

PS 2.2

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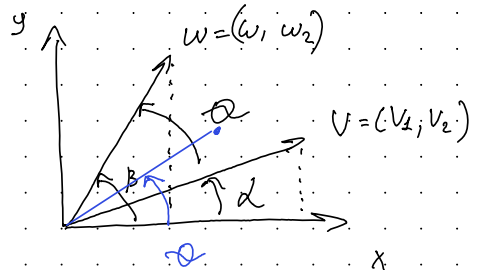
$$\cos \beta = \frac{w_1}{\|w\|}$$

$$\sin \beta = \frac{w_2}{\|w\|}$$

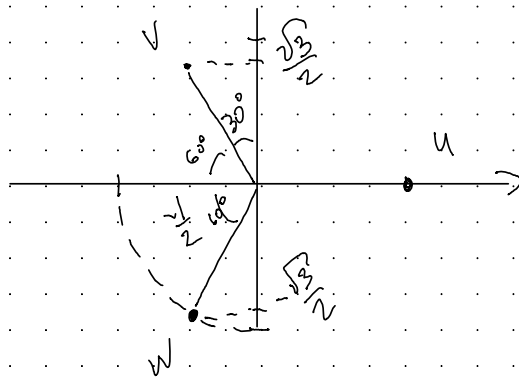
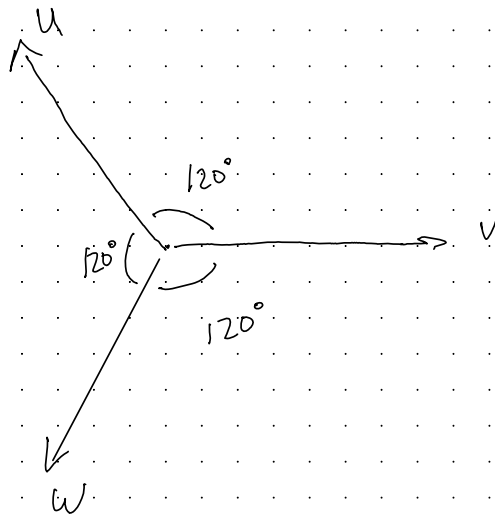
$$\cos(\beta - \alpha) = \cos \beta \cos \alpha + \sin \beta \sin \alpha =$$

$$= \frac{w_1}{\|w\|} \cdot \frac{v_1}{\|v\|} + \frac{w_2}{\|w\|} \cdot \frac{v_2}{\|v\|} =$$

$$= \frac{w_1 v_1 + w_2 v_2}{\|w\| \cdot \|v\|} = \frac{w \cdot v}{\|w\| \cdot \|v\|} = \cos(\theta)$$



28 xy plane



PS 1.3

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$$w_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad w_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad w_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

Find 1. combination that gives zero vector

$$x_1 w_1 + x_2 w_2 + x_3 w_3 = 0$$

$$\begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = 0$$

$$\begin{aligned} x_1 + 4x_2 + 7x_3 &= 0 \\ 2x_1 + 5x_2 + 8x_3 &= 0 \\ 3x_1 + 6x_2 + 9x_3 &= 0 \end{aligned}$$

$$\begin{array}{ccc|l} 1 & 4 & 7 & x_1 \\ 1 & 1 & 1 & x_2 - x_1 \\ 1 & 1 & 1 & x_3 - x_2 \end{array}$$

$$\begin{vmatrix} 1 & 4 & 7 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix} = \begin{vmatrix} x_1 \\ x_2 - x_1 \\ x_3 - 2x_2 + x_1 \end{vmatrix}$$

dependent

$$\begin{array}{ccc} 1 & 3 & 7 \\ 2 & 3 & 8 \\ 3 & 3 & 9 \end{array}$$

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PS 2.1

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$$u_1 = Au_0 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} .8 \\ .2 \end{bmatrix}$$

$$u_2 = Au_1 = \begin{bmatrix} .8^2 + .06 \\ .16 + .14 \end{bmatrix} = \begin{bmatrix} .7 \\ .3 \end{bmatrix}$$

$$u_3 = \begin{bmatrix} .56 + .09 \\ .14 + .21 \end{bmatrix} = \begin{bmatrix} .65 \\ .35 \end{bmatrix}$$

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$$As = s$$

$$\begin{matrix} 0.6 \\ 0.39 \end{matrix}$$

$$.8x + .3y = x$$

$$.2x + .7y = y$$

$$y = \frac{.2x}{.3}$$

$$.8x + .2x = x$$

$$x = x$$

$$x=0$$

$$y=0$$

PS 2.2

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linear  
combination

$$\begin{cases} x + y + z = 0 \\ x - 2y - z = 1 \\ 0 + 3y + 2z = 5 \end{cases}$$



$$x + y + z = 0$$

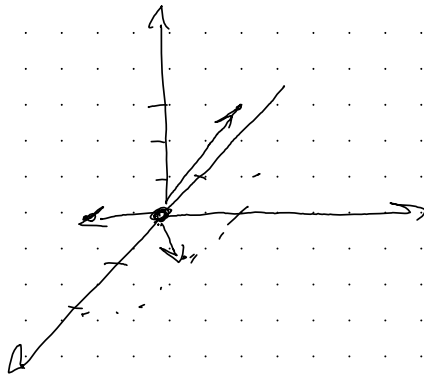
$$0 - 3y - 2z = 0$$

$$0 + 3y + 2z = 5$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} c + \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} l = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

$$c = 2$$

$$l = -1$$



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a) zero

ps 2.3

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$$a) \sum_{j=1}^n a_{3j} x_j$$

$$b) -a_{11} + a_{21}$$

$$c) E_{21}(E_{21}A)$$

$$-a_{11} + (-a_{11} + a_{21})$$

$$-2a_{11} + a_{21}$$

$$d) E_{21} A x = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

(1,1)

$$\sum a_{1j} x_j$$

$$\begin{bmatrix} -1 & 1 & 0 & 0 \end{bmatrix}_{E_{21}} \begin{bmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{bmatrix}_A \begin{bmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{bmatrix} \begin{matrix} (2,1) \\ \downarrow \\ -a_{11} \end{matrix}$$



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$$\left[ \begin{array}{cccc|c} 1 & & & & 1 \\ & 1 & & & 1 \\ & & 1 & & 1 \\ 1 & 2 & 1 & & 1 \\ 1 & 3 & 3 & 1 & 1 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & & & & 1 \\ 0 & & & & -1 \\ 0 & 1 & & & 1 \\ 0 & & 1 & 1 & -1 \\ 0 & 1 & 2 & 1 & 1 \end{array} \right] \xleftarrow{E}$$

$$\left[ \begin{array}{cccc|c} 1 & & & & 1 \\ & 1 & & & -1 \\ & & 1 & & 1 \\ & & & 1 & -2 \\ & & & 1 & -3 \end{array} \right] \xleftarrow{M}$$

PS 2.4

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$$AX = I$$

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$A_{m \times n}$

$B_{n \times p}$

$C_{p \times q}$

$(AB)C$

PS 2.5

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$$a) \begin{bmatrix} \text{row 1} \\ \text{row 2} \\ \text{row 1} + \text{row 2} \end{bmatrix} \cdot x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cccc} 1 & 2 & 3 & 1 \\ 5 & 5 & 6 & 0 \\ 6 & 7 & 9 & 0 \end{array}$$

$$\begin{array}{cccc} 1 & 2 & 3 & 1 \\ 0 & -5 & -9 & -5 \\ 0 & -5 & -9 & -6 \end{array}$$