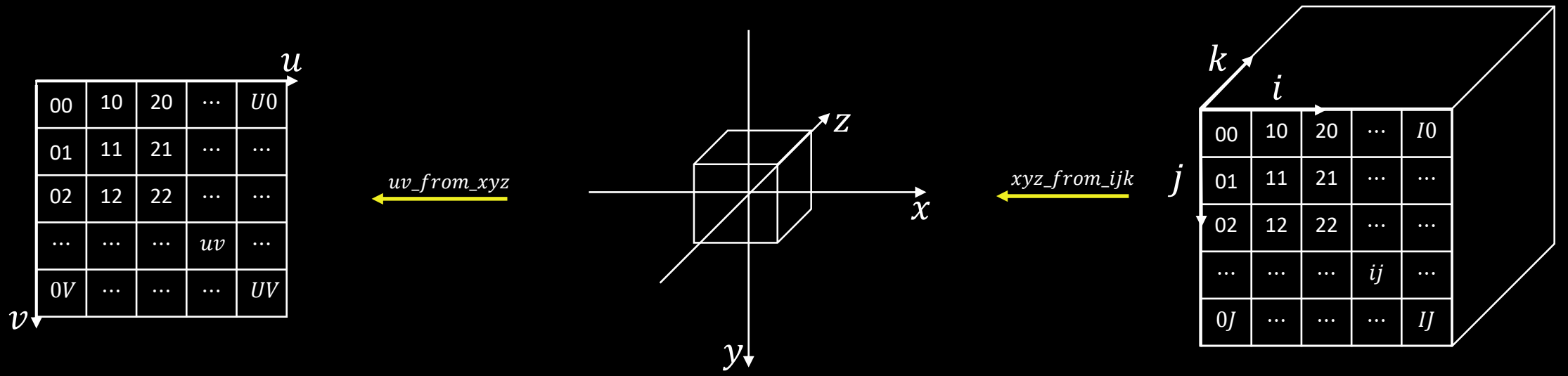
Three reference frames

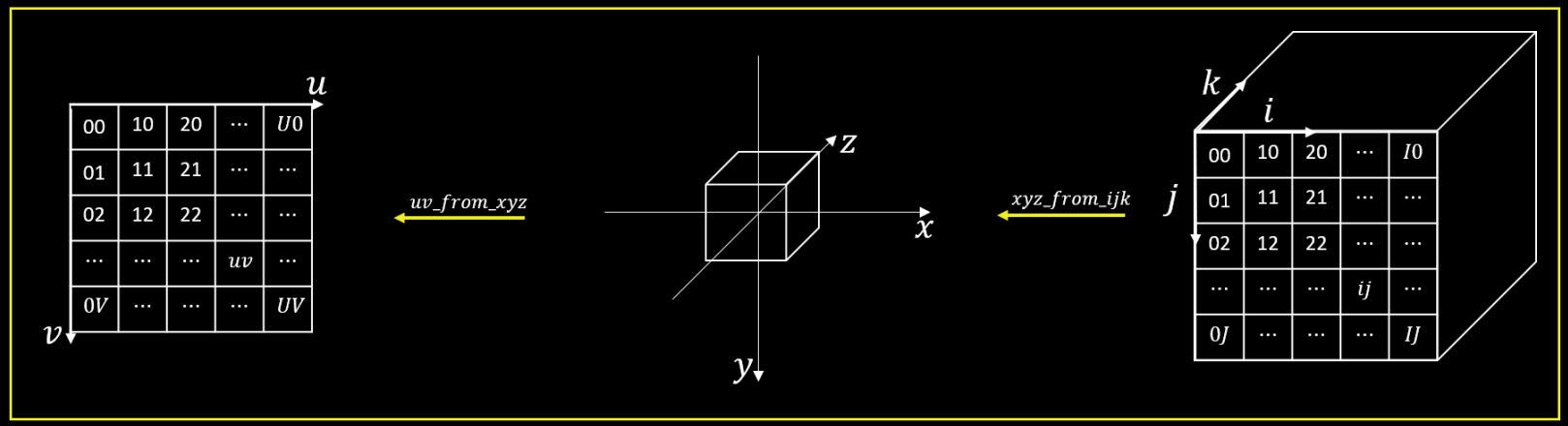


Detector indices
- $u, v \in \{0, 1, 2, \dots, 975\}$
- center (0,0) top left, increasing towards down right

World coordinates
- $x, y, z \in [-80mm, 80mm]$
- centered around iso-center of rotation, with a voxel size of 0.313
- rotation for image acquisition is around z-axis

Volume indices
- $i, j, k \in \{0,1, 2, \dots, 511\}$
- Note, that the orientation of your data in the *ijk* frame depends on how it was saved and loaded (DICOM vs raw etc)
- Typically, ijk aligns with xyz

Frame transforms



$$uv\_from\_xyz = FLIPU \; @ \; P$$

where:

$$FLIPU = \begin{bmatrix} -1 & 0 & 975 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P \in \mathbb{R}^{3\times 4}$$

- P are the projection matrices from the DICOM header of the Siemens Cios Spin
- FLIPU flips the u-axis. Detector read-out direction is reversed to data storage

$$xyz\_from\_ijk = to\_mm \; @ \; to\_iso$$

where:

$$to\_iso = \begin{bmatrix} 1 & 0 & 0 & -c_i \\ 0 & 1 & 0 & -c_j \\ 0 & 0 & 1 & -c_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

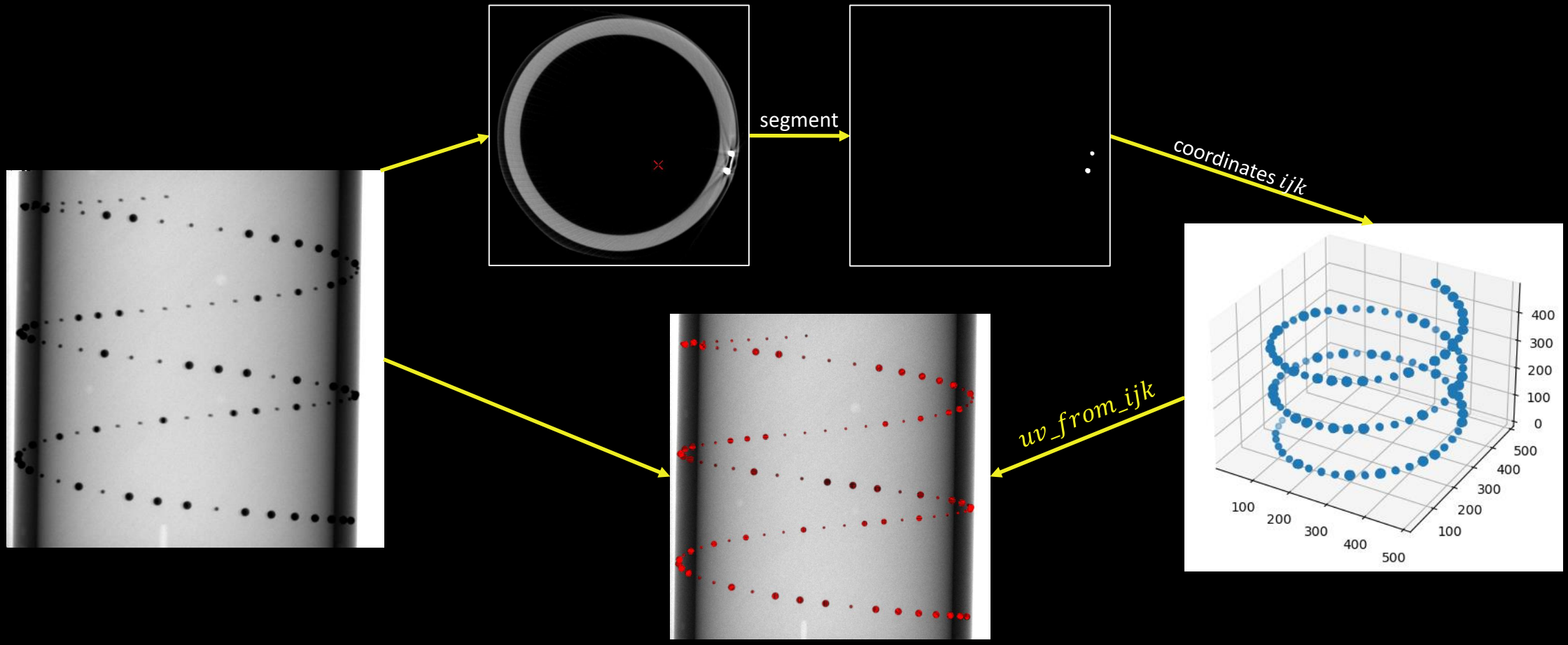
$$to_{iso} = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, s = 0.313, c = \begin{bmatrix} 255.5 \\ 255.5 \\ 255.5 \end{bmatrix}$$

- s is the voxel size (assuming isotropic voxels here)
- c is the rotation center in ijk-space $(512 / 2) - 0.5$ *(the 0.5 is because of voxel center to grid difference)*

Validating Projection Setup

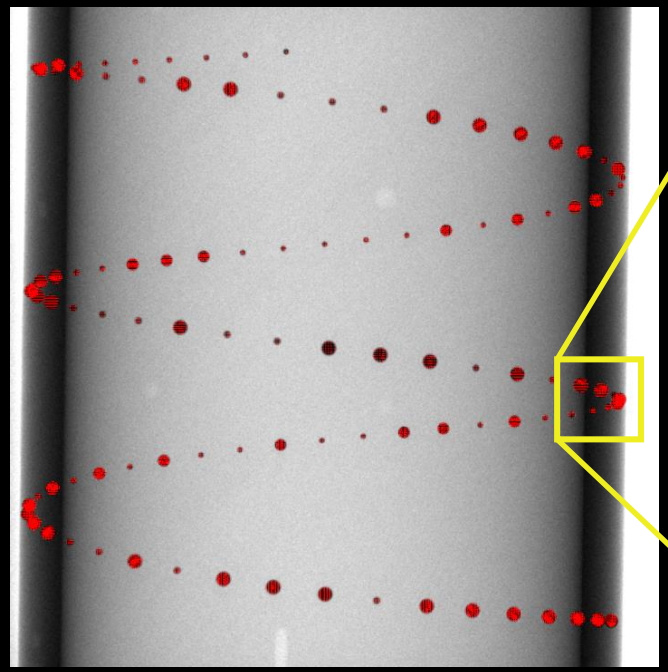
$$uv\_from\_ijk = uv\_from\_xyz \; @ \; xyz\_from\_ijk$$

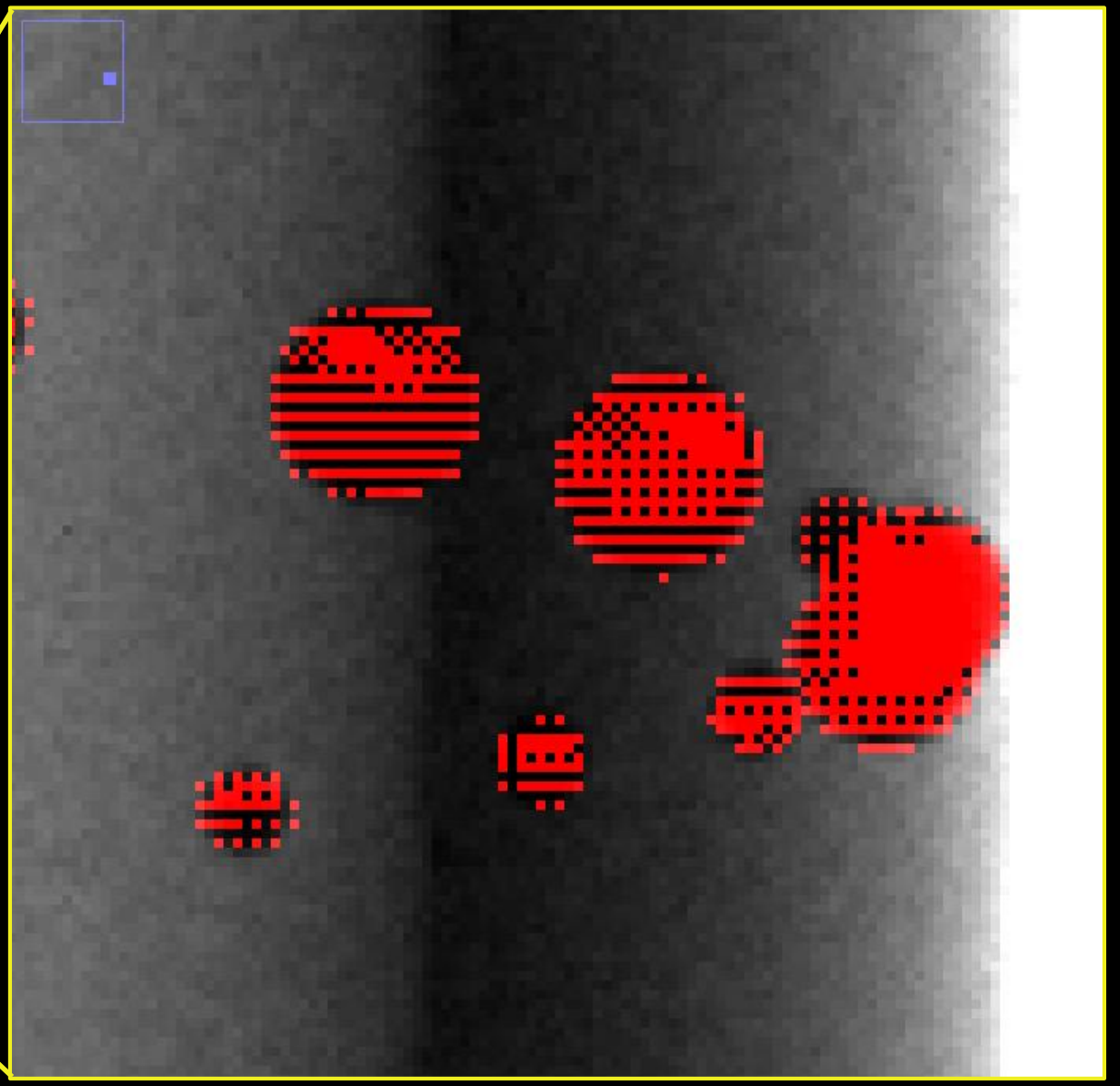
- Using Calibration Phantom to ensure correct alignment



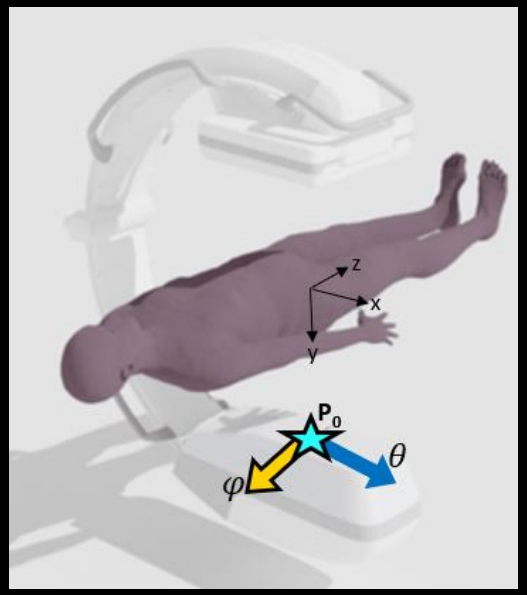
Validating Projection Setup

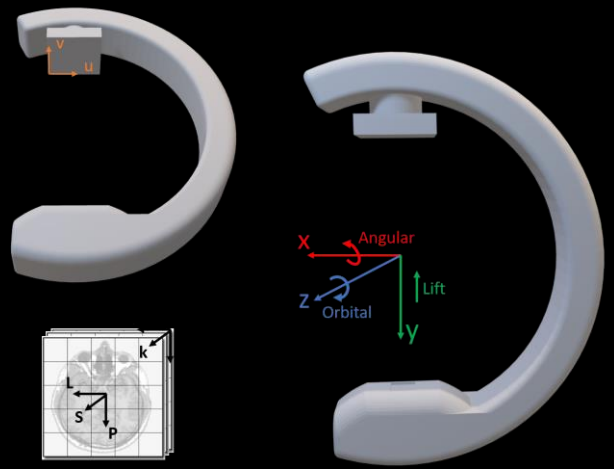
- Using Calibration Phantom to ensure correct alignment





C-Arm System and Relation to Math Convention





CArm Definition Siemens (s)

Orbital (rot z)

$\theta_s \in (-100, 100)$

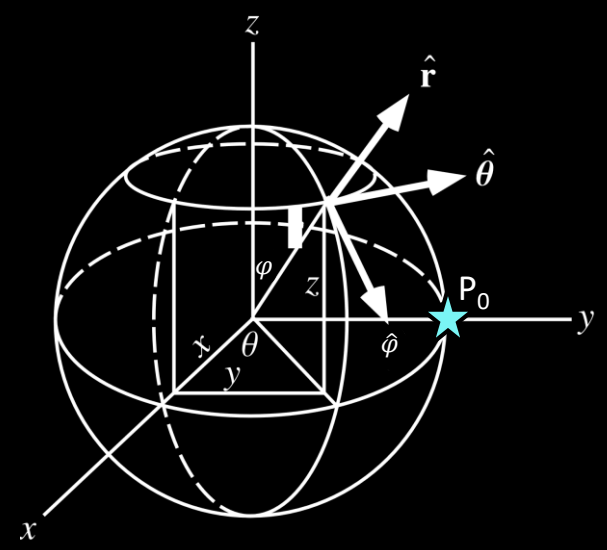
Angular (rot x)

$\varphi_s \in (-30, 30)$

Standard Position $P_{0,s}$

$$\begin{pmatrix} x^0 \\ y^0 \\ z^0 \end{pmatrix} = \begin{pmatrix} 0 \\ s_y \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} r^0 \\ \theta_s^0 \\ \phi_s^0 \end{pmatrix} = \begin{pmatrix} s_y \\ 0 \\ 0 \end{pmatrix}$$



Math ISO norm

Azimuth (rot z)

$\theta_{iso} \in (90, 360)$

Polar (rot x)

$\varphi_{iso} \in (60,120)$

Standard Position $P_{0,iso}$

$$\begin{pmatrix} x^0 \\ y^0 \\ z^0 \end{pmatrix} = \begin{pmatrix} 0 \\ s_y \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} r^0 \\ \theta^0 \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} s_y \\ 90 \\ 90 \end{pmatrix}$$

Clockwise Rotation ≙ Negative Mathematical

$-\widehat{\theta_s} = \widehat{\theta_{iso}}$

$-\widehat{\varphi_s} = \widehat{\varphi_{iso}}$