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Homework Assignment #1
(Due October 4, 2022)

Characteristics of Financial Time Series

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1. Data

You are given a dataset containing 6 indices on the US financial markets, which represent different asset classes:

- stocks: S&P 500 index (total return index) (S&PCOMP(RI))
- government bonds: US Treasury bond index (total return index) (MLGTRSA(RI))
- corporate bonds: US corporate bond index (total return index) (MLCORPM(RI))
- real-estate securities: US real-estate index (total return index) (WILURET(RI))
- commodities: CRB spot index (total return index) (RJEFCRT(TR))
- currencies: USD against major currencies (price index) (JPUSEEN)

Indices are total return indices (including dividends or coupons, except for commodities and currencies) at the daily frequency, from January 2000 to August 2022. The objective of this homework is to investigate the properties of financial returns and in particular the effect of temporal and contemporaneous aggregation.

2. Diagnostic for individual assets: basic observation

Compute the daily and weekly simple return and the daily and weekly continuously compounded (or log) return. Please calculate the weekly returns from Friday to Friday. For both definitions of returns, compute the sample mean, variance, skewness, kurtosis, minimum, and maximum.

2a. Compare simple return and log-returns in daily frequency.
2b. Compare simple return and log-returns in weekly frequency.
2c. How are the above descriptive statistics changed when you change the frequency from daily to weekly? (Consider log-returns only).

3. Diagnostic for individual assets: exploration

Pick only log-returns, and do the following analysis for both daily and weekly returns:

3a. Identify the main crashes and booms (say, the five smallest and largest returns) of the S&P 500 and try to identify if they are related to economic or political events.

3b. For all indices, test if the magnitude of the crashes and booms is consistent with the hypothesis of normality. To perform this test, assume that returns are normally distributed with the sample mean and variance and compute for this distribution the probability of occurrence of the extreme returns that you observed.

3c. Test whether the sample skewness and kurtosis are compatible with the normality hypothesis. That is, test normality using the Jarque-Bera test procedure at the 5% significance level. You are asked to code the Jarque-Bera test yourself.

3d. Compute the first 10 auto-correlations of the return series with the confidence interval. Compute the Ljung-Box test statistics and test the null hypothesis that the return series is not serially correlated over the sample at the 5% significance level. Use the same approach to test the null of no serial correlation of squared returns.

3e. Are your conclusions of points (a) to (d) altered by the change of frequency? Elaborate on the *temporal aggregate normality* feature.

4. Diagnostic for a portfolio

The idea is to construct a portfolio composed of the selected asset classes, using equal weights. To define the daily (weekly) portfolio return, you compute the portfolio return as the average of the daily (weekly) simple return of the six asset classes. Pick only simple returns.

4a. Compute the summary statistics of point 2 (sample mean, variance, skewness, kurtosis, minimum, and maximum) at daily frequency. Compare the statistics with those of the individual stocks (point 2a.). Elaborate on the contemporaneous aggregate normality feature.

4b. Re-do the same exercise at weekly frequency. Elaborate on the relative effect of temporal and contemporaneous aggregate normality features.

5. Conclusion

How would interpret these results from an asset allocation and risk management perspective?