$\boxed{1}$ $A = \{12k + 11 \mid k \in \mathbb{Z}\}$

$B = \{4j + 3 \mid j \in \mathbb{Z}\}$

a) is $A \subset B$?

If $A \subset B$ then $\forall k \in \mathbb{Z}$ $\exists j \in \mathbb{Z}$ such that $12k + 11 = 4j + 3$

$$12k + 11 = 4j + 3$$
$$4j = 12k + 8$$
$$j = \underbrace{3k + 8}_{\in \mathbb{Z}}$$

We proved an existance of integer j for every interger k. Therefor we proved $A \subset B$

b) is $B \subset A$?

if $B \subset A$ then $\forall j \in \mathbb{Z}$ $\exists k \in \mathbb{Z}$ such that $12k + 11 = 4j + 3$

$$12k = 4j - 8$$

$$k = \frac{j - 8}{3}$$

We can show that $\frac{j-8}{3}$ can be non integer if we take $j = 9$ $\frac{9-8}{3} = \frac{1}{3} \approx 0{,}3(3)$

therefor we proved $B \not\subset A$

$\boxed{2}$ $A = \{\emptyset, \{b, \emptyset\}\}$

$B = \{1, \emptyset, \{\emptyset\}\}$

$A \setminus B = \{\{b, \emptyset\}\}$

$(A \setminus B) \times A = \{(\{b, \emptyset\}, \emptyset), (\{b, \emptyset\}, \{b, \emptyset\})\}$

$\mathcal{P}((A \setminus B) \times A) = \{\emptyset, \{(\{b, \emptyset\}, \emptyset)\},$
$\{(\{b, \emptyset\}, \{b, \emptyset\})\},$
$\{(\{b, \emptyset\}, \emptyset), (\{b, \emptyset\}, \{b, \emptyset\})\}\}$

3. We need to prove: 1) "existance": $\exists C \in P(X)$ such that $A \Delta C = B$
2) "uniqueness": there's only one set C whish satisfies given conditions.
In order to prove existance, we assume that C exists, then by def. of symmetric difference we can write $C = A \Delta B$ (1)

Let's take an arbitrary x such that
$x \in (A \Delta C) \overset{\text{by (1)}}{=} x \in (A \Delta A \Delta B) \overset{\text{by def. of Sym. Dif}}{=} x \in (\emptyset \Delta B) \overset{\text{by def. sym. dif}}{=} x \in B$

by this chain of equivalences we showed existance of C.

To prove uniqueness, assume exists two different sets C_1 and C_2 such that $C_1 \in P(X)$, $C_2 \in P(X)$ and $A \Delta C_1 = A \Delta C_2 = B$.
Let's take an arbitrary x such that $x \in C_1$ and $x \notin C_2$ then from $A \Delta C_1$ we can consider two cases:
1) $x \in B \Rightarrow x \notin A$ by $A \Delta C_1 = B$ then there is a contradiction $A \Delta C_2 = B$ $x \notin A$ $x \notin C_2$ $x \in B$
therefore set C is unique

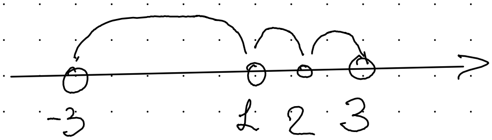
$\boxed{4}$

$A = \{x \in \mathbb{R} : -1 \leq x \leq 5\}$

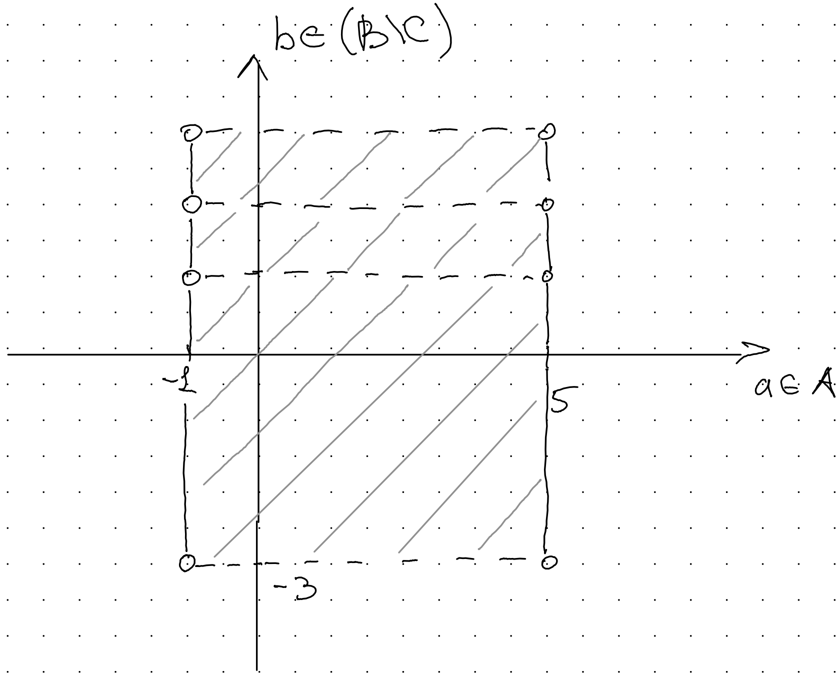
$B = \{y \in \mathbb{R} : |y| < 3\}$

$C = \{z \in \mathbb{N} : 1 \leq z \leq 6\}$

$B \setminus C$



$A \times (B \setminus C)$



$\boxed{5}$ $\bigcup_{n=2}^{\infty} [\frac{1}{n}, n) = \{x \mid x \in \mathbb{R}, x > 0\}$

We need to prove two cases:

1) $\bigcup_{n=2}^{\infty} [\frac{1}{n}, n) \subset \{x \mid x \in \mathbb{R}, x > 0\}$

2) $\{x \mid x \in \mathbb{R}, x > 0\} \subset \bigcup_{n=2}^{\infty} [\frac{1}{n}, n)$

Let's denote $\{x / x \in \mathbb{R}, x > 0\} = K$

Let's take an arbitrary $x \in K$

we notice exists $\ell \in \mathbb{N}$ such that $x < \ell$ and $x \geq 1/\ell$

by def. of Union

$\frac{1}{\ell} \leq x < \ell \iff x \in [\frac{1}{\ell}, \ell) \Rightarrow$

$\Rightarrow x \in \bigcup_{n=2}^{\infty} [\frac{1}{n}, n)$

$$\ell = \begin{cases} \lceil x \rceil + 1 & \text{when } x \geq 1 \\ \lceil x^{-1} \rceil + 1 & \text{when } 0 < x < 1 \end{cases}$$