1.1 Vectors and linear combinations

two dimentional vector

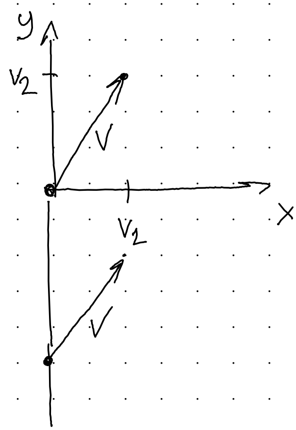
$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ $\qquad 2v$

column vector

component

scalar

LA is buit on top of adding vectors and multiplying by scalars



$\left.\begin{matrix} v+w \\ v-w \\ cv \end{matrix}\right\}$ computed components at a time

By combining these operations we form <u>linear combinations</u>

$cv + dw$ – linear combination

$v + w$ when $c = d = 1$

cv when $d = 0$

Soon i will look in all linear combinations → plane of vectors

$cv + dw = 2 \times \begin{bmatrix} 4 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Column vector is same as arrow goes from origin to the point where vector ends

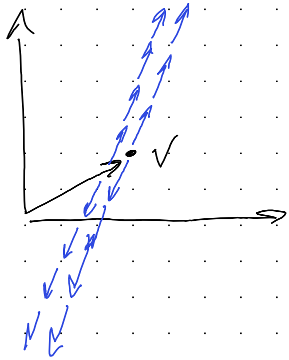
$v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

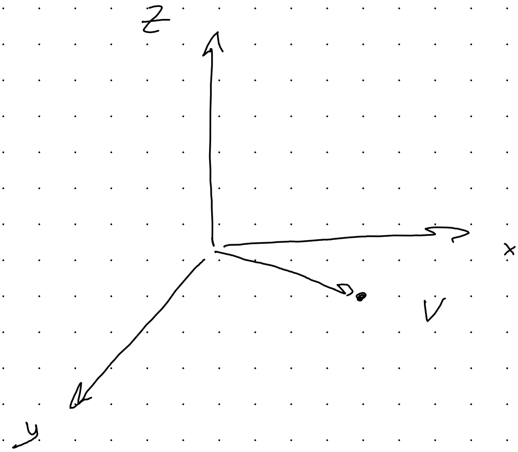
$v = (1, 2, 3)$

Dont mix up with row vector, which is column vector transpose $[1, 2, 3]$

What surface in 3d space do you get from all linear combinations of u v.

In 2d should be a plane


V


z
x
y
V

1.2. Length and dot product

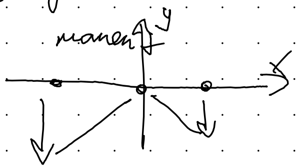
dot (inner) product

$$V = (v_1, v_2) \qquad w = (w_1, w_2)$$

$$V \cdot w = v_1 w_1 + v_2 w_2$$

for dot product 0 means two vectors are perpendicular.

examples



dot product engineering

economics

p – prices

q – buy / sell

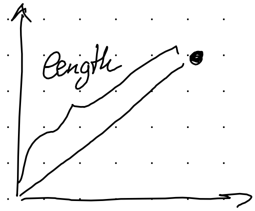
$$p \cdot q = 0$$

Подумай о software engineering

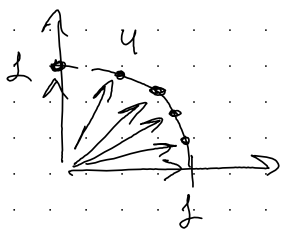
Я думаю & как это программист это уже инструменты. Программисты давно вымерли остались учёные и инженеры. Задача программиста перевести текст в код.

length (norm)

$\|V\| = \sqrt{v \cdot v}$



Unit vector u is vector whose length equals one $u \cdot u = 1$



Divide any non zero vector v by its length $u = \frac{v}{\|V\|}$

* any 3 dimentional vector is a linear combination of i j k

Unit vectors correspond to points on unit sphere.

1.3 Matrices

Combination

$$c x_1 + d x_2 + e x_3 = \begin{bmatrix} c x_1 + d x_2 + e x_3 \\ \cdots \\ \ \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} c \\ d \\ e \end{bmatrix} = -\|-$$

$$A x = b$$

which combination produces b?
(system of linear equations)

This matrix A is "invertable" From b we can recover x

$b = Ax$ and $x = Sb$

$$Ax = b \quad \text{and} \quad x = A^{-1}b = Sb$$

in calculus

$$\frac{dx}{dt} = b \quad \text{and} \quad x(t) = \int_0^t b$$

Cyclic differences

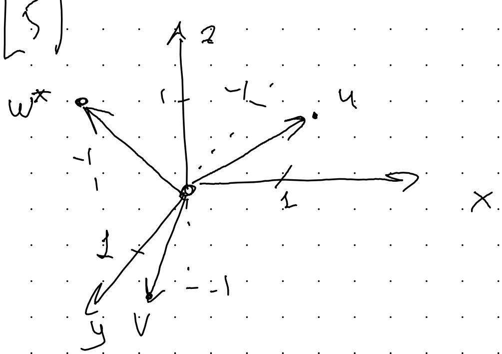
$$u = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \quad w^* = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$Cx = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - x_3 \\ x_2 - x_1 \\ x_3 - x_2 \end{bmatrix} = b$$

not triangular

$b = 0 \Rightarrow$ inf many solutions
$X = C$

if $b = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \Rightarrow$ no solutions



Independence and Dependence

u and v all combinations give a 2d plane

if the third vector w^* is in the plane:

w is not in the plane → independence

w^* is linear combination of u and v

$u + v + w^* = 0$

u, v, w independent if no combination except $0u + 0v + 0w = 0$ gives $b = 0$

u, v, w^* other combinations give $b = 0$

Independend columns $Ax = 0$ has one solution A is invertable

Dependent columns $Ax = 0$ has many solutions A is singular matrix

Worked examples

1.3. A

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \Big[$$