# CFD Assignment 2
# AEM-ADV19

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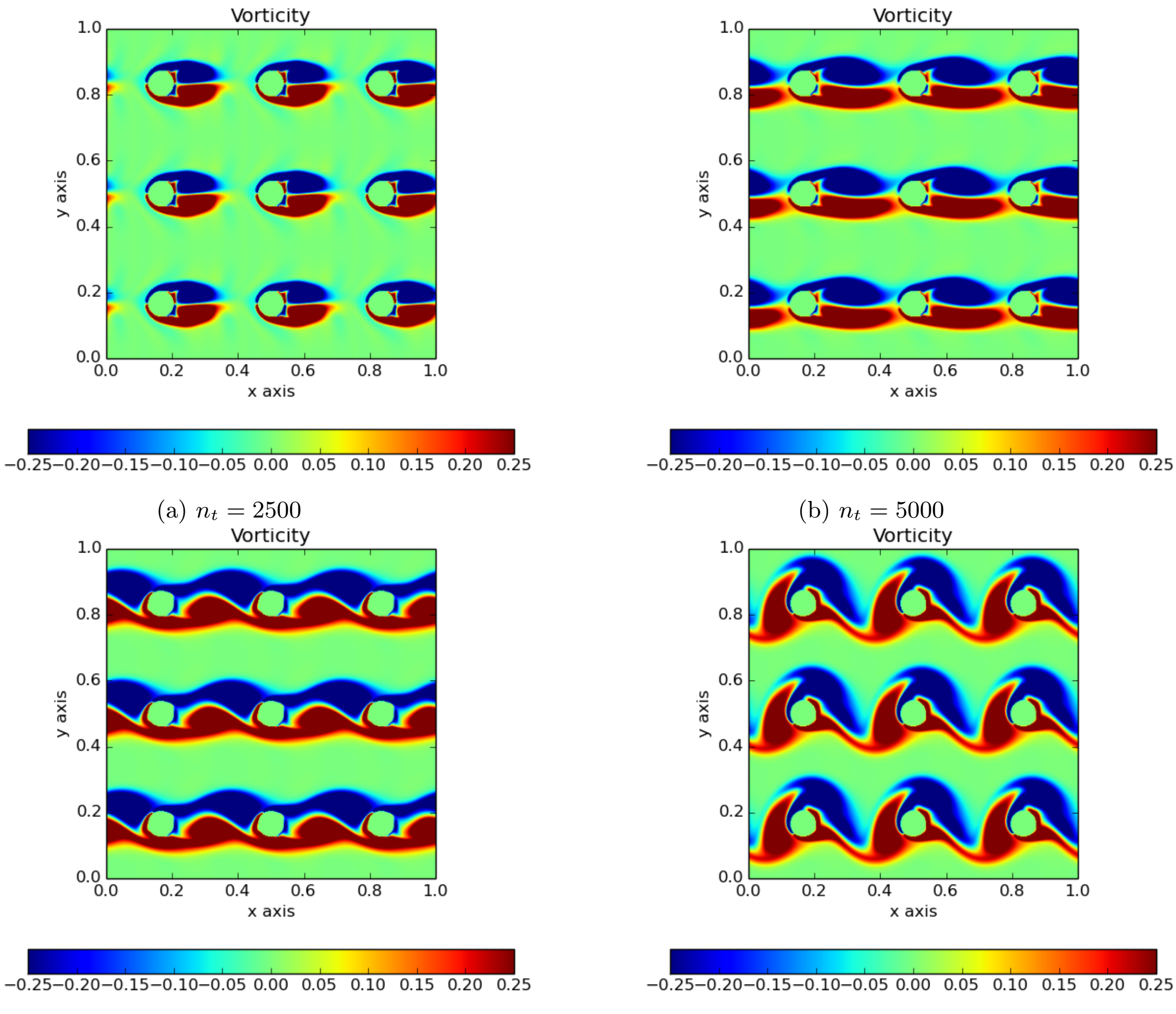
# Question 1

In figure 1 we see the heat exchanger at intervals of 625 units of normalised time. In image $(a)$, the flow enters the left side of the domain at $x = 0$, we see it forms a stagnation point on the cylinder leading edge. Focusing on the rear of the cylinder vortex shedding has begun as there are small pockets of reverse flow on each half of the rear cylinder. In the following image $(b)$ the small vortices have begun to interact with the larger vortical formations which has perturbed the flow and is beginning to cause the shear layer to fluctuate around an imaginary centerline. If the problem were simply a cyclinder in a channel then this would inevitably become a Von Kármán street vortex[1]. However due to the existence of cylinders behind the initial wake, such a flow structure cannot form, instead in image $(d)$ we can see coherent vortical structures forming as the pertubation moves across the domain.




(a) $n_t = 2500$

(b) $n_t = 5000$

(c) $n_t = 7500$

(d) $n_t = 10000$

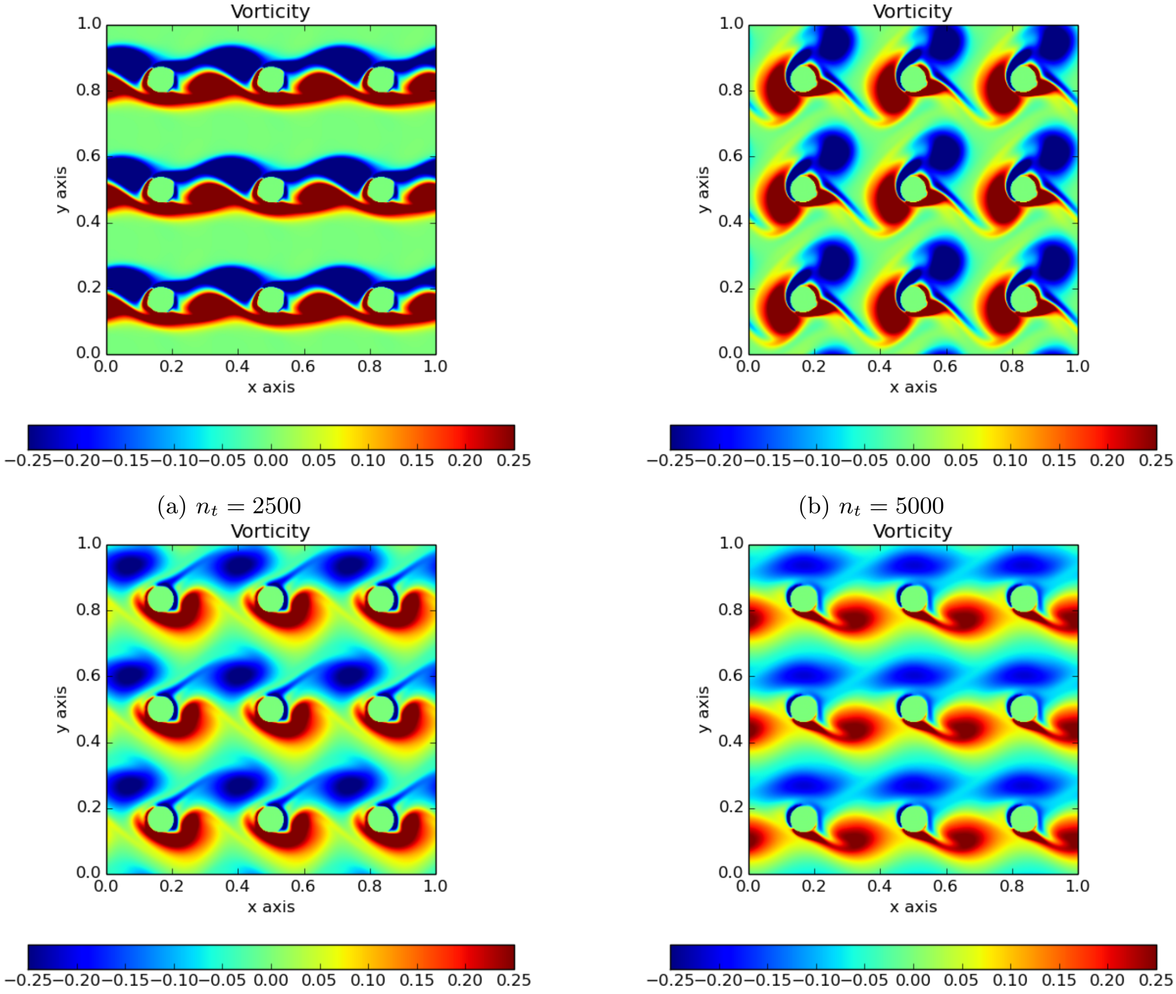
Figure 1: Vorticity plots, CFL = 0.25, Adams-Bashford Scheme, $2^{\text{nd}}$ order accuracy derivatives.

## Question 2

It is not possible to run the simulations with a CFL of 0.75. This is due to the numerical domain no longer containing the physical domain. The solution then accumulates errors which lead to the exponential growth of values throughout the domain and the eventual explosion of the solution.

# Question 3

Using the Runge-Kutta scheme the simulation is now stable for $CFL = 0.75$. This means now for each time step in we would have needed three times steps when using the set up in question 1. Comparing the domains in figure 1 ($c$) to that of figure 2 ($a$), they are identical.




(a) $n_t = 2500$ (b) $n_t = 5000$ (c) $n_t = 7500$ (d) $n_t = 10000$

Figure 2: Vorticity plots, CFL = 0.75, 3$^{\text{rd}}$ order Runge-Kutta Scheme, 2$^{\text{nd}}$ order accuracy derivatives.

In figure 3 we can see the centre-line velocity at $nt = 2500$ and 7500 for the Runge-Kutta simulation with $CFL = 0.75$, and the Adams-Bashforth at $CFL = 0.25$ repsectively. Immediately obvious is that both have the identical velocity therefore we can see that the more stable scheme allows us to progress in time more rapidly. Thus we understand one of the key compromises in CFD, computation time against computational accuracy.

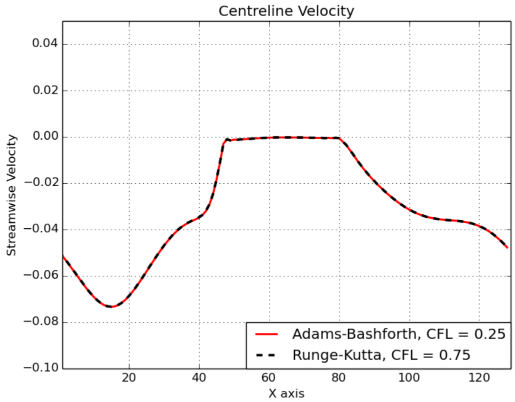
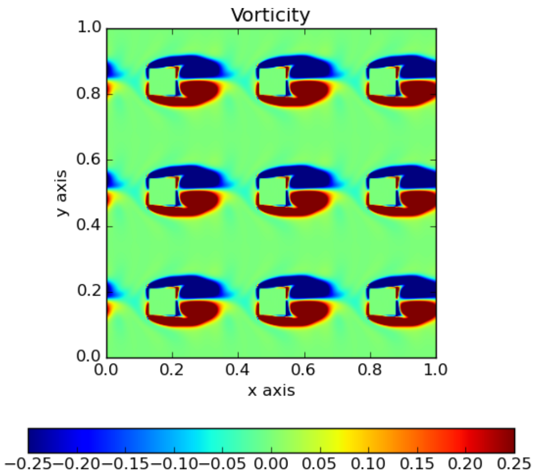


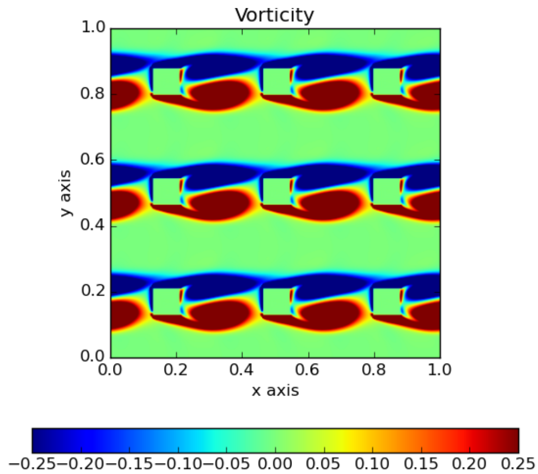

Figure 3: Comparison of centreline velocity at identical simulation time intervals $t = 1875 units$.

# Question 4

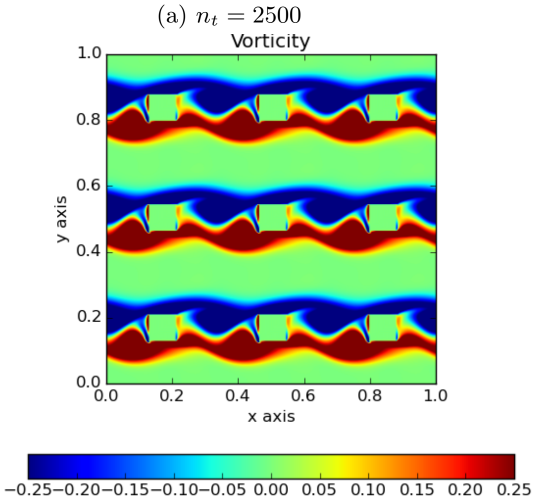
The square cylinders are shown in figure 4. The vorticity plots are very similar to those of the circular cylinders. However we can see the inner trailing vortex is far weaker than for circle. This is as there is no adverse pressure gradient to allow the grouwth of these counter rotating structures. This has the effect of increasing the strength of the large vortices which then begin to interact with the other cylinders downstream. Overall the solution has stronger more distinct vortices, but shares similar general characteristics of shear layer mixing.



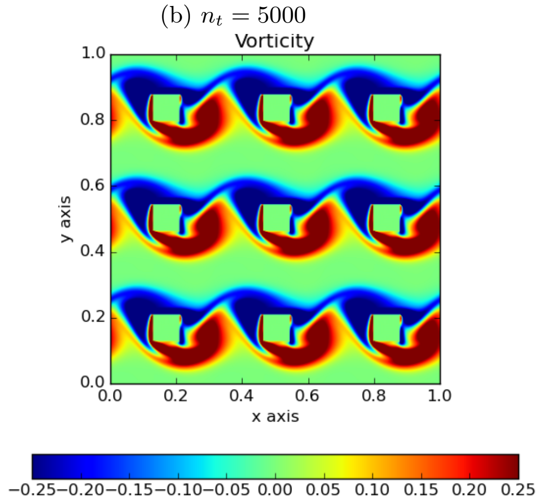

(a) $n_t = 2500$




(b) $n_t = 5000$




(c) $n_t = 7500$




(d) $n_t = 10000$

Figure 4: Vorticity plots, CFL = 0.25, 2$^{\text{nd}}$ order Adams-Bashforth Scheme, 2$^{\text{nd}}$ order accuracy derivatives.

# Question 5

Using the fourth order spatial derivatives shows no discernible difference in scheme when looking at figure 6 below. Looking at the second order Taylor expansion,

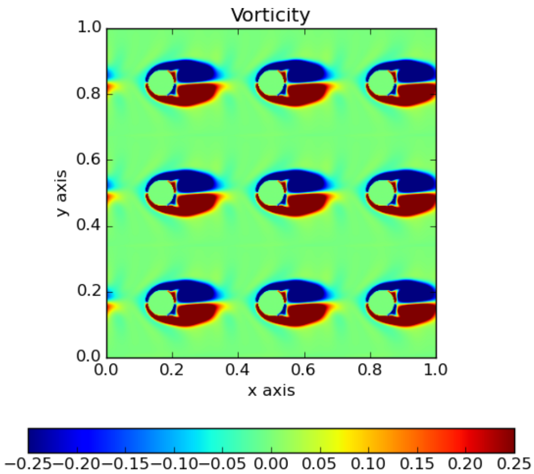
$$\frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2\Delta x} + \frac{u_i''' \Delta x^2}{6} + O(\Delta x^3) \tag{1}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} - \frac{u_i'''' \Delta x^2}{12} + O(\Delta x^3) \tag{2}$$

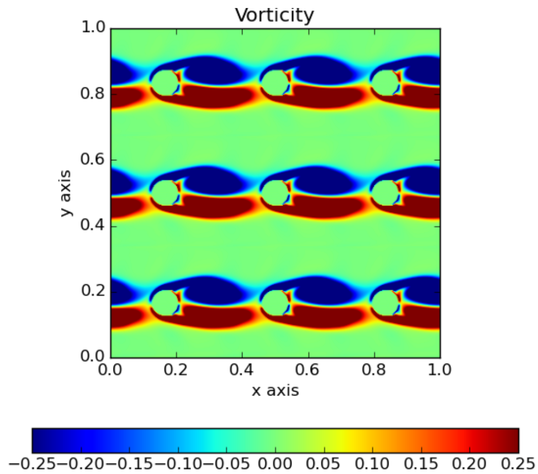
which we can compare with the fourth order derivatives shown below.

$$\frac{\partial u}{\partial x} = \frac{-u_{i+2} + 8u_{i+1} - 8u_{i-1} + u_{i-2}}{12\Delta x} + O(\Delta x^4) \tag{3}$$

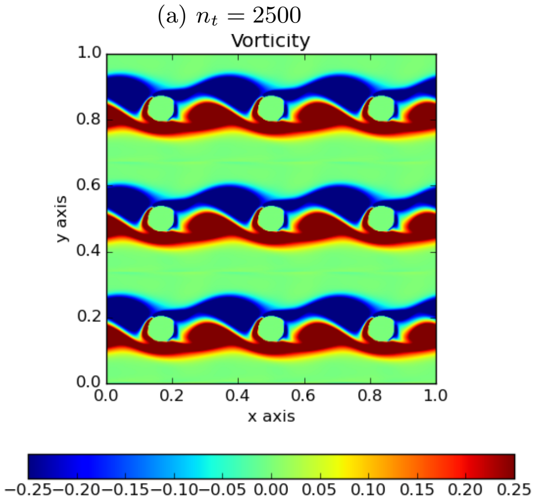
$$\frac{\partial^2 u}{\partial x^2} = \frac{-u_{i+2} + 16u_{i+1} - 30u_i + 16u_{i-1} - u_{i-2}}{12\Delta x^2} + O(\Delta x^4) \tag{4}$$



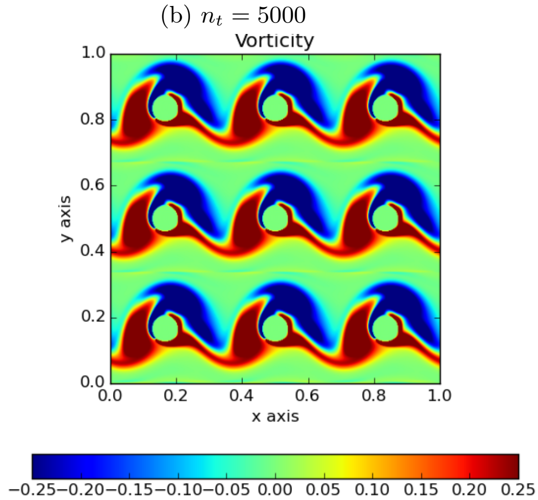

(a) $n_t = 2500$




(b) $n_t = 5000$




(c) $n_t = 7500$



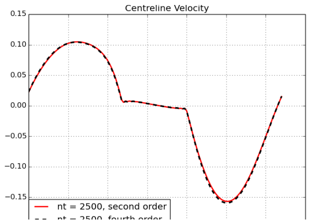

(d) $n_t = 10000$

Figure 5: Vorticity plots, CFL = 0.25, Adams-Bashford Scheme, $4^{\text{th}}$ order accuracy derivatives.

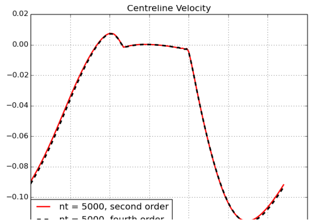
Plotting the centreline velocity of both the second and fourth order schemes against each other

# Question 6

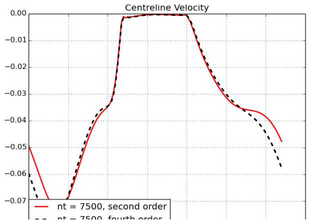
**What will happen to the flow if you run the simulation in question 1 for a very long time? Justify your answer in less than 100 words.**



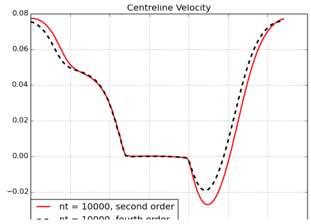

(a) $n_t = 2500$




(b) $n_t = 5000$




(c) $n_t = 7500$




(d) $n_t = 10000$

Figure 6: Vorticity plots, CFL = 0.25, Adams-Bashford Scheme, $4^{\text{th}}$ order accuracy derivatives.

Running the simulation for a long period of time would see the decay of any flows. This due to the truncation behaving as a dissipative term and as the simulation is using periodic boundary conditions there is no velocity added as $t \to \infty$.

## References

[1] T. Von Kármán. *Aerodynamics*. McGraw-Hill, 1963.