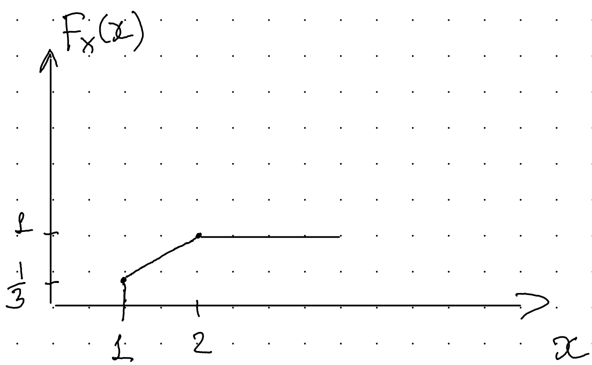
$\boxed{301}$

$$F_X(x) = \begin{cases} 0, & x < a \\ 1/2, & a \leq x < b \\ 1, & x \geq b \end{cases}$$

$\boxed{302}$



$$P_X(X \leq 5/3) = \frac{7}{9}$$

$$P_X\left(X > \frac{3}{2}\right) = 1 - P\left(X \leq \frac{3}{2}\right) = 1 - \frac{2}{3} = \frac{1}{3}$$

$$P_X\left(\frac{4}{3} < X \leq \frac{5}{3}\right) = P\left(X \leq \frac{5}{3}\right) - P\left(X \leq \frac{4}{3}\right) =$$

$$= \frac{7}{9} - \frac{5}{9} = \frac{2}{9}$$

$\boxed{303}$

a) $P_X(9) = 1 - P_X(3) - P_X(4) - P_X(7) - P_X(8)$

$= 1 - \frac{1}{3} - \frac{1}{4} - \frac{1}{6} - \frac{1}{6} = \frac{1}{12}$

b) $F_X(5) = P_X(3) + P_X(4) = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

c) $P(4 \leq X \leq 8) = F_X(8) - F_X(3) =$

$= \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{6} - \frac{1}{3} = \frac{7}{12}$

$P(X \geq 8) = 1 - F_X(8) = 1 - \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{6}\right) = \frac{3}{12} = \frac{1}{4}$

$\boxed{304}$

10 10 50

$X: \{(10,10), (50,10), (10,50)\} \to \{20, 60\}$

$$P_X(x) = \begin{cases} \frac{1}{3}, & x = 20 \\ \frac{2}{3}, & x = 60 \end{cases}$$

$\boxed{305}$

X - number of drawings resulting in black ball

$P(\text{black}) = \frac{6}{10}$ $P(\text{white}) = \frac{4}{10}$

Geometric distribution

a) $P_X(X=k) = P(\text{black})^k \, P(\text{white})$ $k \in \mathbb{N} \cup \{0\}$

b) $P(X \geq 3) = 1 - \left(P(X=0) + P(X=1) + P(X=2)\right) =$

$= 1 - \left(P(\text{white}) - P(B)\, P(W) - P(B)^2 P(w)\right) =$

$= 1 - P(w)\left(1 + P(B) + P(B)^2\right) = 1 - \frac{4}{10}\left(1 - \frac{6}{10} - \left(\frac{6}{10}\right)^2\right) =$

$= \frac{27}{125}$

$\boxed{306}$ fft distribution

Y – total number of drawings

$P_Y(Y=k) = P(B)^{k-1} \cdot P(W) \qquad k \in \mathbb{N}$

$P_Y(Y \geq 2) = 1 - P(Y=1) = 1 - 0{,}4 = 0{,}6$

$\boxed{307}$ Binomial distribution

X – number of white balls among the three balls

$X \sim Bin(3, 0{,}4)$

$P(W) = p \qquad P(B) = 1 - p = q$

$P(X=k) = \frac{n!}{k!(n-k)!} \cdot p^k \cdot q^{n-k}$

$P(X \geq 2) = P(X=2) + P(X=3) = 0{,}288 + 0{,}064$

$= 0{,}352 = \frac{44}{125}$

$\boxed{308}$

Hypergeometric

$\boxed{309}$

Poisson Distribution

$X \sim Po(m)$

$$P_X(k) = e^{-m} \frac{m^k}{k!}$$

$$P_X(0) = e^{-m} \frac{m^0}{0!} = \frac{1}{2}$$

$$e^{-m} = \frac{1}{2}$$

$$e^m = 2$$

$$m = \ln 2$$

$$P(X \geq 2) = 1 - \left(P(X=0) + P(X=1)\right) =$$

$$= 1 - \left(\frac{1}{2} + e^{-\ln 2} \cdot \frac{\ln 2}{1}\right) = 1 - \left(\frac{1}{2} + \frac{1}{2} \cdot \ln 2\right)$$

$$= 0{,}153$$

$\boxed{310}$ $f(x) = c\,x^2$ $\qquad 0 \leq x \leq 6$

$$\int_0^6 c\,x^2\,dx = 1$$

$$\left.\frac{c\,x^3}{3}\right|_0^6 = 1$$

$$\frac{c}{3}\left(6^3 - 0^3\right) = 1$$

$$c = \frac{\cancel{3}}{\cancel{6}_2 \cdot 6 \cdot 6} = \frac{1}{72}$$

$\boxed{311}$

$$f(x) = \frac{c}{\sqrt{x+1}} \qquad -1 \leq x \leq 1$$

$$\int_{-1}^{1} \frac{c}{\sqrt{x+1}}\,dx = 1$$

$$\left.\frac{2c\,(x+1)^{\frac{1}{2}}}{1}\right|_{-1}^{1} = 1$$

$$2c\left(\sqrt{2} - 0\right) = 1 \qquad c = \frac{1}{2\sqrt{2}}$$

$$P(X \geq 0) = 1 - F_X(0) = 1 - \int_{-1}^{0} \frac{1}{2\sqrt{2}\sqrt{x+1}}\, dx =$$

$$= 1 - \frac{1}{\sqrt{2}}(x+1)^{\frac{1}{2}} \Big|_{-1}^{0} = 1 - \left(\frac{1}{\sqrt{2}} - 0\right) =$$

$$= \frac{\sqrt{2} - 1}{\sqrt{2}} \simeq 0{,}293$$

$\boxed{312}$

$$f(x) = c\, x^2 e^{-\beta x^2} \qquad \text{if} \quad x \geq 0$$

$$\int_0^{+\infty} c\, x^2 e^{-\beta x^2}\, dx = 1$$

$c\,(\int$

$\boxed{316}$ uniform distribution on (a,b)

$f_X(x) = \frac{1}{5} \qquad 0 \le x \le 5$

a) if later than 13:06 then $X \ge 3$ (13:06 − 13:03)

$P(X \ge 3) = 1 - F_X(3) = 1 - \int_0^3 \frac{1}{5} dx = \frac{2}{5}$

b) $P(X \ge 4) = 1 - F_X(4) = 1 - \int_0^4 \frac{1}{5} dx =$

$= 1 - \frac{1}{5} x \Big|_0^4 = 1 - \frac{4}{5} = \frac{1}{5}$

$\boxed{317}$ $X \sim Exp(2{,}5)$ − waiting time in min from opening time

$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-0{,}4x} & \text{if } x \ge 0 \end{cases}$

a) $P_X(X \le 3) = F_X(3) = 1 - e^{-0{,}4 \cdot 3} \approx 0{,}699$

b) $P_X(X \ge 4) = 1 - F_X(4) = e^{-0{,}4 \cdot 4} \approx 0{,}202$

c) $P(3 \leq X \leq 4) = F_X(4) - F_X(3) =$

$= (1 - 0{,}202) - 0{,}699 = 0{,}099$

d) $P(X \leq 3 \cup X \geq 4) = F_X(3) + (1 - F_X(4)) =$

$= 0{,}699 + 0{,}202 = 0{,}901$

e) o