Introduction to Reinforcement Learning

The RL Problem

Rewards

- A **reward** R_t is a scalar feedback signal
- It indicates how well the agent is doing at step t
- The agent's job is to maximize the cumulative reward

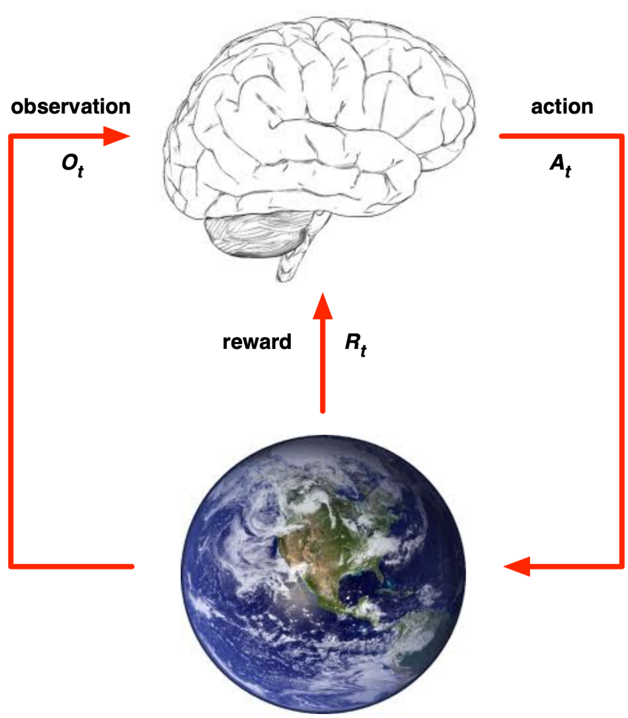
Reinforcement learning is based on the **reward hypothesis**.

> **Definition (Reward Hypothesis)**
> All goals can be described by the maximization of the expected cumulative reward.

Sequential Decision Making

- Goal: select actions to maximize total future reward
- Actions may have long-term consequences
- The reward may be delayed
- It may be better to sacrifice immediate reward to gain more long-term reward

Agent and Environment



- At each step t the agent:
  - Executes action A_t
  - Receives observation O_t
  - Receives scalar reward R_t
- The environmentL
  - Receives action A_t
  - Emits observation O_{t+1}
  - Emits scalar reward R_{t+1}
- t increments at environment step

History and State

- The **history** is the sequence of observations, actions, and rewards

$$H_t = O_1, R_1, A_1, \ldots, A_{t-1}, O_t, R_t \tag{1}$$

- i.e., the history may contain all observable variables up to time t
- What happens next depends on the history:
  - The agent selects actions
  - The environment selects observations/rewards
- **State** is the information used to determine what happens next
- Formally, state is a function of the history (we can customize this function)

$$S_t = f(H_t) \tag{2}$$

Environment State

- The **environment state** S_t^e is the environment's private representation
- i.e., whatever data the environment uses to pick the next observation/reward
- The environment state is not usually visible to the agent
- Even if S_t^e is visible, it may contain irrelevant information

Agent State

- The **agent state** S_t^a is the agent's internal representation
- i.e., whatever information the agent uses to pick the next action
- i.e., it is the information used by reinforcement learning algorithms
- It can be any function of history

$$S_t^a = f(H_t) \tag{3}$$

Information State

- An **information state** (a.k.a. **Markov state**) contains all useful information from the history.

> **Definition**
> A state S_t is Markov if and only if
> $$\mathbb{P}\left[S_{t+1} | S_t\right] = \mathbb{P}\left[S_{t+1} | S_1, \ldots, S_t\right] \tag{4}$$

- "The future is independent of the past given the present"

$$H_{1:t} \rightarrow S_t \rightarrow H_{t+1:\infty} \tag{5}$$

- Once the state is known, the history may be thrown away
- i.e., the state is a sufficient statistic of the future
- The environment state S_t^e is Markov
- The history H_t is Markov

Fully Observable Environments

- **Full observability**: agent directly observes environment state

$$O_t = S_t^a = S_t^e \tag{6}$$

- Agent state = encironment state = information state
- Formally, this is a **Markov decision process (MDP)**

Partially Observable Environments

- **Partially observability**: agent indirectly observes environment
- Now agent state $\neq$ environment state
- Formally, this is a **partially observable Markov decision process (POMDP)**
- Agent must construct its own state representation S_t^a, e.g.
    - Complete history: $S_t^a = H_t$
    - Beliefs of environment state: $S_t^a = \left(\mathbb{P}\left[S_t^e = s^1\right], \ldots, \mathbb{P}\left[S_t^e = s^n\right]\right)$
    - Recurrent nerual network: $S_t^a = \sigma\left(S_{t-1}^a W_s + O_t W_o\right)$

Inside An RL Agent

Major Compoennts of An RL Agent

An RL agent may include one or more of these components:

- Policy: agent's behavior function
- Value function: how good is each state and/or action
- Model: agent's representation of the environment

Policy

- A **policy** is the agent's behavior

- It is a map from state to action, e.g.,
    - Deterministic policy: $a = \pi(s)$
    - Stochastic policy: $\pi(a|s) = \mathbb{P}\left[A_t = a | S_t = s\right]$

Value Function

- **Value function** is a prediction of future reward
- Used to evaluate the goodness/badness of states
- And therefore to selet between actions, e.g.,

$$v_\pi(s) = \mathbb{E}_\pi\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots | S_t = s\right] \tag{7}$$

Model

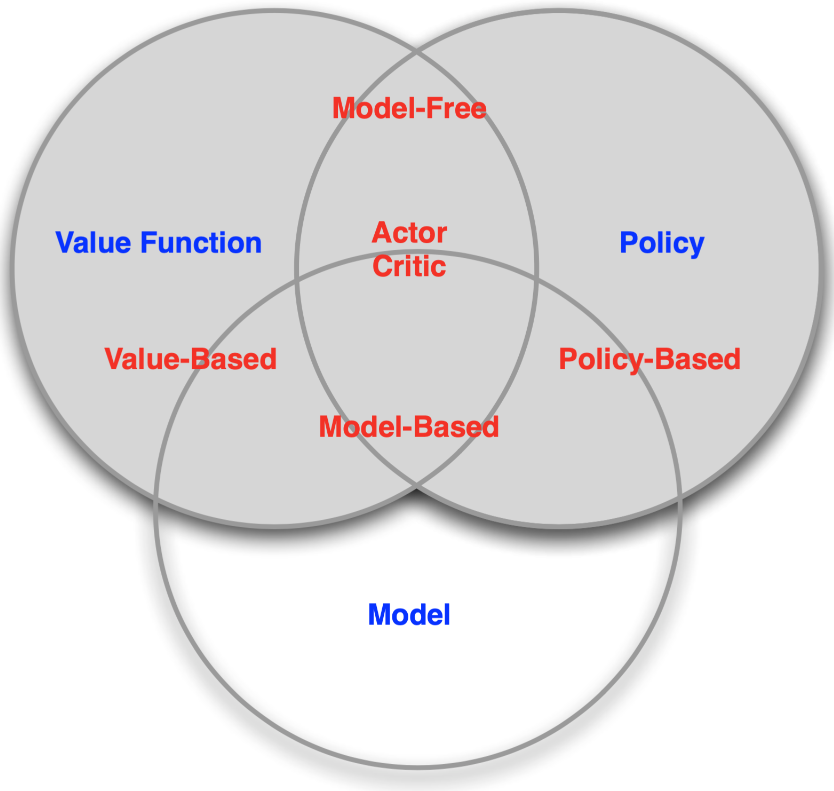
- A **model** predicts what the environment will do next
- $\mathcal{P}^a_{ss'}$ predicts the next state
- $\mathcal{R}$ predicts the next (immediate) reward

$$\begin{aligned} \mathcal{P}^a_{ss'} &= \mathbb{P}\left[S_{t+1} = s' | S_t = s, A_t = a\right] \\ \mathcal{R}^a_s &= \mathbb{E}\left[R_{t+1} | S_t = s, A_t = a\right] \end{aligned}$$

Categorizing RL Agents

- Value based
    - No policy (implicit)
    - Value function
- Policy based
    - Policy
    - No value function
- Actor critic
    - Policy
    - Value function
- Model free
    - Policy and/or value function
    - No model
- Model based
    - Policy and/or value function
    - Model

RL Agent Taxonomy



Problems Within RL

Learning and Planning

Two fundamental problems in sequential decision making

- Reinforcement learning
  - The environment is initially unknown
  - The agent interacts with the environment
  - The agent improves its policy
- Planning
  - A model of the environment is known
  - The agent performs computations with its model (without any external interaction)
  - The agent improves its policy
  - a.k.a. deliberation, reasoning, introspection, pondering, thought, search

Exploration and Exploitaiton

- Reinforcement learning is like trial-and-error learning
- The agent should discover a good policy
- From its experience of the environment
- Without losing too much reward along the way
- **Exploration** finds more information about the environment

- **Exploitation** exploits known information to maximize reward
- It is usually important to explore as well as exploit

Prediction and Control

- **Prediction**: evaluate the future
  - Given a policy
  - What is the value function for the uniform random policy?
- **Control**: optimize the future
  - Find the best policy
  - What is the optimal value function over all possible policies?
  - What is the optimal policy?