Survey / Questions

- Descrete RV

Does PMF is same as distribution function?

What is hyper geometric and Poisson distribution?

- Continous RV

What is distribution (of probability)?

What is PDF? Is it dencity function?

What is skew continous prob.?

What is α quantile?

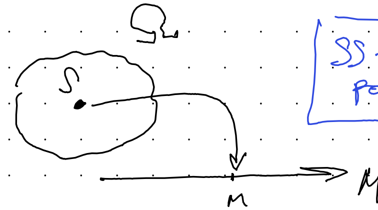
- Expectation

What is measure of location and dispersion, dependance?

RV is function defined on sample space.

$X : \Omega \to E$

SS — Measurable space



SS - set of all possible outcomes

A discrete RV when image (range) of the function is finately or infinitely countable
Continous RV if range is uncountable interval

? what is a SS in multi throw die

RV is var. whose possible values are numerical outcomes of a random phenomenon

The set of possible RV X is S_X

Descrete RV

what is countable inf. number?

Simple RV is RV with finite possible values
Infinite countable set of possible values

Probability distribution - of RV describes how probabilities are distributed over the values of RV.

probability is between 0 and 1 incl.
probability sums to 1

Probability distribution function (cummulative distributed function)

$P(X \le x) = F_X(x)$ distribution function of RV X

Theorem 1

The distr. function $F_X(x)$ of RV X has properties

$$F_X(x) \to \begin{cases} 0 \\ 1 \end{cases} \text{ when } x \to \begin{cases} -\infty \\ +\infty \end{cases}$$

$F_X(x)$ nondecreasing

$F_X(x)$ is continuous from the right of any x

Expected value (mean) of any discrete rv is an average of the possible outcomes, with each outcome weighted by its probability

$$E(X) = \sum_{x_i \in S_X} x_i \, p(x_i)$$

$$E(X) = \mu_X$$

- E is measure of location

Properties:

$E[c] = c$

$E[cX] = c\,E[X]$

$E[X+Y] = E[X] + E[Y]$

$E[X-Y] = E[X] - E[Y]$

$E[X \cdot Y] = E[X] \cdot E[Y]$ (only independent random vars)

Variance measure of location

$$Var(X) = E[(X - E[X])^2]$$

$$= V(X) = D(X) = \sigma^2_X$$

Standard deviation

$$\sigma_X = \sqrt{Var(X)}$$

Properties

$$Var(cX) = c^2 Var(X)$$
$$Var(X) = E[X^2] - (E[X])^2$$

When X and Y are independent:

$$Var(X+Y) = Var(X) + Var(Y)$$
$$Var(X-Y) = Var(X) + Var(Y)$$

Note $Var(X) \geq 0$

Assume $\mathrm{Var}(aX + bY) =$?

$a^2 \mathrm{Var}(X) + b^2 \mathrm{Var}(Y) + 2ab \operatorname{cov}(X, Y)$

$\operatorname{cov}(X, Y) = E\big((X - E[X]) \cdot (Y - E[Y])\big)$
covariance of X and Y

cov > 0 positively correlated
cov < 0 negatively cor.

$E\{X \cdot Y - X\,EY - EX \cdot Y + EX\,EY\} =$

$= E(X \cdot Y) - E[X \cdot EY] - E[EX \cdot Y]$

$= E[XY] - E[X] \cdot E[Y]$

if $\operatorname{cov}(X, Y) = 0$ it does not mean that X and Y are independent

Not necessary, we should check by probability

Bivariate distribution

		Y_i	
		0	1
X_i	-1	0	1/3
	0	1/3	0
	1	0	1/3
$P(Y)$		1/3	2/3

$P(X, Y)$

$$P(X=1, Y=1) = 1/3 \neq P(X=1)\,P(Y=1) = \frac{1}{3} \times \frac{2}{3}$$

Discrete distributions

Bernoulli distr.

$X \sim Be(p)$

$$p(k) = P(X=k) = p^k (1-p)^{1-k} \quad \text{for } k = 0, 1$$

X that counts the success in a random trial is said to be Bernoulli distribution

Binomial distribution

RV X that counts the number of successes k in n trials is said to have a binomial distribution

$X \sim Bin(n, p)$

$$p(k) = C_n^k \, p^k (1-p)^{n-k}$$

Uniform distribution

descrete RV X has k different values

$$p(X = x_j) = \frac{1}{k} \qquad j = 1, \dots, k$$

Poisson distribution

useful in estimating number of occurances over time or space interval

Poisson distribution for infinite descrete RV

k and $\lambda > 0$ constants

$$p(X = x_k) = \frac{\lambda^k}{k!} e^{-\lambda}$$