Natural n. $\mathbb{N}$
Integer n. $\mathbb{Z}$
Rational n. $\mathbb{Q}$
Real n. $\mathbb{R}$
Irrational n. $\mathbb{I}$
Complex $\mathbb{C}$

Chapter 5 Sets

$\emptyset$ - empty set

For finite set we denote $|S|$
as cardinality / size

Subset

$$A \subset B \quad \forall x \ (x \in A \Rightarrow x \in B)$$

Propositions

- Reflexivity For every set A
  it holds that $A \subset A$
- Antisymmetry A and B
  satisfy $A \subset B \wedge B \subset A \Rightarrow A = B$
- Transitivity if sets A, B, C
  satisfy $A \subset B$ and $B \subset C \Rightarrow A \subset C$
- The empty set is a subset of
  every set

Set A is called proper subset of set B if A is subset of B and $A \neq B$. In this case we write $A \subsetneq B$

Fact: The $\emptyset$ is subset of all sets, that is $\emptyset \subseteq B$ for any set B

Def if A is a set, the power set of A is another set denoted $\mathcal{P}(A)$ and defined to be the set of all subsets of A: $\mathcal{P}(A) = \{X \mid X \subseteq A\}$

Fact
if A is finite set, then
$|\mathcal{P}(A)| = 2^{|A|}$

Chapter 6 Set operations

Def
The union of two sets A and B is the set $A \cup B$ that consists of all elements that belong to at least to one of the sets A or B

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

Def The difference of two sets A and B is set $A \setminus B$ that consists of all elements that belong to A but do not belong to B

$$A \setminus B = \{x \mid x \in A \wedge x \notin B\}$$

Def symmetric difference of two sets A and B is the set $A \triangle B$ that consists of all elements that belong to exactly one of the sets A or B

$$A \triangle B = \{x \mid (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\}$$

$A \triangle B = A \setminus B \cup B \setminus A$

Def The complement of a set A is the set A' that consists of all elements in the universal set U that do not belong to A

$A' = \{ x \mid x \in U \wedge x \notin A \} = U \setminus A$

Def Cartesian product of two sets A and B is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$

$A \times B = \{ (a, b) \mid a \in A, b \in B \}$