$$\begin{array}{cccc} & \mathcal{M} & S & U \\ M = & \text{all } 3\times 3 \text{ m.} & \begin{array}{c}\text{symmetric} \\ 3\times 3\end{array} & \begin{array}{c}\text{UT} \\ 3\times 3\end{array}\end{array}$$

Basis for $M$ = all $3\times 3$'

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \dots, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\dim M = 9$

$S$ - symmetric m.

$\dim S = 6$

$$\begin{bmatrix} 1 & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & 1 & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & & \\ & & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{bmatrix} \dots$$

$\dim U = 6$

$S \cap U$ = diagonal

$\dim(S \cap U) = 3$

$\mathcal{S} + \mathcal{U} = S + u = $ all $3 \times 3$

$\dim(\mathcal{S}+\mathcal{U}) = 9$

$\dim \mathcal{S} + \dim \mathcal{U} = \dim(\mathcal{S} \cap \mathcal{U}) + \dim(\mathcal{S} + \mathcal{U})$

Rank 1

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 8 & 10 \end{bmatrix}$$

$\dim C(A) = \text{rank} = \dim C(A^T)$

Rank 1 m

$A = U V^T$ $\quad A = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 & 5 \end{bmatrix}$

$M =$ all $5 \times 17$ m

Subset of rank 1 m. not a subset

Example

In $\mathbb{R}^4$ $\quad v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$

$S =$ all $v$ in $\mathbb{R}^4$ with $v_1 + v_2 + v_3 + v_4 = 0$

$=$ nullspace of $A = [1\ 1\ 1\ 1]$ rank 1

$\dim N(A) = 3$

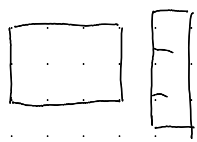
basis $S$ $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

$C(A) = \mathbb{R}^1$

$N(A^T) = \{0\}$

# Recitation

$M = 2 \times 3$ m.



$$v = \begin{bmatrix}2\\1\\1\end{bmatrix}$$

$$A = \begin{bmatrix}+1 & -1 & -1\\ 2 & -2 & -2\end{bmatrix}$$

$$\text{basis } N(A) = \left\{ \begin{bmatrix}1\\1\\0\end{bmatrix} \quad \begin{bmatrix}1\\0\\1\end{bmatrix} \right\}$$

$$A\begin{bmatrix}2\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \qquad B\begin{bmatrix}2\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$(A+B)\begin{bmatrix}2\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \quad \checkmark$$

$$(cA)\begin{bmatrix}2\\1\\1\end{bmatrix} = c\left(A\begin{bmatrix}2\\1\\1\end{bmatrix}\right) = c\begin{bmatrix}0\\0\end{bmatrix} \checkmark$$