Lab 7

Exercise 1

a) A car manufacturer advertises a car that uses 4.7 litres of fuel per 100km. Let μ be the mean fuel consumption for this model. You assume that the dealer will not overrate the consumption, but suspect he may underrate the consumption
- What can be used for H_0?
- What can be used for H_1?

b) A company that manufactures ball bearings claims the average diameter is 6 mm. To check that the average diameter is correct, the company decides to formulate a statistical test.
- What can be used for H_0?
- What can be used for H_1?

c) A consumer group wants to test the truth in a package delivery company's claim that it takes an average of 24 hours to deliver a package. Complaints have led the consumer group to suspect the delivery time is longer than 24 hours.
- What can be used for H_0?
- What can be used for H_1?

Exercise 2

Jack has a biased coin, which he uses to make some money during gambling events. The coin is identical to a regular one (looks and weighs the same), but the probability of getting heads on a coin flip is 1/5. Unfortunately, a regular unbiased coin also got into Jack's pocket. Jack takes a coin from his pocket (he doesn't know whether it is the biased or the unbiased one) and decides to test the following hypotheses:

H_0: the coin is unbiased, P(heads) = 1/2,

H_1: the coin is not unbiased, P(heads) $\neq$ 1/2.

Jack flips the coin from his pocket 10 times, and he gets heads 8 times. Use the number of heads in 10 trials as a test statistic and find the p-value. These tables will help you:

Table 1: x – number of heads; P(X=x) – probability of seeing heads x times if the biased coin is flipped 10 times

x	0	1	2	3	4	5	6	7	8	9	10
P(X=x)	0.107	0.268	0.302	0.201	0.088	0.026	0.006	0.001	0.000	0.000	0.000

Table 2: x – number of heads; P(X=x) – probability of seeing heads x times if the unbiased coin is flipped 10 times

x	0	1	2	3	4	5	6	7	8	9	10
P(X=x)	0.001	0.010	0.044	0.117	0.205	0.246	0.205	0.117	0.044	0.010	0.001

(a) Find the p-value.
(b) Decision of the test (if we use 0.05 as the level of significance).

(c) Find the probability that it is an honest coin (the probability, that H_0 holds) on the condition that we saw 8 heads in 10 trials a.k.a. P (H_0 | 8 heads in 10 trials).

(d) Test the following hypotheses:

H_0: the coin is biased, P(heads) = 1/5,

H_1: the coin is not biased, P(heads) $\neq$ 1/5.

What is the p-value?

Decision?

(e) What would have the p-value been, if Jack had got heads 3 times (instead of 8) in 10 tosses?

p-value for H_0: the coin is unbiased

p-value for H_0: the coin is biased

Exercise 3

We know a scientist made a type I error.

a) Is the null hypothesis actually true or not?
b) Was the p-value scientist obtained smaller or larger than significance level (0,05)?

One of the statements below is true, the other is false. Which one is false, and which is true (why)?

Statement 1: If the p-value is smaller than significance level then one has proven the alternative hypothesis to be true.

Statement 2: If the p-value is greater or equal to significance level then one has proven the null hypothesis to be true.

Mathematical Statistics
Week 7 exercises

Exercise 4*

A manufacturer of sprinkler systems used for fire protection in office buildings claims that the true average system-activation temperature is 130°. A sample of $n = 9$ systems, when tested, yields a sample average activation temperature of 131.08°F. If the distribution of activation times is normal with standard deviation 1.5°F, does the data contradict the manufacturer's claim at significance level $\alpha = 0.01$?

Exercise 5*

A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at $\alpha = 0.05$?

$\boxed{2}$ H_0 : coin is unbiased

H_1: the is not unbiased

$P(X \geq 8 \cup X \leq 2 \mid H_0 \text{ is true}) = P(X = 8 \mid H_0) + \ldots$

$+ P(X = 9 \mid H_0) + \ldots = 0{,}001 + 0{,}01 + 0{,}044 +$

$+ 0{,}044 + 0{,}01 + 0{,}001 = 0{,}11$

b) $0{,}11 > 0{,}05 \Rightarrow H_0$ not reject

c) $P(H_0 \mid 8h \text{ out of } 10 \text{ trials}) =$

$$= \frac{P(X=8 \mid H_0) \cdot P(H_0)}{P(X=8)} = \frac{P(X=8 \mid H_0)\, P(H_0)}{P(X=8 \mid H_0) \cdot P(H_0) + P(X=8 \mid H_1) P(H_1)} =$$

$$= \frac{1/2 \cdot 0{,}044}{1/2\,(0{,}044 + 0)}$$

d) H_0 : the coin is biased

H_1 : the coin is unbiased

$$P(X \geq 8 \mid H_0) = 0$$

$0 < 0{,}05$ reject H_0

e) 3 heads

H_0 unbiased

$$P\left(\begin{matrix} X \leq 3 \cup \\ X \geq 7 \end{matrix} \middle| H_0\right) = (0{,}001 + 0{,}01 + 0{,}044 + 0{,}117) \cdot 2 = 0{,}344$$

H_0 biased

$$P(X \geq 3 \cup X = 0 \mid H_0) = 0{,}107 + 0{,}201 + 0{,}088 + 0{,}026 + 0{,}006 + 0{,}001 = 0{,}429$$

$\boxed{3}$

a) H_0 is true

b) p value obtained is smaller than significance level 0,05

Statement 1 → true

St-2 → false

$\boxed{4}$

$n = 9$ $\qquad$ $X \sim N(m, D^2)$

$\bar{x} = 131,08$

$D = 1,5$

$\alpha = 0,01$

$I_m = \bar{X} \pm \lambda_{\alpha/2} D = 131,08 \pm 2,575 \cdot 1,5$

$=$

$H_0 : \mu = 130$

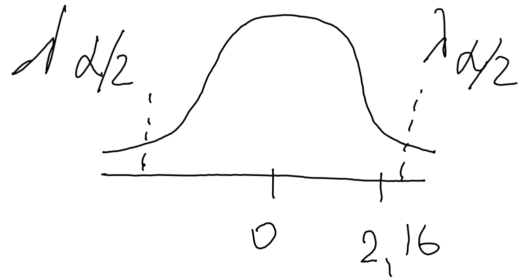
$H_1 : \mu \neq 130$

$X \sim N(\underbrace{\mu_0}_{=130}, \underbrace{\sigma^2}_{=1,5^2})$

$\bar{X} \sim N(\mu_0, \frac{\sigma^2}{n})$

$U = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} =$ $\qquad$ $U \sim N(0,1)$

$$u = \sqrt{9}\,\frac{131{,}08 - 130}{1{,}5} = 1{,}08 \times 2 = 2{,}16$$



$$\lambda_{\alpha/2} = \pm 2{,}575$$