Задача 1

$\bar{a} = (1, -3, 0)$
$\bar{b} = (-1, 2, 1)$

$|\bar{e}_1|, |\bar{e}_2|, |\bar{e}_3| = 3, \sqrt{2}, 4$

$\angle(\bar{e}_1, \bar{e}_2) = \angle(\bar{e}_2, \bar{e}_3) = 45^\circ$

$\angle(\bar{e}_1, \bar{e}_3) = 60^\circ$

$(\bar{e}_1, \bar{e}_1) = 9$

$(\bar{e}_1, \bar{e}_2) = 3\sqrt{2} \cdot \cos 45^\circ = 3$

$(\bar{e}_1, \bar{e}_3) = 12 \cdot \cos 60^\circ = 6$

$(\bar{e}_3, \bar{e}_3) = 16$

$(\bar{e}_2, \bar{e}_1) = 3$

$(\bar{e}_2, \bar{e}_2) = 2$

$(\bar{e}_2, \bar{e}_3) = 4$

$(\bar{a}, \bar{a}) = (\bar{e}_1 - 3\bar{e}_2, \bar{e}_1 - 3\bar{e}_2) = (\bar{e}_1, \bar{e}_1) - 6(\bar{e}_1, \bar{e}_2) + 9(\bar{e}_2, \bar{e}_2)$

$= 9 - 6 \cdot 3 + 9 \cdot 2 = 27 - 18 = 9$

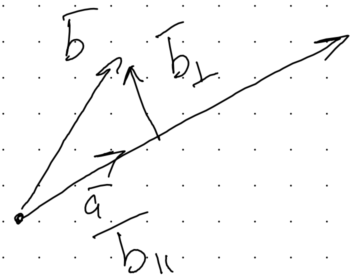
$|\bar{a}| = \sqrt{(\bar{a}, \bar{a})} = 3$

$(\bar{b}, \bar{b}) = (-e_1 + 2e_2 + e_3, -e_1 + 2e_2 + e_3) =$

Задача 2

$\bar{a} = (1; -1; 2)$ ОНБ

$\bar{b} = (1, 1, 2)$



$$\bar{b} = \bar{b}_{\parallel} + \bar{b}_\perp$$

$$\bar{b}_{\parallel} = pr_{\bar{a}} \bar{b} = \frac{(\bar{b}, \bar{a})}{(\bar{a}, \bar{a})} \cdot \bar{a}$$

$$(\bar{a}, \bar{a}) = 1 + 1 + 4 = 6$$

$$(\bar{b}, \bar{a}) = 1 + (-1) + 4 = 4$$

$$\bar{b}_{\parallel} = \left(\frac{2}{3}, -\frac{2}{3}, \frac{4}{3}\right)$$

$$\bar{b}_\perp = \bar{b} - \bar{b}_{\parallel} = \left(\frac{1}{3}, \frac{5}{3}, \frac{2}{3}\right)$$

Свойства $[\bar{c}, \bar{b}]$

1. Антикомутативность

$$[\bar{a}, \bar{b}] = -[\bar{b}, \bar{a}] \quad \forall \bar{a}, \bar{b}$$

2. Линейность (по второму аргументу выполняется в силу антикоммутативности)

$$[\alpha_1 \bar{a}_1 + \alpha_2 \bar{a}_2, \bar{b}] = \alpha_1 [\bar{a}_1, \bar{b}] + \alpha_2 [\bar{a}_2, \bar{b}]$$

$$\forall \bar{a}_1, \bar{a}_2, \bar{b} \quad \forall \alpha_1, \alpha_2 \in \mathbb{R}$$

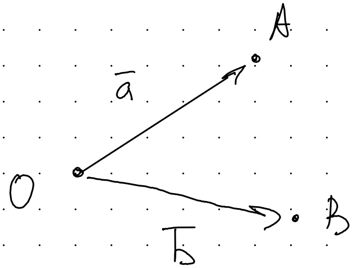
3. $[\bar{a}, \bar{b}] = \bar{0} \iff \bar{a} \parallel \bar{b}$

Утверждение

Задача 3

$\bar{a} = (2, 3, 1) \quad \bar{b} = (-1, 1, 2)$ ОНБ

Найти площадь треугольника $S_{\triangle OAB}$ - ?



$$[\bar{a}, \bar{b}] = \begin{vmatrix} \bar{e}_1 & \bar{e}_2 & \bar{e}_3 \\ 2 & 3 & 1 \\ -1 & 1 & 2 \end{vmatrix} =$$

$$= \left[\bar{e}_1 \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - \bar{e}_2 \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} + \bar{e}_3 \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} \right] =$$

$$= \left[\bar{e}_1 5 - \bar{e}_2 5 + \bar{e}_3 5 \right] = \begin{pmatrix} 5 \\ -5 \\ 5 \end{pmatrix}$$

$$\left| [\bar{a}, \bar{b}] \right| = \sqrt{25 + 25 + 25} = 5\sqrt{3}$$

$$S_{\triangle OAB} = \frac{1}{2} \left| [\bar{a}, \bar{b}] \right| = 2{,}5\sqrt{3}$$

Задача 4

Доказать, что для 3х некол-х векторов $\overline{a}, \overline{b}, \overline{c}$ - неколлинеарные

$$[\overline{a}, \overline{b}] = [\overline{b}, \overline{c}] = [\overline{c}, \overline{a}] \Leftrightarrow \overline{a} + \overline{b} + \overline{c} = 0$$

Δ

1) $(\Rightarrow)$

$$[\overline{a}, \overline{b}] = [\overline{b}, \overline{c}]$$
$$[\overline{a}, \overline{b}] = -[\overline{c}, \overline{b}]$$
$$[\overline{a}+\overline{c}, \overline{b}] = \overline{0}$$
$$\overline{a}+\overline{c} = \lambda \overline{b}$$

$$[\overline{b}, \overline{c}] = [\overline{c}, \overline{a}]$$
$$[\overline{b}+\overline{a}, \overline{c}] = 0$$
$$\overline{b}+\overline{a} = \gamma \overline{c}$$

$$\gamma \overline{c} - \overline{b} + \overline{c} = \lambda \overline{b}$$
$$(\gamma+1)\overline{c} = (\lambda+1)\overline{b}$$

т.к. $\overline{c}$ и $\overline{b}$ неколлинеарны $\Rightarrow \gamma = -1$ и $\lambda = -1 \Rightarrow$
$$\overline{a}+\overline{b}+\overline{c} = 0$$

2) $(\Leftarrow)$

$$\overline{a}+\overline{b}+\overline{c} = 0$$
$$\overline{a}+\overline{b} = -\overline{c}$$
$$[\overline{a}+\overline{b}, \overline{c}] = \cancel{[-\overline{c}, \overline{c}]} \; 0$$
$$[\overline{a}, \overline{c}] + [\overline{b}, \overline{c}] = 0$$
$$[\overline{b}, \overline{c}] = -[\overline{a}, \overline{c}]$$
$$[\overline{b}, \overline{c}] = [\overline{c}, \overline{a}]$$

аналогично для другой пары

Смешанное произведение векторов

Опр:

$$(\bar{a}, \bar{b}, \bar{c}) = (\bar{a}, [\bar{b}, \bar{c}])$$

Утв-е

$(\bar{a}, \bar{b}, \bar{c}) = 0 \Leftrightarrow \bar{a}, \bar{b}, \bar{c}$ – компланарны

Задача 5

Доказать, что если векторы

$[\bar a, \bar b]$, $[\bar b, \bar c]$, $[\bar c, \bar a]$ — компл. то векторы $\bar a$, $\bar b$, $\bar c$ компланарны

$\triangleright$

$$(\bar a, \bar b, \bar c) = (\bar a, [\bar b, \bar c]) =$$ по условию компланарности

$$= (\bar a, \alpha_1 [\bar a, \bar b] + \alpha_2 [\bar c, \bar a]) =$$

$$= \alpha_1 \underbrace{\cancel{(\bar a, [\bar a, \bar b])}}_{0} + \alpha_2 \underbrace{\cancel{(\bar a, [\bar c, \bar a])}}_{0} = 0$$

$\bar a \perp [\bar a, \bar b]$

$\square$