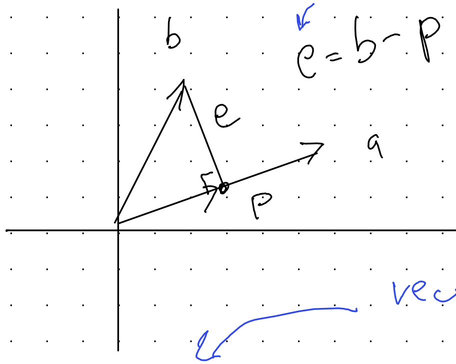
error

$e = b - p$



Projecting vector b onto vector a.

vector p is some multiple of a

$$p = a x$$

$$a^T (b - x a) = 0$$

$$a^T b - a^T x a = 0$$

$$a^T x a = a^T b$$

$$x = \frac{a^T b}{a^T a}$$

$p = a x$ – projection

$$p = a \underbrace{\frac{a^T b}{a^T a}}_{x} = P b$$

projection matrix

if we double b $\Rightarrow$ it doubles its projection p

* Reference to Rank 1 matrix

$$P = \frac{a\,a^T}{a^T a}$$

matrix

number

! nom-r and denom-r don't cancel.

What's the rank of matrix P. It projects to line, so assume 1

$C(P)$ = line through a

$\text{rank}(P) = 1$

If we multiply any vector by the matrix P we land in column space.

Properties of projection m.

$P^T = P$ symmetric

? How to prove it

$P^2 = P$

Why project?

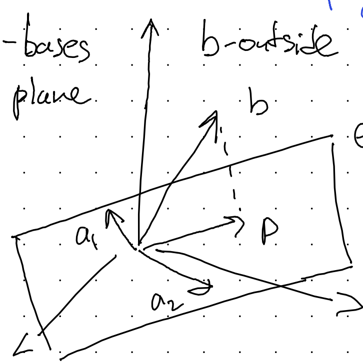
Because $Ax = b$ may have no solutions

solve $A\hat{x} = p$ instead

projection of b onto column space

a_1, a_2 - bases for plane

b-outside plane



$e = b - p$ e is perp to plane

$p = \hat{x}_1 a_1 + \hat{x}_2 a_2$

plane of a_1 a_2 = col space of

$$A = \begin{bmatrix} | & | \\ a_1 & a_2 \\ | & | \end{bmatrix}$$

Main point:

$p = A\hat{x}$ Find $\hat{x}$

key: $b - A\hat{x}$ is perp to plane

$$a_1^T(b - A\hat{x}) = 0$$

$$a_2^T(b - A\hat{x}) = 0$$

$$\begin{bmatrix} - & a_1^T & - \\ - & a_2^T & - \end{bmatrix}(b - A\hat{x}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A^T \underbrace{(b - A\hat{x})}_{e} = 0$$

In which subspaces vector e?

e in $N(A^T) \Rightarrow e \perp C(A)$

$$A^T A \hat{x} = A^T b$$

Now we have matricies

$$\hat{x} = (A^T A)^{-1} A^T b$$

What's the projection?

$$\frac{a a^T}{a^T a}$$

$$p = A\hat{x}$$

in 1D

Here we opened brackets, but not allowed

$$p = \underbrace{A (A^T A)^{-1} A^T}_{\text{that's proj. matrix } P} b$$

$$A A^{-1} (A^T)^{-1} A^T = I$$

What're the properties for any proj matrix

* A is not a square m. ⇒ it has no inverse m.

$$P = P^T$$

$$P = P^2$$

But what if A is square invl matrix? I assure b would be in $C(A)$, so what's the proj. matrix in this case?

$$P^2 = A (A^T A)^{-1} (\underbrace{\cancel{A^T A (A^T A)^{-1}}}_{I}) A^T =$$

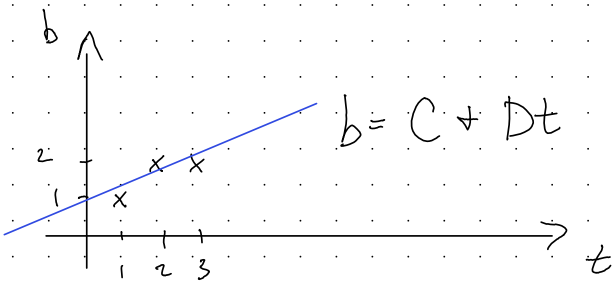
$$= A (A^T A^{-1}) A^T$$

It will be identity, it's actually written above?

When would I have a bunch of equations and yet i want the best possible solution

Least squeres (Application)

fitting by a line $(1,1), (2,2), (3,2)$



$b = C + Dt$

I'm looking for numbers C, D

$$C + D = 1$$
$$C + 2D = 2$$
$$C + 3D = 2$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$A \quad x = b$$

No solutions, 2 unknowns we still looking for best solution.

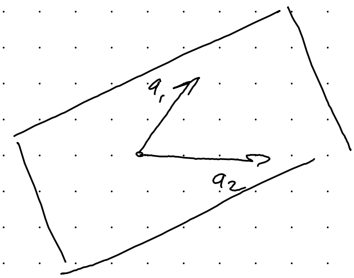
But the eqation which has a solution

$$A^T A \hat{x} = A^T b$$

Recitation

Find the orthogonal projection m. ont the plane

$p = Pb$



$P - ?$

$x + y - z = 0$

$$A = \begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P = A(A^TA)^{-1}A^T$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{array}\right] \to \begin{bmatrix} 2 & -1 & 1 & 0 \\ 0 & 1{,}5 & 0{,}5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & 0{,}5 \\ 0 & 1 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 1 & 0 \\ 0 & 3/2 & 1/2 & 1 \end{bmatrix}$$

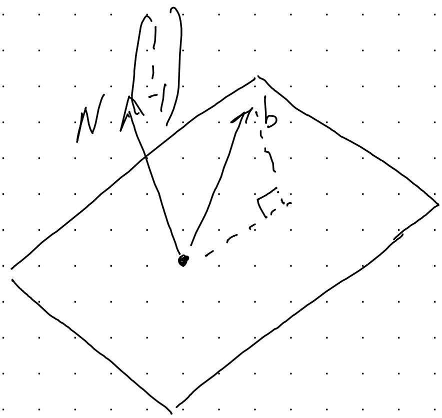
$$\begin{bmatrix} 2 & -1 & 1 & 0 \\ 0 & 1 & 1/3 & 2/3 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & 4/3 & 2/3 \\ 0 & 1 & 1/3 & 2/3 \end{bmatrix} \to$$

$$\to \begin{bmatrix} 1 & 0 & 2/3 & 1/3 \\ 0 & 1 & 1/3 & 2/3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} = \begin{bmatrix} -1/3 & 1/3 \\ 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

$$\begin{bmatrix} -1/3 & 1/3 \\ 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 & 1/3 \\ -1/3 & 2/3 & 1/3 \\ 1/3 & 1/3 & 2/3 \end{bmatrix}$$

$$Ib = Pb + P_N b \Rightarrow I = P + P_N$$



$$P_N = N(N^T N)^{-1} N^T$$

$$\begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = 3$$

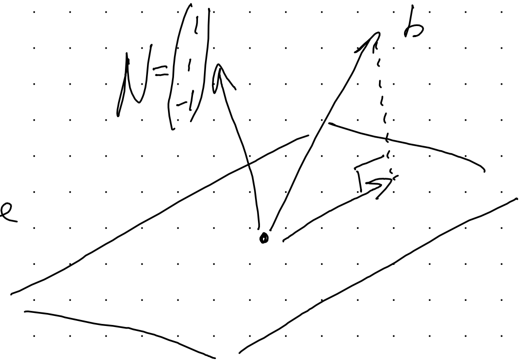
$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

On the page i solved recitation 2nd time

Find the orthogonal proj. matrix onto the plane : $x + y - z = 0$

$$\underset{A}{\begin{pmatrix} 1 & 1 & -1 \end{pmatrix}} \underset{x}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = 0$$



I can find null space of A, which should be the plane.

Rowspace of A and nullspace are orthogonal compliments, so I assume

$$P_A + P_{N(A)} = I$$

It might be simpler to find a proj. matrix for N, because N is vector.

$$P_N = \frac{a a^T}{a^T a} = \frac{\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}}{\begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}} = \frac{\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}}{3}$$

$$P_{N(A)} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$