Discrete Distributions

X discrete uniform distribution

$$P(X = x_i) = \frac{1}{n} \qquad i = 1, \dots, n$$

Denote $a = 1$ and $b = n$

$$EX = \frac{a+b}{2}$$

$$V(X) = ?$$

TODO
prove
equations

Binomial distribution

X counts number of successes k
in the n trials

$$X \sim Bin(n, p)$$

? What is Bernouly numbers?

$$P_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$E[X] = E\left[\sum_{i=1}^{n} y_i\right] = \sum E(y_i) = np$$

$$Var(x) = np(1-p)$$

Poisson distribution

$$P_X(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$EX = VX = \lambda$$ TODO: Prove

Continuos random varable

Continuous random var. can take any value in some part of the real line

Cumulative distribution function

$$F(x) = P(X \le x) = P(X \in (-\infty, x])$$

X is a continous random var if there exists a function $f(x) \ge 0$ such that for any constants a and b with $-\infty \le a \le b \le \infty$

$$P(a < X \le b) = \int_a^b f(x)\,dx$$ PDF

$$F(x) = \int_{-\infty}^{x} f(u)\, du$$

Properties

$$\lim_{x \to -\infty} F(x) = 0$$

$$\lim_{x \to +\infty} F(x) = 1$$

Theorem 2 if $a < b \Rightarrow F_X(b) \geq F_X(a)$

$$P(a \leq X \leq b) = F_X(b) - F_X(a)$$

- $F_X(b) - F_X(a) = \int_a^b f_X(t)\, dt$
- $f(x) \geq 0$
- $\int_{-\infty}^{\infty} f(x)\, dx = 1$

Expectation

$$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx$$

Also, if $Y = g(X)$

$$E[Y] = \int_{-\infty}^{\infty} g(X)\; f_X(x)\, dx$$

Quantiles

X_α is determened such that area under the density function to the left of is equal to α

$$F(X_\alpha) = \alpha$$

$X_{0,95}$ separates the top 5% of probability mass of X.

$F(X_{0,5}) = 0,5$ median

$0,75$ – upper quantile

Uniform distribution

$$f_X = \begin{cases} \frac{1}{b-a} & \text{if } x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$$

Exponential

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$X \sim Exp(\lambda)$$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$$

Distribution models rate or intencity

Memoryless property

Some connection between exp and poisson distr-s

Normal distr.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

$X \sim N(\mu, \sigma^2)$

Standard normal distribution

$N(0, 1)$

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Theorem 3

if $X \sim N(\mu, \sigma^2)$ and $\overbrace{Y = \frac{X-\mu}{\sigma}}^{\substack{\text{transformation} \\ \text{Standardization}}}$, $\Rightarrow$

$\Rightarrow Y \sim N(0, 1)$

$W = X - \mu \qquad W \sim N(0, \sigma^2)$ centering