$\boxed{\text{PS } 2.5}$

$\boxed{24}$

$$UU^{-1} = I$$

$$U = \begin{bmatrix} 1 & a & b \\ & 1 & c \\ & & 1 \end{bmatrix}$$

$$U^{-1} = \begin{bmatrix} 1 & -a & -b+ca \\ & 1 & -c \\ & & 1 \end{bmatrix}$$

$\boxed{40}$

$$A = \begin{bmatrix} 1 & -a & 0 & 0 \\ & 1 & -b & 0 \\ & & 1 & -c \\ & & & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & a & ab & abc \\ & 1 & b & cb \\ & & 1 & c \\ & & & 1 \end{bmatrix}$$

$\boxed{\text{PS } 2.6}$

$\boxed{13}$ easy

$\boxed{18}$ $A = LDU \qquad A_1 = L_1 D_1 U_1$

$$LDU = L_1 D_1 U_1$$

$$L_1^{-1} L D U = I D_1 U_1$$

$$\underbrace{L_1^{-1} L} D I = D_1 \underbrace{U_1 U^{-1}}$$

diagonal

$\boxed{25}$

$\boxed{26}$

$\boxed{\text{PS } 2.7}$

$\boxed{13}$ a)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = I$$

b)

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\hat{P}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \hat{P} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \hat{P} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\boxed{36}$

- Lower triangular m. ✓

$$\begin{bmatrix} 1 & & \\ 2 & 1 & \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 6 & & \\ 8 & 6 & \\ 1 & 2 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 14 & 6 & 0 \\ & & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & & \\ 2 & 1 & \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

- Symmetric matrices

$$A = L_A D_A L_A^T$$
$$B = L_B D_B L_B^T$$

- Positive M.

multiplication should satisfy the group definition

A^{-1} is not positive

- Diagonal invertable m: D: ✓

yes

- Permutation m. ✓

- $Q^T = Q^{-1}$ ✗

$$\overset{P}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}} \overset{P^T}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$$

$$(AB)^{-1} = (AB)^T$$

$$B^{-1}A^{-1} = B^T A^T$$

$\boxed{40}$

$$\|V\| = \sqrt{V \cdot V}$$

$$V \cdot W = 0 \qquad V \perp W$$

$$\frac{V \cdot W}{\|V\| \ \|W\|} = \cos\theta$$

$\boxed{\text{PS 3.1}}$

$\boxed{23}$ b is linear combination of A / unless column space contains b

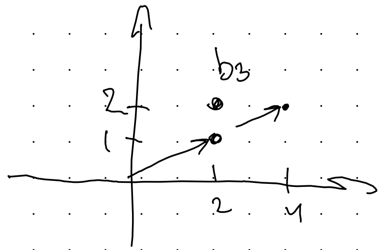
Worked example
3.1A

$Ax = b_1$

if subspace then b_1 is in the subspace

in order to construct A we need a valid subspace $\notin R^n$

b_1 and $b_2 \subseteq C(A)$



$$\overset{b_3}{\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}}$$

$$\cancel{\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}}$$

3.1.B ?

$V_1 - R^3 \qquad V_2 = R^3$

$\boxed{23}$

1. comb.

a) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & 7 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 4 \end{bmatrix}$

$Ax = b$ is solvable when b is in $C(A)$

$\boxed{30}$ a)

$$c(s+t) = cs + ct$$

b) $S + T$ - plane
$S \cup T$ - two lines

$\boxed{18}$ ✓

A)

$$A1 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

symmetric

$A^T = A$

$$A2 = \begin{bmatrix} 6 & 1 & 1 \\ 1 & 6 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$A1 + A2 = \begin{bmatrix} 7 & 1 & 3 \\ 1 & 7 & 5 \\ 3 & 5 & 4 \end{bmatrix}$$

Symmetric m. form a subspace

B) Skew symmetric $A^T = -A$

$$(A+B)^T = A^T + B^T = -A - B$$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ -2 & -3 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & -1 & -1 \\ 1 & 6 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Form a subspace

~~Doesn't form subspace~~

C) Asymetric ($A^T \neq A$) m. doesn't form subspace.

We can find two assymetric m-s which result to symetric m after summation.

$\boxed{32}$

$A = [A \; AB]$ have same column space

column space will become bigger if new column is not in the column s. $C(A)$

Let's prove that $C(AB)$ in $C(A)$.

Let's assume last row in A has no pivot

$$row_n = k_1 row_1 + k_2 row_2 + \dots + k_{n-1} row_{n-1}$$