disjunction $\vee$

implication $\Rightarrow$

- From A we can conclude B
- A is sufficient for B
- B is necessary for A
- A is true only if B is true

equivalence $\Leftrightarrow$

Precedence: $\neg \; \wedge \; \vee \; \Rightarrow \; \Leftrightarrow$

Def: Propositional formula

1. every propositional variable is a propositional formula

2. truth values are propositional formulas

3. if F is prop. f. then $\neg F$ is also a prop. f.

4. if F and G are propositional formulas, then $F \wedge G$, $F \vee G$, $F \Rightarrow G$ and $F \Leftrightarrow G$ are also propositional formulas

5. if F is a propositional formula then (F) is also a propositional formula

last operator in formula is called main component.

Def: the truth value of the propositional formula F in given valuation is found by following rules:

1 if $F = \neg G$, then $F = 1$ iff $G = 0$

2 if $F = G \wedge H$

3 if $F = G \vee H$

4 if $F = G \Rightarrow H$

5 if $F = G \Leftrightarrow H$

Def: A propositional formula is called a tautology if it is true for every valuation of its variables

Formula F is called a contradiction if it's false for every valuation of it's variables

Def: F is called satisfiable if it is true for at least one valuation of variables. F is called invalid if it is false for at least one valuation of its variables

Prop-n: Formula F is a taut. iff $\neg F$ is contr.

Proposition: Form. F is satisfiable iff $\neg F$ is not tautology

Def We say that the formula G logically follows from the formulas $F_1, \dots, F_n$ (or is a logical consequence of $F_1, \dots, F_n$) iff in every valuation of the variables that makes all of $F_1, \dots, F_n$ true, G is also true

$$F_1, F_2, \dots F_n \models G$$

$\models$ concludes

logical entailment (semantic consequence)

Theor. From formulas $F_1, \dots, F_n$ we conclude the formula G iff the formula $F_1 \wedge \dots \wedge F_n \Rightarrow G$ is tautology.

Logical equivalence

Def Formulas F and G are logically equivalent iff for every possible valuation of their variables, F and G have the same truth value. $F \equiv G$

$\equiv$ logical equivalence

Logical eqivalences

Absorbtion

$$F \wedge (F \vee G) \equiv F$$

$$F \vee F \wedge G \equiv F$$

implication

$$F \Rightarrow G \equiv \neg F \vee G$$

$$F \Rightarrow G \equiv \neg(F \wedge \neg G)$$

Principal disjunctive normal form

Def conjunction of variables or their negations is called a basic conjunction

$X \wedge Y$, $X \wedge Y \wedge \neg Y$, $X \wedge \neg Y \wedge Z$

Def principal basic conjunction (full conjunction) is a conjunction where each given variable appears exactly once, either as variable itself or its negation. $X \wedge \neg Y \wedge Z$ (only principal from pevious)

Def A propositional formula that is logically equivalent to formula F, and is disjunction of basic conjunctions is called disjunctive normal form (DNF) of formula F

Def The principal disjunctive normal form (full disjunctive normal form) of propositional formula F is the form equivalent to F that is disjunction of principal basic conjunctions

Theorem: A formula has the principal disjunctive normal form iff it is satisfiable

Precedence: $\neg \; \wedge \; \vee \; \Rightarrow \; \Leftrightarrow$