Assignment 5

Jackson Q6.17(a), 7.1, 7.19(a), 7.19(b)

Bonus: Q6.17(b)

6.17a	6.17b	7.1a	7.1b	Total
3	2	2.5	2.5	10

Assignment 9.

Q6.17

(a) The questions asks for the generalization of the Lorentz force, $\underset{\sim}{F} = q(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B})$.

Maxwell's eqns, written with a magnetic charge & magnetic current density are.

$$\nabla \cdot \underset{\sim}{D} = \rho_e \quad ; \quad \nabla \times H = \frac{\partial \underset{\sim}{D}}{\partial t} + \underset{\sim}{J_e}$$

$$\nabla \cdot \underset{\sim}{B} = \rho_m \quad ; \quad -\nabla \times \underset{\sim}{E} = \frac{\partial \underset{\sim}{B}}{\partial t} + \underset{\sim}{J_m}.$$

The duality transformation given by (6.151) for the fields & (6.152) for the sources leaves the generalized Maxwells equations invariant. That is.

$$\underset{\sim}{E} = \underset{\sim}{E}' \cos\xi + Z_0 \underset{\sim}{H}' \sin\xi \qquad (1)$$
$$\underset{\sim}{B} = -Z_0 \underset{\sim}{D}' \sin\xi + \underset{\sim}{B}' \cos\xi \qquad (2)$$
$$Z_0 q_e = Z_0 q_e' \cos\xi + q_m' \sin\xi = Z_0 q \qquad (3)$$
$$q_m = -Z_0 q_e' \sin\xi + q_m' \cos\xi = 0 \quad 1 \qquad (4)$$

From (3);
$$q\cos\xi = q_e' \cos^2\xi + \frac{q_m'}{Z_0}\sin\xi\cos\xi$$
$$= q_e'\cos^2\xi + q_e'\sin^2\xi = q_e'$$

&
$$q\sin\xi = q_e'\cos\xi\sin\xi + \frac{q_m'}{Z_0}\sin^2\xi$$
$$= \frac{q_m'\cos^2\xi}{Z_0} + \frac{q_m'\sin^2\xi}{Z_0} = \frac{q_m'}{Z_0} \quad 1$$

$$F' = q(\underset{\sim}{E}'\cos\xi + Z_0 \underset{\sim}{H}'\sin\xi) + q(\underset{\sim}{v} \times [-Z_0\underset{\sim}{D}'\sin\xi + \underset{\sim}{B}'\cos\xi])$$
$$= q\cos\xi(\underset{\sim}{E}' + \underset{\sim}{v}\times\underset{\sim}{B}') + q\sin\xi Z_0(\underset{\sim}{H}' - \underset{\sim}{v}\times\underset{\sim}{D}')$$
$$\therefore \underset{\sim}{F}' = q_e'(\underset{\sim}{E}' + \underset{\sim}{v}\times\underset{\sim}{B}') + q_m'(\underset{\sim}{H}' - \underset{\sim}{v}\times\underset{\sim}{D}') \quad 1$$

we note $\underset{\sim}{v}$ does not change under the duality transform.

(b) The questions to show invariance under (6.151) & (6.152).

$$\underset{\sim}{E} = \underset{\sim}{E}'\cos\xi + Z_0\underset{\sim}{H}'\sin\xi \;; \qquad Z_0\underset{\sim}{D} = Z_0\underset{\sim}{D}'\cos\xi + \underset{\sim}{B}'\sin\xi$$
$$Z_0\underset{\sim}{H} = -\underset{\sim}{E}'\sin\xi + Z_0\underset{\sim}{H}'\cos\xi \;; \qquad \underset{\sim}{B} = -Z_0\underset{\sim}{D}'\sin\xi + \underset{\sim}{B}'\cos\xi$$

&

$$Z_0 q_e = Z_0 q_e'\cos\xi + q_m'\sin\xi$$
$$q_m = -Z_0 q_e'\sin\xi + q_m'\cos\xi$$

There are ~~not~~ no currents present in generalized force.

$$\underset{\sim}{F} = q_e(\underset{\sim}{E} + \underset{\sim}{v}\times\underset{\sim}{B}) + q_m(\underset{\sim}{H} - \underset{\sim}{v}\times\underset{\sim}{D}). \quad (5)$$

Under these duality transformations.

$$\underset{\sim}{F}' = \left(q_e'\cos\xi + \frac{q_m'}{Z_0}\sin\xi\right)\left(\underset{\sim}{E}'\cos\xi + Z_0\underset{\sim}{H}'\sin\xi + \underset{\sim}{v}\times\left(-Z_0\underset{\sim}{D}'\sin\xi + \underset{\sim}{B}'\cos\xi\right)\right) +$$
$$\left(-Z_0 q_e'\sin\xi + q_m'\cos\xi\right)\left(-\frac{\underset{\sim}{E}'}{Z_0}\sin\xi + \underset{\sim}{H}'\cos\xi - \underset{\sim}{v}\times\left(\underset{\sim}{D}'\cos\xi + \frac{\underset{\sim}{B}'}{Z_0}\sin\xi\right)\right) \quad (1)$$

$$= q_e'\Big[\underset{\sim}{E}'\cos^2\xi + \cancel{E}'\cos\xi\sin\xi\, Z_0\underset{\sim}{H}' + \underset{\sim}{v}\times(-Z_0\underset{\sim}{D}'\cos\xi\sin\xi + \cos^2\xi\,\underset{\sim}{B}') +$$
$$\underset{\sim}{E}'\sin^2\xi - Z_0\cos\xi\sin\xi\,\underset{\sim}{H}' - \underset{\sim}{v}\times(-Z_0\underset{\sim}{D}'\cos\xi\sin\xi - \underset{\sim}{B}'\sin^2\xi)\Big] +$$

$$q_m'\Big[\frac{\underset{\sim}{E}'}{Z_0}\sin\xi\cos\xi + \underset{\sim}{H}'\sin^2\xi + \underset{\sim}{v}\times\Big(-\underset{\sim}{D}'\sin^2\xi + \frac{\underset{\sim}{B}'}{Z_0}\sin\xi\cos\xi\Big)$$
$$+ -\frac{\underset{\sim}{E}'}{Z_0}\cos\xi\sin\xi + \underset{\sim}{H}'\cos^2\xi \cancel{+} \underset{\sim}{v}\times\Big(\underset{\sim}{D}'\cos^2\xi + \frac{\underset{\sim}{B}'}{Z_0}\cos\xi\sin\xi\Big)\Big]$$

$$\therefore \underset{\sim}{F}' = q_e'\left[\underset{\sim}{E}' + \underset{\sim}{v}\times\underset{\sim}{B}'\right] + q_m'\left[\underset{\sim}{H}' \cancel{+} - \underset{\sim}{v}\times\underset{\sim}{D}'\right] \quad (1)$$

Hence, $\underset{\sim}{F}$ is invariant under (6.151) & (6.152).

7.1a Find polarization ellipse for Stokes parameters

$s_0 = 3, \quad s_1 = -1, \quad s_2 = 2, \quad s_3 = -2$

in linear & circular polarisation basis.

Linear basis:

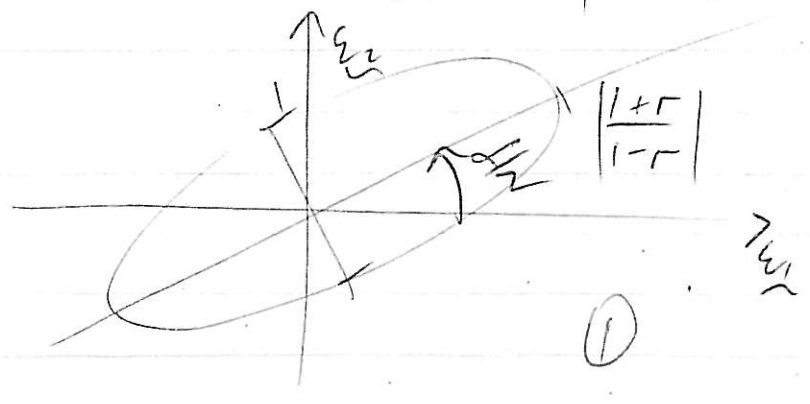
$$s_0 = |\underset{\sim}{\epsilon}_1 \cdot \underset{\sim}{E}|^2 + |\underset{\sim}{\epsilon}_2 \cdot \underset{\sim}{E}|^2 = a_1^2 + a_2^2$$
$$s_1 = |\underset{\sim}{\epsilon}_1 \cdot \underset{\sim}{E}|^2 - |\underset{\sim}{\epsilon}_2 \cdot \underset{\sim}{E}|^2 = a_1^2 - a_2^2$$
$$s_2 = 2\,\mathrm{Re}\left[(\underset{\sim}{\epsilon}_1 \cdot \underset{\sim}{E})^*(\underset{\sim}{\epsilon}_2 \cdot \underset{\sim}{E})\right] = 2a_1 a_2 \cos(\delta_2 - \delta_1)$$
$$s_3 = 2\,\mathrm{Im}\left[(\underset{\sim}{\epsilon}_1 \cdot \underset{\sim}{E})^*(\underset{\sim}{\epsilon}_2 \cdot \underset{\sim}{E})\right] = 2a_1 a_2 \sin(\delta_2 - \delta_1)$$

$$\cancel{E_+ = a_+ e^{i\delta_+}}$$
$$E_1 = a_1 e^{i\delta_1}, \quad E_2 = a_2 e^{i\delta_2}$$

~~with E Tr~~ To transform $a_1, a_2, \delta_1, \delta_2$ to a polarisation ellipse with angle ψ need a matrix rotation ~~etc~~. Simpler to extract polarisation ellipse from circular basis:

$$s_0 = |\underset{\sim}{\epsilon}_+^* \cdot \underset{\sim}{E}|^2 + |\underset{\sim}{\epsilon}_-^* \cdot \underset{\sim}{E}|^2 = a_+^2 + a_-^2$$
$$s_1 = 2\,\mathrm{Re}\left[(\underset{\sim}{\epsilon}_+^* \cdot \underset{\sim}{E})^*(\underset{\sim}{\epsilon}_-^* \cdot \underset{\sim}{E})\right] = 2a_+ a_- \cos(\delta_- - \delta_+)$$
$$s_2 = 2\,\mathrm{Im}\left[(\underset{\sim}{\epsilon}_+^* \cdot \underset{\sim}{E})^*(\underset{\sim}{\epsilon}_-^* \cdot \underset{\sim}{E})\right] = 2a_+ a_- \sin(\delta_- - \delta_+)$$
$$s_3 = |\underset{\sim}{\epsilon}_+^* \cdot \underset{\sim}{E}|^2 - |\underset{\sim}{\epsilon}_-^* \cdot \underset{\sim}{E}|^2 = a_+^2 - a_-^2$$

with $\dfrac{\text{semi major axis}}{\text{semiminor axis}} = \left|\dfrac{1+r}{1-r}\right|$ & $\dfrac{E_-}{E_+} = re^{i\alpha}$



$$E_+ = a_+ e^{i\delta_+}$$
$$E_- = a_- e^{i\delta_-}$$

$$\underset{\sim}{\epsilon}_\pm = \frac{1}{\sqrt{2}}(\underset{\sim}{\epsilon}_1 \pm i\underset{\sim}{\epsilon}_2)$$

$S_0 + S_3 = 2a_+^2 = 1 \Rightarrow a_+ = \frac{1}{\sqrt{2}}$ (1)

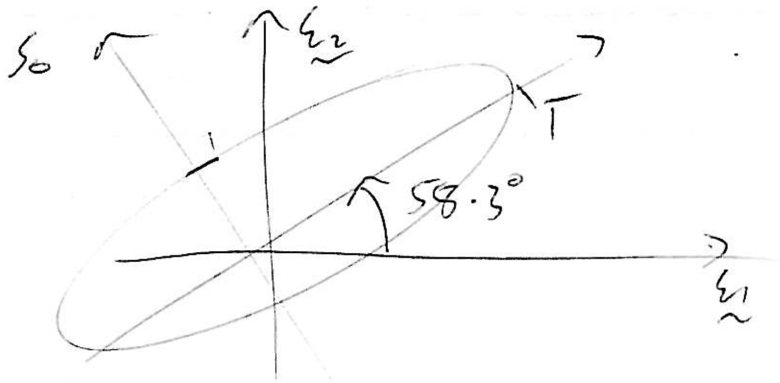
$a_-^2 = S_0 - a_+^2 = 3 - \frac{1}{2} = \frac{5}{2}$ $\therefore$ $a_- = \sqrt{5}/\sqrt{2}$.

$\tan(\delta_- - \delta_+) = S_2/S_1 = -2$ $\therefore$ $\frac{\delta_- - \delta_+}{2} = 58.3^\circ$ (0.5)

(note $\sin(\delta_- - \delta_+) > 0$, $\cos(\delta_- - \delta_+) < 0$).

& $r = |a_-/a_+| = \sqrt{5}$ &

$T = \left|\frac{1+r}{1-r}\right| = \left|\frac{1+\sqrt{5}}{1-\sqrt{5}}\right| = \left|\frac{3.236}{-1.236}\right| = 2.62$ (0.5)



(1)

Note $S_0 + S_1 = 2a_1^2 \Rightarrow a_1 = 1$ (1)

& $2a_2^2 = S_0 - S_1 \Rightarrow a_2 = \sqrt{2}$

$\tan(\delta_2 - \delta_1) = S_3/S_2 = -1$

& $\sin(\delta_2 - \delta_1) < 0$ $\Rightarrow (\delta_2 - \delta_1) = -\pi/4$. (0.5)

$\cos(\delta_2 - \delta_1) > 0$

$\therefore$ $E = E_1 \underset{\sim}{\varepsilon_1} + E_2 \underset{\sim}{\varepsilon_2}$

$E e^{-i\delta_1} = \underset{\sim}{\varepsilon_1} + \sqrt{2} e^{-i\pi/4} \underset{\sim}{\varepsilon_2}$

$E e^{-i\delta_+} = \frac{1}{\sqrt{2}} \underset{\sim}{\varepsilon_+} + \sqrt{\frac{5}{2}} e^{-i\,116.6/360\,\pi} \underset{\sim}{\varepsilon_+}$ (0.5)

7.1(b) $S_0 = 25, \quad S_1 = 0, \quad S_2 = 24, \quad S_3 = 7.$

Circular basis: $S_0 + S_3 = 2a_+^2 = 32 \therefore a_+ = 4$ 0.5

$a_-^2 = S_0 - a_+^2 = 25 - 16 = 9 \therefore a_- = 3.$ 0.5

$S_2/S_1 = \tan(\delta_- - \delta_+) = \infty$

As $S_2 > 0 \Rightarrow \delta_- \cancel{\alpha} - \delta_+ = \pi/2 \therefore \alpha/2 = \pi/4.$ 0.5

$r = |E_-/E_+| = a_-/a_+ = 3/4.$

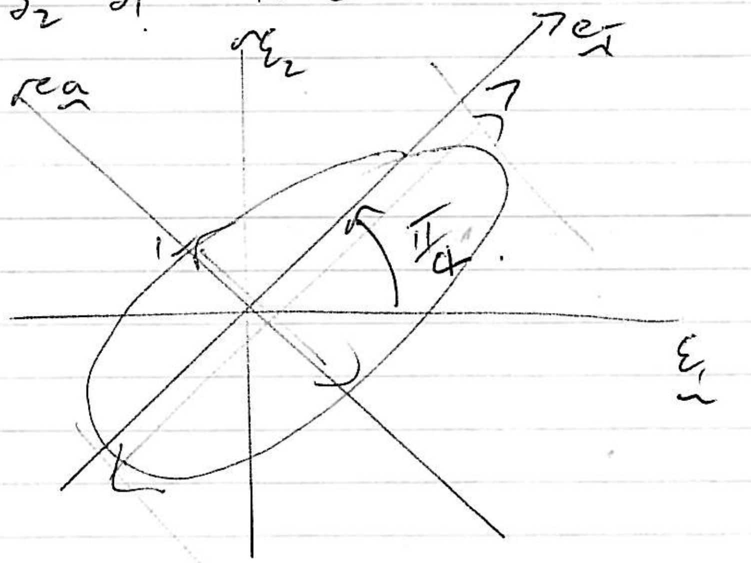
$\therefore \quad T = \frac{\text{semimajor radius}}{\text{semiminor radius}} = \left|\frac{1+r}{1-r}\right| = \left|\frac{1+3/4}{1-3/4}\right| = 7$ 0.5

Linear basis: $S_0 + S_1 = 2a_1^2 \Rightarrow a_1 = 5/\sqrt{2}.$ 0.5

$a_2^2 = a_1^2 - S_1 = 25/2 - 0 \therefore a_2 = \frac{5}{\sqrt{2}}.$ 0.5

$\tan(\delta_2 - \delta_1) = S_3/S_2 = 7/24$

$\therefore \delta_2 - \delta_1 = 16.3^\circ$ 0.5



For u_3^2 soln. $\frac{v_{1B}}{v_{1*}} = -\frac{v_A^2\cos\theta}{u_3^2} = \frac{-v_A^2\cos\theta}{s^2\cos^2\theta}$

$\therefore \frac{v_{1*}}{v_{1B}} = -\frac{s^2\cos\theta}{v_A^2} \simeq 0$

& so the only appreciable component is ~~pa~~ parallel to $\underset{\sim}{B_0}$.

0.5

7.19

Given $u(x,0) = f(x)e^{ik_0x}$ with $f(x)$ = modulation envelope

- calculate $|A(k)|^2$, ~~sketch $u(x$~~
- sketch $|u(x,0)|^2$ & $|A(k)|^2$
- evaluate explicitly rms deviations from means (Δx, Δk)
- test inequality $\Delta x \Delta k \geq \frac{1}{2}$.

(a) $f(x) = Ne^{-\alpha|x|/2}$

$$A(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} u(x,0)e^{-ikx}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} Ne^{-\alpha|x|/2}e^{i(k_0-k)x}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\cdot\frac{4\alpha N}{\alpha^2+4(k-k_0)^2}$$

& so $|A(k)|^2 = \frac{8\alpha^2N^2}{\pi(\alpha^2+4(k-k_0)^2)^2}$ 1

$(\Delta x)^2 = \langle x_p^2\rangle - (\langle x_p\rangle)^2$

~~with~~
with $\langle x_p\rangle = \int_{-\infty}^{\infty} x|f(x)|^2\,dx \Big/ \int_{-\infty}^{\infty}|f(x)|^2\,dx$

$\langle x_p^2\rangle = \int_{-\infty}^{\infty} x^2|f(x)|^2\,dx \Big/ \int_{-\infty}^{\infty}|f(x)|^2\,dx$

For given $f(x)$ $\langle x_p\rangle = 0$ as $f(x)$ is even about $x=0$.

& $\langle x_p^2 \rangle = 2/\alpha^2$

So $(\Delta x)^2 = 2/\alpha^2 \quad \therefore \quad \Delta x = \sqrt{2}/\alpha$. (0.5)

Also $(\Delta k)^2 = \langle k_p^2 \rangle - (\langle k_p \rangle)^2$ with.

$$\langle k_p \rangle = \int_{-\infty}^{\infty} k\,|A(k)|^2\,dk \Big/ \int_{-\infty}^{\infty} |A(k)|^2\,dk$$

$$= k_0$$

$$\& \ \langle k_p^2 \rangle = \int_{-\infty}^{\infty} k^2\,|A(k)|^2\,dk \Big/ \int_{-\infty}^{\infty} |A(k)|^2\,dk$$

$$= \frac{\alpha^2}{4} + k_0^2$$

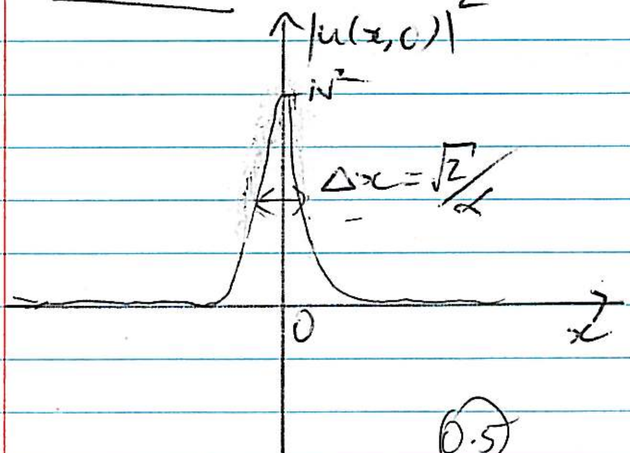
So $(\Delta k)^2 = \alpha^2/4 + k_0^2 - k_0^2 = \alpha^2/4$.

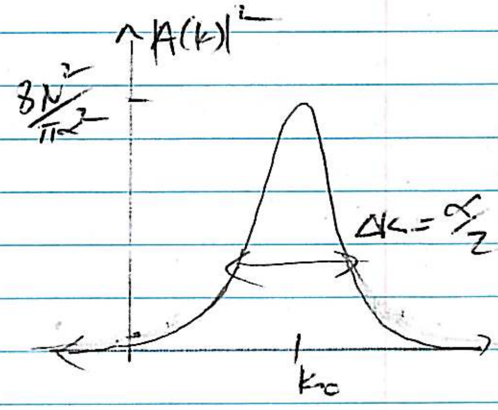
$\therefore \quad \Delta k = \alpha/2$ (0.5)

Thus $\Delta x \Delta k = \dfrac{\sqrt{2}}{\alpha}\dfrac{\alpha}{2} = \dfrac{1}{\sqrt{2}} \geqslant \dfrac{1}{2}$ (1)

& so $\Delta x \Delta k \geqslant \frac{1}{2}$.

<u>Sketches</u>





(0.5) (0.5)

Gaussian Squared Lorentzian.

7.19(b) $f(x) = N e^{-\alpha^2 x^2/4}$

$$A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} N e^{-\alpha^2 x^2/4} e^{i(k_0 - k)x} \, dx$$

$$= \frac{1}{\sqrt{2\pi}} N \, 2 e^{-(k-k_0)^2/\alpha^2} \frac{\sqrt{\pi}}{\alpha}$$

1

& so $|A(k)|^2 = \frac{2N^2}{\alpha} e^{-2(k-k_0)^2/\alpha^2}$

$$\langle x_p \rangle = \int_{-\infty}^{\infty} x \left| N e^{-\alpha^2 x^2/4} \right|^2 dx \Big/ \int_{-\infty}^{\infty} \left| N e^{-\alpha^2 x^2/4} \right|^2 dx$$

$= 0$ as $f(x)$ is symmetric.

$$\langle x_p^2 \rangle = \int x^2 \left| N e^{-\alpha^2 x^2/4} \right|^2 dx \Big/ \int \left| N e^{-\alpha^2 x^2/4} \right| dx$$

$= \frac{1}{\alpha^2}$

0.5

So $(\Delta x)^2 = \langle x_p^2 \rangle - (\langle x_p \rangle)^2 = \frac{1}{\alpha^2} \Rightarrow \Delta x = \frac{1}{\alpha}$

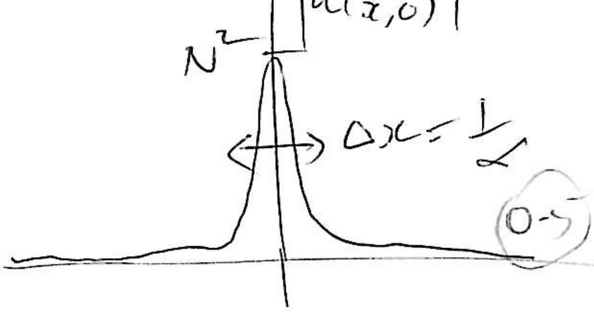
$\langle k_p \rangle = k_0$

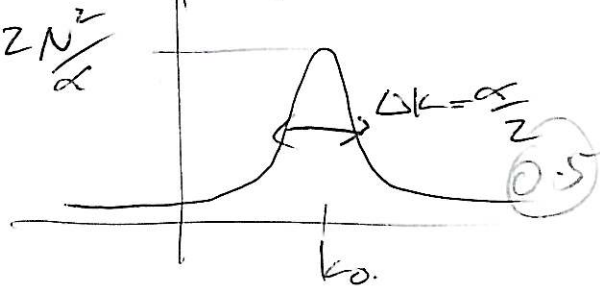
$\langle k_p^2 \rangle = \frac{1}{4}(\alpha^2 + 4k_0^2)$

0.5

$(\Delta k)^2 = \frac{1}{4}(\alpha^2) + k_0^2 - k_0^2 = \frac{\alpha^2}{4} \Rightarrow \Delta k = \frac{\alpha}{2}$

Hence $\Delta x \Delta k = \frac{1}{2}$. 1





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7.19(c) $f(x) = \begin{cases} N(1-\alpha|x|) & \text{for } \alpha|x|<1 \\ 0 & \alpha|x|>1. \end{cases}$

$$A(k) = \frac{1}{\sqrt{2\pi}} \int N(1-\alpha|x|)\, e^{i(k_0-k)x}\, dx$$

$$= \frac{-2N}{\sqrt{2\pi}} \frac{\alpha\left(\cos\left(\frac{k-k_0}{\alpha}\right)-1\right)}{(k-k_0)^2}$$

So $|A(k)|^2 = \frac{2N^2}{\pi} \alpha^2 \frac{\left(\cos\left(\frac{k-k_0}{\alpha}\right)-1\right)^2}{(k-k_0)^4}$ 1

$\langle x_p \rangle = 0$

$\langle x_p^2 \rangle = \frac{\cancel{N}^2}{15|\alpha|^3} \Big/ \frac{2N^2}{3|\alpha|} = \frac{3}{30|\alpha|^2} = \frac{1}{10|\alpha|^2}$.

$(\Delta x)^2 = \langle x_p^2 \rangle - \langle x_p \rangle^2 = 1/10|\alpha|^2 \Rightarrow \Delta x = \frac{1}{\sqrt{10}\,\alpha}$ 0.5

$\langle k_p \rangle = \int k|A(k)|^2 dk \Big/ \int |A(k)|^2 dk$

For $k_0 = 0$

~~NB: For $k_0 \neq 0$~~

$\langle k_p \rangle = 0$

$\langle k_p^2 \rangle = 3|\alpha|^2$

[NB: Can use ~~an offset~~ a change of variable in integral with effect $k_0 \to 0$.]

$\therefore (\Delta k)^2 = 3|\alpha|^2 - 0 = 3|\alpha|^2$ 0.5

$\therefore \Delta x \Delta k = \frac{1}{\sqrt{10}\,\alpha} \sqrt{3}\,\alpha = \sqrt{3/10} = 0.547\sim$.

$> \frac{1}{2}$ 1

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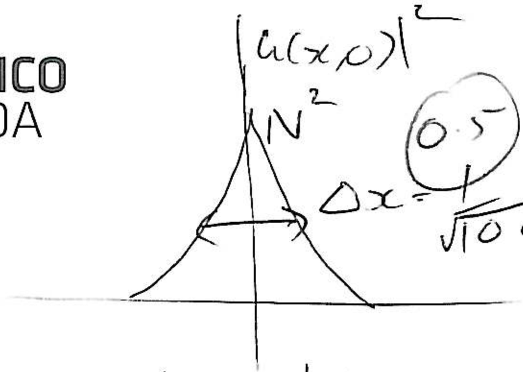
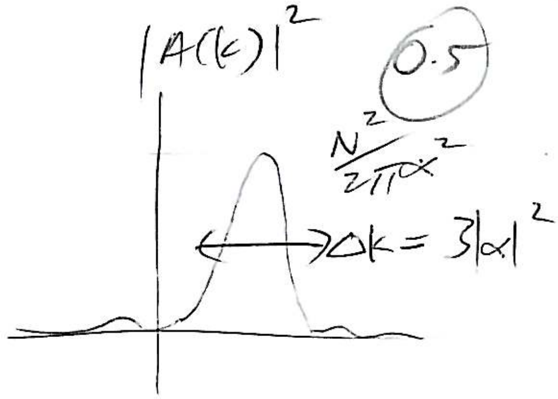




7.19(d) $f(x) = \begin{cases} N & |x| < a \\ 0 & |x| > a \end{cases}$

$$A(k) = \frac{1}{\sqrt{2\pi}} \int N e^{i(k+k_0)x}\, dx \qquad (1)$$

$$= \cancel{\frac{1}{\sqrt{2\pi}}}\, N \sqrt{\frac{2}{\pi}}\, \frac{\sin((k-k_0)a)}{k-k_0}.$$

$$|A(k)|^2 = N^2 \frac{2}{\pi} \frac{\sin^2((k-k_0)a)}{(k-k_0)^2}$$

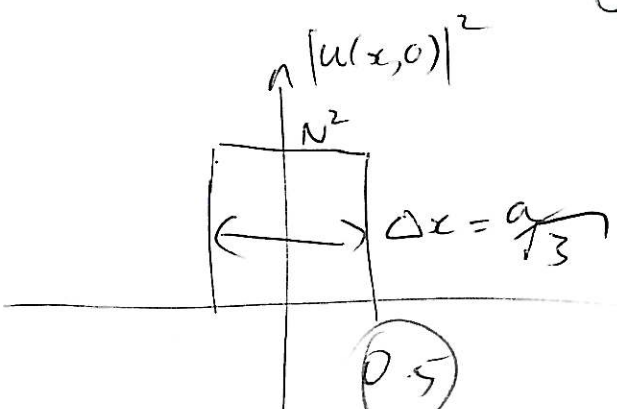
$\langle x \rangle = 0$

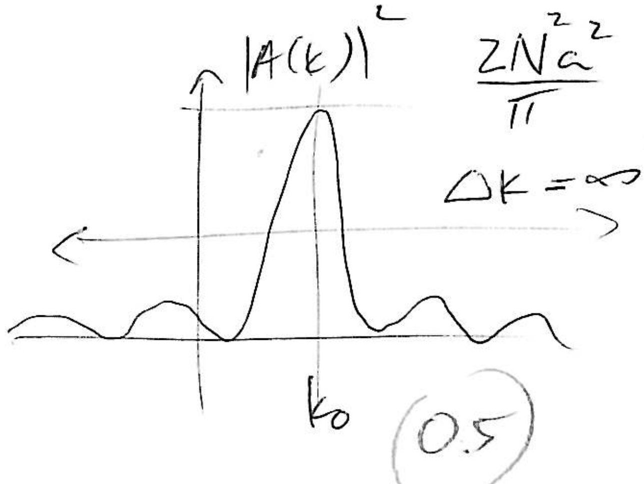
$\langle x^2 \rangle = a^2/3 \quad \therefore (\Delta x)^2 = a^2/3 \quad \Delta x = \frac{a}{\sqrt{3}}.$ (0.5)

$\langle k \rangle = \cancel{\infty}\; k_0$

$\langle k^2 \rangle = \infty \quad \Rightarrow \quad \Delta k = \infty$ (0.5)

& $\Delta x \Delta k = \infty$ (1)





6

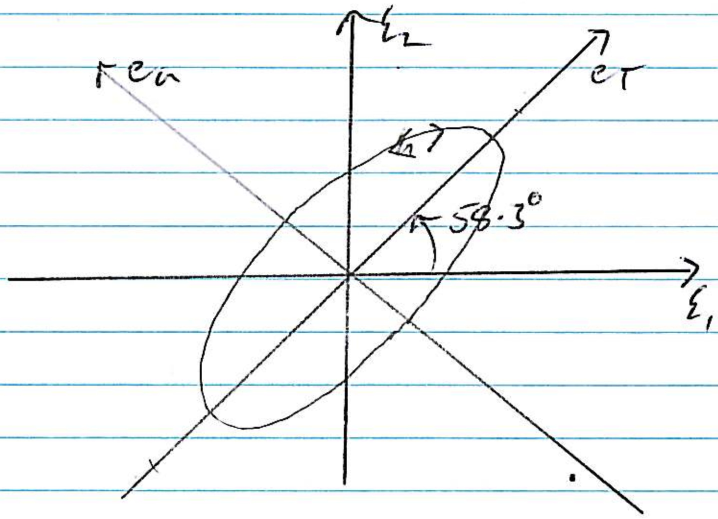
This is the angle ψ.

Also,

$$T = \left| \frac{1 + a_-/a_+}{1 - a_-/a_+} \right| = \left| \frac{1 + \sqrt{5}/2}{1 - \sqrt{5}/2} \right| = 2.618$$

0.25

mag. of T



wave vector out of page

1

$|T| = \frac{\text{major axis}}{\text{minor axis}}$ ~~ratio~~

(d) $S_0 = 25$, $S_1 = 0$, $S_2 = 24$, $S_3 = 7$.

Linear Basis: $S_0 + S_1 = 2a_1^2 = 25 \therefore a_1 = 5/\sqrt{2}$

$a_2^2 = a_1^2 - S_1 = 0$ ~~linearly polarized~~.

$\therefore a_2 = 5/\sqrt{2}$.

0.25

$\therefore S_3/S_2 = \tan(\delta_2 - \delta_1) = \frac{7}{24} \therefore \delta_2 - \delta_1 = 16.3^\circ$

0.25

& $$\tan\psi = \frac{-1 + 1 \pm \sqrt{0 + 4\cos^2(\delta_2 - \delta_1)}}{-2\cos(\delta_2 - \delta_1)}$$

$$= \frac{0 \pm 2\cos(\delta_2 - \delta_1)}{-2\cos(\delta_2 - \delta_1)}$$

$$= \mp 1 \quad \therefore \psi = \cancel{\pm 45^\circ} \mp 45^\circ.$$

& $$\frac{T}{i} = \frac{1 + e^{i(\delta_2 - \delta_1)}\tan\psi}{-\tan\psi + \frac{a_2}{a_1} e^{i(\delta_2 - \delta_1)}}$$

$\therefore \frac{T}{i} = \cancel{0.382i\ (\psi = -45)}$ $= -7i\ (\psi = 45^\circ)$, $-0.143i\ (\psi = -45^\circ)$

0.25

or $T = 7\ (\psi = 45^\circ)$, $0.143\ (\psi = -45^\circ)$

That is, the wave is RH elliptically polarized.

Circular Basis:

$s_0 + s_3 = 2a_+^2 = 32 \therefore a_+ = 4$ 0.25

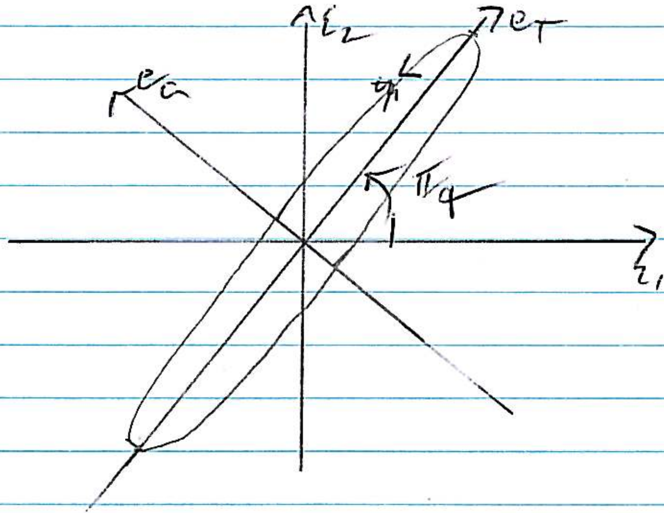
$a_-^2 = s_0 - a_+^2 = 25 - 16 = 9 \therefore a_- = 3$.

$\frac{s_2}{s_1} = \tan(\delta_- - \delta_+) = \infty$. As $s_2 > 0 \therefore \delta_- - \delta_+ = \frac{\pi}{2}$ 0.25

$\therefore \frac{\delta_- - \delta_+}{2} = \pi/4$ which is ψ.

Finally

$$T = \left| \frac{1 + a_-/a_+}{1 - a_-/a_+} \right| = \left| \frac{1 + 3/4}{1 - 3/4} \right| = 7$$ 0.25



wave vector out of page

1

~~For u_3^2 soln. $\frac{v_{1B}}{v_{1*}} = -\frac{v_A^2 c\cos\theta}{u_3^2} = \frac{-v_A^2 c\cos\theta}{s^2 c\cos^2\theta}$~~

~~$\therefore \frac{v_{1*}}{v_{1B}} = -\frac{s^2\cos\theta}{v_A^2} \simeq 0$~~

~~& so the only appreciable component is parallel to $\underset{\sim}{B_0}$.~~ 0.5

7.19

Given $u(x,0) = f(x)e^{ik_0x}$ with $f(x)$ = modulation envelope

- calculate $|A(k)|^2$, ~~sketch $u(x)$~~
- sketch $|u(x,0)|^2$ & $|A(k)|^2$
- evaluate explicitly rms deviations from means (Δx, Δk)
- test inequality $\Delta x \Delta k \geq \frac{1}{2}$.

(a) $f(x) = Ne^{-\alpha|x|/2}$

$$A(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} u(x,0)e^{-ikx}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} Ne^{-\alpha|x|/2}e^{i(k_0-k)x}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\cdot\frac{4\alpha N}{\alpha^2+4(k-k_0)^2}$$

& so $|A(k)|^2 = \dfrac{8\alpha^2N^2}{\pi(\alpha^2+4(k-k_0)^2)^2}$ 1

$$(\Delta x)^2 = \langle x_p^2\rangle - (\langle x_p\rangle)^2$$

~~with~~ with $\langle x_p\rangle = \int_{-\infty}^{\infty} x|f(x)|^2\,dx \Big/ \int_{-\infty}^{\infty}|f(x)|^2\,dx$

$$\langle x_p^2\rangle = \int_{-\infty}^{\infty} x^2|f(x)|^2\,dx \Big/ \int_{-\infty}^{\infty}|f(x)|^2\,dx$$

For given $f(x)$ $\langle x_p\rangle = 0$ as $f(x)$ is even about $x=0$.

& $\langle x_p^2 \rangle = 2/\alpha^2$

So $(\Delta x)^2 = 2/\alpha^2 \quad \therefore \quad \Delta x = \sqrt{2}/\alpha$. 0.5

Also $(\Delta k)^2 = \langle k_p^2 \rangle - (\langle k_p \rangle)^2$ with.

$$\langle k_p \rangle = \int_{-\infty}^{\infty} k\,|A(k)|^2 dk \Big/ \int_{-\infty}^{\infty} |A(k)|^2 dk$$
$$= k_0$$

$$\& \ \langle k_p^2 \rangle = \int_{-\infty}^{\infty} k^2 |A(k)|^2 dk \Big/ \int_{-\infty}^{\infty} |A(k)|^2 dk$$
$$= \frac{\alpha^2}{4} + k_0^2$$

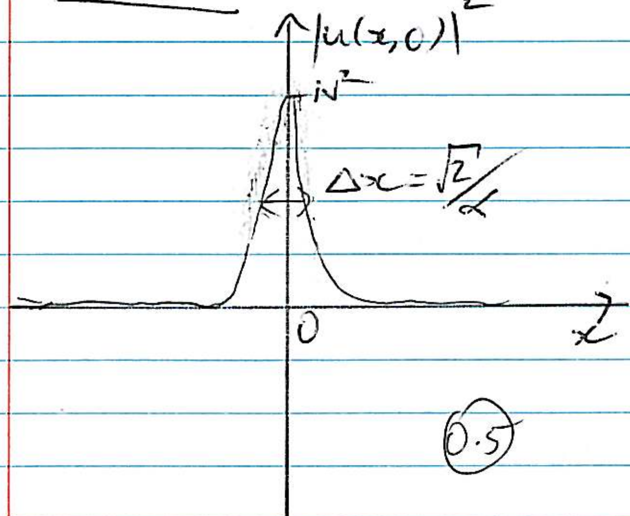
So $(\Delta k)^2 = \alpha^2/4 + k_0^2 - k_0^2 = \alpha^2/4$.

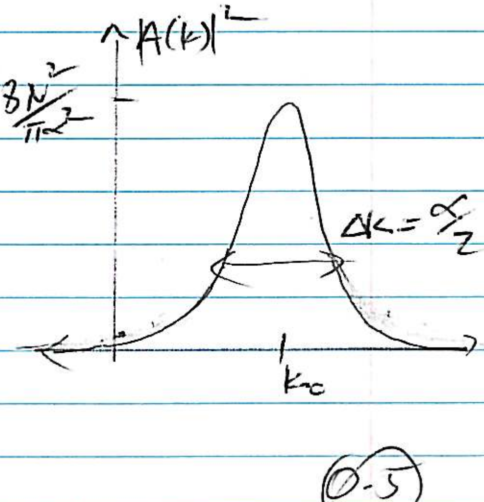
$\therefore \quad \Delta k = \alpha/2$ 0.5

Thus $\Delta x \Delta k = \dfrac{\sqrt{2}}{\alpha}\dfrac{\alpha}{2} = \dfrac{1}{\sqrt{2}} \geq \dfrac{1}{2}$ 1

& so $\Delta x \Delta k \geq \frac{1}{2}$.

<u>Sketches</u>





Gaussian

Squared Lorentzian.