

# Elimination - Success

1st pivot

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix}$$

A      b

$$\xrightarrow{(2,1)} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

$$\xrightarrow{(3,2)} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

U (upper triangular)

Back - Substitution

$$z = -2$$

$$y = 1$$

$$x = -2$$

Matrices Elimination

$$\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{matrix} 3 \times \text{col 1} \\ + \\ 4 \times \text{col 2} \\ + \\ 5 \times \text{col 3} \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 7 \end{bmatrix} \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} = \begin{matrix} 1 \times \text{row 1} \\ + \\ 2 \times \text{row 2} \\ + \\ 7 \times \text{row 3} \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

$E_{31}$   
elimination  
matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

$E_{32}$

$$(E_{-32} (E_{21}) A) = U$$

move parentheses  
(associative law)

Permutation

Exchange rows 1 and 2

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

Exchange columns

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

# What's inverse matrices

## Inverses

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$E^{-1} \quad E \quad I$

## Recitation

$$x - y - z + u = 0$$

$$2x + 2z = 8$$

$$-y - 2z = -8$$

$$3x - 3y - 2z + 4u = 7$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 2 & 0 & 2 & 0 \\ 0 & -1 & -2 & 0 \\ 3 & -3 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & -1 & -2 & 0 \\ 2 & 0 & 2 & 0 \\ 3 & -3 & -2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & -1 & -2 & 0 \\ 2 & 0 & 2 & 0 \\ 3 & -3 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & 2 & 4 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{l} -2 \\ -3 \end{array} \left[ \begin{array}{cccc|c} \boxed{1} & -1 & -1 & 1 & 0 \\ 2 & 0 & 2 & 0 & 8 \\ 0 & -1 & -2 & 0 & -8 \\ 3 & -3 & -2 & 4 & 7 \end{array} \right]$$

$$\frac{1}{2} \left[ \begin{array}{cccc|c} \boxed{1} & -1 & -1 & 1 & 0 \\ 0 & \boxed{2} & 4 & -2 & 8 \\ 0 & -1 & -2 & 0 & -8 \\ 0 & 0 & 2 & - & 7 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} \boxed{1} & -1 & -1 & 1 & 0 \\ 0 & \boxed{2} & 4 & -2 & 8 \\ 0 & 0 & 0 & -1 & -4 \\ 0 & 0 & 2 & 1 & 7 \end{array} \right]$$