Mathematical Statistics
Exam

1. (10p) Definition of mean square error of an estimator. Describe the relation with the variance of an estimator and give the proof.

2. (5p) Definitions of Type I error and Type II error. Which type of error we would like to decrease in hypothesis testing and explain why.

3. (10p) For parameter θ there are proposed two estimates, $\widehat{\theta}_1$ and $\widehat{\theta}_2$. Nothing is known about these estimates. Describe, how we could find out (how or on what basis we could compare them), which of these two proposed estimates is better for parameter θ.

4. (10p) Let us have a random sample $x_1, x_2, \ldots, x_n$ from $N(\mu, \sigma^2)$, where μ is unknown. State and prove the theorem about confidence interval for the mean, for both cases when σ is known and σ being unknown.

5. (10p) Assume that we have a model $X_i \sim f(x_i|\theta)$. Clearly identify the following steps in least square estimation method. What means the generalization of the least square method?

6. (5p) State the theorem about the relation of Chi-square distribution and Normal distribution. Where is Chi-square distribution used in inferential statistics?