High Performance Computing AE3-422

F. M. Grabner - 01220997

Imperial College London - Department of Aeronautics

Question 1

Clearly describe your finite element implementation of the heat conduction problem. Your implementation should allow the domain to be discretized using both four-node and eight-node quadrilateral finite elements.

Domain is defined as follows,

$$q_{i,i} = 0 \quad \text{in} \quad \Omega$$
$$u = g \quad \text{on} \quad \Gamma_g$$
$$-q_i n_i = h \quad \text{on} \quad \Gamma_h.$$

In order to move from the strong form as above we should multiply by an arbitrary weight function and integrate over the domain.

$$0 = \int_\Omega w_i q_{i,i} d\Omega \tag{1}$$

through integration by parts we find this to be,

$$\int_\Omega w_i q_{i,i} d\Omega = \int_\Omega (w_i q_i)_{,i} d\Omega - \int_\Omega w_{i,i} q_i d\Omega$$

applying divergence theorem,

$$\int_\Omega w_i q_{i,i} d\Omega = \int_\Omega w_i q_i \hat{n}_i d\Gamma - \int_\Omega w_{i,i} q_i d\Omega$$

which applying the condition for Γ_h,

$$0 = -\int_{\Gamma_h} w_i h d\Gamma - \int_\Omega w_{i,i} q_i d\Omega. \tag{2}$$

Now substitute $q_i = -\kappa_{ij} u_{,i}$ into equation 2 to find the final weak formulation.

$$\int_{\Gamma_h} w_i h d\Gamma = \int_\Omega w_{i,i} \kappa_{ij} u_{,i} d\Omega \tag{3}$$