_1^*

Prove variance for binomial distn

$$Var(X) = E[X^2] - E[X]$$