Rank

- The rank of A is the number of pivots. This number is r.

Rank One

$$A = \begin{bmatrix} 1 & 3 & 10 \\ 2 & 6 & 20 \\ 3 & 9 & 30 \end{bmatrix}$$

$$A = \underset{\text{column}}{u}\,\overset{\text{row}}{v^T} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 10 \end{bmatrix}$$

$$Ax = u\left(v^T x\right) = 0$$

Thee definitions of rank (including one above).

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 2 & b_1 \\ 2 & 4 & 6 & 8 & b_2 \\ 3 & 6 & 8 & 10 & b_3 \end{array}\right]$$

Augmented m. $[A\ b]$

Solvability condition on b

$Ax = b$ solvable when b is in $C(A)$

If combination of rows of A gives zero row, then same combination of entries of b must give 0

To find complete solution to $Ax = b$

1. $X_{particular}$: set all 3 vars to 0, solve for pivot vars $Ax = b$

2. $X_{nullspace}$

$X = X_p + X_n$

columns

Full column rank $r = n$

no free variables and nullspace would be Z

$X = X_{part}$ (unique if $\exists$)

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 6 & 1 \\ 5 & 1 \end{bmatrix}$$

Full row rank $r = m$

can solve $Ax = b$ for every b

$r = m = n$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \quad R = I \quad N = Z$$

Recitation

$$\begin{bmatrix} \boxed{1} & -2 & -2 & b_1 \\ 2 & -5 & -4 & b_2 \\ 4 & -9 & -8 & b_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -2 & b_1 \\ 0 & -1 & 0 & b_2 - 2b_1 \\ 0 & -1 & 0 & b_3 - 4b_1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -2 & b_1 \\ 0 & -1 & 0 & b_2 - 2b_1 \\ 0 & -1 & 0 & b_3 - 4b_1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -2 & b_1 \\ 0 & -1 & 0 & b_2 - 2b_1 \\ 0 & 0 & 0 & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & 5b_1 - 2b_2 \\ 0 & 1 & 0 & 2b_1 - b_2 \\ 0 & 0 & 0 & -2b_1 - b_2 + b_3 \end{bmatrix}$$