Q3. a) ~~The~~ In the $|m\rangle$ basis we have

$$\rho_{mn} = \langle m|\hat{\rho}|n\rangle$$

$$\Rightarrow \frac{d\rho_{mn}}{dt} = \langle m|\frac{d\hat{\rho}}{dt}|n\rangle$$

since $\hat{M}$ is an observable, $\hat{M}^\dagger = \hat{M}$

$$= 2mn\langle m|\hat{\rho}|n\rangle - m^2\langle m|\hat{\rho}|n\rangle - n^2\langle m|\hat{\rho}|n\rangle$$

$$= -(n-m)^2\rho_{mn}$$

$$\Rightarrow \rho_{mn}(t) = \rho_{mn}(0)\,e^{-(n-m)^2 t}$$

For an initial ~~coherent~~ pure state, $\hat{\rho}(0) = |\psi\rangle\langle\psi|$, where $|\psi\rangle = \sum_m c_m|m\rangle$

$$\Rightarrow \hat{\rho}(0) = \sum_{m,n} c_m c_n^* |m\rangle\langle n|$$

$$\Rightarrow \rho_{mn}(t) = c_m c_n^* e^{-(n-m)^2 t}$$

b) As $t \to \infty$ $\rho_{mn} \to 0$, $m \neq n$

$$\rho_{mm} = |c_m|^2$$

This means that $\hat{\rho}$ becomes a mixed state of all the $|m\rangle$ states with probability of being in that state being given by $|c_m|^2$. Since all off-diagonal terms are zero, there is no coherence between these states.