$$(AB)(B^{-1}A^{-1})$$

$$AA^{-1} = I$$

$$(A^{-1})^T (A)^T = I$$

$$(A^T)^{-1} A^T = I$$

$$\begin{array}{cc} E_{21} & A \\ \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} & \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} \end{array}$$

$$A = L \quad U$$

$$\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

$E_{32} E_{31} E_{21} A = U$ (no row exchanges)

$$A = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} U$$

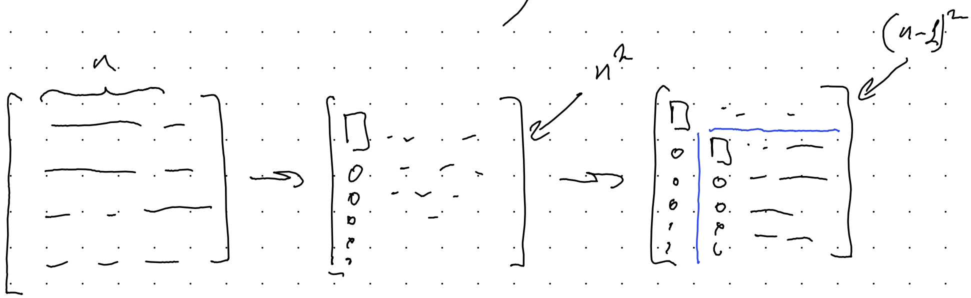
$\boxed{A = LU}$ if no row exchanges, multipliers go to L

How many operations on $n \times n$ matrix A?

$n^2 - \frac{1}{2}n^2$

$$S = \overbrace{\underbrace{n-1}_{a_{n-1}} + n-2 + \dots + 2 + \underbrace{1}_{a_1}}^{n-1}$$

$$S_{n-1} = (n-1)\left(\frac{1 + n-1}{2}\right) = \frac{n^2}{2} - \frac{n}{2}$$



Count

$$n^2 + (n-1)^2 + (n-2)^2 + \dots + 1^2$$

$$\approx \frac{1}{3}n^3$$

from where we take? calculus

Cost of b $\quad n^2$

We are ready to allow row exchanges

Permutations 3x3

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

r1	r2	r3
1	1	1
2	2	2
3	3	3

3 3 3

n^n - with repetions

$\frac{n!}{(n-n)!}$ - w/o rep.

$P^{-1} = P^T$

Recitation

$$A = \begin{pmatrix} 1 & 0 & 1 \\ a & a & a \\ b & b & a \end{pmatrix}$$

Find LU decomposition
For which a, b it exists?

$a \neq 0$

$\cancel{b \neq a}$

$\cancel{b \neq 0}$

$$A = P_{12} \; E_{21} \; E_{31} \; E_{32} \; U$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{a}{b} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & b & a \\ 0 & 0 & a \end{bmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ a & a & a \\ b & b & a \end{pmatrix}$$

$$\underset{P_{31}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}} \begin{bmatrix} 1 & 0 & 1 \\ a & a & a \\ b & b & a \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ b & b & a \\ a & a & a \end{bmatrix}$$

$$\underset{E_{21}}{\begin{bmatrix} 1 & 0 & 0 \\ -b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 0 & 1 \\ b & b & a \\ a & a & a \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & b & a-b \\ a & a & a \end{bmatrix}$$

$$\underset{E_{31}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & b & a-b \\ a & a & a \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & b & a-b \\ 0 & a & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & a & 0 \\ 0 & 0 & a-b \end{bmatrix} \qquad A = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} U$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{b}{a} & 1 \end{bmatrix} \overset{U}{\begin{bmatrix} 1 & 0 & 1 \\ 0 & a & 0 \\ 0 & 0 & a-b \end{bmatrix}}$$

$$\underset{E}{\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & \frac{b}{a} & 1 \end{bmatrix}} \cdot U$$

Singular m. can have a LU decomposition

can be zero because we don't do row exchange