~ B is not compleatly done

1) Let $Z$ be a continuous RV with PDF

$$f_Z(z) \begin{cases} \gamma(1+z^2) & \text{if } -2 < z < 1 \\ 0 & \text{otherwise} \end{cases}$$

a) for what value of $\gamma$ it is posible?

$$\int_{-2}^{1} (1+z^2) = 6 \Rightarrow \gamma = \frac{1}{6}$$

b) * i mixed up variable X and Z should be Z.

$$F_X(x) = P(X \leq x) = \int_{-2}^{x} \underbrace{\frac{1}{6}(1+z^2)}_{f_Z(x) - \text{PDF}} dz$$

$$= \frac{1}{6}\left[\frac{z^3}{3} + z\right]\Bigg|_{-2}^{x} = \frac{1}{6}\left[\frac{x^3}{3} + x - \frac{-8}{3} + 2\right] =$$

$$= \frac{1}{6}\left[\frac{x^3}{3} + x + \frac{14}{3}\right]$$

$$-2 \leq z \leq 1 \quad \int_{-2}^{z} \frac{1}{6}(1+y^2)\,dy = \frac{1}{6}\left[y - \frac{1}{3}y^3\right]_{-2}^{z} =$$

$$= \frac{1}{6}\left[z + \frac{1}{3}z^3 + \frac{14}{3}\right.$$

$\boxed{2}$

A - taxi is waiting

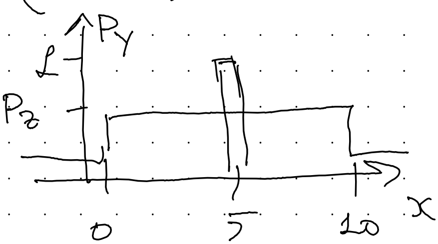
$P(A) = 2/3$

Z - taxi arives in 0 to 10 min
uniform distribution

Y - bus arrives in 5 min

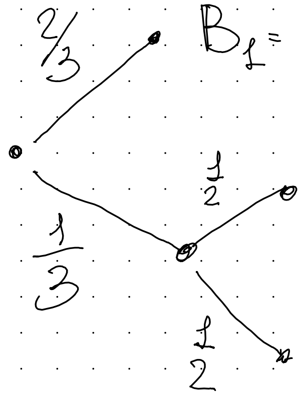
Find CDF of Al's waiting time

$$F_X(x) = P(X \le x) = P(A)\,P(A \le X) + (1 - P(A)) \cdot P(Z \le x)$$




$$F_X(x) = \begin{cases} \frac{2}{3} & \text{if } X = 0 \\ \frac{2}{3} + \frac{1}{3} \cdot \int_0^x \frac{1}{10}\,dx & \text{if } 0 < X < 5 \\ 1 & \text{if } X \ge 5 \end{cases}$$

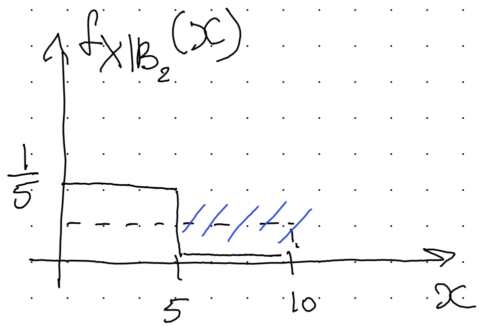
Solution From video recitation:




$B_1 = \{$Al catches waiting taxi$\}$

$B_2 = \{$Al catches next taxi in $[0,5]$ min$\}$

$B_3 = \{$Al catch bus @ $t=5\}$




$$E[X] = P(B_1)\cdot E[X_1|B_1] + P(B_2)E[X/B_2] +$$

$$+ P(B_3)\,E[X|B_3] = \frac{2}{3}\cdot 0 + \frac{1}{3}\,\frac{1}{2}\,\frac{5}{2} + \frac{1}{3}\cdot\frac{1}{2}\cdot 5$$

$$= \frac{5}{12} + \frac{5}{6} = \frac{15}{12}$$

$\boxed{3}$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

a) $F_X(x) = \int_0^x f_X(x)\,dx = -e^{-\lambda z} \Big|_0^x =$

$= -e^{-\lambda x} + 1$

b) $E[X] = \int_0^{+\infty} x\, f(x)\,dx = \frac{1}{\lambda}$

c) $Var(X) = \frac{1}{\lambda^2}$

d) $X_1$ $X_2$ $X_3$ independent exp. RV

with $\lambda$

$$Z = \max\{X_1, X_2, X_3\}$$

$f_Z(z)$ - ?

$$F_Z(z) = P(Z \leq x) = P(X_1 \leq x, X_2 \leq x, X_3 \leq x)$$

by indep.

$$= P(X_1 \leq x)\, P(X_2 \leq x)\, P(X_3 \leq x)$$

$$f_Z = \frac{dF_Z}{dx}(x)$$

$$f_Z = \frac{(1 - e^{-\lambda x})^3}{dx} = 3(1-e^{-\lambda x})^2 \cdot (+\lambda e^{-\lambda x})$$

e) $W = \min\{X_1, X_2\}$

$f_W$ - ?

$F_W(x) = P_W(W \leq x)$