Mathematical Statistics
Week 7 exercises

1. Which of the following hypotheses are stated correctly and why?

   a) $H_0 : \mu \leq 100, H_1 : \mu > 100$;

   b) $H_0 : \sigma = 20, H_1 : \sigma \leq 20$;

   c) $H_0 : p \neq 0.25, H_1 : p = 0.25$;

   d) $H_0 : \mu_1 - \mu_2 = 25, H_1 : \mu_1 - \mu_2 > 100$;

   e) $H_0 : s_1^2 = s_2^2, H_1 : s_1^2 \neq s_2^2$;

   f) $H_0 : \sigma_1/\sigma_2 = 1, H_1 : \sigma_1/\sigma_2 \neq 1$;

2. Templer and Tomeo (2002) reported that the population mean score on the quantitative portion of the Graduate Record Examination (GRE) General Test for students taking the exam between 1994 and 1997 was 558 ± 139 ($\mu \pm \sigma$). Suppose we select a sample of 100 participants. We record a sample mean equal to 585. State the hypotheses.

3. On May 16, 1994, *Newsweek* reported the results of a public opinion poll that asked: "*From everything you know about Bill Clinton, does he have the honesty and integrity you expect in a president?*" (p. 23).
   Poll surveyed 518 adults and 233, or 0.45 of them (clearly less than half), answered yes. Could Clinton's adversaries conclude from this that only a minority (less than half) of the population of Americans thought Clinton had the honesty and integrity to be president? State the hypotheses.

4. Let μ denote the true average radioactivity level (picocuries per liter). The value $5\ pCi/L$ is considered the dividing line between safe and unsafe water. Would you recommend testing $H_0 : \mu = 5$ versus $H_1 : \mu > 5$ or $H_0 : \mu = 5$ versus $H_1 : \mu < 5$? Explain your reasoning. [Hint: Think about the consequences of a type I and type II error for each possibility.]

5. A seismologist claims to be able to predict earthquakes. Whatever data is collected to make these predictions can be viewed as a hypothesis test on whether an earthquake is going to occur.

   (a) What is the null hypothesis of this test?

   (b) What is a type 1 error in this case?

   (c) What is a type 2 error in this case?

   (d) Which would you rather happen while hanging out on University of Tartu campus: type 1 error or type 2 error?

6. We're performing a significance test; someone has claimed that the average age μ of a University of Tartu undergrad is 20.1 years, and we wish to test that hypothesis.

   (a) What is the null hypothesis?

(b) If we choose to do a two-sided test, what's the alternative hypothesis? If we choose to do a one-sided test, what are the possibilities for the alternative hypothesis?

(c) What data might we collect? What would be the test statistic we'd compute?

7. A manufacturer of sprinkler systems used for fire protection in office buildings claims that the true average system-activation temperature is $130°$. A sample of $n = 9$ systems, when tested, yields a sample average activation temperature of $131.08°F$. If the distribution of activation times is normal with standard deviation $1.5°F$, does the data contradict the manufacturer's claim at significance level $\alpha = 0.01$?

1

a) valid   one sided

b) ⌣   one -S.

c) −

d) −

e) −   not about population

f) +

2

$H_0: \mu = 558$

$H_1: \mu \neq 558$

3   Binomial distribution

$H_0: p \geq 0,5$

$H_1: p < 0,5$

⁴

H_0: water is at critical level $\mu = 5$

H_1: water is not safe
$\mu > 0$

	decision	
	return H_0	return H_1
truth H_0	✓	type I error α
H_1	type 2 error β	✓

if we reject H_0 we can make a type I error (false positive)

if we fail to reject H_0 we make type II error (false negative)

5