$\boxed{1}$

$$\neg(\exists x \in \mathbb{R}\ \forall y \in \mathbb{R},\ x^2y > x \Rightarrow xy = 1) \equiv$$
$$\equiv \forall x \in \mathbb{R}\ \exists y \in \mathbb{R},\ \neg(x^2y > x \Rightarrow xy = 1) \equiv$$
$$\equiv \forall x \in \mathbb{R}\ \exists y \in \mathbb{R},\ x^2y > x \land xy \neq 1$$

Consider a negation statement

- if $x = 0$ then $0 \cdot y > 0$ is false for any y therefore conjunction is false and we can conclude ~~that~~ negation statement is false. Original statement is true

$\boxed{2}$

$S =$ student studies
$P =$ student has a party
$H =$ student is happy
$T =$ S. talks a lot

Facts:
a) $(S \land P) \Rightarrow (H \lor T)$
b) $(\neg S \land H \lor T) \Rightarrow P$
c) $P \Rightarrow H$

Given H can we say P ?

Fact c: doesn't allow us to make conclusion $H \Rightarrow P$ by implication truth table

Fact b: $(\neg S \land H \lor T)$ by precedence rule we do not depend on T, but we depend on S. If we assume $S \overset{?}{=} 0 \Rightarrow (\neg S \land H) = 1 \Rightarrow P$
If $S = 1$ we can't conclude P

Fact a: H given 1 then $H \land T = 1$

similar to fact c we cannot draw a conclusion on P by implication truth table

3 $(\neg X \Rightarrow Y) \Leftrightarrow X \wedge Y \equiv$

by 9a

$\equiv \neg(\neg X \wedge \neg Y) \Leftrightarrow X \wedge Y \equiv$

remove by 7 double negation

by 11a

$\equiv \neg(\neg X \wedge \neg Y) \wedge X \wedge Y \vee \neg\neg(\neg X \wedge \neg Y) \wedge \neg(X \wedge Y) \equiv$

DM law

$\equiv (X \vee Y) \wedge X \wedge Y \vee \neg X \wedge \neg Y \wedge (\neg X \vee \neg Y) \equiv$

by 5a

$\equiv X \wedge Y \vee \neg X \wedge \neg Y \equiv$

DM law

$\equiv \neg(\neg(X \wedge Y) \wedge \neg(\neg X \wedge \neg Y))$

4

$$\mathcal{F} \equiv (X \overset{1}{\vee} Y) \overset{5}{\Rightarrow} (Y \overset{2}{\Rightarrow} Z) \overset{4}{\wedge} (X \overset{3}{\Rightarrow} Z)$$

	X	Y	Z	$(X \vee Y)$	$\Rightarrow$	$(Y \Rightarrow Z)$	$\wedge$	$(X \Rightarrow Z)$
→	1	1	1	1	1	1	1	1
	1	1	0	1	0	0	0	0
→	1	0	1	1	1	1	1	1
	1	0	0	1	0	1	0	0
→	0	1	1	1	1	1	1	1
	0	1	0	1	0	0	0	1
→	0	0	1	0	1	1	1	1
→	0	0	0	0	1	1	1	1

$\mathcal{F}$ is true in only those five evaluations

$\mathcal{F} \equiv X \wedge Y \wedge Z \vee X \wedge \neg Y \wedge Z \vee \neg X \wedge Y \wedge Z \vee \neg X \wedge \neg Y \wedge Z \vee \neg X \wedge \neg Y \wedge \neg Z$