Solutions Manual

Linear Algebra Done Right - 4th Edition

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This solution manual was created by the MathLearners study group.

Contents

Chapter 1

Vector Spaces

1.1 A: $\mathbb{R}^n$ and $\mathbb{C}^n$

4 Show that $\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta$ for $\lambda, \alpha, \beta \in \mathbb{C}$.

Proof. Assume $\lambda, \alpha, \beta \in \mathbb{C}$. Thus $\lambda = a + bi, \alpha = c + di$ and $\beta = f + gi$. Thus $\lambda(\alpha + \beta) = (a + bi)(c + di + f + gi)$. From this we get $a(c + di + f + gi) + bi(c + di + f + gi) = (ac + adi + af + agi) + (bci + bdi^2 + bfi + bgi^2)$. And finally $[(ac - bd) + (ad + bc)i] + [(af - bg) + (ag + bf)i] = \lambda\alpha + \lambda\beta$. This completes our proof. ■

7 Show that $\frac{-1+\sqrt{3}i}{2}$ is a cube root of 1 (meaning that its cube equals 1).

Proof.

$$\begin{aligned}
\left(\frac{-1+\sqrt{3}i}{2}\right)^3 &= \frac{(-1+\sqrt{3}i)^3}{2^3} \\
&= \frac{(-1+\sqrt{3}i)(-1+\sqrt{3}i)^2}{8} \\
&= \frac{(-1+\sqrt{3}i)\left[(1-3)+(-\sqrt{3}-\sqrt{3})i\right]}{8} \\
&= \frac{(-1+\sqrt{3}i)(-2+-2\sqrt{3}i)}{8} \\
&= \frac{(2+6)+(2\sqrt{3}-2\sqrt{3})i}{8} \\
&= \frac{8+0i}{8} = 1.
\end{aligned}$$

■

9 Find $x \in \mathbb{R}^4$ such that

$$(4, -3, 1, 7) + 2x = (5, 9, -6, 8).$$

Solution.

$$\begin{aligned}(4, -3, 1, 7) + 2x &= (5, 9, -6, 8)\\ 2x &= (5, 9, -6, 8) - (4, -3, 1, 7)\\ &= (1, 12, -7, 1)\\ x &= (0.5, 6, -3.5, 0.5).\end{aligned}$$

10 Explain why there does not exist $\lambda \in \mathbb{C}$ such that

$$\lambda(2 - 3i, 5 + 4i, -6 + 7i) = (12 - 5i, 7 + 22i, -32 - 9i).$$

Solution. For λ to exist, it would have to satisfy $2\lambda = 12$ and $5\lambda = 7$ (real part of the first two coordinates). But in the first case we get $\lambda = 6$, and in the second case we get $\lambda = 1.4$. Thus λ can't exist.

1.2 B: Definition of Vector Space

1 Prove that $-(-\mathbf{v}) = \mathbf{v}$.

Proof. We use direct proof. Thus let $\mathbf{v} \in V$ and $-\mathbf{v}$ be the additive inverse of $\mathbf{v}$. Thus $\mathbf{v}$ is the additive inverse of $-\mathbf{v}$. Thus $-(-\mathbf{v}) = \mathbf{v}$, which completes our proof. ■

2 Suppose $\alpha \in \mathbb{F}, \mathbf{v} \in V$ and $\alpha\mathbf{v} = \mathbf{0}$. Prove that $\alpha = 0$ or $\mathbf{v} = \mathbf{0}$.

Proof. We will use proof by contradiction. Thus assume $\alpha\mathbf{v} = \mathbf{0}$, $\alpha \neq 0$ and $\mathbf{v} \neq \mathbf{0}$. Since $\alpha \neq 0$, we can multiply both sides of $\alpha\mathbf{v} = \mathbf{0}$ by $\frac{1}{\alpha}$. From this we get

$$\begin{aligned}\frac{\alpha}{\alpha}\mathbf{v} &= \frac{1}{\alpha}\mathbf{0}\\ 1\mathbf{v} &= \frac{1}{\alpha}0\mathbf{v}\\ \mathbf{v} &= \frac{0}{\alpha}\mathbf{v}\\ &= 0\mathbf{v}\\ &= \mathbf{0}.\end{aligned}$$

But this contradicts $\mathbf{v} \neq \mathbf{0}$, which completes our proof. ■

4 The empty set is not a vector space. The empty set fails to satisfy only one of the requirements listend in the definition of a vector space. Which one?

Solution. Since the empty set $\varnothing$ contains no elements, it fails the condition that $0 \in \varnothing$.

5 Show that in the definition of a vector space, the additive inverse condition can be replaced with the condition that

$$0\mathbf{v} = \mathbf{0}$$

for all $\mathbf{v} \in V$.

Proof. We will show that the definitions can be replaced by showing their equivalency. As we've seen in the chapter, the condition $\mathbf{v} + \mathbf{0} = \mathbf{v}$ implies $0\mathbf{v} = \mathbf{0}$.

For the converse, suppose $0\mathbf{v} = \mathbf{0}$. Adding $\mathbf{v}$ to both sides, we get $0\mathbf{v} + \mathbf{v} = 0\mathbf{v} + 1\mathbf{v} = \mathbf{0} + \mathbf{v}$, and so $(0+1)\mathbf{v} = 1\mathbf{v} = \mathbf{v} = \mathbf{0} + \mathbf{v}$, which completes our proof. ∎

7 Suppose $\mathcal{S}$ is a nonempty set. Let $V^{\mathcal{S}}$ denote the set of functions from $\mathcal{S}$ to V. Define a natural addition and scalar multiplication on $V^{\mathcal{S}}$, and show that $V^{\mathcal{S}}$ is a vector space with these definitions.