Mathematical Statistics
Week 5 exercises

1. Industrial engineers who specialize in ergonomics are concerned with designing workspace and devices operated by workers so as to achieve high productivity and comfort. The article "Studies on Ergonomically Designed Alphanumeric Keyboards" (Hum. Factors, 1985: 175–187) reports on a study of preferred height for an experimental keyboard with large forearm–wrist support.

   A sample of $n = 31$ trained typists was selected, and the preferred keyboard height was determined for each typist. The resulting sample average preferred height was $\overline{x} = 80$ cm. Assuming that the preferred height is normally distributed with $\sigma = 2.0$ cm (a value suggested by data in the article), obtain a Confidence Interval for μ, the true average preferred height for the population of all experienced typists.

   Find CI, if σ is not known and the sample standard deviation is 2.0 cm.

2. A finite mathematics course has recently been changed, and the homework is now done online via computer instead of from the textbook exercises. How can we see if there has been improvement? Past experience suggests that the distribution of final exam scores is normally distributed with mean 65 and standard deviation 13. It is believed that the distribution is still normal with standard deviation 13, but the mean has likely changed. A sample of 40 students has a mean final exam score of 70.7.

   Calculate a confidence interval for the population mean using a confidence level of 90%.

3. A random sample of 50 patients who had been seen at an outpatient clinic was selected, and the waiting time to see a physician was determined for each one, resulting in a sample mean time of 40.3 minutes and a sample standard deviation of 28.0 minutes. Find an upper confidence bound for true average waiting time with a confidence level of 95%.

   How the CI changes with a confidence level of 99%?

4. A sample of size 15 is taken from a larger population; the sample mean is calculated as 12 and the sample variance as 25.

   What is the 95% confidence interval for the population mean μ?

5. A machine is producing metal pieces that are cylindrical in shape. A sample of pieces is taken and the diameters are 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, and 1.03 centimetres.

   Find a 99% confidence interval for the mean diameter of pieces from this machine, assuming an approximate normal distribution.

6. Consider the unknown pH value m. Four measurements of m were performed and resulted in

$$8.24 \quad 8.18 \quad 8.15 \quad 8.23.$$

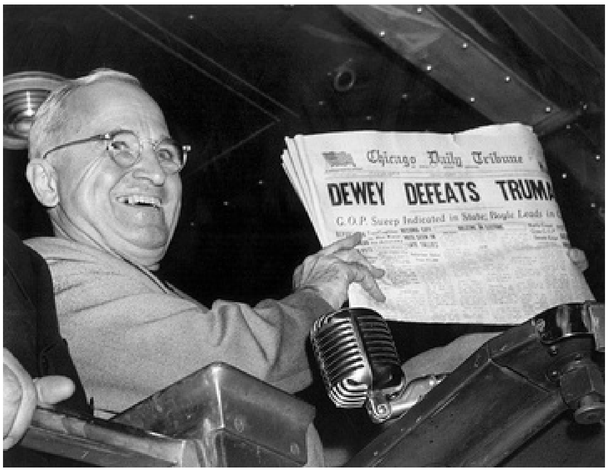
Model: The pH meter has a systematic error $\triangle$ and a random error which is $N(0, \sigma^2)$, hence the four measurements form a random sample from $N(m + \triangle, \sigma^2)$. It is known that $\triangle = 0.10$ and $\sigma = 0.05$.

Find a 99% confidence interval for m.

7. Researcher wants to estimate the mean weight of girls in Liverpool. She assumes that the distribution of weights is normal with unknown mean μ, but known standard deviation σ = 24.6 pounds.

   How many girls should she sample with 95% confidence, if she wants the width of interval to be at most 5 pounds?

8. Interesting case where faulty sampling caused a scandal: USA presidential elections Dewey vs Truman.



$\boxed{1}$

$n = 31$

$\bar{x} = 80$ cm

1) $\sigma = 2{,}0$ cm (assume is known)

$$I_\mu = \left(\bar{x} - \lambda_{0.025} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + \lambda_{0,025} \frac{\sigma}{\sqrt{n}}\right)$$

$$I_\mu = 80 \pm 1{,}96 \cdot \frac{2{,}0}{\sqrt{31}} = 80 \pm 0{,}704$$

2) σ is unknown $S = 2{,}0$ cm

$$I_\mu = \bar{x} \pm t_{\alpha/2}(f) \cdot d$$

where

$$d = \frac{S}{\sqrt{n}}$$

$$f = n - 1$$

$$I_\mu = 80 \pm 2{,}04 \cdot \frac{2}{\sqrt{31}} = 80 \pm 0{,}733$$

$\boxed{2}$

$\mu = 65$

$\sigma = 13$ (known is the same)

$n = 40$

$\bar{x} = 70,7$

$1 - \alpha = 0,9$

$\alpha = 0,1$

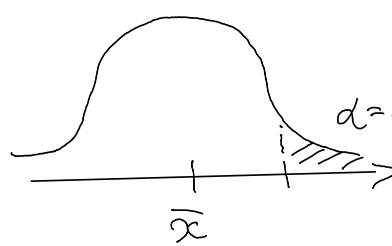
$$I_\mu = \bar{x} \pm \lambda_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$I_\mu = \left(70,7 - 1,645 \cdot \frac{13}{\sqrt{40}},\ 70,7 + 1,645 \cdot \frac{13}{\sqrt{40}}\right)$$

$$I_\mu = (70,7 - 3,381,\ 70,7 + 3,381)$$

$$I_\mu = (67,319,\ 74,081)$$

$\boxed{3}$



$\bar{x} = 40,3$ min

$S = 28,0$ min

$n = 50$

a)

$$I_\mu^{\text{Upper}} = \bar{x} + t_{\alpha,f} \cdot \frac{S}{\sqrt{n}} = 40,3 + 1,675 \cdot \frac{28}{5\sqrt{2}} =$$

$$= 40,3 + 6,633 = 46,933 \text{ min}$$

b) $\alpha = 0{,}01$

$$I_{\mu}^{upper} = \bar{x} + t_{\alpha,f} \cdot \frac{S}{\sqrt{n}} = 40{,}3 + 2{,}425 \cdot \frac{28}{5\sqrt{2}} =$$

$$= 40{,}3 + 9{,}563 = 49{,}863 \text{ min}$$

$$P\left(\mu < \bar{x} + t_{\alpha}(f) \cdot \frac{S}{\sqrt{n}}\right) = 1 - \alpha$$

$$P\left(\mu < \bar{x} + t_{0{,}05}(f) \frac{S}{\sqrt{n}}\right) = 0{,}95$$

$$I_{\mu} = \left(-\infty, \bar{X} + t_{0{,}05}(f) \cdot \frac{S}{\sqrt{n}}\right)$$

$\boxed{4}$

$n = 15$ $\quad 1 - \alpha = 0{,}95$

$\bar{x} = 12$ $\quad \alpha = 0{,}05$

$S^2 = 25$

$$I_{\mu} = \bar{x} \pm t_{\alpha/2}(f) \cdot \frac{S}{\sqrt{n}} = 12 \pm 2{,}14 \cdot \frac{5}{\sqrt{15}} =$$

$$= 12 \pm 2{,}763$$

$\boxed{5}$

$\overline{x} = 1{,}006$ $\qquad$ $1-\alpha = 0{,}99$

$S = 0{,}025$ $\qquad$ $\alpha = 0{,}01$

$n = 9$

$$I_\mu = \overline{x} \pm t_{\frac{\alpha}{2}}(f) \frac{S}{\sqrt{n}} = 1{,}006 \pm \underbrace{3{,}36 \cdot \frac{0{,}025}{3}}_{\approx 0{,}027}$$

$\boxed{6}$

$\overline{x} = \dfrac{x_1 + \dots + x_4}{4} - \Delta$ $\qquad$ $\alpha = 0{,}01$

$\Delta = 0{,}1$

$\sigma = 0{,}05$

$$I_\mu = \overline{x} \pm \lambda_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$$\overline{x} = \frac{8{,}24 + 8{,}18 + 8{,}15 + 8{,}23}{4} - 0{,}1 = 8{,}2 - 0{,}1$$

$$I_\mu = 8{,}1 \pm 2{,}576 \cdot \frac{0{,}05}{2} = 8{,}1 \pm 0{,}0644$$