$\boxed{\text{PS } 3.5}$

$\boxed{2}$ ✓

3 vectors

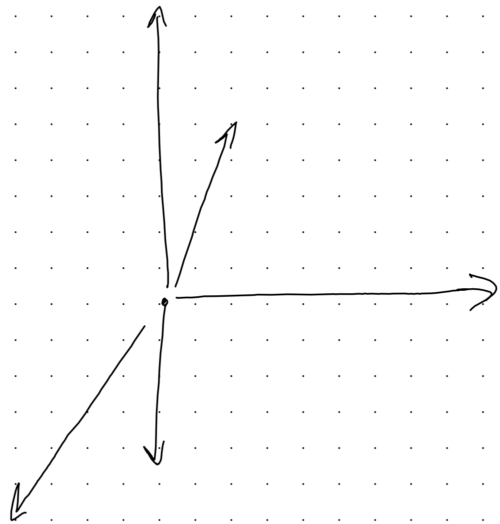
$$\begin{matrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{matrix} \Rightarrow \begin{matrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{matrix} \rightarrow \begin{matrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{matrix}$$

$\boxed{20}$?

$$x - 2y + 3z = 0$$

$$v_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 0 \\ -3 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1{,}5 & 1 \\ 0 & 0{,}5 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & \frac{3}{2} & 1 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix}$$



$$N(A) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

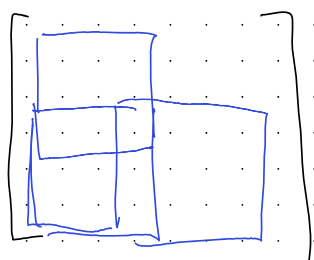
$$\begin{bmatrix} x & 2 & -3 \\ y & 1 & 0 \\ z & 0 & 1 \end{bmatrix} = 0$$

$\boxed{37}$ v

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ \underset{0}{\cancel{d}} & e & f \\ \underset{0}{\cancel{g}} & \underset{0}{\cancel{h}} & i \end{bmatrix} = \begin{bmatrix} d & e & f \\ g & h & i \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ 0 & d & e \\ 0 & g & h \end{bmatrix}$$

3



$a = e = i$

$f = b$

$\boxed{41}$ v

$$c_1 \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + c_4 \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} + c_5 \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} c_3 & c_2 + c_5 & c_1 + c_4 \\ c_1 + c_5 & c_4 & c_2 + c_3 \\ c_2 + c_4 & c_1 + c_3 & c_5 \end{bmatrix}$$

$A = 0$ when all c are 0

$$c_1 = -c_4 = -c_5 = -c_3$$

$$c_2 = c_1 = c_1$$

$\boxed{44}$

$\boxed{\text{PS 3.6}}$ $\boxed{11}$ ✓

a)

$r < m$

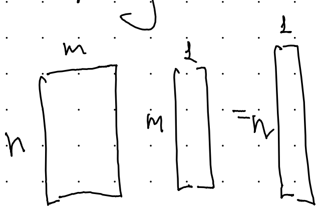
$r \le n$

b) $A^T y = 0$ has solutions other than $y = 0$

if $r < m$

$\boxed{24}$ ✓

a) $A^T y = d$



$y^T A = d$ d is in $C(A^T)$

row space

b) $A^T y = 0$

$\boxed{28}$ $+-$

$\dim C(B) = 2$

$\dim C(C) = 2$

$$\text{basis } B^T = \left\{ \begin{matrix} 1 \\ 0 \\ 1 \\ 0 \\ \vdots \end{matrix} , \begin{matrix} 0 \\ 1 \\ 0 \\ \vdots \end{matrix} \right\}$$

$$\text{basis } C^T = \left\{ \begin{matrix} r \\ n \\ b \\ \vdots \end{matrix} \quad \begin{matrix} p \\ p \\ p \\ \vdots \end{matrix} \right\}$$

$$y^T B = [\quad n \quad]$$

$$R = \begin{bmatrix} \boxed{1} & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & \boxed{1} & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix}$$

$$R_C = \begin{matrix} r & p \\ n & p \\ b & p \\ q & p \\ k & p \\ b & p \\ \cancel{n} & p \end{matrix} \qquad \begin{matrix} p & r \\ p & n \\ p & b \\ p & q \\ p & k \\ p & b \\ p & \cancel{n} \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ \vdots & 0 \\ & \vdots \end{matrix} \qquad \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$$

$$
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & -1 & 0 \\
1 & & & & & \\
& 1 & & & & \\
& & 1 & & & \\
& & & 1 & & \\
& & & & 1 & \\
& & & & & 1
\end{array}
$$

$$
C \to R = \begin{array}{cccccccc}
r & n & b & q & k & b & n & r \\
p & p & p & p & p & p & p & p
\end{array}
$$

$$
p - \frac{pn}{r}
$$

basis $N(C)$

$$
\begin{array}{cccc}
-\frac{b-r}{r} & -\frac{q-r}{r} & \cdots & \\
1 & & & \\
& 1 & & \\
& & 1 & \\
& & & 1 \\
& & & & 1 \\
& & & & & 1
\end{array}
$$

30 ?

$\dim C(A) = 1$

$\dim C(A^T) = 1$

$\dim N(A) = 1$

$\dim N(A^T) = 1$