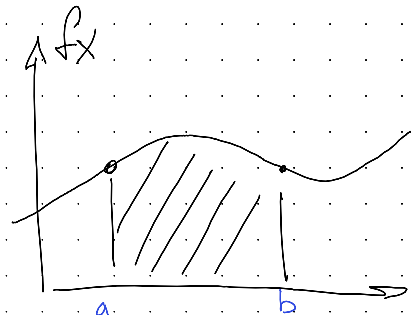
Continuous RV

Section 3.1 – 3.3

A contin. RV is described by a probability density function f_X



$\sum_{x \in [a,b]} P_X(x)$ — discrete case

$$P(a \le X \le b) = \int_a^b f_X(x)\,dx$$

- $\boxed{f_X \ge 0}$
- $\int_{-\infty}^{+\infty} f_X(x)\,dx = 1$

$$P(X=a) = 0$$

Interpretation of dencity fun

$$P(x \le X \le x+\delta) = \int_x^{x+\delta} f_X(x)\,dx \approx f_X(x)\cdot\delta$$

So units of dencity are probability per unit length.

$$P(X \in B) = \int_B f_X(x)\,dx$$

Means and variance

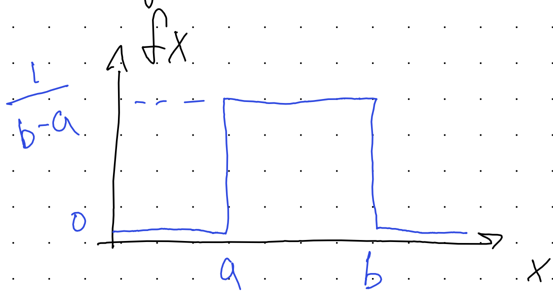
$$E[X] = \int_{-\infty}^{+\infty} x\, f_X(x)\,dx$$

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x)\, f_X(x)\,dx$$

$$\mathrm{var}(X) = \sigma_X^2 = \int_{-\infty}^{+\infty} (x - E[X])^2 f_X(x)\,dx =$$

$$= E[X^2] - (E[X])^2$$

Example: C. Uniform RV



$$f_X(x) = \frac{1}{b-a} \quad a \le x \le b \qquad 0 \text{ otherwise}$$

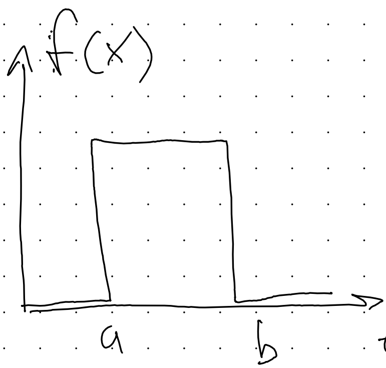
$$E[X] = \int_a^b x \frac{1}{b-a}\, dx = \frac{a+b}{2}$$

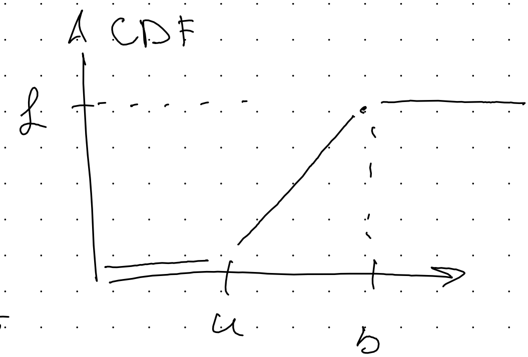
$$\sigma_X^2 = \int_a^b \left(x - \frac{a+b}{2}\right)^2 \frac{1}{b-a}\, dx = \frac{(b-a)^2}{12}$$

$$\sigma_X = \frac{b-a}{\sqrt{12}}$$

Cumulative distr. function

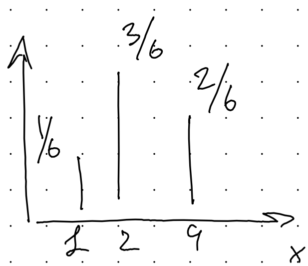
$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(t)\,dt$$

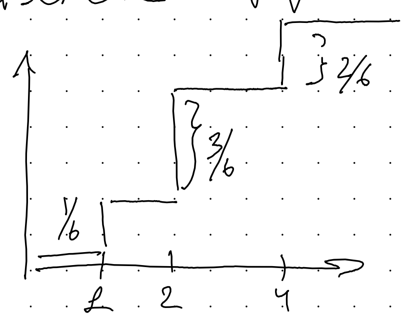




$$\frac{dF_X(x)}{dx} = f_X(x)$$

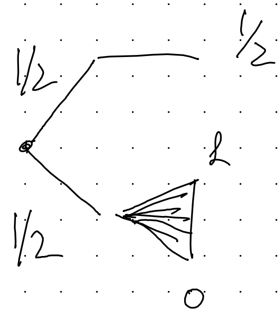
- Also for discrete RV

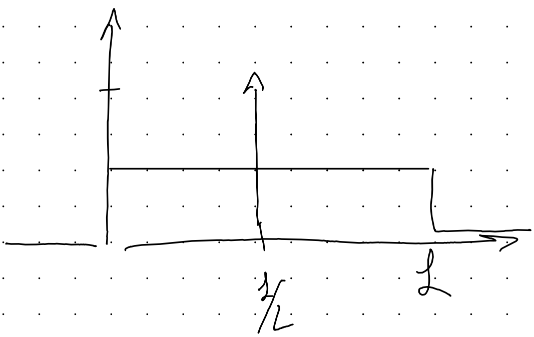




$$F_X(x) = P(X \le x) = \sum_{k \le x} P_X(k)$$

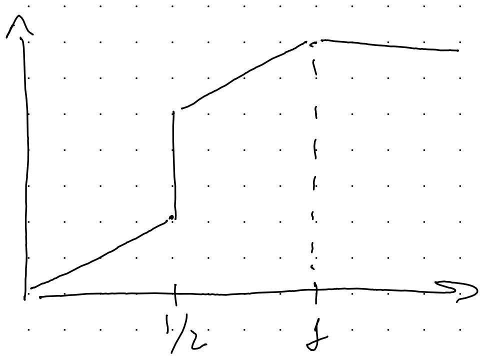
Mixed RV





Corresponding CDF

$$F_X(x) = P(X \le x)$$

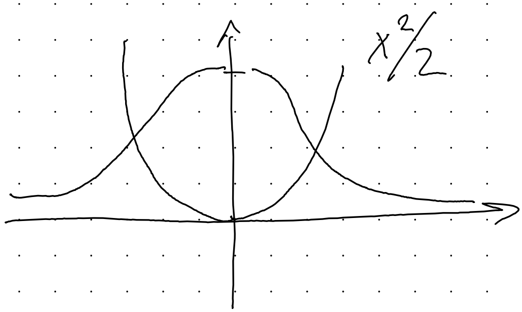


Gaussian (normal) PDF

Important in the theory of prob.
- central limit theorem

Standard normal $N(0,1)$

$$N(0,1): \quad f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$



$$\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi}$$

constant to ensure 1 t. probability

$E[X] = 0$

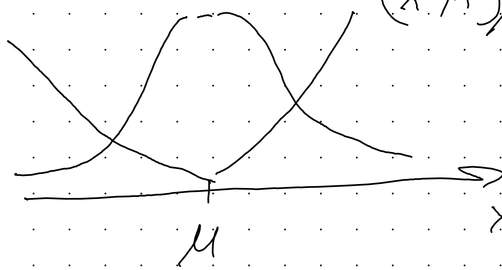
$\mathrm{var}(X) = 1$

General normal (Gaussian) RV

General normal

$N(\mu\ \sigma^2)$: $\quad f_X(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$

$(x-\mu)^2/2\sigma^2$



$E[X] = \mu$

$var(X) = \sigma^2$

When σ is small

narrower PDF

Linear function of a NRV

Let $Y = aX + b \qquad X \sim N(\mu, \sigma^2)$

$E[Y] = a\mu + b$

$Var(Y) = a^2\sigma^2$

Fact (provided later in the course)

$Y \sim N(a\mu + b, a^2\sigma^2)$

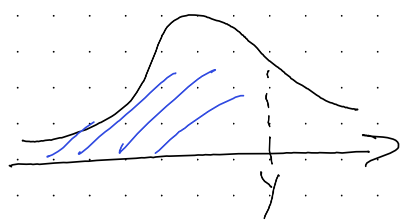
Y is also normal

Standard normal tables

no closed form for CDF, but we have tables for standard normal

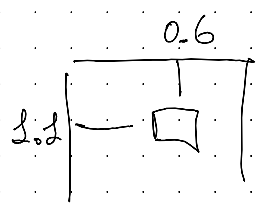
$Y \sim N(0, 1)$

$\Phi(y) = F_Y(y) = P(Y \leq y)$



$\Phi = F(0) = P(Y \leq 0) =$

$\Phi = F(1.16) = 0,8770$



$\Phi(-2) = 1 - \Phi(2)$

Standardizing a Random var.

Let X have mean μ and variance $\sigma^2 > 0$

Let $Y = \frac{X - \mu}{\sigma}$

$E[Y] = 0$

$Var(Y) = \frac{1}{\sigma^2} Var(X) = 1$

if also X is normal then:

$Y \sim N(0, 1)$