Mathematical Statistics
Lab 6

1. Suppose you wish to find the average annual salary for school teachers in the United States. Suppose you took a random sample and found that $\bar{x} = 53145\$$ and you estimated $\sigma = 16451\$$. Find a 95% con

   dence interval for the mean salary, given that your sample had size

   (a) n = 36,

   (b) n = 64,

   (c) n = 144.

   (d) Does the margin of error increase or decrease as n gets larger. What effect did quadrupling n have on the margin of error?

   (e) Does the confi

   dence interval get longer or shorter as the sample size increases? Is that what you would expect?

2. A lady wanted to buy a pair of new fancy shoes. She started to shop around on-line and noticed that prices vary for exactly the same pair of shoes. She wrote down the prices for the shoes from visited on-line stores (109, 120, 120, 110, 100, 125). Assuming that every on-line store determines their own price for given pair of shoes, help the lady to find standard deviation and what would be the 95% confidence interval for the mean price of the fancy shoes.

3. Avid gardener measured length of his 4 favourite flowers and the results were 15; 11; 20; 14 cm. Now the gardener is interested in the population mean – the mean after infinitely many experiments, i.e what would be the average length of his favourite flowers in his garden. Find sample mean, standard deviation for sample mean (what is the difference from standard error?) and finally 95%-confidence interval for the mean!

4. Let's assume that we know the true variation of the weight loss ($\sigma^2 = 100$). What should be the sample size so that we could be 95% sure, that mean weight loss in the population would not be further from the sample mean more than 4 units? 2 units? What would be results be if $\sigma^2 = 25$? You can approximate quantile value for the convenience of the calculation.

5. A physician wanted to examine the change of blood pressure (unit: mm Hg) when using a certain medicine. First, the blood pressure was measured for each of 10 persons, then each of them received a certain dose of the medicine and finally after 20 minutes the blood pressure was measured once again. Model:
   The measured blood pressure of person i before and after is $N(\mu_i, \sigma_1^2)$ and $N(\mu_i + \triangle, \sigma_2^2)$, respectively.
   (a) Interpret the parameters $\mu_l, \mu_2, \ldots, \mu_{10}$ and $\triangle$.
   (b) The result of the measurements was

Person no.	1	2	3	4	5	6	7	8	9	10
Blood pressure before	75	70	75	65	95	70	65	70	65	90
Blood pressure after	85	70	80	80	100	90	80	75	90	100

Construct a 95% confidence interval for $\triangle$.

[1]

$I_{\bar{x}}$ - ? $\alpha = 5\%$

a) $n = 36$

$$\underline{I}_m = (\bar{x} - t_{\alpha/2}(f)\, d,\ \bar{x} + t_{\alpha/2}(f)) =$$

$$= \left(53145 \mp 2{,}03 \cdot \underbrace{\frac{16451}{\sqrt{36}}}_{\approx 5566},\right) =$$

$$= (47575,\ 58711)$$

b) $$\underline{I}_m = \left(53145 - \underbrace{2.00 \cdot \frac{16451}{8}}_{\approx 4112{,}75}\right.$$

c) $$\underline{I}_m = \left(53145 - \underbrace{1{,}98\ \frac{16451}{12}}_{\approx 2714}\right.$$

d) margin of error decreases as n gets larger
if n quandruples then error decreses twice

e) conf. interval decreses.

$\boxed{2}$

$$I_{\sigma^2} = \left(\frac{\sum^{n} (x_i - \bar{x})}{\chi^2_{\alpha/2}(f)} , \frac{\sum^{n} (x_i - \bar{x})}{\chi^2_{-\alpha/2}(f)} \right)$$

$$\bar{x} = \frac{109 + 120}{6} = 114$$

$$s^2 = \frac{1}{n-1} \sum^{n} (x_i - \bar{x})^2 = 86$$

$$t_{5/2}(5) = 2{,}57$$

$$I_\mu = 114 \pm 2{,}57 \frac{\sqrt{86}}{\sqrt{5}} = (104, 124)$$

$\boxed{3}$

$m - ?$

$\bar{X} = 15$

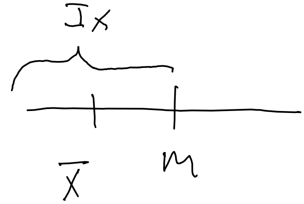
$S = 14$

$t_{\frac{0,05}{2}, 3} = 3,182$

$I_{\mu} = 15 \pm \quad = (9, 20)$

$SE_{\bar{X}} = \frac{S}{\sqrt{n}}$

$\boxed{4}$



$$I_m = \bar{X} \pm \underbrace{\lambda_{\alpha/2}\frac{\sigma}{\sqrt{n}}}_{4}$$

$$4 \geq \lambda_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$n_4 \geq \left(\lambda_{\alpha/2}\frac{\sigma}{4}\right)^2 \geq \left(1{,}96\cdot\frac{\cancel{10}^{\cancel{5}\ 2{,}5}}{\cancel{4}_{\cancel{2}}}\right)^2 \geq (4{,}9)^2 \geq 24{,}01 \gtrsim 25$$

$$n_2 = (49\cdot 2)^2 \geq 96{,}4 \gtrsim 97$$

$\underline{\sigma^2 = 25}$

$$n_4 \geq \left(\frac{4{,}9}{2}\right)^2 \geq 6{,}025 \gtrsim 7$$

$$n_2 \gtrsim 25$$

$\boxed{5}$

$X \sim N(\mu_i, \sigma_1^2)$

$Y \sim N(\mu_i + \Delta, \sigma_1^2)$

$Z \sim Y - X \sim N(\Delta, 2\sigma_1^2)$

$I_\Delta = \bar{\Delta} \pm t_{\alpha/2}(f) \frac{s}{\sqrt{n}}$

$t_{5/2}(9) = 2{,}26$

$\bar{X} = \frac{10 + 0 + 5 + 15 + 5 + 20 + 15 + 5 + 25 + 10}{10}$

$= 11$

$s^2 = \frac{1}{9}(1^2 + 11^2 + 6^2 + 4^2 + 6^2 + 9^2 + 14^2 + 6^2 + 14 + 1) =$

$= \frac{1}{9}(1 + 121 + 36 + 16 + 36 + 81 + 196 + 36 + 196 + 1) \approx 60$

$$I_D = 11 \pm 2{,}26 \cdot \frac{\sqrt{60}}{\sqrt{10}} = (5{,}5 \, , \, 16{,}5)$$

$$\approx 5{,}5$$