

b)

We have found the Grand Partition Function to be:

$$\text{In}[15]:= Z[\beta_-] = 4 \frac{\pi}{n!} \left( \frac{2 \pi m}{\beta h^2} \right)^{3 n/2} \text{Exp}[\mu \beta n] \left( \frac{\text{Sinh}[H \beta \mu_0]}{H \beta \mu_0} \right)^n ;$$

The Grand Potential is given by:

$$\Omega = \frac{-1}{\beta} \text{Ln}(Z)$$

Expanding out all of the terms we get:

$$\begin{aligned} \text{In}[27]:= \Omega[\beta_-] &= \text{Expand} \left[ \text{PowerExpand} \left[ \frac{-1}{\beta} \text{Log}[Z[\beta]] \right] \right] \\ \text{Out}[27]= & -n \mu - \frac{2 \text{Log}[2]}{\beta} - \frac{3 n \text{Log}[2]}{2 \beta} + \frac{3 n \text{Log}[h]}{\beta} + \frac{n \text{Log}[H]}{\beta} - \frac{3 n \text{Log}[m]}{2 \beta} - \\ & \frac{\text{Log}[\pi]}{\beta} - \frac{3 n \text{Log}[\pi]}{2 \beta} + \frac{5 n \text{Log}[\beta]}{2 \beta} + \frac{n \text{Log}[\mu_0]}{\beta} + \frac{\text{Log}[n!]}{\beta} - \frac{n \text{Log}[\text{Sinh}[H \beta \mu_0]]}{\beta} \end{aligned}$$

c)

Finding the magnetism:

$$\begin{aligned} M &= -\frac{\partial \Omega}{\partial H} = -\frac{\partial}{\partial H} \left( \frac{n \text{Log}[H]}{\beta} - \frac{n \text{Log}[\text{Sinh}[H \beta \mu_0]]}{\beta} \right) \\ M &= -\frac{n}{H \beta} + n \mu_0 \text{Coth}[H \beta \mu_0] \end{aligned}$$