CFD Assignment 2 AEM-ADV19

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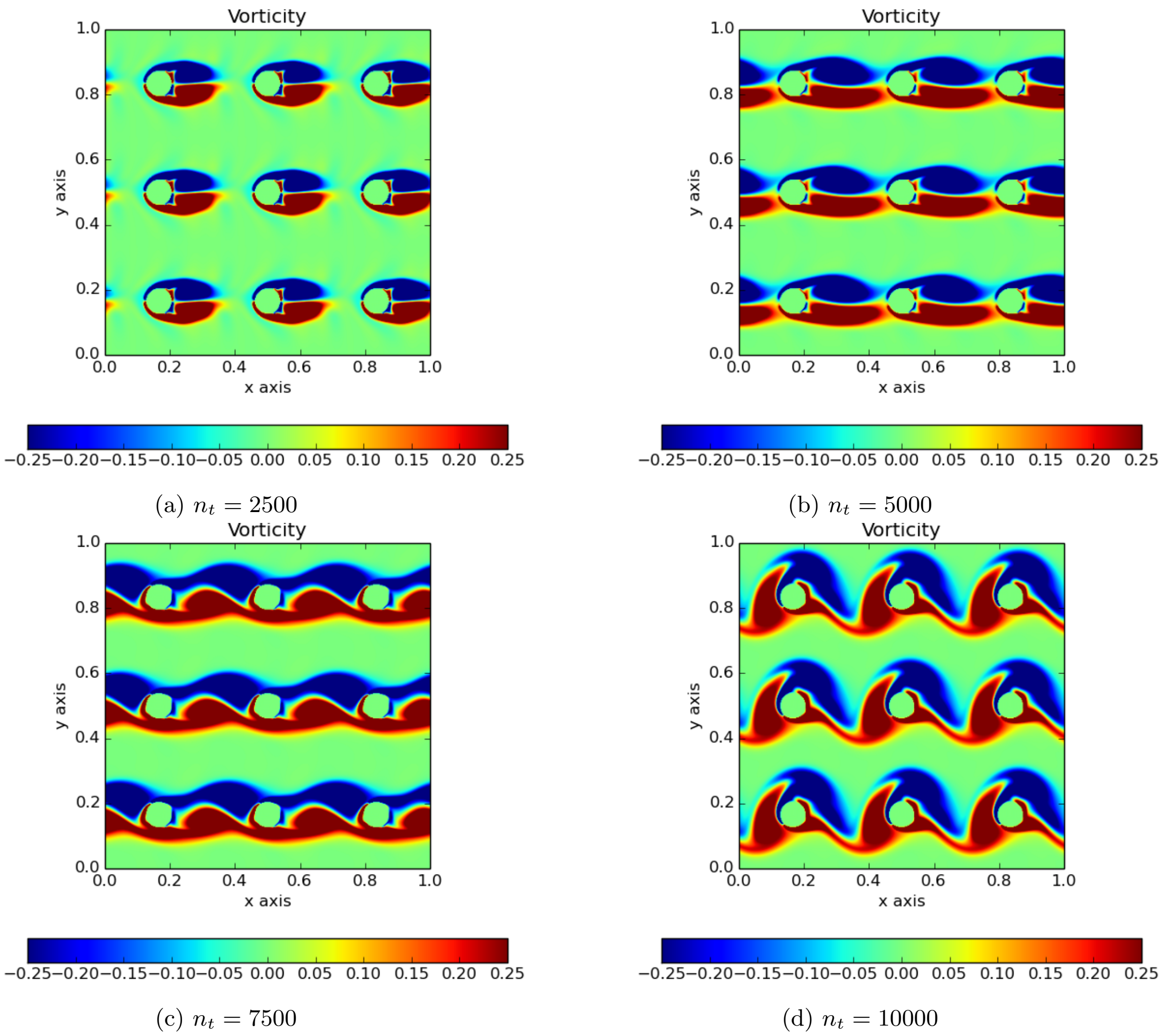
Question 1

In figure 1 snapshots of the heat exchanger at intervals of 625 units of normalised time are shown. Image (a), the flow enters the left side of the domain at $x = 0$, a stagnation point is seen to form on the cylinder leading edge. As time passes and the flow begins to have large-scale coherent structures such as in a Von Kármán street vortex[1]. This can be identified in images (a) and (b), with small pockets of reverse flow present at each trailing edge.

The vortices now traverse the domain and inevitably collide with the cylinders that are situated downstream, image (c). Then in the following image (d) they are seen to wrap around the heat exchangers.

The highly vortical structures will have consequences for the convection and conduction of heat between the elements and the fluid within in the exchanger. Strong vortices will enhance the transport of fluid to the elements increase the conduction. Then the destruction of strong coherent formations will enhance the mixing of the heat throughout the fluid.



(a) $n_t = 2500$

(b) $n_t = 5000$

(c) $n_t = 7500$

(d) $n_t = 10000$

Figure 1: Vorticity plots, CFL = 0.25, Adams-Bashford Scheme, 2[nd] order accuracy derivatives.

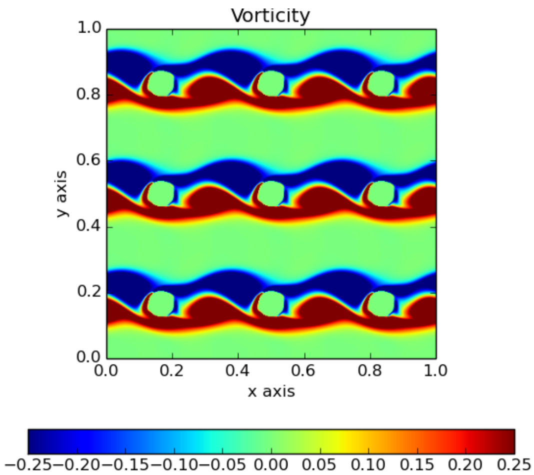
Question 2

It is not possible to run the simulations with $CFL = 0.75$ using the Adams-Bashforth scheme. This is due to the numerical domain no longer containing the physical domain. The solution then accumulates errors which lead to the exponential growth of values throughout the domain. Hence NaN is found at all points within the domain after 44 iterations.

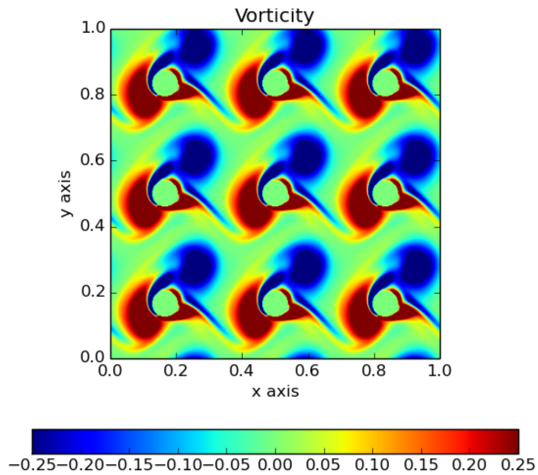
Question 3

Using the Runge-Kutta scheme the simulation is now stable for $CFL = 0.75$. Hence for each time-step completed using this order of CFL, we would have required three iterations if the CFL were at 0.25. This is noted in comparison of the figure 1 (c) to that of figure 2 (a) which occur after the same period of time is simulated.

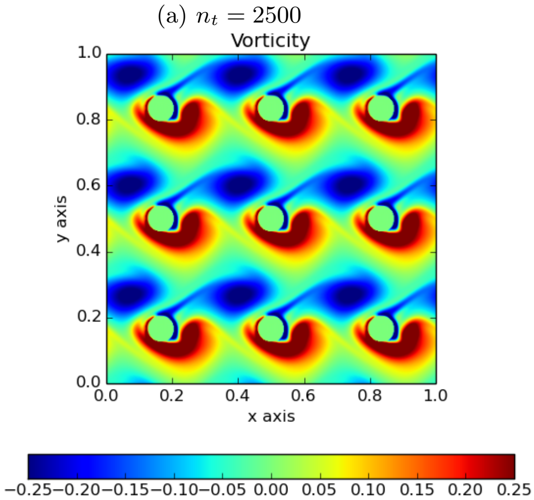
In figure 3 we can see the centre-line velocity at $nt = 2500$ and 7500 for the Runge-Kutta simulation with $CFL = 0.75$, and the Adams-Bashforth at $CFL = 0.25$ repsectively. Both have the identical velocity therefore we can see that the more stable scheme allows us to progress in time more rapidly. Thus we understand one of the key compromises in CFD, computation time against computational accuracy.



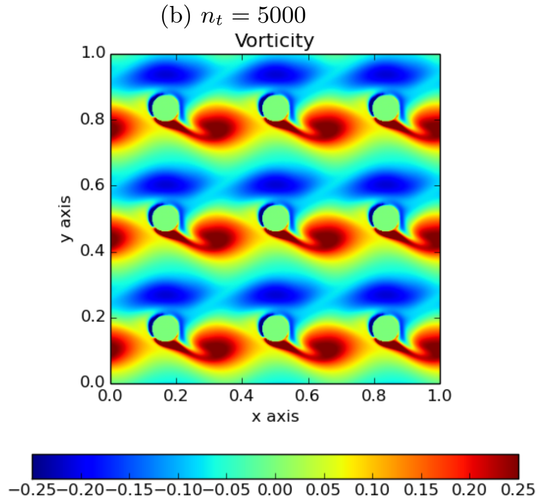
(a) $n_t = 2500$



(b) $n_t = 5000$



(c) $n_t = 7500$



(d) $n_t = 10000$

Figure 2: Vorticity plots, CFL = 0.75, 3$^{\text{rd}}$ order Runge-Kutta Scheme, 2$^{\text{nd}}$ order accuracy derivatives.

Remarking on the development of the flow the strength of vortices decays significantly between images (a) and (d). This is caused by the destruction of the large coherent vortices by the heating elements. The vortices then spread and mix throughout the domain. The negative clock-wise vortex is travelling upwards in the y axis and will ultimately come into contact with the counter-clockwise vortex where they will interact further losing kinetic energy.

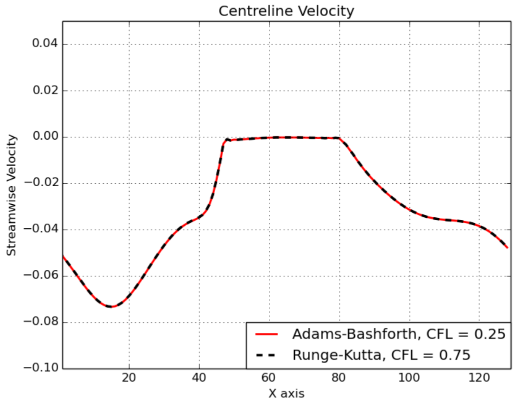
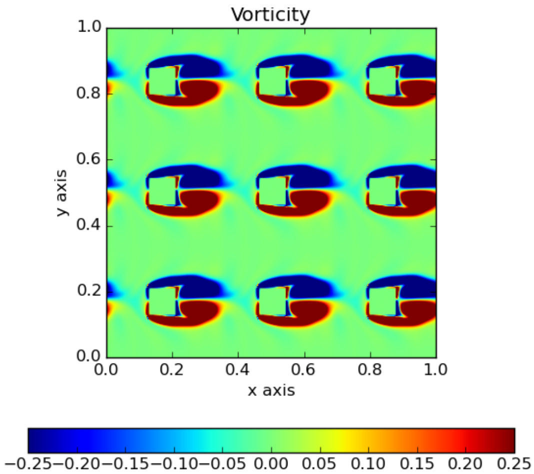


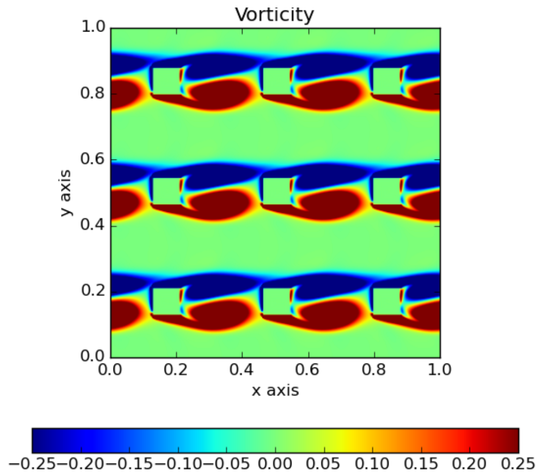
Figure 3: Comparison of centreline velocity at identical simulation time intervals $t = 1875 units$.

Question 4

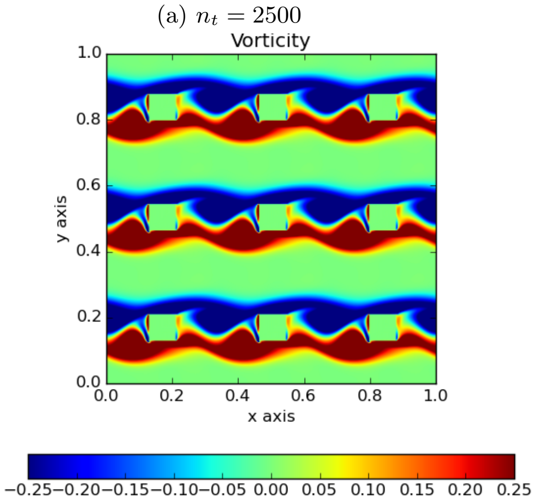
The square cylinders are shown in figure 4. The vorticity plots are very similar to those of the circular cylinders. However we can see the inner trailing vortex is far weaker than for circle. This is as there is no adverse pressure gradient to allow the grouth of these counter rotating structures. This has the effect of increasing the strength of the large vortices which then begin to interact with the other cylinders downstream. Overall the solution has stronger and more distinct separated vortices, but shares similar general characteristics of shear layer mixing as with the circular case.



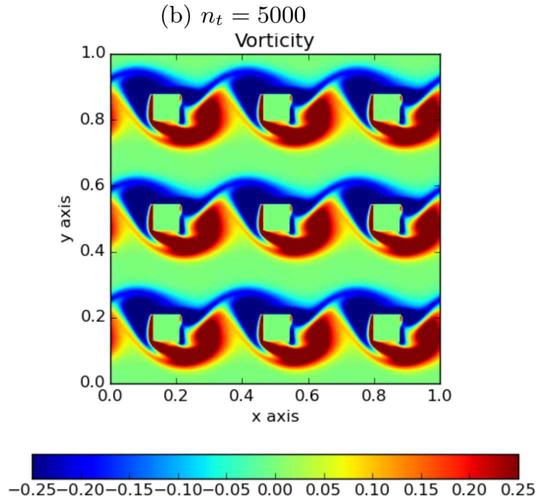
(a) $n_t = 2500$



(b) $n_t = 5000$



(c) $n_t = 7500$



(d) $n_t = 10000$

Figure 4: Vorticity plots, CFL = 0.25, 2[nd] order Adams-Bashforth Scheme, 2[nd] order accuracy derivatives.

Question 5

Using the fourth order spatial derivatives shows no discernible difference in scheme when looking at figure 5 below. Looking at the second order Taylor expansion,

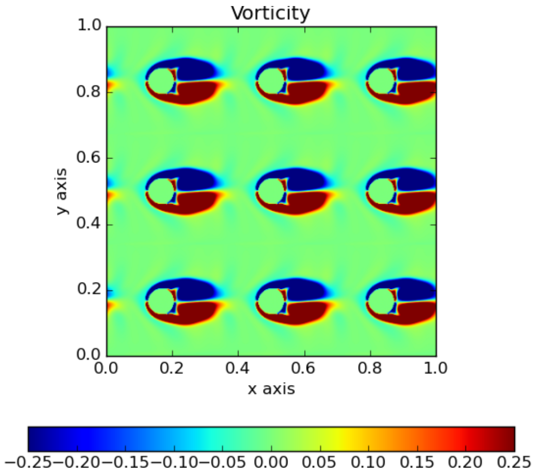
$$\frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2\Delta x} + \frac{u_i''' \Delta x^2}{6} + O(\Delta x^3) \tag{1}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} - \frac{u_i'''' \Delta x^2}{12} + O(\Delta x^3) \tag{2}$$

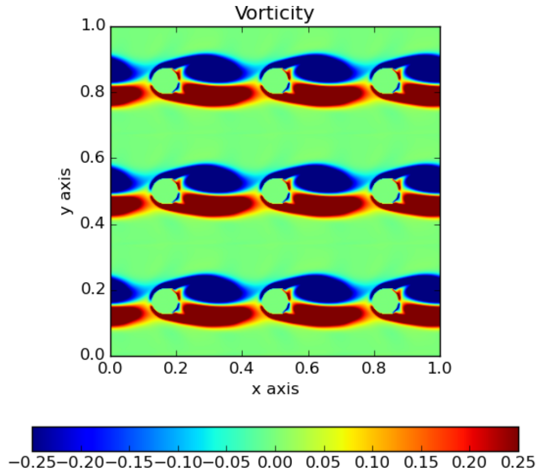
which we can compare with the fourth order derivatives shown below.

$$\frac{\partial u}{\partial x} = \frac{-u_{i+2} + 8u_{i+1} - 8u_{i-1} + u_{i-2}}{12\Delta x} + O(\Delta x^4) \tag{3}$$

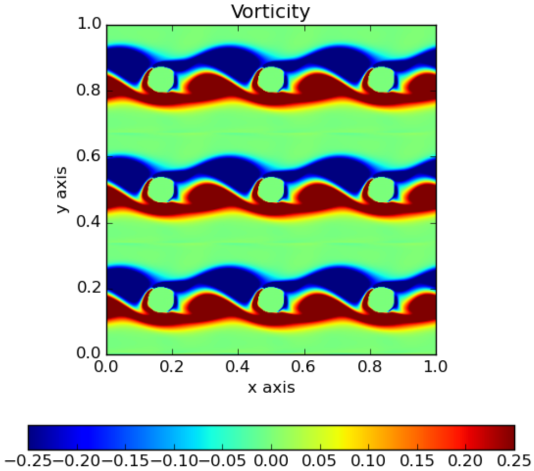
$$\frac{\partial^2 u}{\partial x^2} = \frac{-u_{i+2} + 16u_{i+1} - 30u_i + 16u_{i-1} - u_{i-2}}{12\Delta x^2} + O(\Delta x^4) \tag{4}$$



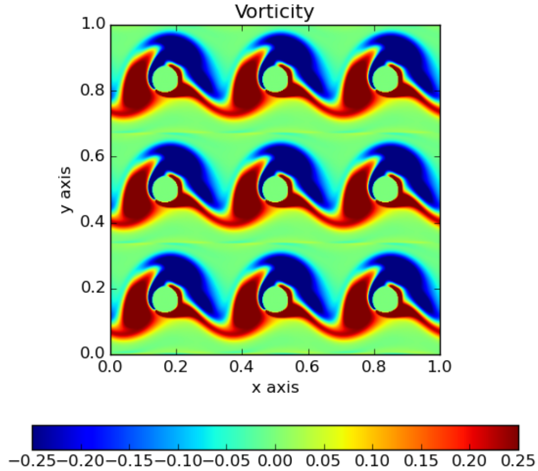
(a) $n_t = 2500$



(b) $n_t = 5000$



(c) $n_t = 7500$

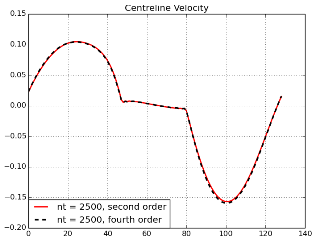


(d) $n_t = 10000$

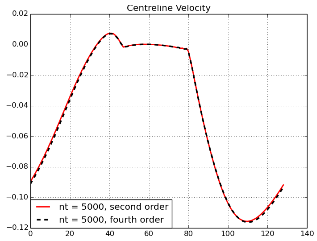
Figure 5: Vorticity plots, CFL = 0.25, Adams-Bashford Scheme, 4^{th} order accuracy derivatives.

Plotting the centreline velocity of both the second and fourth order schemes against each other

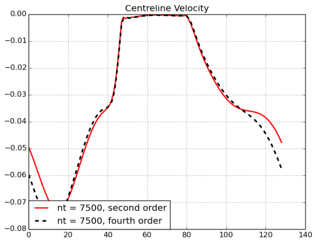
Comparison of the centerline velocity fields of the cylinders between 2^{nd} and 4^{th}, we see that there is a small difference in velocity profile however it only becomes apparent in figure 6 (c) onwards. The profiles are somewhat smoothed, due to the inclusion of more points in the calculation. However from the centerline velocity alone it is not possible to say whether the 4^{th} or 2^{nd} order are more dissipative in reality. Though it is likely to be the 2^{nd} order version as it truncates more of the solution, and there are no discontinuities across which the larger stencil of the fourth order derivatives may lose energy mistakenly.



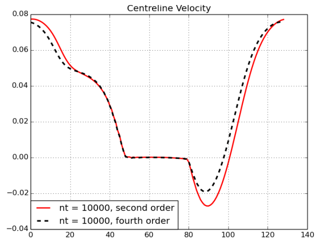
(a) $n_t = 2500$



(b) $n_t = 5000$



(c) $n_t = 7500$



(d) $n_t = 10000$

Figure 6: Centerline Velocity, CFL = 0.25, Adams-Bashford Scheme.

Question 6

Running the simulation for a long period of time would see the decay of any flows. As the simulation adds velocity only upon initialising of the domain, the use of periodic boundary condition means there is no velocity input at inlet or outlet. Corollary to this the truncation errors behaves as a dissipative term, hence energy is removed from the domain upon each iteration. In CFD a pressure gradient is often added to periodic boundary condition problems to keep TKE (total kinetic energy) constant within the domain at each timestep.

In regards to the development of the flow, the large coherent structures would break with each interaction of the heating elements. This would cause the development of a fully turbulent flow within the domain, though the grid resolution is under-resolved to capture this. Ergo the flow would simply dissipate for this case.

References

[1] T. Von Kármán. *Aerodynamics*. McGraw-Hill, 1963.