The Interplay between Wealth and Human Capital Inequality - Implications for the UK's post-Covid19 recovery **PRELIMINARY AND INCOMPLETE**

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Abstract

Wealth, and human capital are strongly intertwined. But what effect does the distribution of one have on the other? In this paper, I develop a general equilibrium heterogeneous agent incomplete market model with endogenous wealth and human capital to analyse the interactions between these two factors. I calibrate the model to the UK economy in the pre-Covid19 period and analyse the interaction of wealth and human capital in the stationary equilibrium. I find that there are important nonlinearities in human capital investments, with workers with low levels of wealth investing considerably less in accumulating human capital than their counterparts with more wealth. I then analyse the economic dynamics of the distribution of human capital in the aftermath of an unexpected economic shock, showing that wealth poorer households are more exposed to these shocks, implying that the distribution of wealth matters for the recovery of the economy following recessions.

Finally, I assess the impact of the Covid19 pandemic and associated support measures in the UK. The model predicts that the UK economy will likely suffer a significant reduction in human capital in the aftermath of the Covid19 pandemic, but targeted policy action has helped to reduce the impact of the crisis particularly for low wealth households.

Keywords: Wealth, Human capital, Incomplete markets, Covid19
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1 Introduction

Wealth and human capital are closely intertwined (Lazear (1975)). Accumulated wealth allows individuals to invest in costly education and training, which in turn increases their potential labour earnings. It also provides a safety net in the event of unforeseen circumstances or economic shocks, allowing individuals to maintain their human capital investments. Additionally, wealth provides the opportunity to take risks and invest in skills that may not provide immediate returns but can pay off in the long run. In turn, human capital aids in the accumulation of wealth, by enabling more skilled workers to earn higher wages in the labour market, or use their financial sophistication to make better investment choices and grow their savings.

This relationship between wealth and human capital has far-reaching implications for economic development, inequality and social mobility. Wealth disparities can perpetuate and amplify existing inequalities, as those with higher levels of wealth can access more expensive education, training and other opportunities, while those with lower wealth may be unable to invest in such activities, leading to a widening of the wealth and income gaps. As those with little wealth struggle to increase their skillset they fall behind their better-off peers in terms of earnings and wealth accumulation, leaving them not only materially poorer, but also more vulnerable in the face of economic shocks, and less able to take advantage of opportunities. In time, this creates a vicious cycle that reduces the overall supply of talent in the economy hampering growth and prosperity for all.

Understanding the complex interactions between wealth, income and human capital is key to developing policies that are tailored to the needs of different groups and promote economic development and social mobility. For example, recent university graduates who are wealth-poor (due to student debt) but rich in human capital may require different policies than those who are more affluent at the moment but have a worse future outlook. Policies that focus on increasing wealth, such as tax breaks for the wealthy, may do little to address the unique needs of this group, as they already have high levels of human capital. Alternatively, policies that focus on increasing access to affordable education and training, or providing financial support to graduates struggling with student debt, may have a greater impact on this group's economic prospects.

Quantitative economic models that produce realistic distributions of human capital, income and wealth are well-suited to address the questions and challenges arising from the relationship between wealth, income and human capital. These models can provide insights into the relationship between these factors and help identify the best policy interventions to reduce wealth disparities and ensure that everyone has access to the resources and opportunities needed to build and maintain their human capital. Such models can also be used to quantify the effects of wealth disparities, economic shocks and other factors on human capital and wealth accumulation and the interactions and feedback loops between the two.

The heterogeneous agent incomplete market model based on the works of Aiyagari (1994), Bewley (1986), Huggett (1993) and Imrohoroglu (1989) has become the workhorse model of the economic analysis of wealth inequality and has been applied to study a vast range of economic questions (see e.g. Quadrini & Rios-Rull (2015) for a survey). A key feature of these models is that labour earnings are assumed to be exogenously given so that the households' problem focuses on the consumption-savings decision given a stochastic sequence of earnings. This means that most of these models do not consider the feedback effect that wealth inequality has on the distribution of earnings. Recent research has emphasized the role of family wealth for the educational outcomes of young adults. Scions from wealthy families are more likely to complete high school, enrol in university and complete their degree giving them a clear educational advantage in the labour market (see Lovenheim (2010), Karagiannaki (2017), Dräger (2022)). But, the link between wealth and human capital accumulation is not limited to (higher) education decisions. Both theoretical and empirical contributions to the study of human capital emphasize the importance of learning throughout the active working years (see Becker (1964), Ben-Porath (1967), Acemoglu (1998), Ma et al. (2020)). With longer working lives and increasing speed of technological change, such "on the job" or "post-formal education" training and skill accumulation is becoming increasingly relevant as evidenced by an increasing focus on "lifelong learning". Contrary to traditional higher education environments, financing for individuals interested in investing in such skills is much more limited making the role of pre-existing wealth more relevant for its study.

In this paper I provide an extension to the standard incomplete markets model, by allowing workers to invest in "risky" human capital in the spirit of Krebs (2003), resulting in an endogenously generated joint distribution of wealth, human capital and income. Specifically, I develop a general equilibrium model featuring endogenous wealth and human capital to illustrate some of the interactions between wealth and human capital. Labour earnings depend on human capital and idiosyncratic productivity shocks similar to Guvenen et al. (2014), giving rise to an endogenous earnings distribution. I calibrate the model to the UK economy in the pre-Covid-19 period using detailed household microdata from the UK's longitudinal household survey.

I analyse the interaction of wealth and human capital in the stationary equilibrium and find that there are important non-linearities in human capital investments. At low levels of human capital, when investments should yield high marginal returns, workers with low levels of wealth invest considerably less in accumulating human capital than their counterparts with more wealth. This pattern also holds true for workers with considerably higher levels of human capital. This suggests the existence of low-wealth poverty traps, where individuals with low wealth struggle to improve their skillset and therefore lag behind comparable individuals who have higher levels of wealth through previous savings or inheritance.

The steady state results provide evidence to the effect, that aside from ability and opportunity, wealth plays a key role in influencing the distribution of human capital and earnings. Further, it allows me to quantify these effects and assess the long-term impact of small initial differences in household wealth. I show these to result in quantitatively large differences in human capital accumulation that can persist for years and therefore entrench inequalities across generations.

I use the model to analyse the economic dynamics of the distribution of human capital in the aftermath of an unexpected economic shock and find a clear nonlinear pattern, with households close to the borrowing constraint exhibiting the strongest negative reaction to the shock. I show that this can lead to persistent increases in human capital and earnings inequality with consequences for future economic growth and development. This suggests that the initial distribution of wealth and human capital matters considerably for the aggregate response to an aggregate shock, with higher numbers of borrowing-constrained households leading to a bigger, more persistent drop in human capital and a slower recovery. Large recessions can be associated with persistent scarring effects, both for the individual and the larger economy (c.f. Ouyang (2009), Huckfeldt (2022)). The mechanism showcased by the model hints at a possible role for savings (or rather the absence of the same) in helping to explain these effects, but also in addressing other aggregate phenomena, such as the UK productivity puzzle (see Crafts & Mills (2020).

This highlights the need for public insurance policies to protect the most vulnerable in society during large economic shocks like the ones the UK has been experiencing in recent years. Providing support and insurance to low-wealth, low-income households is thus shown to not just be a charitable act to protect individual welfare, but rather an investment into the future stock of national human capital that has important implications for economic recovery and progress going forward.

I explore this point further by evaluating the medium-run impact of the Covid-19 pandemic through the lens of the model. The Covid-19 crisis in the UK combined a highly nonlinear economic shock, that affected workers across the income distribution in different ways (see Blundell et al. (2022), Marmot & Allen (2020), Stancheva (2021) and references therein), with a large-scale policy intervention by the government aimed at protecting lives and livelihoods across the UK. I calibrate a Covid-19 scenario, that captures both the highly unequal nature of the economic impact of the pandemic, as well as the emergency benefits intended to offset the loss of earnings and employment. This allows me to make model-based predictions of the medium-run impact of the pandemic along the lines of human capital, wealth and earnings inequality, as well as evaluate the effectiveness of the policy response and compare and contrast other possible intervention scenarios.

The analysis shows that Covid-19 is likely to lead to a persistent loss of human capital for the aggregate economy over the coming decades. While a strong intervention has prevented an even more substantial reduction, growth will likely slow as a result of a lower overall skill supply, exacerbating a decade of low productivity growth (Crafts & Mills (2020)).[1]

Comparing different policy interventions I show that in the absence of additional support measures low wealth households would have been most affected by the negative implications of the Covid-19 pandemic, leading to higher levels of human capital and wealth inequality. Successful policy intervention thus prevented welfare losses on the order of around 1.15% of lifetime consumption for the average household. An alternative UBI-type support system is shown to be slightly more welfare efficient but does not command a democratic majority amongst households.

The rest of this paper is organized as follows: Section 2 presents the model; Sections 3 and 4 present the calibration and the stationary economy; Section 5 showcases the response of the model economy to an unexpected productivity shock; Section 6 expands the analysis by analysing the medium run effects of the Covid-19 pandemic in the UK under different public insurance regimes; and Section 7 concludes.

[1] While the analysis in this paper focuses on the skills of individuals of prime working age, there is a growing worry about the impact of the pandemic on the skill formation of children and young adults in full-time education (c.f. Blundell et al. (2022)). As these cohorts enter the labour market in the coming years and decades they will likely amplify these trends.

2 Model

The economy is comprised of a continuum of infinitely lived household dynasties, distributed on the interval $I = [0, 1]$. Time is discrete and indexed by $t = 1, 2, 3, ...$ with each step corresponding to a year. At each point in time, the household is represented by a prime-aged worker (25-55 years) who supplies labour in a competitive labour market and makes decisions about consumption, savings and human capital accumulation during their working life. Households derive utility from consumption and receive income from labour earnings net taxes and transfers and the return on their savings which they supply to firms in a competitive capital market.

Human capital in the model refers to a unidimensional index that summarizes the total of all the worker's skills, talents and experiences that are relevant for general labour productivity. As the model focuses on prime-age workers, the model abstracts from higher and further education decisions. The accumulation of human capital in the model therefore refers primarily to skills that are acquired throughout the working life, through training or other means. Human capital accumulates based on the worker's human capital investment decisions and is subject to a small rate of depreciation over time.

A worker's labour productivity depends on two factors: their level of human capital and some exogenous productivity factor that captures how well their skillset is matched to the demands of the job, as well as other idiosyncrasies. Over time, the workers' productivity state does fluctuate stochastically creating uncertainty about future productivity. This means that human capital is *risky* in the sense that two workers with identical levels of human capital can have different levels of labour earnings depending on their exogenously assigned productivity state. Since the assignment of productivity states follows a random process, household dynasties differ in their history of labour productivity, leading to inequality in wealth, income and human capital.

Workers are subject to a risk of leaving the labour force ("dying"), which is realised through an exogenous random process. When workers leave the labour force, the household produces a new worker who steps into the fray.[2] The offspring inherits their parent's wealth[3] and part - but not all - of their human capital and labour ptoductivity. This process creates intergenerational persistence in both wealth and human capital.

2.1 The households' preferences and constraints

The household dynasties desire to maximize their expected lifetime utility:[4]

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

[2]To simplify the computation of the model I do not specify a full life cycle model, and instead stick with a "hybrid" model of stochastic "death" and replacement (see for example Rios-Rull (1996)).

[3]I also do not model bequests and inheritance dynamics explicitly, and instead assume that the bequest motive is sufficiently strong, that the terminal value function of a hypothetical finite horizon life cycle model is equivalent to the equivalent continuation value in the infinite horizon case. For a discussion of the implications of using a dynastic framework relative to a life cycle model see for example Hubmer et al. (2021). See Telemo (2022) for a life cycle model with bequests.

[4]I drop household-specific subscripts for ease of exposition.

where $\beta \in (0,1)$ is the discount factor[5] and $c_t > 0$ is consumption in period t. The utility function takes the standard CRRA form:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \tag{2}$$

where $\sigma \neq 1$ is the coefficient of relative risk aversion. The CRRA form ensures that $u(c_t)$ is twice continuously differentiable, strictly increasing and strictly concave.

The household receives income from its asset holdings a_t in line with the risk-free interest rate r_t as well as labour income from selling its labour services in the labour market. These labour earnings are subject to a progressive income tax regime by the government which taxes earnings from high earners and provides some transfers to low-earning individuals. Net labour earnings are defined following Heathcote et al. (2017):

$$y_t = \tau_1 \left(l_t w_t\right)^{1-\tau_2}, \tag{3}$$

where τ_1 and τ_2 are two parameters describing the tax and benefits system; w_t is the wage rate and l_t is the worker's labour productivity, which will be described further in the next subsection. As described in Heathcote et al. (2017), $\tau_2 \in (0,1)$ governs the progressivity of the tax and transfer system[6], while $\tau_1 > 0$ sets the average level of tax. The government's sole objective is to redistribute income in this manner. Any shortfall in revenue is covered by the government in an unmodelled way; similarly, any excess tax revenue is spent in a way that does not affect firms or households.[7]

The households budget constraint is therefore as follows:

$$c_t + a_{t+1} + x_t = (1+r_t)a_t + y_t, \tag{4}$$

where $a_{t+1} \in [a_{\min}, +\infty)$ denotes total end-of-period assets, which are subject to a borrowing limit[8] $a_{\min}$ and $x_t \geq 0$ denotes the worker's investment into his human capital.

2.2 Human capital accumulation

Workers accumulate human capital, by investing in a human capital investment good x_t the price of which is normalised to 1 for convenience. The investment good refers to expenditures that increase skills and do not count towards pleasurable consumption, such as training courses, professional certification exams or data science boot camps, but also activities that improve social and interpersonal skills, such as club memberships, and expenses to attend conferences, mentorship, and networking events.[9] More generally, x_t implicitly represents a utility cost that might be inherent in learning new skills, by absorbing resources that could have been used for consumption.

[5] The discount factor in this model accounts for the fact that the individual head of the household may retire ("die") at the end of a given period.

[6] If $\tau_2 = 0$ this corresponds to a flat tax rate of $1-\tau_1$ while in the case of $\tau_2 = 1$ there would be complete redistribution.

[7] I assume this minimal government set up for simplicity. Having a more active fiscal policy, particularly in the context of training and education would be an interesting extension of the model.

[8] In principle, assets are unbounded above, but for the computational implementation of the model I use a grid with a finite upper bound on assets. For the calibrations, I consider the upper bound is chosen sufficiently large that agents never attain it.

[9] I abstract from the fact that some expenditures (e.g. books) might be both pleasurable (c_t) and skill-improving (x_t).

The household does not have an explicit budget of allocated time, but x_t can also be understood as a time cost, standing in for lost earnings that the household has to forgo in order to pursue skill improvement activities. In this case x_t stands for resources that are earned by the household, but then have to be paid back to their employer to cover training costs. [10]

Human capital accumulation is governed by the following law of motion, which is a modified version of the standard human capital accumulation equation that is used in the literature (see Ben-Porath (1967), Heckman et al. (1998), Krebs (2003)):

$$h_{t+1} = \delta(\varkappa)h_t + \chi x_t^{\upsilon} \tag{5}$$

where $\delta(\varkappa) \in (0,1)$ is the depreciation rate of human capital and $\chi > 0$ and $\upsilon \in (0,1)$ are two parameters governing the transformation of the investment good x_t into human capital. The depreciation rate of human capital can take two distinct values, depending on whether the worker is exiting the labour force in that period. Whether the worker stays in the labour force or leaves is summarised by an indicator variable $\mathbf{1}_{\varkappa}$. If $\mathbf{1}_{\varkappa} = 1$, the worker ends his working life at the beginning of the period, and the household's human capital depreciates at rate $\delta(1) = \delta^{exit}$. A worker may decide to exit the labour force for a number of reasons, such as early or scheduled retirement, redundancy, or as a result of a severe illness or other impactful life event. The probability of such an event is given exogenously and has a probability ω. In this case, the depreciation rate effectively determines what share of the worker's human capital gets passed on to the next generation. If $\mathbf{1}_{\varkappa} = 0$ on the other hand, the worker remains in the labour force and the household's human capital depreciates at rate $\delta(0) = \delta^{stay}$.

I assume that workers cannot have negative levels of human capital (skill), so that $h_{t+1} \in [0, h_{\max}]$, where $h_{\max}$ is an upper bound determined by the limitations of human nature. In practice, however, the choices of functional form ensure that households will always choose values of h_{t+1} that lie within the boundaries of the interval, as long as $h_{\max}$ is chosen to be sufficiently large. Additionally, the assumption that workers "die" and lose a majority of their human capital helps to bound the problem here (see also Khun (2008) for a discussion of the case of permanent income shocks).

2.3 Labour productivity

As mentioned above, a worker's labour productivity depends on two factors: i) his stock of human capital h_t; and ii) factors exogenous to the worker that determine how productive his human capital is, e.g. his industry and occupation. These factors are summarised in the random variable e_t. Combining these, a worker's effective labour supply is given by:

$$l_t = e_t * \log(1 + h_t), \tag{6}$$

where $e_t > 0$ is the value of the exogenous productivity state, and h_t is the worker's current stock of human capital. The functional form $g(h) = \log(1 + h_t)$ is chosen to provide an increasing and concave function that transforms raw human capital into labour services.[11]

[10] Note that this is the reverse of the argument made in Heckman et al. (1998), but the argument works this way as well.

[11] Most studies in labour economics consider logged earnings or earnings residuals as their object of interest making this a reasonable choice (see for example Mincer (1974)). Adding 1 is merely a rescaling of the human capital measure that ensures that labour earnings cannot be negative.

This implies that exogenous labour productivity does not exclusively determine labour earnings, but rather is a contributing factor together with human capital (see for example Guvenen et al. (2014), for a similar mechanism). This also distinguishes the model from other models that include human capital and asset inequality (see Krebs (2003), Huggett et al. (2011)), where human capital is the sole ingredient generating labour income.

Exogenous productivity follows an $AR(1)$ process in logs:

$$\log(e_t) = \phi \log(e_{t-1}) + \varepsilon, \tag{7}$$

with $\varepsilon_t \sim N(0, \eta^2)$ and $|\phi| < 1$. Assuming this process for labour productivities is standard in much of the heterogenous agent incomplete market literature, and ensures that e_t follows a stationary process and therefore has an ergodic stationary distribution.

Analogously to the case of human capital discussed above it is assumed that new entrants into the labour market inherit some of their parents labour productivity through for example famillial connections or other unmodelled processes. Accordingly ϕ is a function of $\mathbf{1}_{\varkappa}$, and takes the values $\phi(0) = \phi^{stay}$ and $\phi(1) = \phi^{exit}$ respectively.

2.4 Aggregate quantities and market clearing

The economy has a single representative firm, which rents both capital and labour services from the workers in competitive labour and capital markets. The final output is produced by an aggregate Cobb-Douglas function:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} \tag{8}$$

where A is total factor productivity and $\alpha \in (0, 1)$ is the capital share of income. Capital depreciates at a constant rate $\zeta \in (0, 1)$.

The representative firm's problem is given by:

$$\max_{K_t, L_t} AK_t^{\alpha} L_t^{1-\alpha} - w_t L_t - (r_t + \zeta) K_t \tag{9}$$

Assuming perfect competition, factor prices equal their respective marginal products:

$$w_t = (1-\alpha) A \left(\frac{K_t}{L_t}\right)^{\alpha} \tag{10}$$

$$r_t = \alpha A \left(\frac{L_t}{K_t}\right)^{1-\alpha} - \zeta. \tag{11}$$

In order to satisfy the necessary condition for the existence of an equilibrium with finite assets, I assume that the real interest rate lies in the interval $r_t \in (-1, \frac{1-\beta}{\beta})$ (see Acikgoz (2018) and references therein).

The government raises taxes and issues transfers to households according to the progressive tax and transfers schedule described in (3). The net tax take of the government is given by T_t. Apart from redistributing income, the government has a number of other spending commitments G_t (e.g. defence and infrastructure) that do not affect the utility of the household or the problem of the firm. I assume that the government can always find ways of spending extra tax revenue or can borrow costlessly in international markets without impacting the national economy. Hence the governments budget is perfectly balanced in every period:

$$T_t = G_t. \tag{12}$$

2.5 Stationary equilibrium

I assume that the economy is initially in a stationary equilibrium, in which changes to the asset and human capital positions of individual households cancel each other out, so that all relevant aggregate variables remain constant (see Hubmer et al. (2021)). In this equilibrium the aggregate capital stock and the effective labour supply are constant across all periods: $K_t = K_{ss}, L_t = L_{ss}$. This means that the factor prices and the agents' value and policy functions are time-invariant, as long as the economy remains in the stationary state. The standard definition of a recursive stationary equilibrium applies and is found when the quantities of labour and capital demanded by the aggregate firm are consistent with the quantities of capital and labour services supplied by the households, given the factor prices.[12]

Specifically, the stationary equilibrium of the model economy is defined as follows:

1. The representative firm solves its profit maximization problem, and factor prices equal their respective marginal products, $w_{ss} = (1-\alpha)A\left(\frac{K_{ss}}{L_{ss}}\right)^{\alpha}, r_{ss} = \alpha A\left(\frac{L_{ss}}{K_{ss}}\right)^{1-\alpha} - \zeta$.
2. Given w_{ss}, r_{ss} the agents solve the stationary version of the households problem, giving rise to a stationary distribution Ψ.
3. Capital and labour markets clear, $L_{ss} = \int l_i d\Psi(i), K_{ss} = \int a_i d\Psi(i)$.
4. The government's budget clears, $T_{ss} = \int (w_{ss}l_i - y_i)d\Psi(i) = G_{ss}$.

I solve the household's problem using dynamic programming. For this purpose, I rewrite the households' problem in its recursive form:

$$V(a_t, h_t, e_t) = \max_{\{c_t, a_{t+1}, x_t\}} \left\{ \frac{(c_t)^{1-\sigma}}{1-\sigma} + \beta E_t \left[V(a_{t+1}, h_{t+1}, e_{t+1}) \mid e_t \right] \right\} \tag{13}$$

$$\begin{gathered} s.t. \\ c_t + a_{t+1} + x_t = (1 + r_{ss})a_t + y_t \\ h_{t+1} = h_t\delta_t + \chi x_t^{\nu} \\ y_t = \tau_1(w_{ss}e_t \log(1 + h_t))^{1-\tau_2} \\ c_t \geq 0, h_t \geq 0, \; a_t \geq a_{\min}, x_t \geq 0 \end{gathered}$$

The main added difficulty of this problem is the existence of human capital as a second state in addition to assets. Models with two state variables have become increasingly common in macroeconomic modelling frameworks, but these tend to focus on a liquid and an illiquid asset class (see Kaplan & Violante (2014), Kaplan et al. (2018)). Human capital differs from physical assets in that agents can augment it, but cannot sell it off, implying an asymmetry in the possible margins of adjustment (see also Angelopoulos et al. (2021), who solve a model with health and assets).

I solve the household's problem using a modified version of the endogenous gridpoint algorithm for two state variables presented in Auclert et al. (2021). The main trick I exploit is the fact that the human capital accumulation function can be thought of

[12] For details see Appendix.

as providing a convex adjustment cost function, which allows me to use some of the techniques developed in their paper.[13]

The main outcomes of interest are the value function $V(a_t, h_t, e_t)$ and the policy functions for assets, consumption and human capital investment, which I denote as $g^a(a_t, h_t, e_t)$, $g^c(a_t, h_t, e_t)$ & $g^x(a_t, h_t, e_t)$ respectively. The policy functions, together with the transition function for the exogenous productivity state e_t, and the retirement shock $\varkappa_t$ define a transition function Q^{ss} that maps current period states into next period states. The distribution Ψ is the stationary distribution associated with Q^{ss}.

I find the stationary distribution of all endogenous and exogenous variables, using a non-stochastic simulation (histogram) approach (see Heer & Maussner (2009), Young (2010), Angelopoulos et al. (2021)). For this I begin by creating the histogram of the joint state space across all endogenous and exogenous variables: $\tilde{\Psi}(a, h, e) \geq 0$ with total probability mass 1. I then define an approximation $\tilde{Q}^{ss}$ to the transition function Q^{ss} using linear interpolation to map next period states that fall between gridpoints back onto gridpoints (see Angelopoulos et al. (2021) for a description of the interpolation procedure with two endogenous state variables). Initializing $\tilde{\Psi}_0$ with uniform mass, I calculate the next period distribution as:

$$\tilde{\Psi}_{n+1} = \tilde{\Psi}_n \tilde{Q}^{ss}. \tag{14}$$

I continue updating $\tilde{\Psi}_n$ until the difference between successive histograms becomes less than a pre-set convergence criterion. I use the final histogram $\tilde{\Psi}$ from this procedure as an approximation of Ψ. Using enough grid points across the state space, this procedure can approximate Ψ to any desired degree of accuracy.

After describing the model and its solution method in this section, in the next section, I will describe the calibration procedure, and show that the model can capture key features of the empirical distributions of human capital, wealth and income in the UK.

3 Calibration

The calibration of the stationary equilibrium follows a mixed approach. I set a number of parameters to standard values that are used across a number of similar calibration exercises. Further parameters are set prior to solving the model using information specific to the economic situation that I study in this paper. All remaining parameters are estimated using simulated methods of moments from key features of the pre-Covid-19 economy in the UK over the period 2009 - 2019.

For this purpose I use a sample of households from the UKLHS "Understanding Society" (UnSoc.) between 2009 and 2019. UnSoc is a comprehensive and representative panel survey and has been used to study a wide range of economic and social relationships. For this paper I focus on households comprised of an employed (part- or full-time) household head aged between 25 and 55 years and their partner.[14]

[13] For details on the solution algorithm, see Appendix.

[14] I trim the top and bottom 1% of gross labour earnings in each year. For more details on sample selection see Appendix.

3.1 Deep parameters

A number of parameters are set to standard values that are commonly used in the literature. These include the value of the coefficient of relative risk aversion σ in the household's utility function and a number of parameters related to the production function of the representative firm - α, A and ς.

I set the coefficient of relative risk aversion $\sigma = 1.5$ which is a standard value for the UK (see Angelopoulos et al. (2020) and references therein). I set total factor productivity $A = 1$ as a convenient normalisation and set the capital share $\alpha = 0.3$ (see Angelopoulos et al. (2020) and references therein) and the depreciation rate of capital $\zeta = 0.1$ to match the 10% annual depreciation rate used in Faccini et al. (2013).

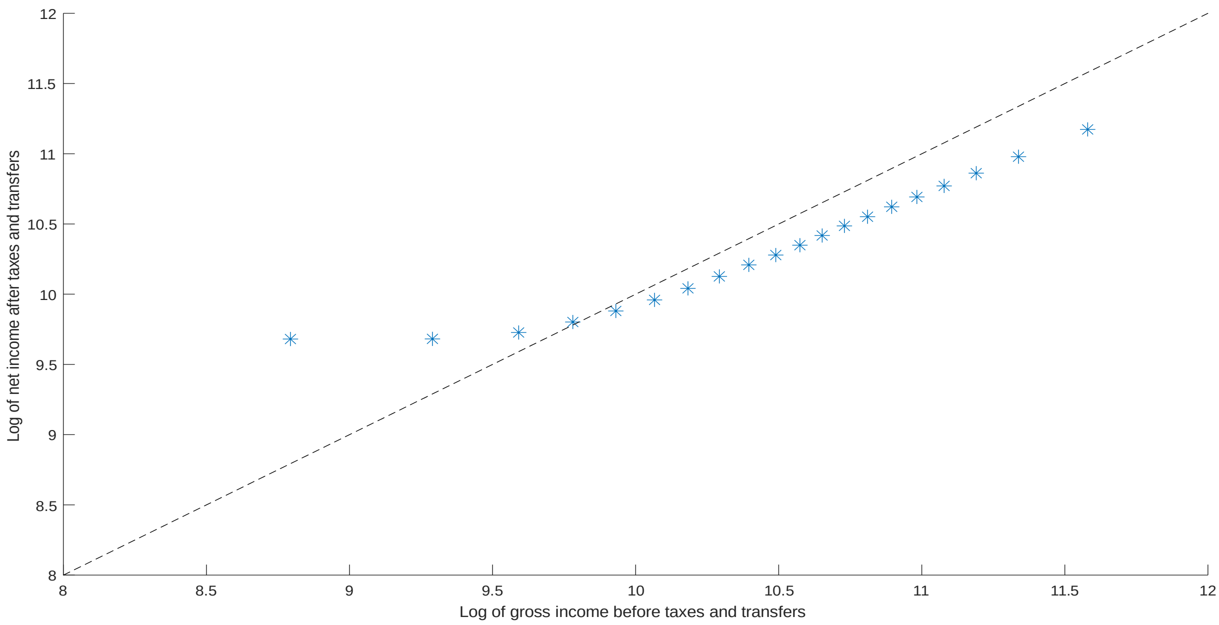
3.2 Taxes and transfers

The UK has an active taxes and transfers policy which achieves a considerable level of redistribution - at least compared to the United States - from high earners to low earners (see for example Belfield et al. (2016), Blundell et al. (2018), Blundell & Etheridge (2010)). Evidence for this can also be found in my sample, as is presented in Figure 1 below. Figure 1 plots disposable income after taxes and transfers against market income for the UnSoc sample. As can be seen, a considerable portion of observations lie above the 45-degree line, indicating that these individuals receive more income through transfers than they pay in taxes. On the other hand, more affluent workers tend to make a net contribution to the welfare system. A progressive tax system has important implications for human capital accumulation. Progressive labour income taxes reduce the incentive to accumulate human capital, particularly for individuals who already have high earnings, as marginal tax rates decrease the return to high skills. On the other hand, transfers to the bottom of the earnings distribution provide valuable resources to low-income individuals who can use these resources to invest in skills. For these reasons, it is important to include a stylized tax and transfer system as described in equation 3 in the previous section.

To calibrate τ_1 and τ_2 I follow the strategy of Heathcote et al. (2017): Assuming that the tax and transfer system takes the form described in equation 3, I estimate the equation in logs using data on gross labour income and net labour income plus social benefits. The data estimates are $\hat{\tau}_1 = 3.915$ and $\hat{\tau}_2 = 0.387$ respectively. The second of these parameters can be used without modification, but since $\hat{\tau}_1$ governs the average level of taxes, I need to normalise it for the level of income in the stationary economy.[15] As noted in Heathcote et al. (2017) the break-even level of income using this modelling approach is given by $y^{breakeven} = \hat{\lambda}^{(\frac{1}{\hat{\tau}_2})}$, where $\hat{\lambda} = \exp(\hat{\tau}_1)$. Given the estimates, this break-even level of income is roughly $\pounds 25,000$ which is quite close to the annual salary of a couple working full-time on minimum wage. I calculate the ratio of this break-even income to the average income level in the sample resulting in a ratio of 0.85. I then set the calibrated value for $\hat{\tau}_1$ to match this ratio assuming an average income level of 1 in the stationary model economy, resulting in a calibrated value of 0.943.

[15] I set this to 1 as a convenient normalisation.

Figure 1: Representation of the UK's Tax/Transfer system for the UnSoc sample



Note: Each datapoint represents the average in a quantile of the pre-redistribution income distribution. For details on variable construction and sample selection see this section and Appendix.
Source: Pooled UnSoc sample (2009 - 2019).

3.3 Stochastic labour force exit

The model features stochastic exit of the workers from the labour force. Every period workers exit with probability $0 < \omega < 1$ after which they are replaced by their household with a fresh worker. This approach is motivated by a desire to introduce turnover in the labour force into the model without adding the additional computational complexities of specifying a full lifecycle model. In my calibration, ω is set to $\frac{1}{30}$ to match the 30-year work life of the sample, and δ^{exit} is set as part of a minimum distance procedure to match some relevant data moments. Setting the expected work life to 30 allows me to focus on the savings and human capital investment behaviour of prime-aged workers. It is well understood that workers close to retirement have different incentives with regard to their savings and human capital investment behaviour (see Ben-Porath (1967), Huggett et al. (2012), Lachance (2012)).

An exit results in a worker being replaced by their offspring. An exiting worker can pass on all of their wealth but only part of their human capital to their child. This transfer of human capital is modelled by a higher level of human capital depreciation in the period of the death: $0 < \delta^{exit} < \delta^{stay} < 1$. The modelling choice in this case is driven by two considerations: 1. The model focuses on prime-age workers and hence abstracts from education choices made prior to labour market entry. It is likely that the children of parents with high levels of human capital have had more opportunities to accumulate skills prior to entering the labour market, for example by attending university. 2. There is a growing empirical literature that documents high intergenerational persistence of human capital across a variety of settings (see for example Black et al. (2005), Lundborg et al. (2018), Adermon et al. (2021)). Among other considerations, parents with different levels of income and/or education are likely to impart to their children different non-cognitive skills and attitudes, shaping their future labour market outcomes (see Acemoglu (2020)).

3.4 Data targets & model fit

The remaining parameters are calibrated as part of a minimum distance procedure to match a number of relevant data targets from the data. Targets represent some moments of the distribution of labour earnings, human capital and wealth in the data. For this purpose, I first purge all earnings and income variables of predictable variations by running a mincerian regression, controlling for a third-order polynomial of age, sex and region and year fixed effects. I retain the residuals from the Mincerian and use their exponent as my income measures. I also construct a proxy of human capital using measures of cognitive skills which are available in the data.[16] As UnSoc does not contain any information on wealth, I further use some standard values from the aggregate distribution of wealth.

I still require values for the exogenous productivity process e_t - namely the (intergenerational) persistence $\phi^{stay}(\phi^{exit})$ and standard deviation of innovations η. As well as the parameters relating to the law of motion of human capital: δ^{stay} and δ^{exit} which are the values of the human capital depreciation for workers staying in or exiting the labour force; and χ and ν which are the linear and nonlinear parameters in the human capital production function, transforming the investment good into human capital.

I calibrate these remaining parameters, by minimizing the distance between a number of relevant data moments and their model counterparts. Specifically, I choose the P10-P90 values of gross earnings and human capital in the UnSoc. sample, as well as the sahre of Households with less than 0 assets from WAS. For a detailed description of all data targets, their model counterparts and their construction, see Appendix. I choose the parameters $\theta = \{\phi^{stay}, \phi^{exit}, \eta, \upsilon, \chi, \delta^{stay}, \delta^{exit}, a_{\min}\}$ to minimize the weighted sum of the model implied moments $\hat{m}(\theta)$ from their data counterparts $\hat{m}^{Data}$.

$$\min_{\theta}[\hat{m}^{Data} - \hat{m}(\theta)]^T W[\hat{m}^{Data} - \hat{m}(\theta)] \tag{15}$$

W here is a diagonal weighting matrix that is equal to the reciprocal of the variance of the corresponding data moment.[17]

The whole set of target moments as well as the model fit for the calibrated model is presented in Table 1.

As can be seen, the final calibrated model matches the selected data moments well. Figure 2 below highlights the fit of the calibrated model with respect to these two dimensions by plotting the cross-sectional distributions of gross labour earnings and human

[16] For more details on variable construction see Appendix.
[17] Variances for the data moments are based on 10,000 bootstrap samples.

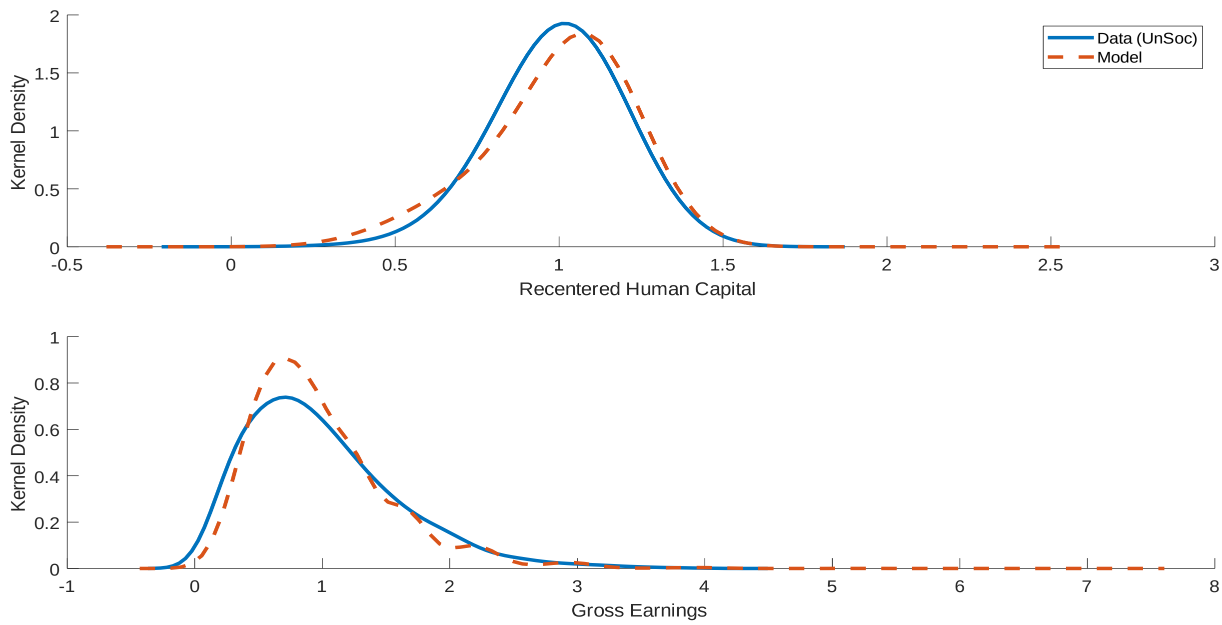
Table 1: Calibration Targets & Model Fit

Moment	Model	Data	% Deviation	Weight
y_{p10}	0.44	0.34	-29.58	25793.59
y_{p20}	0.55	0.49	-12.69	25943.45
y_{p30}	0.68	0.62	-8.34	26665.05
y_{p40}	0.74	0.75	2.11	16319.53
y_{p50}	0.91	0.89	-2.70	10782.28
y_{p60}	0.97	1.03	5.83	8976.68
y_{p70}	1.20	1.21	0.81	7647.66
y_{p80}	1.31	1.45	9.80	5821.10
y_{p90}	1.69	1.81	6.98	3930.55
h_{p10}	0.71	0.81	12.44	93268.63
h_{p20}	0.85	0.88	3.93	99470.37
h_{p30}	0.97	0.94	-3.71	109680.81
h_{p40}	1.01	0.98	-2.86	211552.41
h_{p50}	1.04	1.01	-2.66	272553.17
h_{p60}	1.07	1.05	-2.58	230921.64
h_{p70}	1.11	1.08	-2.69	223022.21
h_{p80}	1.15	1.12	-2.29	186512.59
h_{p90}	1.19	1.18	-0.85	149612.83
$a_{<0}$	0.14	0.19	24.00	40000.00

Note: UnSoc pooled sample and model simulation. Weight refers to the weighting of the moment in the W matrix. For details see Appendix.

capital for both the UnSoc data and the model simulation.

Figure 2: Human capital and gross earnings in data and model



Note: Human capital measures are rescaled to mean 1 for comparability. Data gross earnings refer to exponentiated mincerian residual. For details on data construction see Appendix.
Source: Pooled UnSoc sample (2009 - 2019). and Model.

In terms of wealth, the left tail of the wealth distribution is matched quite well, with the model predicting a share of 18.4% borrowing-constrained households compared to 19% in the WAS data. For the purposes of this investigation, capturing a realistic measure of wealth-poor households is crucial in order to assess the quantitative effect of the borrowing friction.

Table 2 summarizes some key features of the stationary economy, providing an overview of key endogenous variables including moments that were not explicitly targeted by the calibration. As we have seen in the calibration section, the model captures the features of the UK earnings distribution quite well. Apart from earnings and human capital, the distributions of which are directly targeted by the calibration procedure, wealth is a key endogenous variable in the model. It is therefore important to evaluate the model fit with respect to household assets. The gini of assets for example is over 60 gini points, which is slightly below the empirical value of wealth inequality in the UK which lies around 0.71 (see Angelopoulos et al.(2020), (2021)) within the typical range for these types of models.

Table 3 provides a closer look at the model fit with respect to some important data moments. In addition to the UnSoc data which I used for the calibration, I further use information on the distribution of assets from the Wealth and Asset Survey (WAS), and consumption from the Living Cost and Food Survey (LCFS). To make the data comparable with the model output, I restrict the respective sample populations to households whose head is aged between 25 and 55 years.[18]

With respect to untargeted moments of the distributions of gross earnings and human capital the fit with respect to the data is quite good. The mean-to-median ratio, gini coefficient, CoV, and upper and lower tail measures of inequality are tracking the data quite closely, even though I only targeted the first and second moments or the second and third moments respectively. The fit with respect to the shares of earnings and human capital held by specific parts of the distribution is also quite well matched including for the shares of the top 10%.

With respect to assets and consumption, the model generally generates too little inequality. Particularly looking at measures of upper tail inequality, such as the P90-P50 ratio or the share of wealth and consumption of the top 80 or 90%, the model shows a lot less concentration of consumption or wealth. This might be a feature of the sample selection in these cases, but the difficulty of matching the concentration of wealth at the very top using HIM-type models is well-known.

Overall the model fit with respect to the targeted moments is quite good, particularly with respect to the distributions of human capital and earnings.

3.5 Discussion of calibrated parameters

The calibrated parameters are presented in Table 4 below. A main point of interest are the calibrated parameters related to the human capital accumulation function. As human capital is often difficult to measure in practice, empirical estimates of the parameters are rare and often context specific. It is however possible to contextualise the parameters with some comparable values used in other works, as well as simple common sense. Beginning with the depreciation rate during normal times δ^{stay}, the calibrated model suggests an annual depreciation of around 8%. In a recent study, Dinerstein et al. (2022) estimate

[18] For more details on variable construction see Appendix C in Angelopoulos et al. (2021). I am grateful to Dr Spyridon Lazarakis for providing me with some of the summary statistics in this case.

a skill depreciation of 4.3% for Greek individuals who are out of work for the duration of a year which is close to the value I find in the calibration, assuming that even unemployed teachers invest a little into their human capital. Other evidence comes from more quantitative modelling exercises that are closer to this paper: Krebs (2003), whose work probably most closely mirrors this paper uses a depreciation rate of 6% which is in line with the calibration employed here. Huggett et al. (2011), estimate a lifecycle model using US data and find average human capital depreciation rates of around 2.5% using a "flat spot technique" for workers close to retirement, but it is important to note that since they use earnings as a proxy, their measure of human capital conflates exogenous productivity with human capital. Blundell et al. (2016), estimate a lifecycle model for women in the UK. They find depreciation rates between 5.7% and 8.1% annually, which fits into the value found here. Finally, Fan et al. (2022) also estimate a lifecycle model on US data and find a baseline estimate of 8.6%, but argue the number should probably be lower.

Turning to the next parameter δ^{exit}, which measures the depreciation rate for workers who exit the labour force. Weighing in at roughly 63% this appears rather large, but given that it signifies the beginning of a new generation of workers it does not seem particularly excessive. There is little consensus on the exact magnitude of intergenerational transmission of human capital, other than broad agreement that it can be sizeable (see for example Black et al. (2005), Lundborg et al. (2018), Adermon et al. (2021)). Clark (2014) finds intergenerational correlation coefficients of between 0.7 and 0.8, however, these numbers are disputed and other estimates suggest that values of around 0.45 are more appropriate (see the discussion in Solon (2018)), which is in line with the number found here.

Finally, the nonlinear term in the function transforming the investment good into human capital. The calibrated value of 0.618 is comfortably below 1 and fits into the interval $(0.5, 0.9)$ which is suggested by Huggett et al. (2011) as reasonable values. In the same paper, they suggest that values around 0.65 provide the best model fit for their application, which is very close to the value estimated here.

Table 2: Model Summary Statistics

	Gross Earnings	Net Earnings	Gross Income	Net Income	Human Capital	Assets	Consumption	HC Investment	Net Transfers
Mean/Median	1.097	1.025	1.125	1.046	0.962	1.459	1.016	1.059	5.646
Gini	0.287	0.179	0.28	0.187	0.099	0.565	0.14	0.165	1.525
CoV	0.547	0.325	0.528	0.338	0.185	1.108	0.253	0.319	3.165
P10/P50	0.478	0.637	0.504	0.635	0.682	-0.07	0.709	0.734	-9.548
P90/P50	1.851	1.457	1.886	1.518	1.141	3.661	1.342	1.465	26.719
$Q_{<50}$	0.284	0.345	0.298	0.361	0.648	0.266	0.339	0.06	-0.008
$Q_{>80}$	0.352	0.281	0.358	0.295	0.443	1.208	0.23	0.046	0.017
$Q_{>90}$	0.22	0.166	0.211	0.165	0.248	0.828	0.124	0.026	0.013

Note: Calibrated Model. Earnings refer to labour earnings only. Income refers to labour earnings plus returns on asset holdings. Net Transfers are taxes paid minus transfers received. $Q_{<50}$ denotes the share of the bottom 50 percent. Similarly, $Q_{>80}$ refers to the share of the population above the 80th percentile.

Table 3: Untargeted Moments

	Gross Earnings		Human Capital		Assets		Consumption	
	UnSoc	Model	UnSoc	Model	WAS	Model	LCFS	Model
$\frac{Mean}{Median}$	1.127	1.097	0.988	0.962	2.921	1.459	1.135	1.016
Gini	0.325	0.287	0.083	0.099	0.747	0.565	2.780	0.140
CoV	0.598	0.547	0.149	0.185	2.397	1.108	0.540	0.253
$\frac{P10}{P50}$	0.379	0.478	0.799	0.682	-0.060	-0.070	0.524	0.709
$\frac{P90}{P50}$	2.044	1.851	1.162	1.141	7.456	3.661	1.910	1.342
$Q_{<50}$	0.241	0.284	0.448	0.450	0.016	0.084	0.308	0.339
$Q_{>80}$	0.345	0.352	0.241	0.308	0.742	0.384	0.368	0.230
$Q_{>90}$	0.201	0.220	0.125	0.173	0.548	0.263	0.219	0.124

Note: Comparison of calibrated model with model with selected data moments.
Source: Model; UnSoc pooled Sample (2009-2019);
Wealth and Asset Survey (WAS) (2014 - 2016);
Living Cost and Food Survey pooled sample (2014 - 2016).
For variable construction see Appendix and Angelopoulos et al. (2021).

Table 4: Calibrated Parameters

Parameter	Description	Value	Standard Error	Calibration Method
ϕ^{stay}	Persistence of Prod. Shock	0.783	(0.004)	Minimum Distance
ϕ^{exit}	Intergenerational transmission of Prod.	0.564	(0.017)	Minimum Distance
η	Variance of Prod. Shock	0.478	(0.004)	Minimum Distance
τ_1	Linear Taxes	0.943	-	Outside Calibration
τ_2	Progressivity of Taxes	0.388	-	Outside Calibration
δ^{stay}	Depreciation of HC	0.912	(0.002)	Minimum Distance
δ^{exit}	Intergenerational transmission of HC	0.374	(0.012)	Minimum Distance
χ	Linear HC Production	0.470	(0.008)	Minimum Distance
υ	Decreasing Returns in HC	0.618	(0.005)	Minimum Distance
β	Discount Factor	0.96	-	Standard Value
α	Labour Share	0.3	-	Standard Value
σ	CRRA Coefficient	1.5	-	Standard Value
ζ	Depreciation Rate of Capital	0.1	-	Standard Value
a_{min}	Borrowing Limit	-0.340	(0.011)	Minimum Distance
ω	Probability of Labor Force Exit	0.03333	-	Outside Calibration
A	Total Factor Productivity	1	-	Normalization

Note: Numerical standard errors in parentheses.

4 Stationary Economy

Having solved the model and calibrated it to match key features of the UK economy between 2009 and 2019, this section analyses the stationary distribution in more detail. The main focus of the analysis is on exploring how wealth and human capital interact and shape the incentives and constraints of the households. I analyse how wealth (or lack thereof) enables (prevents) households to build up human capital quickly and thereby (not) taking advantage of profitable labour market opportunities and how this mechanism generates large persistent differences in human capital and ultimately income and consumption between households. This analysis is then extended in the following section, where I analyse the behaviour of the economy in response to an unexpected aggregate productivity shock, which lays the groundwork for the application to the Covid-19 crisis in the penultimate section.

In the rest of this section, I explore some properties of the stationary economy, focussing on the agent's optimal decision rules and how these are both influenced by and contributing to wealth and human capital inequality.

4.1 Policy function analysis

In the model workers invest in both physical assets and human capital, implying the existence of trade-offs. Wealth provides a constant income stream due to interest payments and can be used to smooth consumption in the case of a drop in income. Human capital on the other hand provides generally larger returns than savings, but the return is risky since it depends on the stochastic productivity state e_t.[19] Furthermore, human capital is inalienable and cannot be used directly to smooth consumption in response to income shocks.

Figure 3 plots the policy functions of agents with different levels of wealth and human capital. The first row shows the level of investment into an agent's human capital $g^x(a_t, h_t, e_t)$, by their current human capital and wealth, as well as by different exogenous productivity levels. Moving from left to right, the subplots represent the policy functions conditional on different values of the exogenous productivity state e_t - namely the minimum, mean and maximum values of e_t. As the marginal product of human capital depends on the exogenous productivity state, we expect workers with higher human capital productivity to invest more resources into acquiring additional human capital. Indeed this is confirmed by comparing the levels of the different conditional policy functions. At the same current level of human capital and asset holdings, workers in higher productivity states, invest more into their human capital, in line with predictions based on their increased marginal return from doing so.

Slightly more unexpected is a striking pattern of nonlinearity emerging from these policy functions, which exist *conditional* on a given level of exogenous productivity. Nonlinearities along this dimension indicate differing incentives and constraints based on different values of human capital and wealth. With an increasing and concave function, mapping human capital into earnings, investment into human capital should be highest for low levels of human capital and then decrease in line with human capital's marginal

[19] Since the return to human capital is risky, its average return must be greater than that of the riskless assets, or no household would ever invest in human capital. See also the general result and a discussion of this point in Krebs (2003).

product.[20] However, since workers with low levels of human capital also have low levels of labour earnings, their available resources are potentially quite restricted. Here wealth enters as a crucial factor, relaxing the budget constraint, so that human capital investment can be maintained at the desired level. This effect becomes obvious, particularly if one focuses on low levels of human capital. As a general trend, human capital investment x_t decreases as the initial level of human capital increases - a pattern that is consistent with the decreasing marginal returns of human capital due the concavity of $\log(1 + h_t)$. However, at very low levels of h_t workers with low levels of wealth invest much less into their human capital, than their more prosperous counterparts.[21]

This simple graphical analysis shows how the borrowing friction in the model creates an interdependency between wealth and human capital, thereby making wealth inequality relevant for human capital accumulation and thus future earnings inequality.[22] It further shows that this interaction is much more impactful at low levels of human capital. At high levels of human capital, and across different productivity states, the policy functions are much more similar across the asset dimension than is the case at lower levels of human capital. This is likely driven by two factors: i) higher human capital implies higher labour earnings, and therefore even wealth-poor households are likely able to finance their investment plans out of current earnings ("income effect"); ii) as the marginal product of human capital decreases, investment plans drop so that they can be easier attained by resource-constrained workers ("price effect").

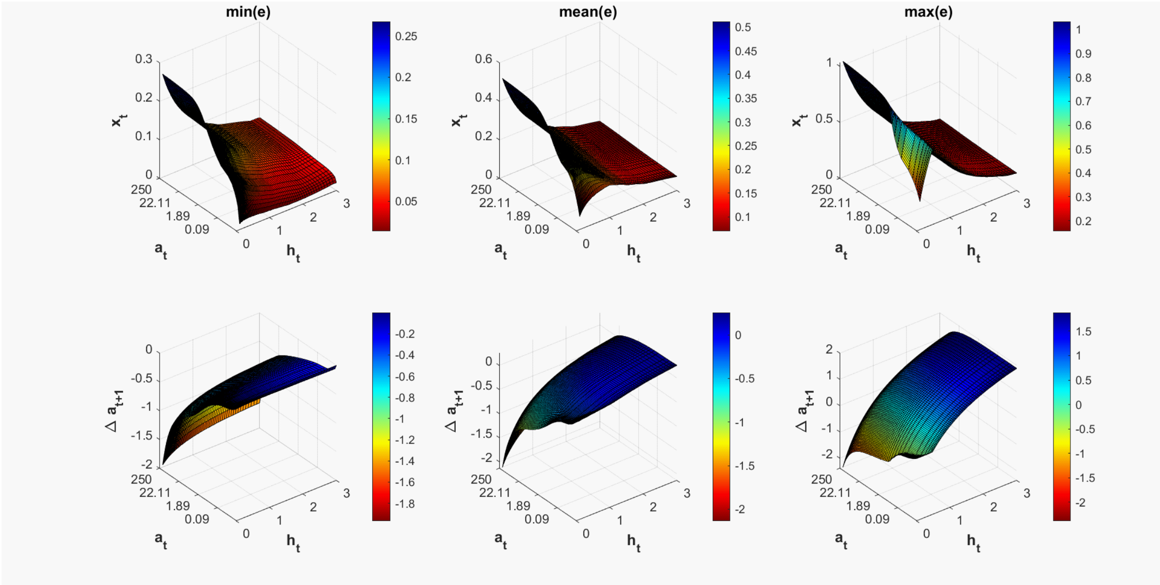
These observations are reinforced by looking at the changes in the worker's net asset positions $\triangle a_{t+1} = g^a(a_t, h_t, e_t) - a_t$ in the second row of Figure 3. Workers with low levels of human capital, but high levels of wealth draw down their assets to finance not only consumption but also a rapid expansion of their human capital. Agents close to the borrowing constraint on the other hand see very little change in their asset position until they have built up a reasonable amount of human capital.

[20] It might be useful to imagine the case of a perfectly frictionless version of the model as an example. In that case, investment in human capital should follow a "waterslide" type shape.

[21] In empirical studies the role of credit constraints on human capital accumulation can often be substantial (see Lochner & Monge-Naranjo (2012) for a survey).

[22] This point will be explored further below in this section.

Figure 3: Policy functions for endogenous variables



Note: Policy functions of endogenous variables for minimum, mean and maximum level of productivity. Assets are plotted on a semi-log scale.

4.2 The role of initial conditions

As the analysis of the policy functions has shown, wealth enables workers to accumulate human capital more quickly which has important implications for future earnings. Forward-looking, rational workers in this model make taking into account future expected costs and benefits of their present actions. Actions taken by the worker today affect their position in the next period, leading to a dynamically changing set of incentives and constraints. The value and policy functions that constitute the solution of the model encode this information in a way that is accessible to the agent, but not necessarily the economist trying to glean an insight.

To give a more detailed account of the interaction between wealth and human capital inequality in this model, this subsection provides some further analysis of how these interactions play out dynamically across time. In order to assess these dynamic effects, I simulate hypothetical households which differ only with respect to their initial level of wealth and track relevant measures of their human capital and earnings potential over time.

Figure 4 plots the evolution of human capital and gross earnings for workers starting with assets equal to the 5th, 25th, 75th and 95th percentile of the stationary distribution, conditional on different initial levels of human capital and the *same* level of initial productivity. I present the results normalized relative to a comparable worker with assets equal to the median of the stationary distribution of assets. This normalisation is provided for convenience, as in the long run the expectation of all households will converge to the unconditional average irrespective of initial conditions. This *ergodic* behaviour of the system means that across all households there is a small "drift" towards the population average, however, in the short and medium run, interesting trends in divergences emerge.

Specifically, each line in the top row of Figure 4 corresponds to the sequence:

$$\frac{E_0(\{h_t\}_0^T|h_0^{pc}, a_0^{pc}, e_0) - E_0(\{h_t\}_0^T|h_0^{pc}, a_0^{50}, e_0)}{E_0(\{h_t\}_0^T|h_0^{pc}, a_0^{50}, e_0)}, \tag{16}$$

where h_0^{pc}, a_0^{pc} indicate initial conditions equal to specific percentiles of the stationary distributions of human capital and wealth respectively. So for example, the lines in the first subplot of Figure 4 are obtained by setting h_0^{pc} to the 20th percentile of the human capital distribution, and then calculating the expected mean percentage deviations, setting in turn a_0^{pc} to the 5th, 25th, 75th and 95th percentile of the stationary distribution of assets. The subplots in the second row are obtained similarly, using gross earnings instead of human capital.

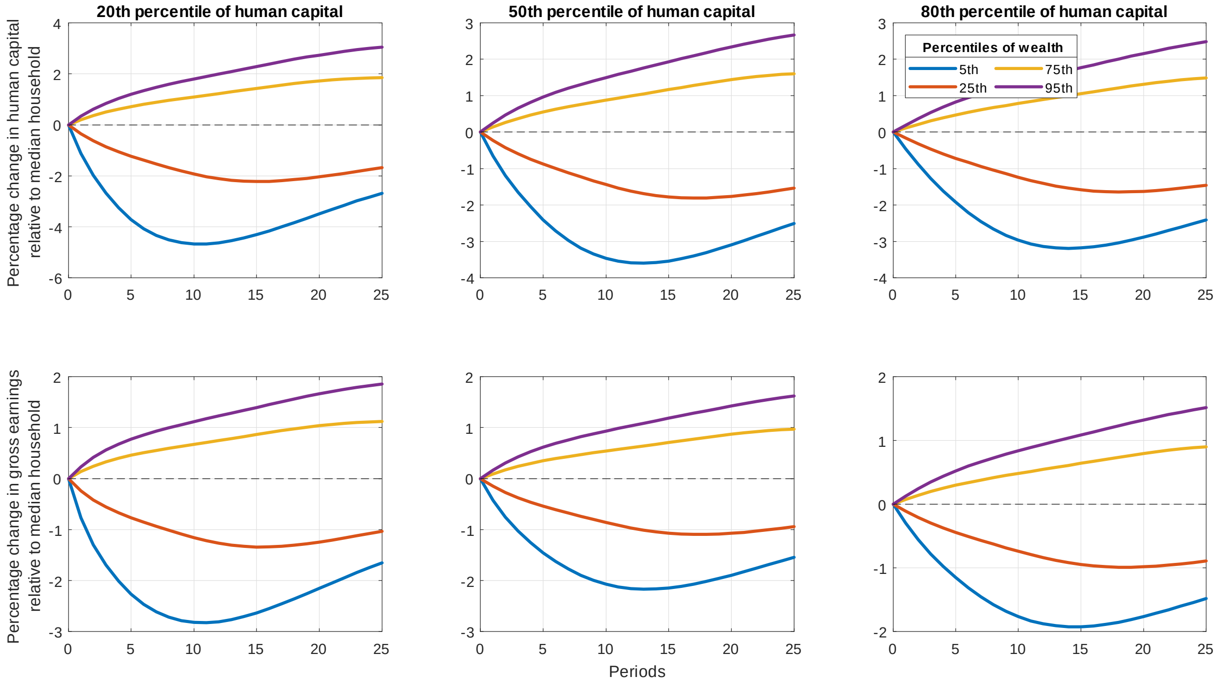
Turning to the subplots in row one, the most striking outcome is a large negative gap that opens up between workers starting with comparatively low assets and comparable workers with more savings. A worker starting close to the borrowing constraint at the 5th percentile of wealth, and the 20th percentile of human capital, has after 10 years accumulated about 4% less human capital than his twin who started with median wealth, even though both started with the same amount of human capital in the first period. This 4% gap in human capital translates into a sizeable effect on labour earnings, with a mean difference in gross earnings of roughly 3%. Compared to this a worker at the 25th percentile of assets has only lost just over 2% in human capital relative to the median wealth household. This nonlinear effect is emphasized by looking at the comparatively small differences between workers starting with the 75th and 95th percentiles of wealth, compared to workers at the 75th and the 25th or the 25th and the 5th percentiles.

This pattern of very strong negative effects for low-wealth households is consistent across different initial levels of human capital, even though it is attenuated somewhat at higher levels of h_t. For example, the difference between the 5th and 50th percentile of wealth households reduces to around 3.5% and 3% if we consider workers that start with human capital equal to the 50th or 80th percentiles of the stationary distribution respectively.

These observations confirm some of the earlier analysis: i) the resource constraint binds particularly strongly for households close to the borrowing constraint; and ii) at higher levels of human capital, the impact of initial differences is weakened somewhat.

An additional, striking feature, which this graphical analysis has uncovered, is the long persistence of these differences in initial conditions. Comparing for example 25th and 75th percentile households relative to the median wealth household, we see that the differences in terms of human capital and gross earnings are initially small, but over time small differences accumulate and a gap opens up. This hints at the existence of very persistent impacts of small differences in initial wealth as well as a strong role in the intergenerational transmission of human capital mediated by asset wealth.

Figure 4: Mean differences in human capital accumulation and gross earnings, by wealth quantiles



Note: Workers begin with mean level of productivity in the initial period.
All series are normalized relative to a household with 50th pctile of wealth in the initial period.

Focussing on average outcomes can sometimes understate another important function that wealth fulfils in the context of the incomplete markets model, namely helping the household weather bad shocks, and reduce the negative consequences of risk by providing a form of self-insurance. The workers in the model face risk in the form of exogenous productivity shocks. The standard analysis in the context of the HIM model suggests that savings allow households to smooth consumption across periods with high and low productivities. The model presented in this paper adds an additional dimension to this, as workers not only strive to smooth consumption but also maintain human capital at an optimal level. This adds to the agents' precautionary savings motives and provides an additional avenue for initial differences in wealth to have long-term consequences.

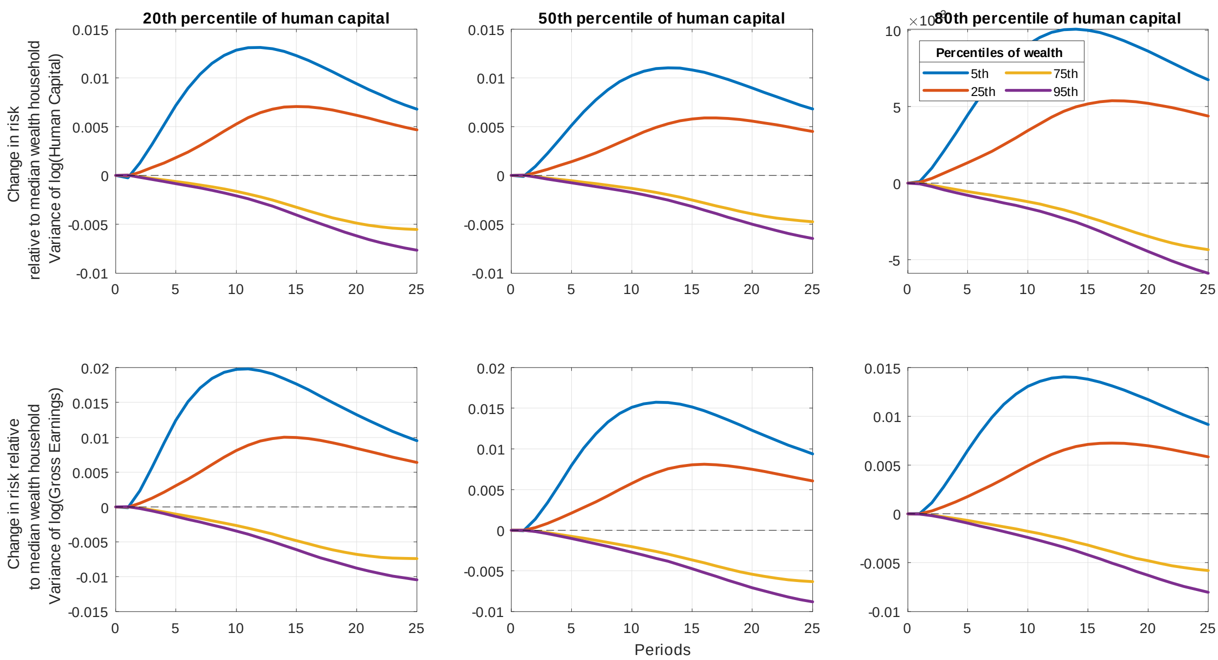
Figure 5 provides an illustration of the impact of initial differences in wealth on future human capital and by extension earnings risk, by plotting the variance of logarithms of human capital and gross earnings for different initial levels of wealth and human capital in a similar manner to Figure 4 above, only that I am plotting absolute differences instead of percentage differences. For a household starting at $t = 0$ the value of the coefficient of variation with respect to some outcome at some time t in the future provides a summary measure of the distribution of the different realizations of that outcome that the household might face - hence it provides a measure of risk with respect to that outcome.

Focussing on the first row in Figure 5, with respect to human capital, higher levels of risk indicate more uncertainty about future earnings and available resources. Naturally, as all households begin from the same initial level of human capital, the difference in the variance of logarithms begins at 0 and then slowly rises as the exogenous shocks accumulate over time. Accordingly, an increase in human capital risk is expected, but it is notable that the rate of increase is largest for the households with the lowest levels of initial wealth. Households starting with wealth equal to the 5th or 25th percentiles see a remarkable increase in their human capital and income risk relative to the median wealth household. Households beginning with assets around the 75th and 95th percentile, see an actual reduction of their risk, relative to the median wealth household in line with the insurance value of their assets. Again, the effect is highly nonlinear, with the difference between the 5 and 25th percentile households, far exceeding the difference between the 75th and 95th percentile households. This suggests that the poorest households are most exposed to human capital and therefore future income risk, as a result of their inability to use wealth as a tool to maintain human capital levels through times of low productivity. As before, higher initial levels of human capital help reduce the importance of these initial differences in wealth somewhat, but differences remain quantitatively large.

For a risk-averse household, the variance of future earnings is an important factor in determining welfare and well-being. The second row in Figure 5 provides an analysis of the future earnings risk that households face, based on their initial level of wealth. Overall the patterns are more similar compared to the case of human capital, as earnings risk is partly a function of the exogenous productivity process which is the same for all households. But still, after a couple of periods, there is a notable elevation of earnings risk for households with lower initial wealth which is a consequence of higher human capital risk.

In a standard incomplete markets model, the main function of precautionary savings is to enable consumption smoothing and thus counteract the negative consequences of income risk. By including human capital in the model, agents gain the ability to directly affect the stream of future incomes. This gives wealth an important additional function effectively allowing rich households to directly affect their future income risk. This potentially increases the value of savings for households and emphasizes the importance of accounting for wealth inequality when assessing income and earnings inequality.

Figure 5: Dynamic risk in human capital and earnings, by wealth quantiles



Note: Workers begin with mean level of productivity in the initial period.

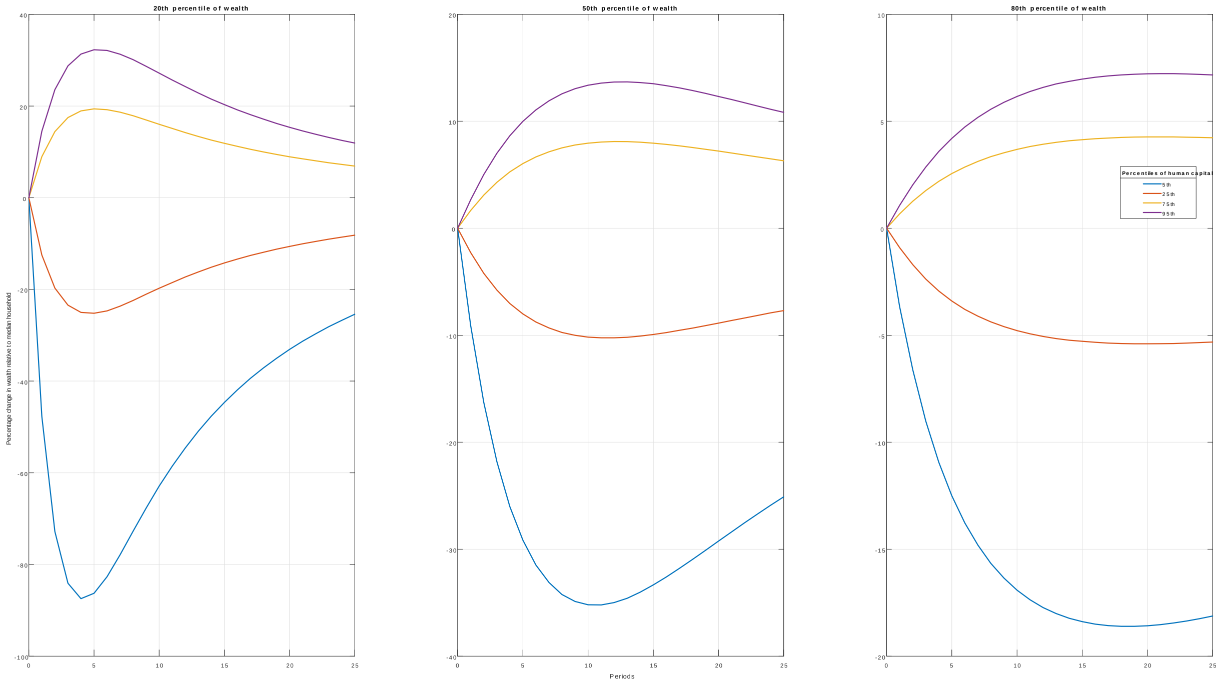
The preceding analysis has shown, that initial differences in wealth can lead to large and persistent differences in human capital accumulation by otherwise identical households. But the relationship between wealth and human capital is not limited to flow in this direction. The perhaps more intuitive relationship between the two quantities suggests that workers with more human capital obtain higher earnings and are thus able to accumulate more wealth.

Figure 6 quantifies the effect of initial differences in human capital on wealth accumulation. The logic of the sequences plotted in Figure 6 is similar to those plotted in Figure 4, with the difference that the dimensions of wealth and human capital are flipped - we are following households that start with the same amount of initial assets but with different endowments of human capital.

The plots reveal a clear wealth inequality enhancing role for human capital. Across different initial levels of wealth, households with lower initial human capital accumulate wealth much slower than their more skilled counterparts. These differences can be substantial: for households with close to zero assets, those with human capital equal to the 5th percentile will after 10 years only have accumulated 15% of the wealth of a household that started with the median amount. In contrast, highly skilled households starting with the 95th percentile of human capital will have accumulated 30% more assets than the median household. These effects are large and highly persistent, even though the absolute difference does diminish as the initial level of assets increases.

Taken together with the previous analysis suggests that the model generates an important feedback loop from wealth to human capital back to wealth. Wealth and human capital inequality are therefore mutually reinforcing, suggesting that any analysis that aims to understand one of the two should not disregard the other.

Figure 6: Mean differences in wealth accumulation, by human capital quantiles



Note: Workers begin with mean level of productivity in the initial period.

In this section, I presented an analysis of the policy functions of the model in the stationary equilibrium. I showed that workers with higher human capital productivity optimally invest more resources into acquiring additional human capital. Furthermore, non-linearities in the policy functions emerged, indicating differing incentives and constraints based on different levels of human capital and wealth.

Subsequently, I presented an analysis of the importance of wealth, by analysing the dynamic behaviour of hypothetical households with different initial conditions. This analysis showed that workers starting with low assets accumulate up to 6% less human capital than their twins who started with median wealth, which translates into a sizeable effect on labour earnings over time. Moreover, these differences were found to be highly persistent, leading to potentially large differences in intergenerational inequality in human capital and income.

Further, I discussed the role of risk in the model. Higher levels of risk indicate more uncertainty about future earnings and available resources. I find, that the rate of increase is largest for the households with the lowest levels of initial wealth. Meaning that the poorest households are most exposed to human capital and therefore future income risk and as a result face the most uncertainty about their future consumption.

Finally, I illustrated the effect of human capital on wealth, showing that amongst households with the same initial level of wealth, higher-skilled households accumulate assets much faster leading to a positive feedback loop between wealth and human capital.

The analysis in this section has provided an insight into the dynamics of human capital accumulation in the model, and how initial differences in wealth can lead to persistent differences in human capital and consumption across households and vice versa. In the next section, I will complement this analysis by looking at the reaction of the economy to an unexpected aggregate shock. This approach allows a more detailed analysis of how different initial conditions interact with aggregate shocks to shape the dynamics of the economy and to show how differences in wealth interact with human capital inequality.

5 Aggregate dynamics

The last section analysed the policy functions of the model in the stationary equilibrium, showing that differences in human capital and wealth can have an impact on incentives and constraints, leading to an interaction between wealth and human capital inequality. I also looked at the importance of initial conditions, showing that even small differences in wealth can lead to persistent differences in human capital and earnings across households, further finding that the poorest households are most exposed to human capital and future income risk. In all this it was assumed that the economy is in a steady, stationary state and not subject to aggregate disturbances, large or small that affect the economic fortunes of all households at the same time.

However, in reality, the economy is rarely in a state of perfect stationarity. At the aggregate level, unexpected economic shocks are an unavoidable reality. These shocks can arise from a variety of sources and can have a significant impact on the economy. For the UK, some notable recent examples of economic shocks include the Great Financial Crisis of 2007/08, the Brexit referendum, the Covid-19 pandemic, and the war in Ukraine with its associated energy and cost of living crisis.

In this section, I will analyse the response of the model economy to an unexpected, one-off, aggregate productivity shock.[23] This analysis complements the steady state analysis in the previous section and also lays the groundwork for the evaluation of the Covid-19 pandemic in the following section.

5.1 Dynamic equilibrium

To implement the shock, I assume that the economy is in its stationary equilibrium, when at time $t = 0$, an unexpected productivity shock hits the economy, reducing total factor productivity by 1%. The shock lasts for one period and then disappears, never to return again. The households observe the shock and perfectly forecast the path of all aggregate variables in the future and make their decisions based on their knowledge on the future evolution of factor prices.

This defines a perfect-foresight transition path for the economy.[24] Specifically, such a path for the economy is characterised by:

1. A time horizon $T > 0$ large enough that if the economy is hit by an aggregate shock at $t = 0$ it will have settled back into its stationary state by time $t = T$.

2. A path for the aggregate variables $\{K_t, L_t, A_t, G_t\}_{t=0}^T$.

3. Given $\{K_t, L_t, A_t\}_{t=0}^T$, the representative firm solves its optimization problem, and factor prices are given by $\{r_t, w_t\}_{t=0}^T$.

[23] In general these are called "MIT" shocks by the literature.
[24] For details of the implementation, see Appendix.

4. Given $\{r_t, w_t\}_{t=0}^{T}$, the households solve the dynamic version of their decision problem, and make choices consistent with optimal behaviour given the path of the aggregates.

5. Markets clear in every period.

6. The government's budget is balanced in every period.

The households optimization problem is now defined by a finite horizon problem, where the terminal value function at time T is equivalent to the value function of the stationary problem. The Bellman equation of this problem is defined as follows:

$$V_t\left(a_t, h_t, e_t\right) = \max_{\{c_t, a_{t+1}, x_t\}_{t=0}^{T}} \left\{ \frac{\left(c_t\right)^{1-\sigma}}{1-\sigma} + \beta E_t\left[V_{t+1}\left(a_{t+1}, h_{t+1}, e_{t+1}\right) \mid e_t\right] \right\} \tag{17}$$

$$\begin{gathered} s.t. \\ c_t + a_{t+1} + x_t = (1+r_t)a_t + y_t \\ h_{t+1} = h_t\delta_t + \chi x_t^{\nu} \\ y_t = \tau_1(w_t e_t \log(1+h_t))^{1-\tau_2} \\ c_t \geq 0, h_t \geq 0,\ a_t \geq a_{\min}, x_t \geq 0 \\ V_T\left(a_t, h_t, e_t\right) = V_{ss}\left(a_t, h_t, e_t\right) \end{gathered}$$

I solve the household's problem by backward induction using a modified version of the algorithm used to solve the stationary household problem.

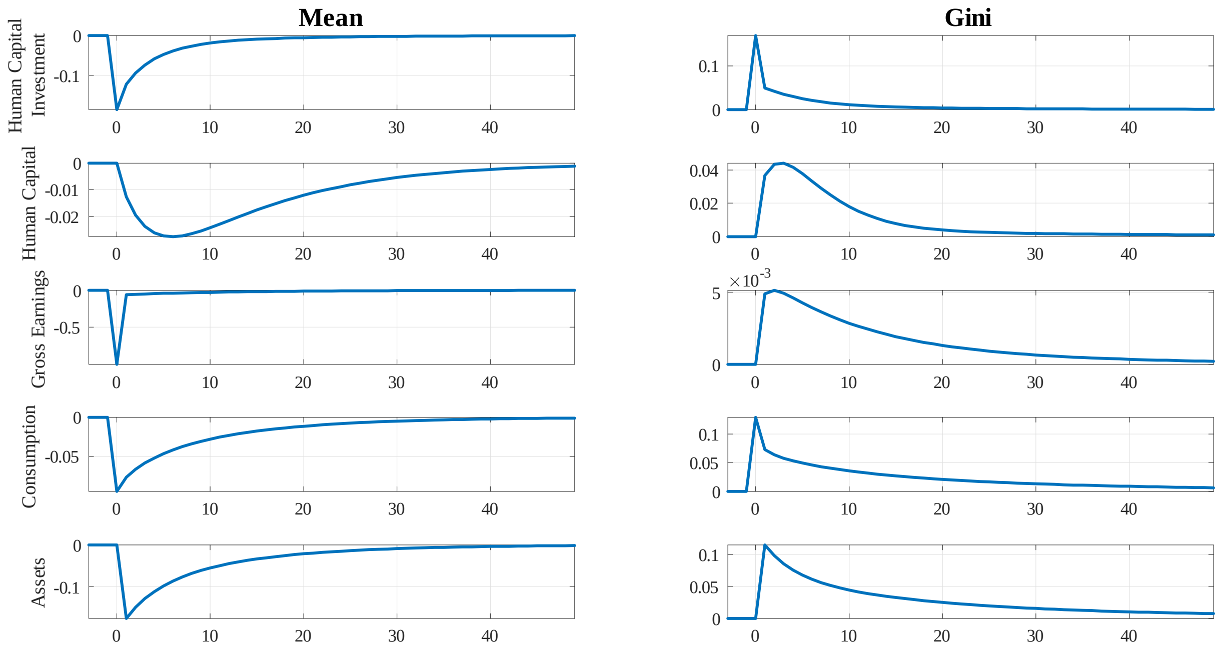
5.2 Aggregate effects

Figure 7 plots both the average and the distributional effects of the MIT shock on human capital investment, human capital, gross earnings, consumption and wealth. Investment into human capital drops sharply in the period of the shock, falling by about 0.15% relative to its steady state value. This is driven by both, a tightening of the households' resource constraint ("income effect"), as well as a reduction of the effective return to human capital going forward ("price effect"). Human capital investment recovers quickly thereafter, reaching a value of 99.95% of its steady state level 5 years after the shock. The drop in investment, affects the aggregate stock of human capital, although the reaction is much more sluggish. Human capital reaches a low of 0.025% below steady state after around 8 years, after which a protracted recovery begins. Human capital inequality follows suit, peaking at around 0.04% above its initial level. Gross labour earnings drop by 1% on arrival of the shock and then mostly recover, but then remain below the initial level for a prolonged period. Similarly earnings inequality rises and remains elevated for around 15 years. This suggests that human capital in the model can generate an internal propagation mechanism, that leads to a persistent "scarring" effect of recessions in terms of both average earnings and inequality.[25] Consumption falls on average, but only slightly by 0.1%, as households draw down asset savings to protect themselves. Assets themselves fall by about 0.15%, and then recover slowly together with average consumption levels.

[25] Since the shock itself has no persistence, the entire dynamic is driven by the internal proagation mechanism of the model.

Consumption inequality exhibits a considerable spike in the period of the shock, likely as a result of borrowing-constrained households being unable to borrow to bridge their temporary earnings shortfall.

Figure 7: Response of aggregate variables to an unexpected 1% drop in TFP



Note: All values are normalised to 0 in the stationary equilibrium.

The deviations analysed above must necessarily appear small, given the size of the MIT shock. However, what is remarkable is how even small shocks can have long, persistent impacts on human capital and earnings. To provide a better evaluation of the impact that these shocks can have over the lifetime of the agents, Table 6 below provides elasticities of a number of key endogenous variables of the model over different time horizons. Elasticities are calculated as the sum of percentage deviations from steady state divided by the magnitude of the shock. So for example, for endogenous outcome z the elasticity with respect to time horizon T is calculated as follows:

$$E_z^T = \frac{\sum_{t=0}^{T} \frac{z_t - z_{ss}}{z_{ss}}}{\sum_{t=0}^{T} \frac{A_t - A_{ss}}{A_{ss}}}, \tag{18}$$

where the subscript ss denotes steady state values.

Focussing first on the elasticities of the means, it can be noted that the impact effects of the shock are initially small, except for the case of earnings, which are directly affected by the fall in total factor productivity. Human capital and wealth stocks are fixed at the beginning of the period, so the shock has no immediate pass-through to these quantities, but even human capital investment reacts fairly inelastically, with an elasticity of around 0.19, whilst consumption has an even lower impact elasticity of 0.09. Looking at the medium-run impact of the shock, the 5 and 10-year elasticities the impact becomes more pronounced. Taken over a 5-year horizon, human capital investment exhibits a response of around 0.54, while the elasticity of gross earnings goes up to 1.28 suggesting that around an additional 28% of the initial shock has been added to earnings losses over that time period. Human capital and assets only respond with a lag to the shock, as was also indicated by Figure 6 above. After 10 years the elasticity of human capital has grown to around 0.22, while that of assets exceeds 0.95. In the long run, elasticities continue to grow, albeit at a slower rate, as the aggregate quantities slowly approach their stationary values. After 30 years, which is the expected working life of a household, several variables exhibit an elasticity of above 1, suggesting that the internal dynamics of the model have amplified the initial shock over time.

Moving on to the elasticities of the Gini coefficient, the initial impact of the shock is less pronounced, with an elasticity of -0.17 for human capital investment and -0.13 for consumption. This suggests that the inequality measured by Gini increases in response to the shock. The medium-term elasticities also remain strong, with a slight increase in the absolute magnitude of the response over a 5 and 10-year horizon. The elasticity of human capital inequality increases to -0.42 at the 10-year mark, and -0.49 after 20 years, indicating that the initial inequality shock has worked to increase human capital inequality over time. The elasticity of gross earnings remains low, at -0.04 after 10 years and only increases slightly to -0.06 after another 10 years. The response of consumption and assets to the shock is also negative and increasing over time, with the elasticity of assets exceeding 1 after 20 years, while consumption inequality shows an elasticity of -0.88 after 20 years. However, overall the impact of the MIT shock on inequality remains less pronounced for human capital and earnings, but has siezable effects on consumption and assets.

Table 5: MIT Shock Elasticities

Mean	Impact	5 years	10 years	20 years	30 years	∞
HC Investment	0.186	0.536	0.703	0.811	0.846	0.867
Human Capital	0	0.082	0.216	0.4	0.486	0.551
Gross Earnings	1	1.208	1.386	1.578	1.661	1.716
Consumption	0.094	0.348	0.537	0.727	0.807	0.867
Assets	0	0.566	0.955	1.319	1.465	1.553
Gini	**Impact**	**5 years**	**10 years**	**20 years**	**30 years**	∞
HC Investment	-0.17	-0.326	-0.419	-0.489	-0.518	-0.569
Human Capital	0	-0.165	-0.312	-0.407	-0.435	-0.491
Gross Earnings	0	-0.02	-0.038	-0.058	-0.068	-0.08
Consumption	-0.129	-0.378	-0.594	-0.875	-1.045	-1.383
Assets	0	-0.375	-0.661	-1.004	-1.207	-1.601

Note: Elasticities in response to an unexpected, one-off 1 % TFP shock.

5.3 Effects across the distribution

The last subsection explored the aggregate consequences of an unexpected economic shock. The analysis showed that even a perfectly symmetric shock can have long-lasting consequences for the distribution of human capital, wealth and consumption. As was explored in the previous section, households in this model behave differently, depending on their levels of wealth and human capital, and as a result, we should expect households to also react differently to the arrival of the shock. In this subsection, I explore how the impact of the MIT shock affects households at different parts of the wealth and human capital distribution.

Figure 8 plots the MIT shock response of human capital and gross earnings for households with different levels of pre-shock wealth and human capital. In spirit this reproduces Figure 4 above, with the modification that the reference group for each household is now a household with identical wealth, living in a world, where the shock never materializes. Specifically, the lines in Figure 8 correspond to the following sequences:

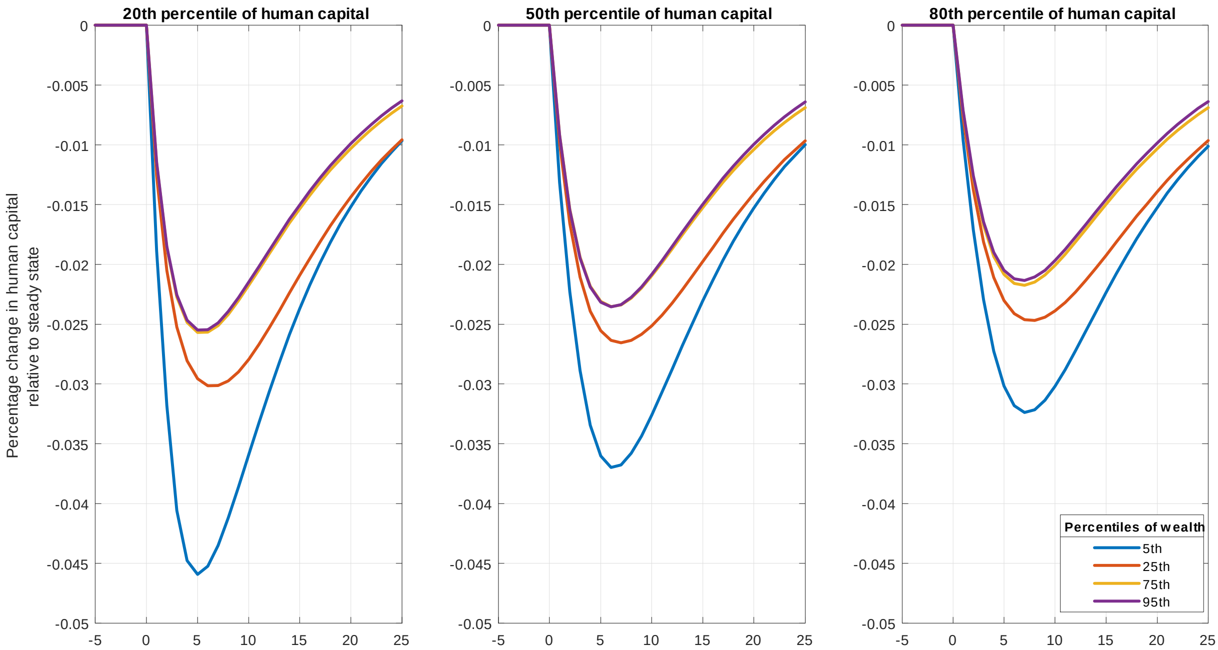
$$\frac{E_0(\{h_t\}_0^T|h_0^{pc}, a_0^{pc}, e_0) - E_0(\{h_t^{ss}\}_0^T|h_0^{pc}, a_0^{pc}, e_0)}{E_0(\{h_t^{ss}\}_0^T|h_0^{pc}, a_0^{pc}, e_0)}, \tag{19}$$

where the superscript ss indicates that the sequence is derived under steady state conditions. The other sequences in Figure 9 are calculated in the same manner, replacing h_t with the relevant endogenous variable in each case. For the interpretation of the findings of Figure 8, this means that the plotted sequences capture the heterogenous effect of the MIT shock on human capital and earnings of households. Given that considerable dynamic heterogeneity exists, even in the absence of a shock (see Figure 4 and analysis in the last section), the overall effect is likely to be exacerbated by the heterogeneous response.

We begin our analysis by looking at each graph in the first row of Figure 8 in turn. Each subplot reveals a smooth deviation from steady state human capital levels, irrespective of initial conditions. This pattern matches the one observed for aggregate human capital of the economy, with deviations reaching a trough of between 0.02 and 0.045 percent around 6 or 7 periods after the arrival of the shock. Moving from left to right, two observations become apparent: i) as we increase the initial human capital of households, the effect of the shock becomes less pronounced for all households, irrespective of initial wealth; ii) the difference in the impact on the lowest wealth households and the remainder becomes smaller. Both of these are a consequence of higher labour earnings, which help to relax the budget constraint for wealth-poor households.

Continuing on the last point, one of the most striking observations from each panel is the huge difference in the human capital response of households with the 5th percentile of wealth versus households with higher initial levels of savings. These wealth-poor households experience a dramatically bigger drop in human capital amounting to around 90% more than that experienced by their more affluent peers. The effects are highly persistent for around 20 years, suggesting high overall cumulative losses. Moving up the distribution of human capital does reduce the absolute size of the drop for all households, but the relative gap remains. Echoing the analysis in the previous section, the nonlinearity in the MIT shock response seems to be highly concentrated at the left tail of the wealth distribution. Households with wealth equal to the 75th or 95th percentiles, appear to experience smaller relative reductions compared to their poorer contemporaries.

Figure 8: Human capital response to MIT shock, by wealth quantiles



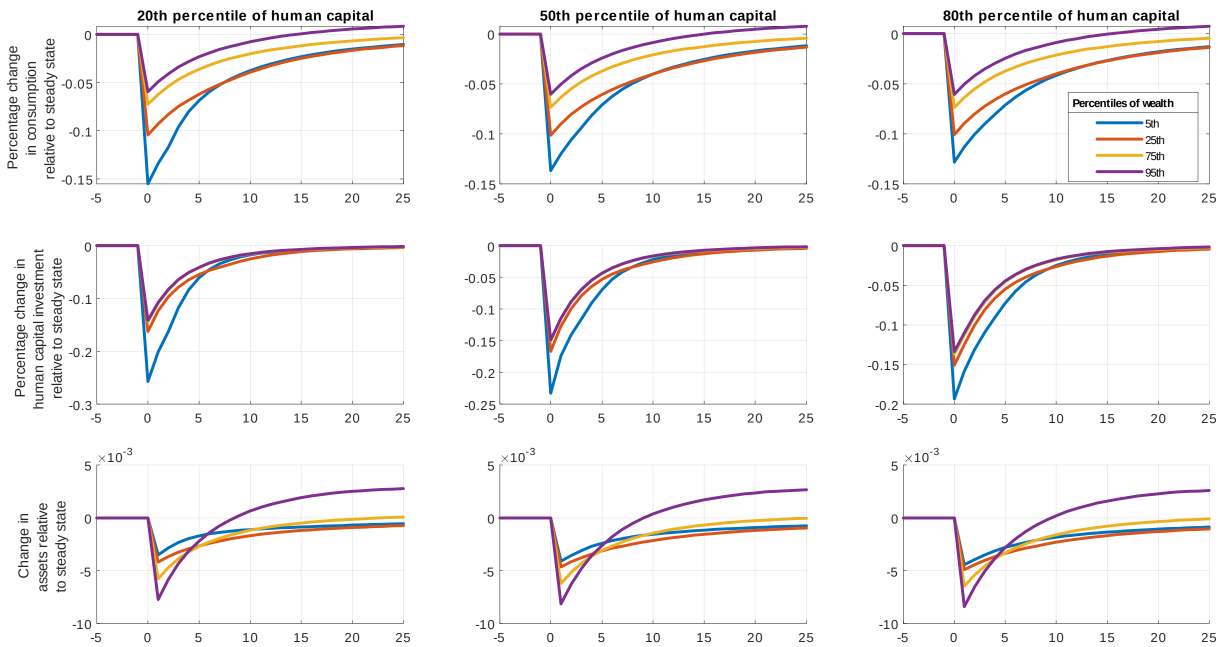
Note: Workers begin with mean level of productivity in the initial period.
Series normalised to expected progression in stationary equilibrium.

The main driver of this behaviour is the lack of ready savings among the poorest housholds. Figure 9 reveals the effect of the MIT shock on the constituent parts of the households budget, namely consumption, human capital investments and savings. Across different levels of initial human capital, the poorest households experience the sharpest and most persistent reductions in their level of consumption and human capital investment as respectively depicted in rows 1 and 2 of Figure 9. In terms of consumption, the losses experienced by households at the 5th percentile of the wealth distribution are about 3 times those experienced by the households at the 95th percentile - a loss sharpened by the much lower initial level of consumption of the former group. Human capital investment also falls sharpest for the poorest households with the effects being most pronounced at low levels of human capital, where earnings are not large enough to relax the budget constraint sufficiently. At higher levels of human capital, the response heterogeneity across the wealth distribution becomes slightly less pronounced, but the poorest households still experience the most persistent falls in their human capital investment, resulting in the persistent drop in human capital shown in Figure 8.

The final row in Figure 9 shows the impact of the MIT shock on the absolute[26] level of savings. The general pattern is roughly the inverse of the pattern for consumption in the first row: the richest households draw down their savings the most allowing them to protect their consumption and human capital investments. As we move down the wealth ladder, households become less able to draw on their savings for these purposes and households have to reduce spending on consumption and human capital investments instead. Naturally, households close to the borrowing constraint are most affected by this effect since they have limited access to additional funds they could borrow.

[26] I use levels rather than percentages to avoid issues with zero and negative asset levels.

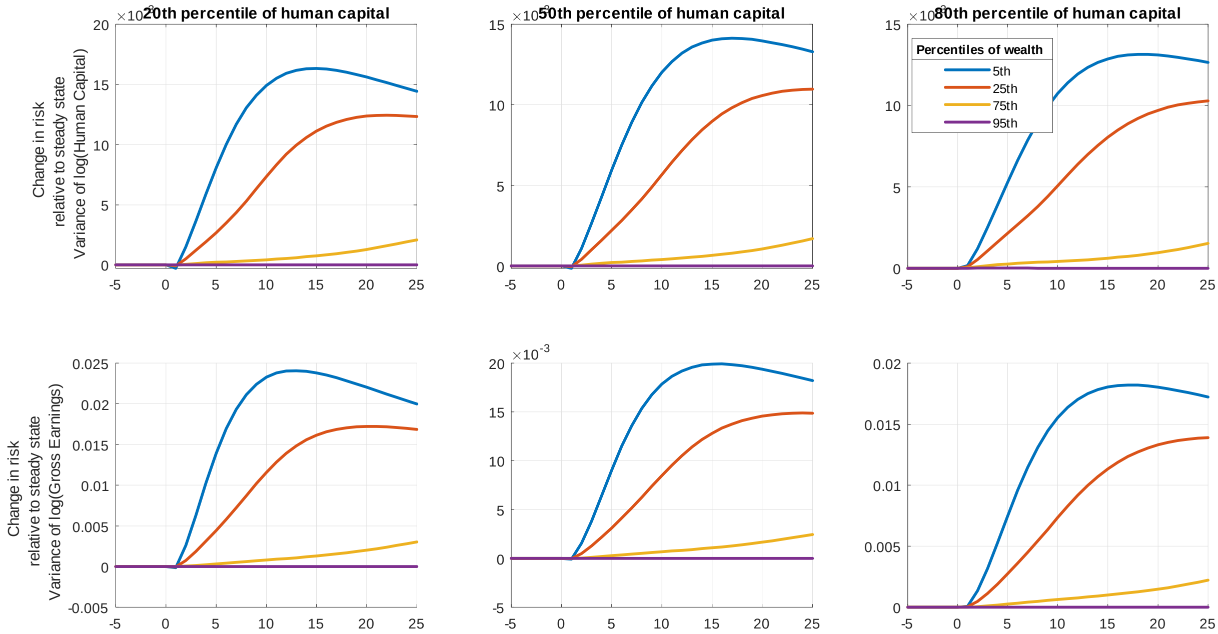
Figure 9: Consumption, human capital investment and assets response to MIT shock, by wealth quantiles



Note: Workers begin with mean level of productivity in the initial period. Series normalised to expected progression in stationary equilibrium.

So far this analysis has focussed on the average impact of the MIT shock across the distribution of household wealth and human capital. However, as was highlighted in the last section wealth also plays a key role in allowing households to mitigate the level of future human capital and earnings risk. Figure 10 plots the absolute change in the variance of logarithms of human capital and gross earnings relative to the steady state experience for each household group. Since the results are similar across both rows I will focus on the analysis of the change in gross earnings risk in the second row. The most notable observation is the start difference in the effect of the MT shock on earnings risk across the household wealth distribution. While households at the 5th and 25th percentile experience rapid and large increases in earnings risk, households at the 75th and 95th percentiles experience only relatively small and gradual increases in the case of the 75th percentile or no noticable increase in risk at all for the richest set of households. This is particularly relevant since we know that poorer households are already more exposed to human capital and earnings risk than their richer counterparts - a tendency that is now being amplified by the effect of the shock. While the magnitudes of the increase slightly at higher levels of human capital, the overall patterns remain the same, highlighting the importance of savings in not only mitigating the immediate effects of a negative income shock, but also in avoiding risk and uncertainty in its aftermath.

Figure 10: Human capital and earnings risk response to MIT shock, by wealth quantiles



Note: Workers begin with mean level of productivity in the initial period.
Series normalised to expected progression in stationary equilibrium.

The analysis in this section has focussed on the effect of an unexpected, symmetric productivity shock on aggregate and distributional outcomes of the model. The analysis showed that even small shocks can have consequences for aggregate variables of the model when assessed over the medium to long run. Further, it was shown that households close to the borrowing constraint have the strongest reaction to the shock, losing more human capital than even only moderately wealthy households, leading to persistent increases in human capital and earnings inequality with consequences for current and future consumption and welfare. This means that the initial distribution of wealth and human capital matters considerably for the aggregate response to an aggregate shock. A higher share of borrowing-constrained households implies a bigger, more persistent drop in human capital and thus a slower recovery.

All these conclusions are derived in a situation where the recession affects all households equally, irrespective of individual circumstances. However, economic shocks are hardly ever this blind. Often the effects of a recession fall disproportionately on those workers who are already struggling with their economic fortunes. In these cases, the impact of the shock is not distributed evenly but rather concentrated on a specific part of the pre-crisis productivity distribution. It is easy to extrapolate that under such circumstances the effect of a shock is going to be exacerbated, necessitating a policy response to protect the most vulnerable in society. The next section will address some of these points by exploring one of the biggest economic shocks in living memory which lead to unprecedented levels of public insurance - the Covid-19 pandemic.

6 Covid-19

The Covid-19 pandemic can justifiably be seen as the defining economic crisis of our age. At the very least, the pandemic caused a massive reduction in economic activity, largely as a result of lockdown measures put in place to protect public health. The corresponding loss of incomes and employment for millions of households necessitated unprecedented levels of intervention on the part of governments all across the world. As we slowly emerge from the pandemic, the question of how effective these policies were in avoiding catastrophe as well as the potential medium- and long-run impacts of the pandemic are at the forefront of a large research agenda.

In this last substantive section, I apply the model to these questions, by simulating the effect of the Covid-19 shock on the UK economy. In doing so I explore the effectiveness of the actual insurance policies applied by the government as well as an alternative, untargeted redistribution mechanism. I also use the model to assess the potential medium- and long-run impacts of the pandemic on human capital and wealth inequality. As I have shown in previous sections, the model is well suited to exploring the economy's response across these two dimensions, which are particularly relevant for the questions related to the economy's recovery from the pandemic. Given the recency of the pandemic, empirical research on the topic is still partially hampered by a data lag, and as a result, the impact of Covid-19 on human capital inequality in the UK is not yet fully understood. Initial research suggests that it has had a negative effect on education, and might have amplified human capital inequalities (Blundell et al. (2022)). However, any medium- and long-run impacts of these turbulent years will only be revealed with time.

In this situation, a model-based quantitative analysis can add value by providing a consistent story of how the economy might evolve over the coming years and help us to assess whether current policies are sufficient to address the long-term impacts of the pandemic, and to identify effective strategies for future crises. We have seen in previous sections, that the interaction between wealth and human capital inequalities, provides an amplification mechanism for aggregate shocks, which has the potential to deepen pre-existing inequalities and slow down a potential economic recovery. Given both, the size of preexisting inequalities in the UK at the beginning of the Covid-19 pandemic, and the size of the shock, the effects are likely to be large and long-lasting. On the other hand, unprecedented policy intervention provided insurance to millions of workers and firms alike - a level of state-led welfare provision unmatched in any previous downturn. These measures were intended to stave off the worst effects of the pandemic and to ensure that the economy could bounce back quickly when conditions improved. Whilst ensuring consumption levels in a period of unprecedented economic turmoil was evidently in the interest of politicians and society at large, there remains the question of unintended consequences. A negative productivity shock creates disincentives to invest in human capital, *even if* incomes are perfectly insured. This means that it is possible, that even though the immediate effect of the pandemic caused a compression of the income distribution, the near future will see a rise in inequality along the lines of wealth, human capital, and income.

The model developed in this paper is uniquely suited to perform this analysis: it distinguishes between the level and productivity of human capital and can therefore account for the "price effect" of a negative, non-uniform productivity shock; and it also allows households to use their savings to invest in human capital and therefore takes account of the role of wealth inequality for the propagation of the Covid-19 shock. In this section I simulate the response of the model economy to an unexpected, non-uniform productivity shock, under different policy responses, providing valuable insight into the effectiveness of these policies.

6.1 Covid shock & policy responses

Covid-19, and the response intended to contain it lead to massive drops in economic activity, that reoccurred across much of 2020/21 and the spring of 2022. For 2020 alone the Bank of England estimated a drop in aggregate output of 9.7% (Harari & Keep (2021)), constituting a massive loss of earnings and employment for the average household. Generally, the effect was not uniform across the distribution of income. Much of the research on the economic impact of Covid-19, documents that households on lower incomes were hit harder than their contemporaries further up the income distribution (see Adams-Prassl et al. (2020), Blundell et al. (2020)). One main reason why initially poorer households were more affected by the pandemic is that they were more likely to work in industries that had been hit hardest by the pandemic, such as hospitality, tourism, and retail which experienced the closure of many businesses and the reduction in demand for services. As a result, these workers have been more likely to suffer from job losses and earnings reductions.

At the same time, richer households were more likely to be in industries that are less affected by the pandemic, such as finance and technology. As a result, their job security and earnings have been much more stable, particularly since remote working capabilities enabled many office-type professions to continue working throughout the pandemic. The overall picture that has emerged was one of a highly skewed income shock, that exacerbated existing inequalities along the lines of age, gender, health, education, professions and social class (see Marmot & Allen (2020), Marmot et al. (2021) and references therein).

Fortunately for many, a large part of the earnings losses were insured away by the government in an unprecedented public insurance effort, instituting the furlough scheme and business support initiatives across much of the pandemic years. Overall, this had the counterintuitive effect of the pandemic resulting in an actual fall in income inequality (see Blundell et al. (2022), Stancheva (2021)).

To accurately assess the effect of the Covid-19 pandemic the shock to the model economy will need to accurately reflect both the large, unequal reduction in pre-tax earnings and the generous income support schemes. My baseline calibration will take into account the large non-uniform drop in pre-tax earnings, as well as the relatively low reduction in after-tax incomes. Then I will calibrate two additional counterfactual insurance policies to assess the effectiveness of the government's response.

The starting point is going to be the numbers reported by Brewer & Tasseva (2021), who analyse the economic effect of the first wave of Covid-19 using data from the Family Resource Survey, the Understanding Society COVID-19 Study, and the UKMOD tax-benefit system. I use the numbers reported in Figure 1 in their paper. Specifically, I will use the drop in earnings (dark blue bar),[27] and net income after all taxes and transfers have been accounted for (black line with circles) as targets in my shock calibration procedure.[28]

[27] I exclude self-employed earnings, in line with my sample selection.

[28] Figure 1 in Brewer & Tasseva (2021), shows average monthly estimates. Since they focus on April and May of 2020 that means that extrapolating to an annual frequency might slightly overestimate the

To implement the shock across the distribution, I first assess which state (h_t, a_t, e_t) puts a household into which decile of the stationary (aka pre-Covid-19) earnings distribution. Then to calibrate the shock, recall that pre-tax earnings (q_t) are given by:

$$q_t = w_t e_t * \log(1 + h_t) \tag{20}$$

and post taxes and transfers income is:

$$y_t = \tau_1 q_t^{1-\tau_2}. \tag{21}$$

I define a new pandemic distribution of productivity e_t^{shock} and also a lump sum pandemic payment from the government $b_t^{shock} > 0$. As a result, during the pandemic households' pre- and post-policy earnings are calculated using modified versions of the previous equations: Gross labour earnings are calculated as

$$q_t^{shock} = w_t e_t^{shock} * \log(1 + h_t) \tag{22}$$

and post taxes and transfers labour income as follows:

$$y_t^{shock} = \tau_1 q_t^{shock^{1-\tau_2}} + b_t^{shock}. \tag{23}$$

Note that b_t^{shock} is a payment received in addition to the automatic tax and transfer stabilizers that are already embedded in the model. I assume that the government can finance its budget throughout the pandemic via additional borrowing without raising additional taxes - indeed this is broadly what happened in 2020/21.[29]

Both e_t^{shock} and b_t^{shock} are assumed to be functions of the exogenous productivity state e_t which effectively is a proxy for a worker's pre-pandemic occupation. As was noted above, the impact of Covid-19 was highly skewed towards workers in low-income (productivity) occupations, making pre-pandemic productivity a relevant predictor of how impacted a household would be by the pandemic.

I approximate both e_t^{shock} and b_t^{shock} using polynomial functions, and try and minimize the squared percentage difference between the reported drops and those implied by the model, taking into account the general equilibrium effect of changes in the wage rate.[30]

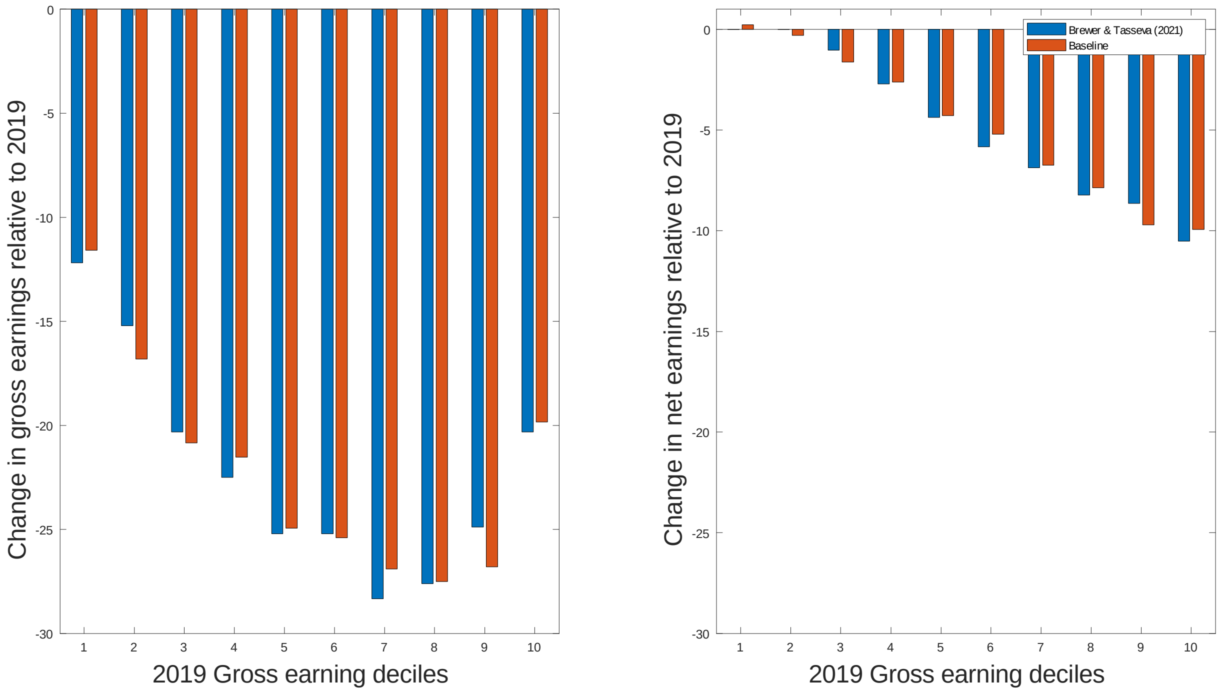
Figure 11 shows the model-implied changes in pre- and post-policy incomes for the calibrated values of the pandemic productivity shock and the emergency subsidies and compares these changes with those reported by Brewer & Tassseva (2021). The fit of the calibration is quite good across the distribution of pre-pandemic gross earnings, reproducing a slightly skewed "U" shape in gross earnings losses and a monotonously decreasing, almost linear shape for net earnings.

impact of the pandemic.

[29] The aim of this paper is to study the impact of Covid-19 and the government's response in the short to medium run. For simplicity, I ignore the implications of this increase in government debt in the remainder of this analysis. However, I acknowledge the importance of this question and provide some discussion of the limitations that this brings in the later parts of this section.

[30] See Appendix for a detailed description of the calibration.

Figure 11: Pre- and post-taxes and transfers earning drops



Note: Data points transcribed from Brewer & Tasseva (2021), Figure 1.

To complement the baseline calibration, I devise two additional policy scenarios: i) no intervention and support measures and ii) a uniform lump-sum payment. The no-intervention scenario is a crucial counterfactual to evaluate the effectiveness of the observed intervention in 2020/21. For this purpose, I assume that the government did not provide any additional Covid-19 relief to households, but simply relied on the automatic stabilizers to offset the loss in employment and earnings. For this, I set $b_t^{shock} = 0$.

Emergency support measures linked to the Covid-19 pandemic were heavily targeted and income-contingent. The Coronavirus Job Retention Scheme ("furlough") for example provided employers with government grants in order to retain workers at 80% of their usual wages (albeit subject to a maximum payment). This policy clearly benefitted high-wage workers, as the absolute subsidy they received would have been considerably larger than for low and medium-wage peers. Other interventions, such as the £20/week universal credit uplift were targeted at the lower end of the income distribution. On balance, these interventions were designed to provide relief to those that required it most. I counterpoint this by assuming that an agnostic non-contingent benefits policy had been put in place instead, which just provided every household with the same amount of relief irrespective of circumstances. To implement this, I calibrate the emergency benefit, so that the total amount that was spent on b_t^{shock} in the baseline case gets distributed equally across households. The corresponding net income changes for both policies can be found in Figure A1 in the Appendix.

One of the most noticeable impacts of the Covid-19 pandemic were restrictions on the way that households could spend their money. Frequent lockdowns meant that nonessential shops were closed for large parts of the year, socialising in bars, cafes and restaurants was heavily curtailed leading to large reductions in consumer spending. Evidence for the

UK and other economies imposing lockdown restrictions suggest, that these policies lead to large reductions in household consumption, particularly for higher-income households (see for example Hacioglu-Hoke et al.(2021); Bank of England (2020); Tenreyro (2021) for the UK; Dossche and Zlatanos (2020) for the EU; and Miescu and Rossi (2021) for the US). Specifically, Davenport et al. (2020) found that among the two highest income quintiles, consumption decreased by about 25% in the early months of the crisis, with smaller changes for lower income groups (consistent with patterns reported in Bank of England (2020) and Tenreyro (2021)). In the context of the model such forced savings are likely to lead to a reallocation by richer households who are constrained in their consumer spending towards financial savings (see on this point Angelopoulos et al. (2021)) and human capital investments. In order to capture this important mechanism, I add a consumption ceiling to the model which applies during the initial phase of the pandemic.[31]

6.2 Impact of Covid-19 on aggregate economic variables

I take the calibrated shocks and apply them fully to 2020, and then again half of the productivity shock and half the extra benefit payment to 2021.

I first discuss the medium-run predictions for the aggregate economy under the baseline calibration and compare them to the no insurance and uniform insurance counterfactuals. After this, I analyse the differences in the response of the aggregate economy through the lens of the heterogenous responses of households at different points of the pre-Covid-19 wealth and human capital distributions. The exercise shows that the initial state of households plays an important role in determining the households' response to the Covid-19 pandemic, and interacts with the policy to drive the dynamics of the aggregate economy.

The Covid-19 pandemic had large and immediate consequences on the UK economy. Many sectors and industries were partially, or fully locked down, leading to the closures of thousands of businesses and forcing millions of laid-off workers to rely on state-provided benefits. But while the prospect of future full-scale lockdowns appears unlikely, there remains some uncertainty of when, or if, the economy will truly recover. Large economic downturns can leave lasting scars on an economy, by destroying burgeoning businesses or causing mass layoffs leading to the destruction of match-specific human capital (see for example Ouyang (2009), Huckfeldt (2022)). Job retention programmes such as the furlough scheme, were designed to maintain employment relations and avoid the painful effects of job separations on a large scale. Yet still, a loss of "business as usual" for large parts of the years will have certainly affected the opportunities and incentives for learning, training and skill acquisition, which are crucial in building human capital.[32]

Figure 12, plots the response of the main endogenous outcomes of the model in response to the Covid-19 shock using the three different policy calibrations. I focus primarily on investment into and the stock of human capital since this is the mechanism through which

[31] The consumption ceiling imposes an additional utility cost on consumption above a certain threshold. I calibrate both the consumption ceiling and the penalty to match as closely as possible the relative mean decrease in consumption for the five pre-Covid-19 income quintiles as reported in Davenport et al. (2020). For details see Figure A7 in the Appendix. For the computational algorithm, see Appendix.

[32] Admittedly, for many of us the pandemic necessitated learning many new skills, such as remote working, teaching and collaborating. Whether these adaptations merely helped us buffer the worst excesses of the recessions, or will make us more productive in the future is an interesting and open question.

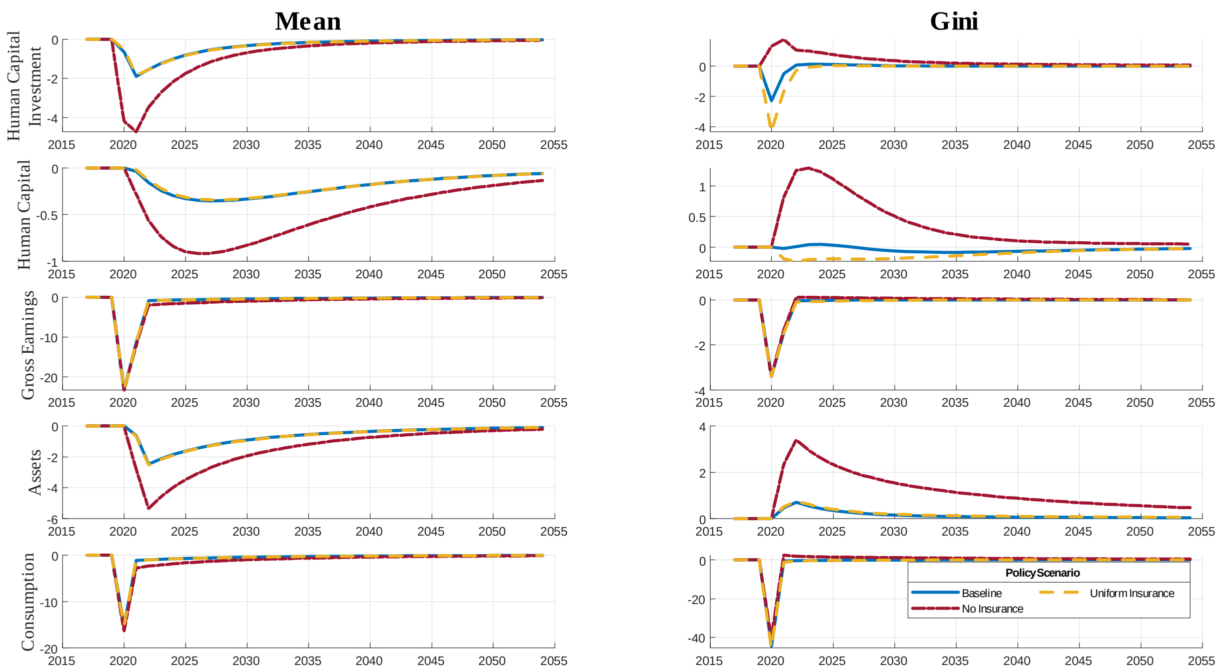
the pandemic might have long-run effects on future economic prosperity, but also wealth due to its relevance to human capital accumulation which was showcased in previous sections.

I begin with an analysis of the baseline calibration, which approximates as closely as possible the actual policy response that occurred in 2020-21. A year after the impact investment into human capital falls sharply, reaching a low of -2% relative to pre-Covid-19 levels. Recovery to steady state levels takes around 10 to 15 years, even though most of the initial investment level is reclaimed by 2030. Average human capital on the other hand falls much more gradually, reaching a trough of -0.4% in the second half of the 2020s. The recovery is even more gradual, and human capital levels are projected to be around 0.2% below 2019 levels until at least 2040. This hints at the likelihood of a very long-winded recovery, with economic potential below pre-crisis levels for a generation.[33]

On a slightly more positive note, it appears that the projected path of the economy points towards greater levels of equality - at least in the medium run. The gini of human capital investment spending drops sharply in 2020, by 2 percent. While investment inequality recovers fairly quickly, the distribution of the stock of human capital does not change noticably throughout the mid-to late 2020s.

Mean assets drop by around 2%, wealth inequality increases by around 0.5% and remains elevated for about 5 years (see also Angelopoulos et al. (2021) for evidence of increased wealth inequality post-Covid-19). Average consumption and consumption inequality drop dramatically in 2020 as a result of the imposed consumption restrictions but recover relatively quickly to their pre-Covid-19 levels after that.

Figure 12: Response of aggregate variables to Covid-19, all policies



Note: Baseline, uniform and no insurance Covid-19 shock calibration.
Series normalised to expected progression in stationary equilibrium.

[33]The model is based on the human capital of working-age individuals and therefore cannot speak to the skills and abilities of future cohorts of labour market entrants. However, given the large-scale disruptions to normal teaching schedules at all levels, the projections of the model are likely on the optimistic side.

At the aggregate level, the model predicts a loss of human capital in the medium run, leading to a loss in economic output and growth potential. Consumption restrictions lead to more equal consumption - at least for a brief period - but the resulting response of high-income households leads to increased wealth inequality.

But how much of this is due to the nature of the economic shock, and the way it has affected workers across the distribution of incomes differently, and which part is played by the government's targeted Covid-19 relief policies? To assess the effectiveness of targeted intervention to help cushion the blow of the Covid-19 shock, and distribute the associated economic pain more equitably, we turn to the other two policy scenarios depicted on Figure 12.

Under the no insurance scenario, the government did not provide any additional payments to households but merely allowed the automatic stabilizers of the tax and transfer system to operate as in normal times. As a result, the economic impact of the pandemic is much bigger for all households, but additionally, the removal of the broadly regressive Covid-19 benefits leads to a bigger pass-through of the shock to lower-income households. The proposed policy of providing benefits uniformly across the distribution of households, has very similar dynamics relative to the baseline calibration (see Figure 12, Table 7, and Figure A2 in the Appendix), whilst potentially being easier to administer. Since the uniform insurance scenario is qualitatively and quantitatively similar to the baseline case for the aggregate behaviour of the economy, I will focus on the comparison of the baseline case and the no insurance scenario for now.

Comparing the two scenarios, it is evident that the impact of the Covid-19 pandemic would have been a lot more severe in the absence of additional policy measures. The projected impacts on human capital, and assets are both more negative and much more persistent in the absence of emergency support measures. More importantly, the slightly equality-enhancing tendencies that were documented for the baseline case, reverse in sign, with inequality increasing particularly for human capital and wealth over the short and medium run. This qualitative difference is very suggestive of the success of the government's targeted approach in avoiding a large potential increase in inequality after the pandemic.

As I have illustrated with the analysis of Figures 12 above, many of the negative consequences of the Covid-19 pandemic are highly persistent. Given the slow return to steady state conditions, simply documenting the peaks and troughs of the endogenous variables does not provide a complete picture of the true impact of the crisis. In order to assess the cumulative effects, Table 7 provides a selection of cumulative effects of the Covid-19 shock across different time horizons and policy scenarios. The values presented in Table 7 are expressed as cumulative percentage deviations of the target outcome relative to an annual value of that outcome in the pre-Covid-19 stationary distribution. Specifically, the cumulative effect of the Covid-19 pandemic on outcome z at time horizon T is calculated as:

$$C_z^T = 100\% \sum_{t=0}^{T} \frac{z_t - z_{ss}}{z_{ss}}, \tag{24}$$

In the case of slow-moving variables, such as the aggregate stock of human capital, the impact of the Covid-19 pandemic will show itself over the course of many years and even decades. This means that the impact might be less detectable, as the effect is spread out across many years, but that does not mean the total effect is necessarily negligible. We know from the MIT shock experiments conducted in the previous section, that human capital responds slowly to the shock, but carries a significant degree of inertia. This property is transferred to the case of the Covid-19 shock. In 2020, human capital investment remains roughly constant given the baseline calibration, while after 5 years the reduction is equal to 6% of annual human capital investment pre-Covid-19. This is likely due to the fact that in 2020, consumption restrictions incentivise households to channel resources into skill accumulation rather than consumption, balancing out the shortfall in the short run. Aggregate human capital responds slowly to this fall - reaching a total reduction of 2% after 10 years which then accelerates to -5% after 20 years. In the long-run this cumulative loss of human capital is far from trivial. After 30 years the economy has lost human capital worth 6% of the aggregate capital stock of the pre-Covid-19 economy amounting to over $\frac{1}{5}$ of a percent per annum.[34]

The lesson to be drawn from this analysis is that much of the adjustment of human capital lies in the future. Even if many other indicators of economic activity appear to be converging back to normal, we should be wary of the long-term implications of this slow-moving variable, as any further shocks to the economy can compound the loss of human capital and have lasting effects on economic outcomes. It is therefore important to plan ahead and implement policies that can mitigate the potential long-term effects of the Covid-19 pandemic on human capital, to ensure that the economy can recover and thrive in the years to come.

Trying to reduce the immediate impact of the pandemic was the main aim of the Covid-19 support policies, but as the comparison of the baseline calibration to the no insurance counterfactual suggests, it also helped alleviate some of the negative long-run effects. In the short- and medium-run, human capital investment drops 4% on impact and a cumulative 17% over the first 5 years, in the absence of additional Covid-19 support measures. This additional reduction in investment into human capital leads to a much larger and more persistent drop in the aggregate stock of human capital, falling 2% after 5 years, and 7% after 10 years. A generation after the initial onslaught of the pandemic, the economy would have lost an *additional* 10% of the annual pre-pandemic human capital stock, in the absence of the additional Covid-19 benefit payments.

[34] A baseline calibration of the model without the consumption restriction predicts losses almost exactly twice that order, suggesting that the forced savings in the short run pay off in terms of much smaller losses to aggregate human capital in the long run. See also Figure A8 for a comparison.

Table 6: Cumulative effects of the Covid-19 shock

Baseline

Mean	Impact	5 years	10 years	20 years	30 years	∞	Gini	Impact	5 years	10 years	20 years	30 years	∞
HC Investment	-1	-6	-9	-11	-12	-12	HC Investment	-2	-2	-2	-2	-2	-2
Human Capital	0	-1	-2	-5	-6	-7	Human Capital	0	0	0	-1	-1	-1
Gross Earnings	-23	-37	-40	-43	-44	-45	Gross Earnings	-3	-5	-5	-5	-5	-5
Consumption	-15	-19	-22	-25	-26	-27	Consumption	-45	-46	-47	-47	-47	-46
Assets	0	-7	-14	-20	-22	-24	Assets	0	2	3	4	5	6

No Insurance

Mean	Impact	5 years	10 years	20 years	30 years	∞	Gini	Impact	5 years	10 years	20 years	30 years	∞
HC Investment	-4	-17	-23	-27	-29	-29	HC Investment	1	6	9	11	12	14
Human Capital	0	-2	-7	-13	-16	-19	Human Capital	0	5	9	11	12	15
Gross Earnings	-23	-41	-47	-54	-57	-58	Gross Earnings	-3	-4	-4	-3	-3	-2
Consumption	-16	-25	-32	-39	-41	-43	Consumption	-40	-33	-26	-17	-11	4
Assets	0	-17	-31	-43	-49	-51	Assets	0	11	21	33	40	57

Uniform Insurance

Mean	Impact	5 years	10 years	20 years	30 years	∞	Gini	Impact	5 years	10 years	20 years	30 years	∞
HC Investment	0	-6	-9	-11	-12	-12	HC Investment	-4	-6	-6	-6	-6	-6
Human Capital	0	-1	-2	-5	-6	-7	Human Capital	0	-1	-2	-3	-4	-4
Gross Earnings	-23	-37	-40	-43	-44	-45	Gross Earnings	-3	-5	-5	-5	-5	-5
Consumption	-15	-19	-22	-25	-26	-27	Consumption	-45	-48	-49	-49	-49	-47
Assets	0	-7	-14	-20	-23	-24	Assets	0	2	4	5	6	9

Note: Cumulative differences in key endogenous outcomes due to the Covid-19 pandemic under different policy scenarios.
All numbers expressed as percentage differences relative to 2019 baseline.

For the dynamics of human capital in the UK economy following the Covid-19 pandemic and therefore the potential for future economic growth and development, the preceding analysis provides two key insights: First, while many other economic indicators, such as earnings and consumption begin to recover immediately after the pandemic shock subsides, human capital is much more sluggish, and thus the biggest effects in that regard might still lie ahead of us. The prospect of aggregate human capital staying one percent below its pre-Covid-19 level for around 20 years does not bode well for the future dynamism of the UK economy and its ability to adapt successfully to future challenges. On the other hand, while the outlook is somewhat bleak, decisive policy action has likely delivered us from an even worse fate. The policy counterfactual suggests that in the absence of additional Covid-19 support measures, the fall in aggregate human capital would have been much larger, and crucially coupled with an increase in human capital inequality, exacerbating pre-existing divides in society.

In the next section, I focus in on how households across the distribution of wealth and human capital react to the pandemic, in order to shine a light on some of the aggregate dynamics we have seen in this section, but also to assess where policy has been successful if it has, and what lessons can be learned for the future.

6.3 Impact of Covid-19 across the distribution

There are two main channels through which the Covid-19 pandemic interacts with preexisting inequalities: i) the productivity shock is unequally distributed across the distribution of pre-Covid-19 labour productivities; and ii) households with different initial levels of wealth and human capital have different capacities to deal with a surprise shock, even if it is symmetric in its impact. In this subsection I explore the role of initial conditions in response to the Covid-19 shock, providing some insight on the individual household behaviours that constitute the aggregate response of the economy that we discussed in the last subsection.

Figure 13 presents the impact of the Covid-19 shock on the human capital levels of households with different initial levels of wealth, human capital and productivity in the period preceding the shock. The figure is analogous to Figures 8 & 9 in the preceding section with two modifications to allow a better understanding of the complex interactions between the nonlinear shock, the policy response and wealth and human capital inequality. Firstly each row in the following figures refers to a policy scenario, so a comparison across rows will allow for an assessment of differences due to different policy responses, allowing a counterfactual evaluation of a policy's effectiveness. Secondly, in order to account for the nonlinear impact of the shock across different pre-Covid-19 productivities, each column represents a fixed level of the pre-Covid-19 productivity distribution. Comparison across columns therefore allows an assessment of the impact by pre-Covid-19 productivity. To account for the correlation between human capital and productivity, I fix the initial level of human capital to the mean level conditional on pre-Covid-19 labour productivity and wealth quantiles. Table 8 provides some summary statistics describing relevant features of the households conditional on e_t percentile.

Through the model, the endogenous variables - human capital, earnings and wealth are highly correlated with exogenous human capital productivity. Workers within the 10th percentile of productivity have on average human capital that would put them into the 30th percentile of human capital, compared to the 50rd percentile for the top 10%

Table 7: Summary statistics conditional of productivity percentiles

$pc(e_{2019})$	$\bar{h}_{2019}$	$pc(\bar{h}_{2019})$	$\bar{q}_{2019}$	$pc(\bar{q}_{2019})$	Q1	Q2	Q3	Q4	Q5
10th pctile	0.957	0.3	0.493	0.13	0.049	0.206	0.182	0.18	0.154
50th pctile	1	0.35	0.882	0.48	0.044	0.204	0.203	0.216	0.195
90th pctile	1.043	0.5	1.576	0.85	0.032	0.179	0.215	0.257	0.254

Note: Summary statistics conditional of productivity percentiles pre-Covid-19.
A bar refers to the average value within the productivity percentile (pc = percentile).
Earnings and Human capital normalised relative to population average.
"Qx" refers to the population shares that fall into the respective wealth quantiles.

of productivity. In terms of gross earnings, these differences are even more striking. The productivity percentiles chosen correspond almost perfectly to the 10th, 50th and 90th percentiles of the gross earnings distribution. In terms of wealth, there are also clear disparities between productivity percentiles, with lower productivity percentiles exhibiting a larger share of households in the first two quantiles of the wealth distribution, with the distribution reversing as we consider the highest productivity percentiles.

Focussing on the baseline calibration, moving from left to right the first thing to notice is that the negative impact of the Covid-19 shock on the evolution of human capital is fairly similar for households from the 25th percentile of the pre-Covid-19 wealth distribution upward. Households beginning with assets equal to the 25th percentile of the pre-Covid-19 wealth distribution, reduce their human capital by a maximum of around 0.4% relative to their steady state trajectory by 2030 and then begin a slow recovery. The richer 75th and 95th percentile households lose approximately the same ammount with similar trajectories to the 25th percentile households. The average effect - depicted by the black broken line in each subplot - is slightly progressive in the sense that across higher initial productivity states and levels of human capital, the losses for the average household become incrementally more negative.

Big differences are noticeable when we focus on households close to the borrowing constraint. Across most of the distribution of initial productivity and human capital, households beginning with assets equal to the 5th percentile behave distinctly from their richer counterparts - an expression of the nonlinear behaviour that was encountered in the case of the MIT shock as well.

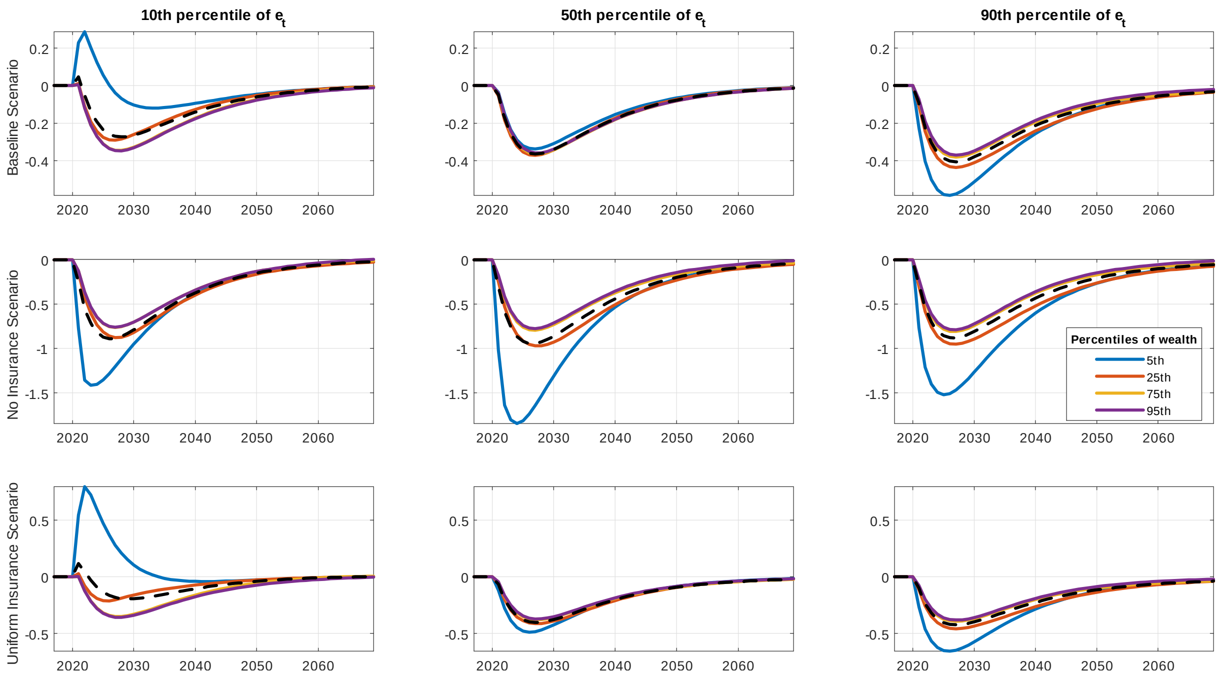
At the lower end of productivity, the poorest households are the only group to increase their human capital slightly relative to their steady state counterfactual. In the medium run, their trajectory seems to be slightly advantageous relative to richer households, but the effect is marginal and likely driven by strong emergency benefits - a view that is confirmed when one compares the trajectory in the absence of additional benefits. For median and higher productivities, these households lose considerably more human capital compared to their pre-Covid-19 trajectory than their richer peers. Their losses amount to about twice what even moderately wealthy households lose, suggesting that the borrowing constraint significantly affects their ability to respond to the pandemic. At the higher end of the productivity distribution particularly, the relative change in human capital becomes strongly negative, exceeding -1.5% indicating that at the higher end of the earnings distribution borrowing-constrained households come off considerably worse than the rest. This is unsurprising, as at this point in the earnings distribution the net income effects of Covid-19 are at their most severe, implying that the self-insurance mechanism via personal and household savings plays a larger role.

In order to interpret these findings it is helpful to remind ourselves that what Figure 13 implies is not that wealth-poorer households accumulate more human capital than asset-rich households. Rather, the figures depict deviations from steady state trajectories (see Figure 4) for which we know that - *ceteris paribus* - richer households accumulate human capital at much faster rates than poorer ones. The broad patterns observed in the subplots of the first row of Figure 13, therefore hint at a slow-down of wealth-based inequality in human capital accumulation, even though there also appears to be a slight pattern of expansion of human capital accumulation at the top end of the productivity distribution. This could explain the pattern that was observed with respect to the aggregate development of human capital inequality in Figure 12.

The second row presentes the evolution of human capital assuming no additional Covid-19 benefits. Immediately it is obvious that as a result the decline in human capital relative to the steady state is considerably larger, on the order of an additional 0.3% to 0.6% across the distribution. This additional loss of human capital is reflected on the aggregate level in the big decline in aggregate human capital that was observed in this case in Figure 12. As expected, the absence of additional benefit payments does affect borrowing-constrained households most severely. Generally speaking, wealth-poorer households experience larger relative declines in human capital than those with more assets, with borrowing-constrained workers experiencing particularly large relative falls. This tendency will on balance increase the correlation between wealth and human capital in the medium run. Furthermore, if we take account of the composition of households in each productivity percentile - e.g. much more low-wealth households in low productivity percentiles, and a higher share of high-wealth households in high productivity percentiles - this also accounts for the large raise in human capital inequality that was predicted in this case. An additional observation in this case, is that the average effect is approximately "U" shaped across the productivity distribution, mirroring the impact of the Covid-19 productivity shock.

Under the uniform insurance policy, there is a strongly progressive redistributive element, as low-income households benefit disproportionately from the uniform payment (see Figure A1 in Appendix). The effect of this policy can be seen in the third row of Figure 13. For households in the lowest productivity percentiles, there is a strong positive effect for households at the 5th percentile of wealth, compared to the baseline policy. These households benefit particularly from the additional resources provided by the Covid-19 benefits. Interestingly richer households do not appear to change their behaviour relative to the baseline calibration. These households are likely already unconstrained as a result of their savings and thus the additional resources from the policy do not significantly alter their human capital accumulation decisions. At the median level of productivity, the uniform policy is actually slightly worse in terms of post taxes and transfers earnings and as a result, wealth-poorer households reduce their human capital slightly more relative to the baseline policy. Finally, at the highest levels of productivity, net labour earnings are fairly similar between both policies, and as a result, the projected human capital trajectories are very similar.

Figure 13: Human capital response to Covid-19, by wealth quantiles



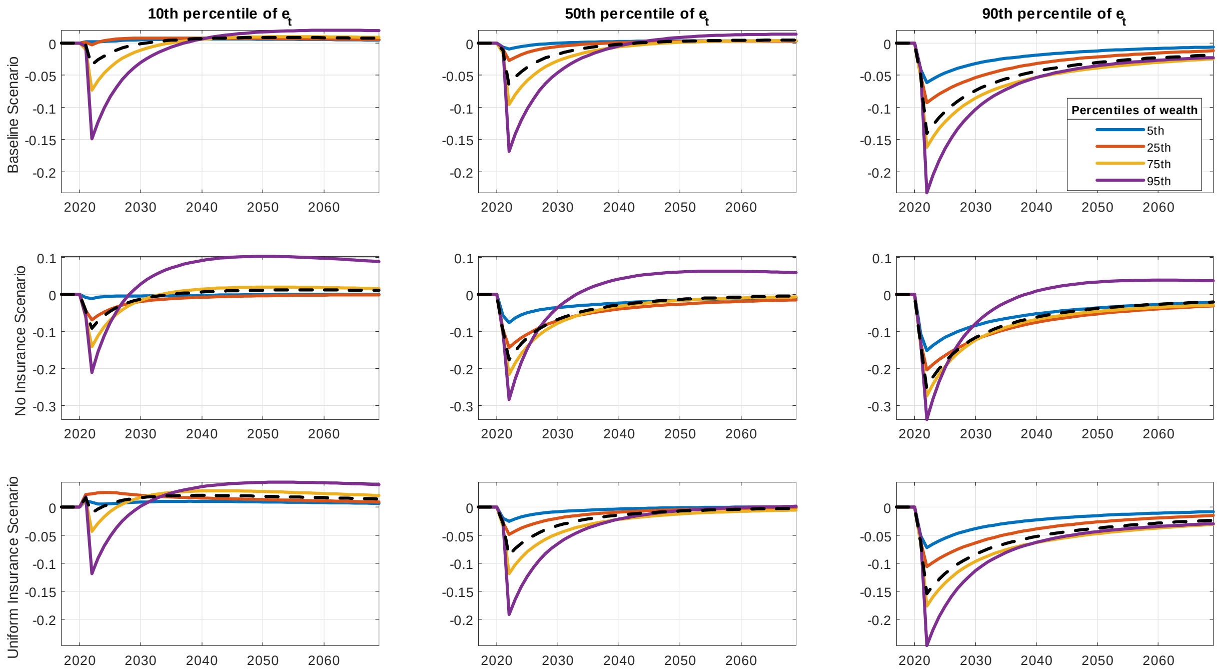
Note: Series normalised to expected progression in stationary equilibrium. Workers begin with human capital equal to the mean level of human capital within their wealth-productivity percentile. See Table 8 for details.

The analysis in this subsection has shown how the aggregate dynamics of human capital post-Covid-19 arise from the differential response of households across the distribution of productivity, human capital and wealth. Under the baseline scenario, I find that the response to Covid-19 is progressive in the sense that on balance households with lower levels of human capital, who begin in low productivity occupations and industries reduce their human capital less than households in higher productivity states who also have higher levels of human capital. This provides a compression of the human capital distribution in the medium run, leading to lower skill inequality. In the long run, however, a strengthening of the correlation between wealth and human capital leads to wealthier households recovering much quicker from the pandemic shock, using their substantial resources to rebuild lost skills, leading to an increase in human capital inequality that proves highly persistent.

Wealth, in this case, provides an important dimension of heterogeneity, as I find that the impact on households human capital accumulation depends strongly on their initial asset holdings, even when conditioning on the same initial productivity levels. Particularly low-wealth households seem to be vulnerable to the Covid-19 shock, particularly when they are not receiving additional benefit payments. Higher-wealth households on the other hand appear less sensitive, likely since they have adequate personal resources to respond to the crisis.

Figure 14 highlights the differential response in asset accumulation across different levels of initial productivity at the onset of the pandemic, by reproducing Figure 13, with a focus on households' wealth positions rather than human capital. Beginning with the baseline case in the top row, the most prominent feature is a disproportionate reduction in the asset positions of households with high wealth. These households are able to draw down their savings to reduce the impact of the economic shock on their human capital investments. This option isn't open to households close to the borrowing constraint, meaning that they have to rely on additional benefit payments from the government.

Figure 14: Wealth response to Covid-19, by wealth quantiles



Note: Series normalised to expected progression in stationary equilibrium. Workers begin with human capital equal to the mean level of human capital within their wealth-productivity percentile. See Table 8 for details.

The changes in the relative asset positions of households are strongly inequality enhancing, as was also initially observed in Figure 12. For the medium and long-run dynamics of the economy this has two important implications: First, increased wealth inequality sows the seeds of increases in human capital and therefore earnings inequality in the future. The analysis of previous sections has shown that wealth-poorer households accumulate human capital at much slower rates than richer households, meaning that an increase in wealth inequality will lead to a slower economic recovery. Second, as low-wealth households fail to accumulate substantial buffer stock savings throughout the pandemic and post-pandemic period, they are not only in a weaker position when it comes to taking advantage of future opportunities, such as high-productivity jobs, but they are also much more vulnerable to future economic shocks.

The importance of self-insurance is particularly highlighted when we consider the counterfactual without additional Covid-19 benefit payments. If households were to solely rely on the automatic stabilizers, households with low wealth perform distinctly worse compared to the baseline case. Whilst households all across the distribution of assets do worse under this scenario, it is particularly borrowing-constrained households who are forced to reduce their desired human capital plans in response to the shock, ultimately leading to higher human capital inequality and also a stronger association of wealth and human capital. This suggests that while the Covid-19 benefits were designed to benefit low-income households, they particularly benefitted low-wealth households. This by itself raises some interesting questions about the design of future policies in response to aggregate shocks, and also the dismal topic of paying for the associated costs. I will discuss these questions and some other points in the next subsection.

6.4 Welfare effects and policy implications

The Covid-19 pandemic was much more than simply an economic shock. The loss of countless lives, the disruption of normal social relations for the majority of two years, and a general sense of uncertainty and anxiety have imposed psychological costs far in excess of any measurable loss of income, employment or consumption. Yet it is an economist's job to try his best and quantify the impact of the pandemic so that society might better understand what was lost, and how to decide what to do in the future.

In this section I will use the model predicted transitions post-Covid-19, to calculate the distribution of welfare losses over the course of the pandemic and its aftermath. For this purpose, I obtain the value functions at the beginning of the pandemic period. Since the value function encodes the solution to the infinite horizon problem of a forward-looking household, the value function at the beginning of the pandemic period summarizes not only the immediate welfare impact of Covid-19 but also the experience of the household along the transition path.

The value function is known at every point in the state space, so I can obtain the distribution of welfare across the dimensions of wealth, human capital and exogenous productivity. To make the welfare measure more interpretable, I convert the welfare differences into percentage differences of *lifetime equivalent consumption* in the stationary equilibrium, using the homogeneity of the utility function:

$$\lambda(a_t, h_t, e_t) = \left[\frac{V_0(a_t, h_t, e_t)}{V_{ss}(a_t, h_t, e_t)}\right]^{\frac{1}{1-\sigma}} - 1, \tag{25}$$

where $\lambda(a_t, h_t, e_t)$ is a number equivalent to the percentage of additional consumption that an agent in state (a_t, h_t, e_t) would require in every state to be indifferent between going through the Covid-19 crisis and post-pandemic period and staying in the stationary world without Covid-19 forever.[35] V_0 and V_{ss} refer to the value functions at the beginning of the pandemic and in the stationary equilibrium respectively.

Figure 15 shows the welfare losses due to the Covid-19 pandemic, both on aggregate and across the distributions of pre-pandemic earnings and wealth holdings. Overall the pandemic had a significant and negative effect on the welfare of households. Under the baseline scenario, the average household would have been happy to trade in a permanently lower level of consumption by 0.9% to avoid the pandemic. Under a uniform benefits policy, the number is marginally smaller, implying that on average a uniform policy would have been preferable. Indisputable however is that additional policy intervention is preferable to simply relying on automatic stabilizers. The no-insurance scenario is associated with an average welfare loss of over 1.45% of lifetime equivalent consumption - over one and a half times that of the interventionist scenarios.

Societies do not consist of an average household, or a benevolent social planner who makes decisions on behalf of such an entity. Instead, the distribution of gains and losses

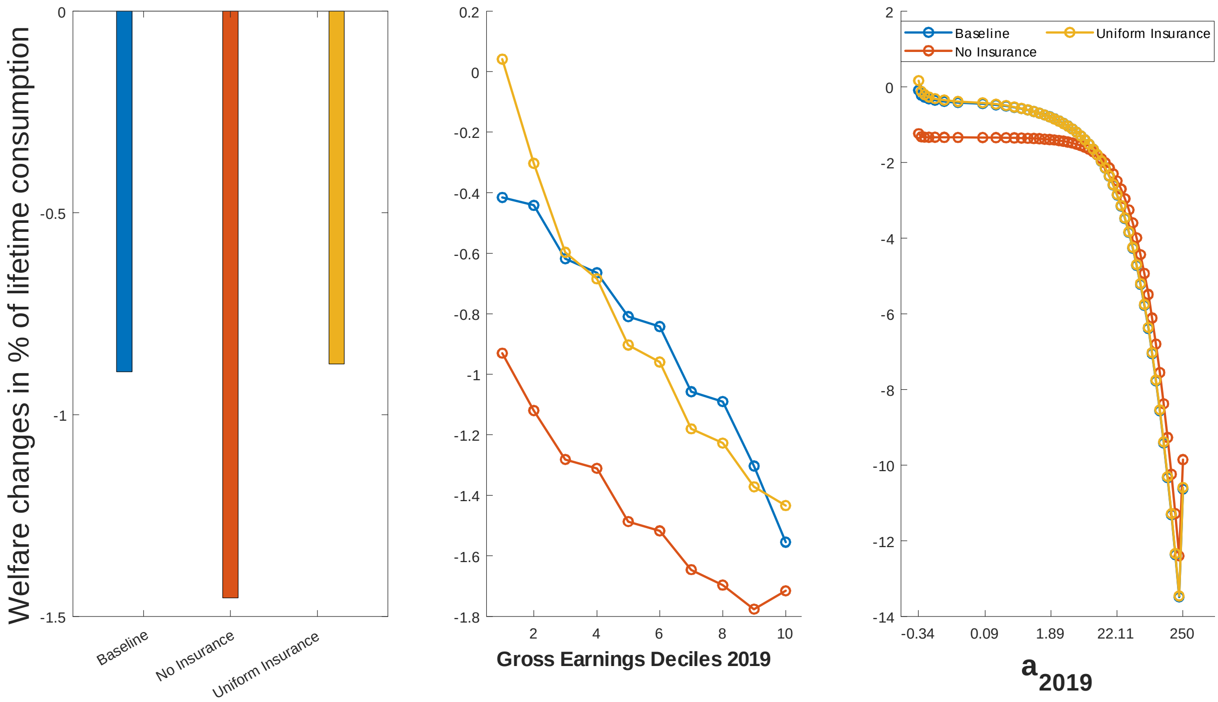
[35] It is no surprise that households generally do not like going through the Covid-19 pandemic, so they require negative additional consumption to remain indifferent.

from any crisis and from any policy will play a crucial part in how society decides to evaluate past actions and proposes to meet new challenges. The middle plot in Figure 15 shows the distribution of welfare losses along the pre-Covid-19 gross earnings distribution. As I have discussed throughout this section, both the impact of Covid-19 itself as well as the policy response were highly nonlinear along the dimensions of pre-pandemic incomes. Under the baseline policy welfare losses form an almost perfect downward sloping line, with an incline of -0.18% per additional decile. This suggests that the highest losses from the pandemic are concentrated amongst the highest pre-Covid-19 earners - who presumably are also best placed to bear the brunt of these losses. The uniform insurance policy is similar to the baseline policy, although it is a lot steeper at low initial income levels, which is unsurprising given the regressive nature of the transfer. Both policies strictly dominate the no-insurance option.

The last subplot plots the distribution of welfare losses by initial wealth. The baseline policy sees a steep fall in welfare as initial assets increase. Throughout this section, we saw that under the baseline calibration wealthier households appeared to be more impacted by the shock, while particularly borrowing-constrained individuals benefitted - at least relatively - from the policy response. Very wealthy households are also most likely to be affected by the consumption restrictions, leading to a large direct fall in their utility. Figure A9 in the Appendix shows the welfare effects of a counterfactual without the consumption cap. Comparing the two figures suggests that the consumption cap mainly affects the welfare of very rich and/or wealthy households, by forcing them to consume less than they would have liked. Under the uniform insurance policy, the distribution of welfare losses is similar, with the exception of very poor or very rich households who experience slightly lower welfare losses under the uniform policy.

Under the absence of additional benefit payments, welfare losses are exacerbated across the board, as expected but there are some interesting insights to be gleaned: Namely that there is a small proportion of households who actually preferred no intervention to any particular policy. These households are located at the very top of the wealth distribution, suggesting that these are super high net worth households who benefit from future higher interest rates brought about by a scarcity of capital.

Figure 15: Welfare effects of the Covid-19 pandemic



Note: Welfare effects expressed in percentages of lifetime consumption equivalents in the stationary distribution. Assets plotted on a semi-log scale.

Apart from this small plutocracy, the majority of households prefer some form of government intervention in response to the pandemic. The uniform insurance has a slight edge in terms of average welfare and is preferred in general by very poor households and also households at the very top of the pre-Covid-19 earnings distribution. In order to assess which policy choice is consistent with majority decision making I calculate the share of households that prefer each policy option. A small majority of 53% of households prefer the baseline policy, while 44% prefer the uniform insurance option. As suggested before, only 3% of households prefer no additional insurance at all.

In order to provide the additional Covid-19 relief payments the UK government at the time borrowed over £300 billion, adding substantially to a large pre-existing public debt. While an assessment of the implications of different strategies to pay back the incurred debt is beyond the scope of this paper, it is evident that short of sovereign default, paying back the debt will require increased taxation at some point in the future and these costs need to be weighed against the benefits of intervention. Whilst this paper cannot speak to the optimal tax schedule it may provide some guiding thoughts on what might and what might not be advisable to consider.

In light of rising wealth inequality, governments might be tempted to introduce wealth taxation in order to pay down the national debt. Whilst such schemes can raise large revenues (e.g. Saez & Zucman (2019)) and may even increase efficiency (Guvenen et al. (2019)) it is important to ensure that these schemes are sufficiently progressive as to not disincentivise saving by households with low or moderate wealth. Throughout this paper, analysis has shown that wealth-poor households face additional challenges with respect to the accumulation of human capital, and are also much more vulnerable to unexpected economic shocks. Wealth-poor households have benefitted from the existing Covid-19 support structures precisely because they would have been ill-placed to face the impact of Covid-19 without them. This suggests that policies that support the wealth poorest households to build up some even moderate savings, would have positive consequences for skill accumulation, intergenerational mobility and resilience to economic shocks.

7 Conclusion

In this paper, I have explored the interaction between wealth and human capital in a novel heterogeneous agent incomplete market model. The model features endogenous human capital accumulation as well as savings, capturing two important dimensions of household heterogeneity. I calibrate the model to the UK economy pre-Covid-19 using microdata.

I find that there are important non-linearities in human capital investments, with workers with low levels of wealth investing considerably less in accumulating human capital than their counterparts with more wealth. This suggests the existence of low-wealth poverty traps, where individuals with low wealth struggle to increase their skillset and therefore lag behind comparable individuals with higher initial levels of wealth. Even small initial differences in household wealth can lead to sizeable and long-lasting gaps in human capital accumulation and therefore earnings inequality.

Low-wealth households are also particularly vulnerable to unexpected economic shocks, unable to draw on savings, these households reduce their investment into skills leading to persistent scarring effects in terms of human capital and incomes. The nonlinearity of the households' response to aggregate shocks means that the distribution of wealth plays an important role in determining how the economy recovers after a recession.

The model was used to analyse the economic dynamics of the distribution of human capital in the aftermath of the Covid-19 pandemic in the UK. Using a baseline calibration, the model predicts that aggregate human capital will fall significantly over the course of the next decades, leading to reduced growth prospects for the foreseeable future. Using counterfactual policy scenarios, I find that overall the Covid-19 support measures put in place throughout 2020/21, were effective in preventing a much more severe economic fallout from the pandemic and were particularly effective in protecting low-skilled and low-wealth households and workers.

The analysis thus highlights the need for decisive and targeted policy action to ensure that the UK economy can recover from the Covid-19 pandemic with a minimum of lasting damage. Measures such as targeted job support, targeted training and skills development programs and policies to promote flexible working should all be considered as potential ingredients in the UK's recipe for economic recovery. These should be complemented by measures to ensure that the gains from economic growth are widely shared and that human capital inequality does not increase further. By taking these steps, the UK can ensure that its economy is not only able to recover from the effects of the pandemic but is also better positioned to cope with future shocks and to continue to grow and develop in a sustainable way.

The model developed in the paper opens up several interesting avenues for future research: For example, it would be interesting to explore the effects of policies that aim to reduce wealth inequality and promote human capital investments, such as education policies, or Universal Basic Income. Further, much of the literature on heterogenous agent models with aggregate shocks operates under the assumption that the distribution of wealth does not matter significantly for the economy's response to aggregate shocks (see Krusell & Smith (1998)). The findings of this paper suggest that perhaps including human capital accumulation might be a fruitful avenue for making wealth inequality matter for aggregate dynamics. I leave these and other questions for future research.

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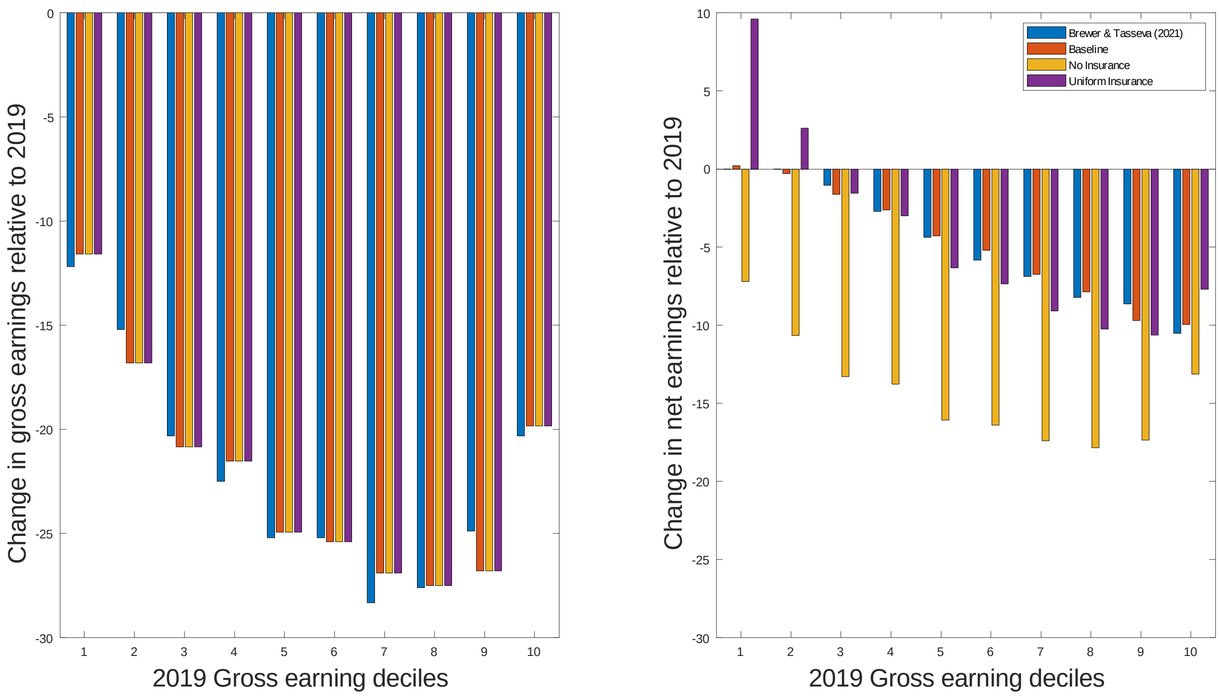
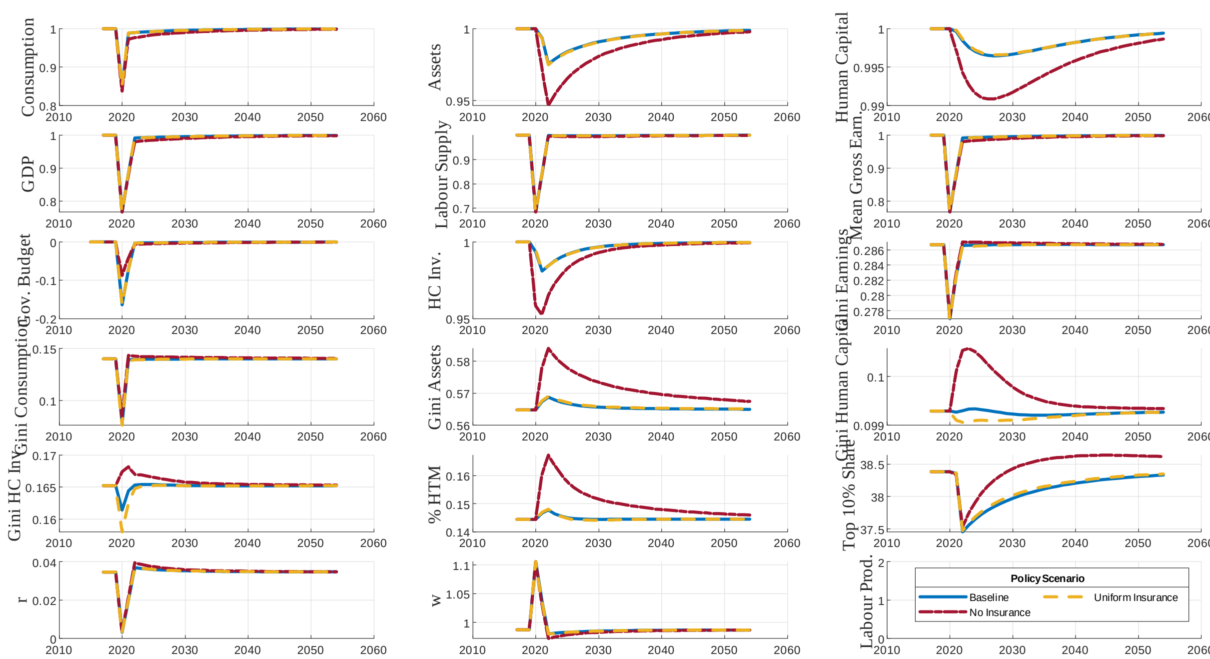
8 Appendix A

Figure A1: Impact of alternative policy interventions on gross and net earnings



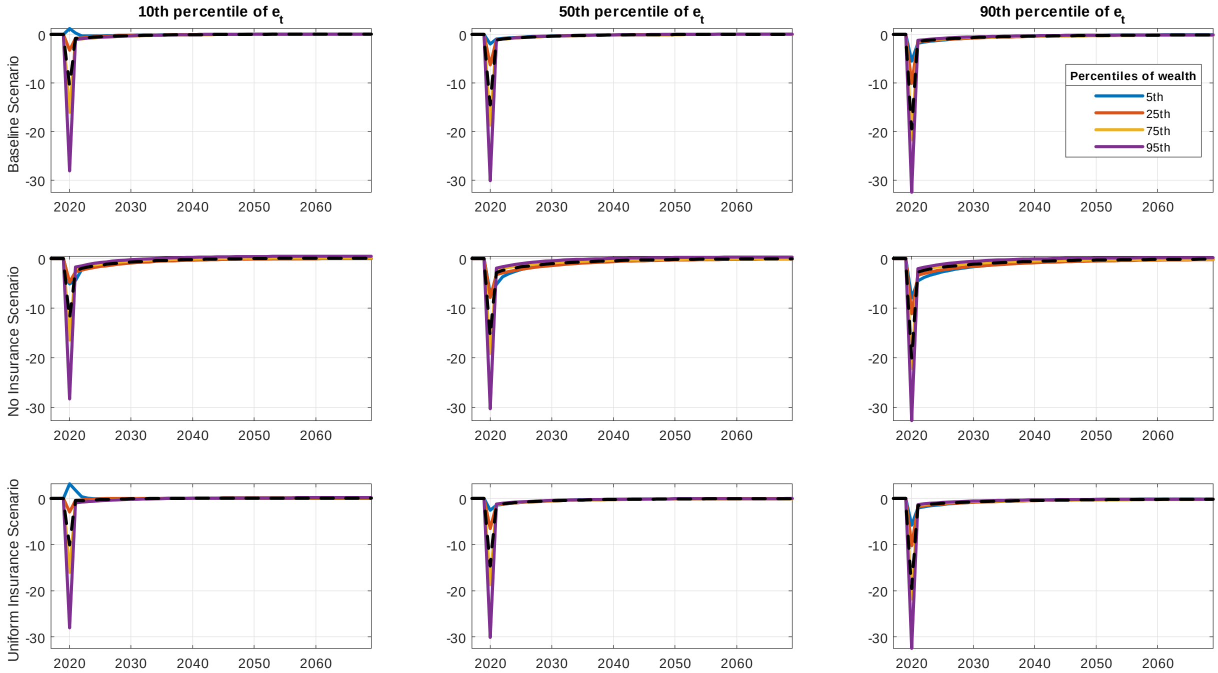
Note: Data points transcribed from Brewer & Tasseva (2021), Figure 1.

Figure A2: Response of aggregate variables to Covid-19



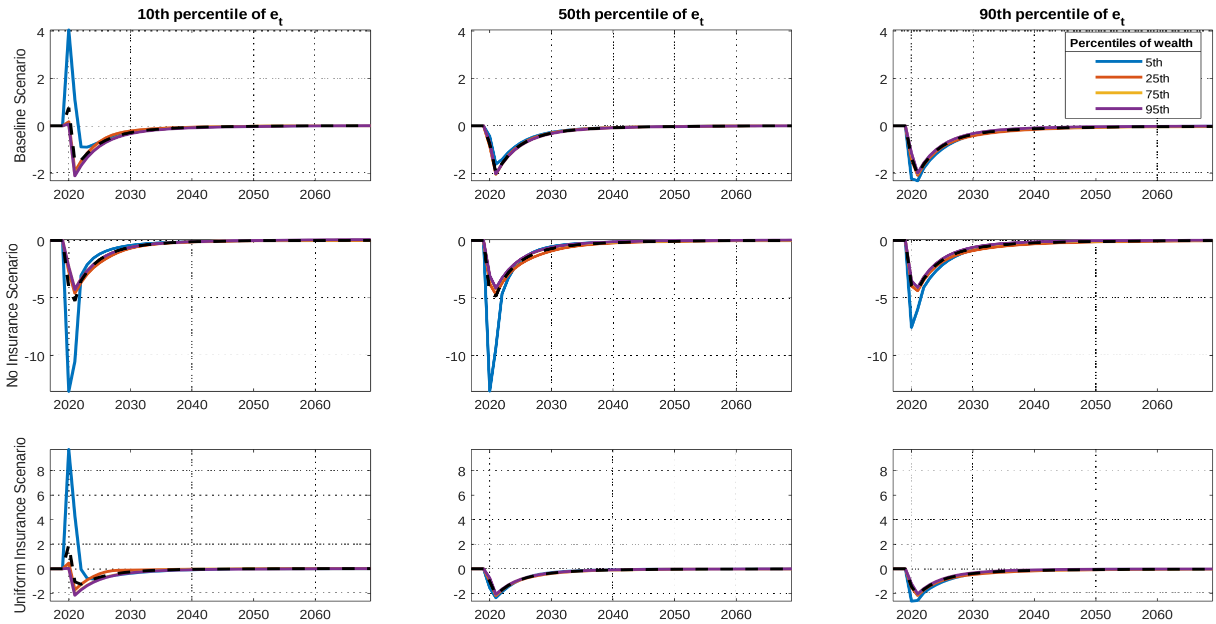
Note: Baseline, no insurance and uniform insurance Covid-19 shock calibration.

Figure A3: Consumption response to Covid-19, by wealth quantiles



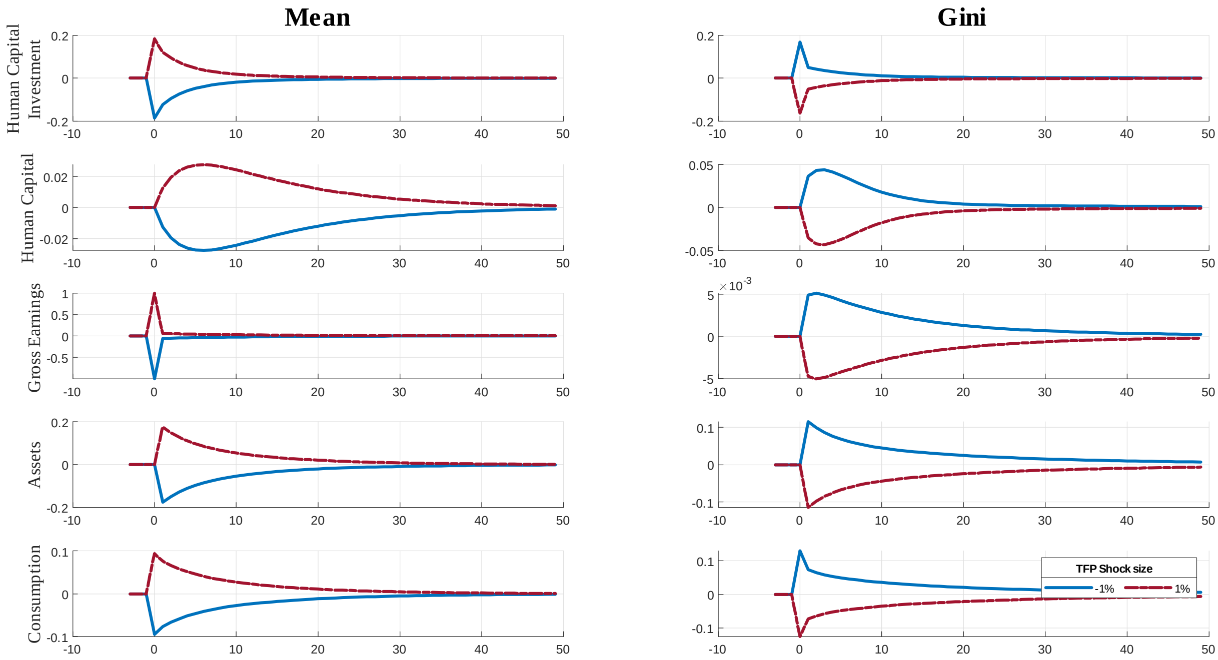
Note: Series normalised to expected progression in stationary equilibrium. Workers begin with human capital equal to the mean level of human capital within their wealth-productivity percentile. See Table 8 for details.

Figure A4: Human capital investment response to Covid-19, by wealth quantiles



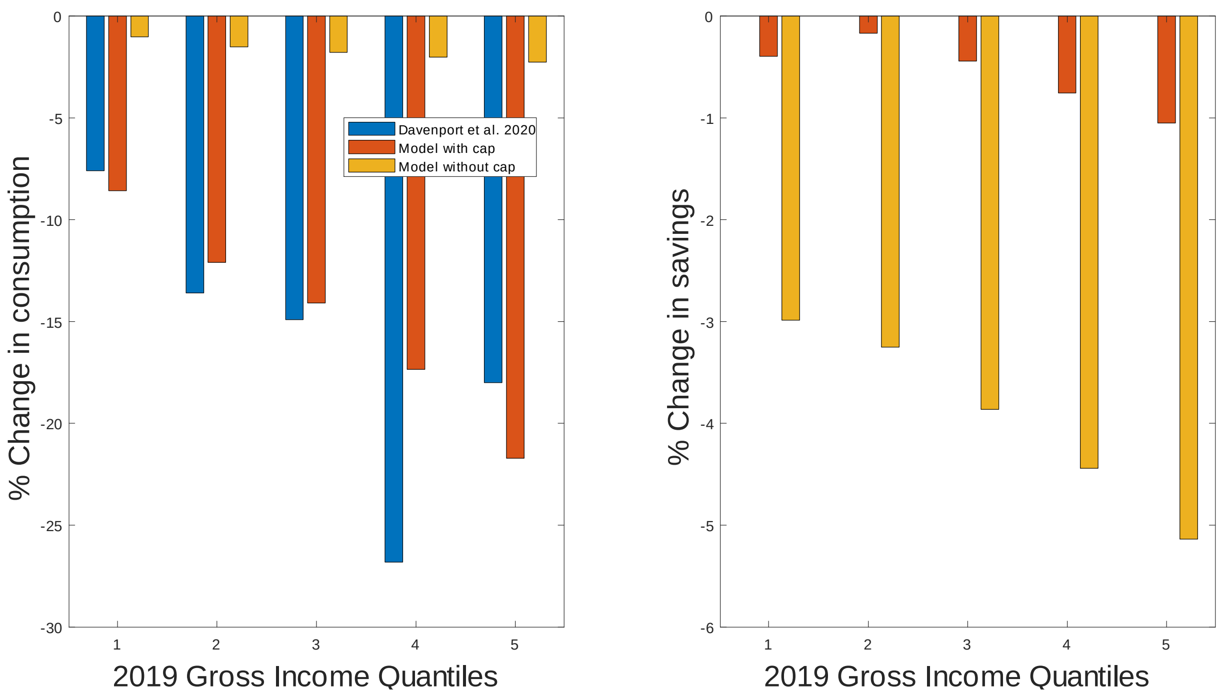
Note: Series normalised to expected progression in stationary equilibrium. Workers begin with human capital equal to the mean level of human capital within their wealth-productivity percentile. See Table 8 for details.

Figure A5: Comparison of positive and negative TFP shocks



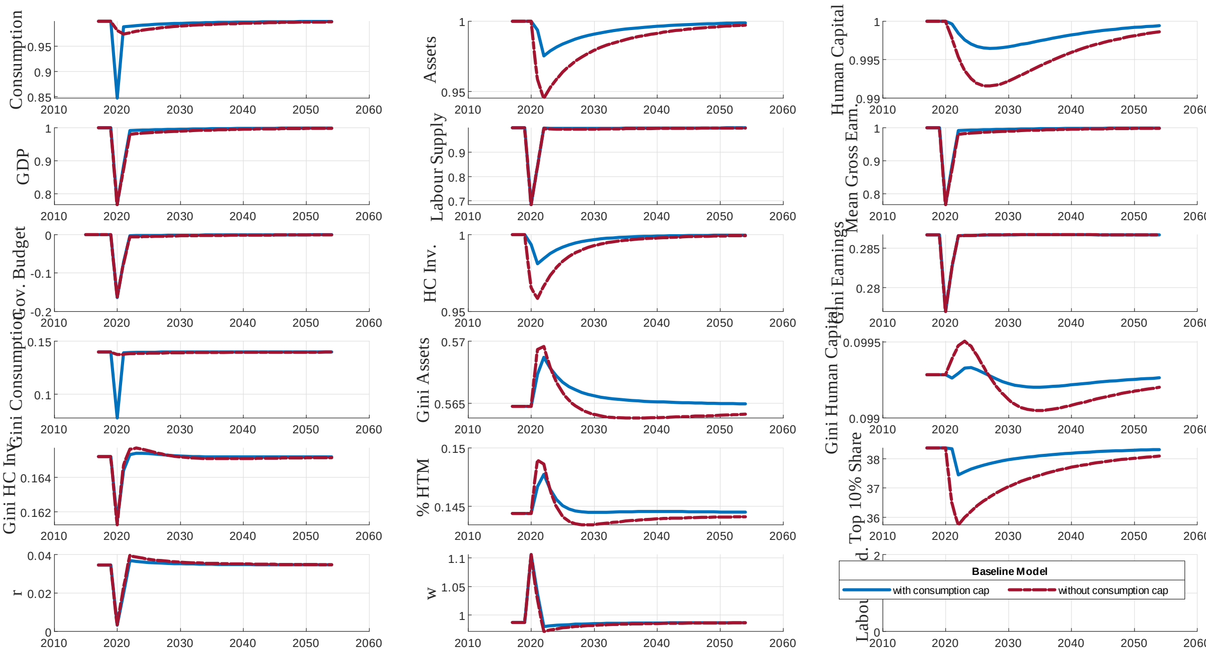
Note: Response of aggregate variables to different TFP shocks.

Figure A6: Impact of consumption cap on household expenditure and savings



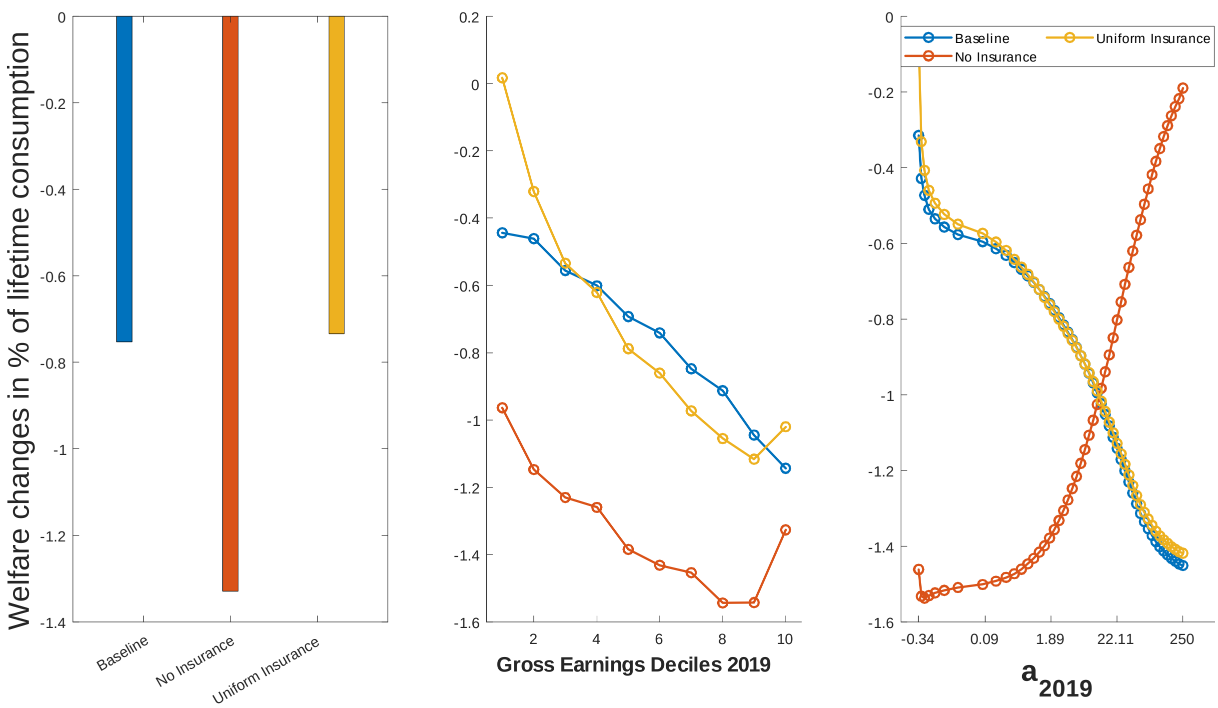
Note: Data points transcribed from Davenport et al. (2020), Figure 4.1.
Baseline model with and without consumption restrictions. Changes relative to averages in pre-Covid-19 period.

Figure A7: Baseline model with and without consumption restrictions



Note: Baseline Covid-19 shock calibration. Series normalised to expected progression in stationary equilibrium.

Figure A8: Welfare effects of the Covid-19 pandemic, no consumption cap



Note: Welfare effects expressed in percentages of lifetime consumption equivalents in the stationary distribution. Assets plotted on semi-log scale.

9 Appendix B

9.1 EGM with 2 state variables

This algorithm is an extended version of Caroll's EGM algorithm (Caroll (2006)). It's based on the extension of the method presented in Auclert et al. (2021), but I have adapted it for the case of human capital.

9.1.1 The problem of the household

The households problem is given by:

$$
\begin{gathered}
V\left(a_t, h_t, e_t\right)=\max_{\left\{c_t, a_{t+1}, x_t\right\}}\left\{\frac{\left(c_t\right)^{1-\sigma}}{1-\sigma}+\beta E\left[V\left(a_{t+1}, h_{t+1}, e_{t+1}\right) \mid e_t\right]\right\} \\
s.t. \\
c_t+a_{t+1}+x_t=(1+r_t) a_t+g(h_t, e_t, w_t, \tau_1, \tau_2) \\
h_{t+1}=h_t \delta_t+\chi x_t^\nu \\
g(h_t, e_t, w_t, \tau_1, \tau_2)=\tau_1(w_t e_t \log(1+h_t))^{1-\tau_2} \\
c_t \geq 0, h_t \geq 0,\ a_t \geq 0, x_t \geq 0 \\
\text{endogenous states }: h_t, a_t \\
\text{exogenous states }: \delta_t, e_t \\
\text{choice variables }: c_t, a_{t+1}, h_{t+1}, x_t
\end{gathered}
$$

First, rewrite the human capital investment function in implicit form:

$$
x_t=m(h_{t+1}, h_t)=\left[\frac{(h_{t+1}-\delta_t h_t)}{\chi}\right]^{\frac{1}{v}}.
$$

To impose the non-negativity constraint on $x_t \geq 0$, further impose:

$$
x_t=m(h_{t+1}, h_t)=\left[\frac{abs\left(h_{t+1}-\delta_t h_t\right)}{\chi}\right]^{\frac{1}{v}}.
$$

Then substitute out consumption and human capital investment to rewrite the bellman equation:

$$
V\left(a_t, h_t, e_t\right) \max_{\left\{h_{t+1}, a_{t+1}\right\}}\left\{\begin{array}{c}
u(g(h_t, e_t, w_t, \tau_1, \tau_2)+(1+r) a_t-a_{t+1}-m(h_{t+1}, h_t)) \\
+\lambda_t(a_{t+1}-\underline{a})+\mu_t(h_{t+1}-\delta_t h_t)+\beta E\left[V\left(a_{t+1}, h_{t+1}, e_{t+1}\right) \mid e_t\right]
\end{array}\right\}
$$

The first order conditions are:

$$
u^{\prime}(c_t)=\lambda_t+\beta E\left[\partial_{a_{t+1}} V\left(a_{t+1}, h_{t+1}, e_{t+1}\right) \mid e_t\right]
$$

and

$$
u^{\prime}(c_t) m_1(h_{t+1}, h_t)=\mu_t+\beta E\left[\partial_{h_{t+1}} V\left(a_{t+1}, h_{t+1}, e_{t+1}\right) \mid e_t\right]
$$

and the envelope conditions are:

$$\partial_{a_t} V\left(a_t, h_t, e_t\right) = (1+r)u'(c_t)$$

and

$$\partial_{h_t} V\left(a_t, h_t, e_t\right) = \left(g_1(h_t, e_t, w_t, \tau_1, \tau_2) - m_2(h_{t+1}, h_t)\right) u'(c_t)$$

9.1.2 Algorithm

The algorithm follows Auclert et al. (2021).

1. Initial guess: Make two guesses for the derivatives of the value function $V_a(a', h', e')$ and $V_h(a', h', e')$.

2. Common $e' \longrightarrow e$. Calculate the discounted expectation using the exogenous transition matrix Π:

$$W_a(a', h', e) = \beta \Pi V_a(a', h', e')$$
$$W_h(a', h', e) = \beta \Pi V_h(a', h', e')$$

3. Unconstrained $h' \longrightarrow h$. Assume neither constraint is binding, then $\lambda_t = \mu_t = 0$, and the FOCs are

$$c^{-\sigma} = W_a(a', h', e)$$
$$c^{-\sigma} m_1\left(h', h\right) = W_h(a', h', e)$$

   Divide each side to get:

$$\frac{1}{m_1\left(h', h\right)} = \frac{W_a(a', h', e)}{W_h(a', h', e)}.$$

   Define $F(h, h', a', e)$ by setting the previous expression equal to 0:

$$F(h, h', a', e) = \frac{W_a(a', h', e)}{W_h(a', h', e)} - \frac{1}{m_1\left(h', h, e\right)} = 0.$$

   Note that $m_1\left(h', h\right) > 0$ **iff** $\mu_t = 0$. $F(h, h', a', e)$ is a mapping that characterizes $h'(a', h, e)$. Use this mapping to map $W_a(a', h', e)$ into $W_a(a', h, e)$ using linear interpolation. Compute consumption as

$$c(a', h, e) = W_a(a', h, e)^{-\frac{1}{\sigma}}.$$

4. Unconstrained $a' \longrightarrow a$. Now use the budget constraint and $h'(a', h, e)$ and $c(a', h, e)$ from the previous step to obtain:

$$a(a', h, e) = \frac{a' + c(a', h, e) + m\left(h'(a', h, e), h\right) - g(h, e, w)}{(1+r)}.$$

   Invert this function via interpolation to get $a'(a, h, e)$ and the same interpolation weights can be used to do $h'(a', h, e) \longrightarrow h'(a, h, e)$.

5. Liquidity constrained $h' \longrightarrow h$. This branch proceeds analogously to the unconstrained case. Assuming that the liquidity constraint is binding, $\lambda_t > 0$, but not the minimum human capital investment constraint, $\mu_t = 0$. The FOCs become:

$$c^{-\sigma} = \lambda + W_a(\underline{a}, h', e)$$
$$c^{-\sigma} m_1 (h', h) = W_h(\underline{a}, h', e).$$

For scaling purposes, define $\kappa \equiv \lambda / W_a(\underline{a}, h', e)$ and rewrite the first FOC as:

$$c^{-\sigma} = (1 + \kappa) W_a(\underline{a}, h', e)$$

Divide side by side to get:

$$\frac{1}{m_1 (h', h, e)} = \frac{(1 + \kappa) W_a(\underline{a}, h', e)}{W_h(\underline{a}, h', e)}$$

$$F(h, h', \kappa, e) = \frac{(1 + \kappa) W_a(\underline{a}, h', e)}{W_h(\underline{a}, h', e)} - \frac{1}{m_1 (h', h, e)} = 0.$$

Solve this for $h'(\kappa, h, e)$ and compute consumption as:

$$c(\kappa, h, e) = [(1 + \kappa) W_a(a', h, e)]^{-\frac{1}{\sigma}}.$$

6. Liquidity constrained $\kappa \longrightarrow a$. Now using $h'(\kappa, h, e)$ and $c(\kappa, h, e)$ from the previous step, use the budget constraint to obtain

$$a(\kappa, h, e) = \frac{\underline{a} + c(\kappa, h, e) + m (h'(\kappa, h, e), h) - g(h, e, w)}{(1 + r)}$$

Invert this function via interpolation to get $\kappa(a, h, e)$. The same interpolation weights can be used to map $h'(\kappa, h, e)$ into $h'(a, h, e)$. We already know that $a'(a, h, e) = \underline{a}$.

7. Binding human capital investment constraint. Suppose that both constraints bind $\lambda_t \neq 0, \mu_t \neq 0$. Then the FOCs are:

$$c^{-\sigma} = \lambda + W_a(\underline{a}, h', e)$$
$$c^{-\sigma} m_1 (h', h) = \mu + W_h(\underline{a}, h', e).$$

We already know that $m_1 (h', h) > 0$ **iff** $\mu_t = 0$. We also know that $m_1 (h', h) \geq 0$.It follows that if $\mu_t > 0$ then $m_1 (h', h) = 0$. So from the second FOC:

$$\mu = -W_h(\underline{a}, h', e).$$

This means that the gradient of the value function with respect to human capital is negative, which is inconsistent with the economic logic of the problem. Hence, the constraint of human capital investment never binds, except for the trivial case when the household has zero resources. Intuitively, convex human capital investment function implies that it is always profitable for the agent to invest an incremental amount into human capital.

8. Update guesses. The final $a'(a,h,e)$ is the element-wise maximum of its unconstrained and liquidity-constrained counterparts. Replace the unconstrained $h'(a,h,e)$ with constrained one at the exact same points. Compute consumption from the budget constraint as

$$c(a,h,e) = (1+r)a + g(h,e,w) - m\left(h'(a,h,e),h\right) - a'(a,h,e)$$

Finally use the envelope conditions and to update the guesses

$$V_a(a,h,e) = (1+r)c(a,h,e)^{-\sigma}$$

$$V_h(a,h,e) = \left[g_1(h,e,w) - m_2\left(h'(a,h,e),h\right)\right]c(a,h,e)^{-\sigma}$$

Go back to step 2, repeat until convergence.

9.2 Algorithm with consumption cap

Assume that occasionally - due to external events - agents are restricted in their consumption behaviour. This is implemented by invoking a consumption cap, and a penalty function that penalizes households for consuming above the capped level. The penalty function is given by:

$$\frac{\xi}{3}\{\min(\bar{c}-c_t),0\}^3$$

where ξ is a parameter that determines the strength of the penalty. Households can still consume beyond the threshold, but they do incur additional utility costs when they do. In times when the consumption cap is not active $\bar{c}$ is just set to $+\infty$.

We skip straight to the bellman, where we insert the penalty function:

$$V\left(a_t,h_t,e_t\right) \max_{\{h_{t+1},a_{t+1}\}} \left\{ \begin{array}{c} u(c_t) + \frac{\xi}{3}\{\min(\bar{c}-c_t),0\}^3 \\ +\lambda_t(a_{t+1}-\underline{a}) + \mu_t(h_{t+1}-\delta_t h_t) + \beta E\left[V\left(a_{t+1},h_{t+1},e_{t+1}\right) \mid e_t\right] \end{array} \right\}$$

The first order conditions are now:

$$(u'(c_t) - \min\{\bar{c}-c,0\}^2) = \lambda_t + \beta E\left[\partial_{a_{t+1}} V\left(a_{t+1},h_{t+1},e_{t+1}\right) \mid e_t\right]$$

and

$$(u'(c_t) - \min\{\bar{c}-c,0\}^2) m_1\left(h',h\right) = \mu_t + \beta E\left[\partial_{h_{t+1}} V\left(a_{t+1},h_{t+1},e_{t+1}\right) \mid e_t\right]$$

and the envelope conditions are:

$$\partial_{a_t} V\left(a_t,h_t,e_t\right) = (1+r)(u'(c_t) - \min\{\bar{c}-c,0\}^2)$$

and

$$\partial_{h_t} V\left(a_t,h_t,e_t\right) = \left(g_1(h_t,e_t,w_t,\tau_1,\tau_2) - m_2(h_{t+1},h_t)\right)\left(u'(c_t) - \min\{\bar{c}-c,0\}^2\right)$$

Algorithm with consumption cap

The algorithm follows Auclert et al. (2021).

1. Initial guess: Make two guesses for the derivatives of the value function $V_a(a', h', e')$ and $V_h(a', h', e')$.

2. Common $e' \longrightarrow e$. Calculate the discounted expectation using the exogenous transition matrix Π:

$$W_a(a', h', e) = \beta\Pi V_a(a', h', e')$$
$$W_h(a', h', e) = \beta\Pi V_h(a', h', e')$$

3. Unconstrained $h' \longrightarrow h$. Assume neither constraint is binding, then $\lambda_t = \mu_t = 0$, and the FOCs are

$$(c^{-\sigma} - \min\{\bar{c} - c, 0\}^2) = W_a(a', h', e)$$
$$(c^{-\sigma} - \min\{\bar{c} - c, 0\}^2) m_1(h', h) = W_h(a', h', e)$$

Divide each side to get:

$$\frac{1}{m_1(h', h)} = \frac{W_a(a', h', e)}{W_h(a', h', e)}.$$

Define $F(h, h', a', e)$ by setting the previous expression equal to 0:

$$F(h, h', a', e) = \frac{W_a(a', h', e)}{W_h(a', h', e)} - \frac{1}{m_1(h', h, e)} = 0.$$

Note that $m_1(h', h) > 0$ **iff** $\mu_t = 0$. $F(h, h', a', e)$ is a mapping that characterizes $h'(a', h, e)$. Use this mapping to map $W_a(a', h', e)$ into $W_a(a', h, e)$ using linear interpolation. Compute consumption by solving the nonlinear equation:

$$c(a', h, e) = (W_a(a', h, e) + \min\{\bar{c} - c, 0\}^2)^{-\frac{1}{\sigma}}.$$

4. Unconstrained $a' \longrightarrow a$. Now use the budget constraint and $h'(a', h, e)$ and $c(a', h, e)$ from the previous step to obtain:

$$a(a', h, e) = \frac{a' + c(a', h, e) + m(h'(a', h, e), h) - g(h, e, w)}{(1 + r)}.$$

Invert this function via interpolation to get $a'(a, h, e)$ and the same interpolation weights can be used to do $h'(a', h, e) \longrightarrow h'(a, h, e)$.

5. Liquidity constrained $h' \longrightarrow h$. This branch proceeds analogously to the unconstrained case. Assuming that the liquidity constraint is binding, $\lambda_t > 0$, but not the minimum human capital investment constraint, $\mu_t = 0$. The FOCs become:

$$(c^{-\sigma} - \min\{\bar{c} - c, 0\}^2) = \lambda + W_a(\underline{a}, h', e)$$
$$(c^{-\sigma} - \min\{\bar{c} - c, 0\}^2) m_1(h', h) = W_h(\underline{a}, h', e).$$

For scaling purposes, define $\kappa \equiv \lambda / W_a(\underline{a}, h', e)$ and rewrite the first FOC as:

$$(c^{-\sigma} - \min\{\bar{c} - c, 0\}^2) = (1 + \kappa) W_a(\underline{a}, h', e)$$

Divide side by side to get:

$$\frac{1}{m_1\left(h', h, e\right)} = \frac{(1+\kappa) W_a(\underline{a}, h', e)}{W_h(\underline{a}, h', e)}$$

$$F(h, h', \kappa, e) = \frac{(1+\kappa) W_a(\underline{a}, h', e)}{W_h(\underline{a}, h', e)} - \frac{1}{m_1\left(h', h, e\right)} = 0.$$

Solve this for $h'(\kappa, h, e)$ and compute consumption by solving:

$$c(\kappa, h, e) = \left[(1+\kappa) W_a(a', h, e) + \min\{\bar{c} - c, 0\}^2\right]^{-\frac{1}{\sigma}}.$$

6. Liquidity constrained $\kappa \longrightarrow a$. Now using $h'(\kappa, h, e)$ and $c(\kappa, h, e)$ from the previous step, use the budget constraint to obtain

$$a(\kappa, h, e) = \frac{\underline{a} + c(\kappa, h, e) + m\left(h'(\kappa, h, e), h\right) - g(h, e, w)}{(1+r)}$$

Invert this function via interpolation to get $\kappa(a, h, e)$. The same interpolation weights can be used to map $h'(\kappa, h, e)$ into $h'(a, h, e)$. We already know that $a'(a, h, e) = \underline{a}$.

7. Binding human capital investment constraint. Suppose that both constraints bind $\lambda_t \neq 0, \mu_t \neq 0$. Then the FOCs are:

$$\begin{aligned} c^{-\sigma} &= \lambda + W_a(\underline{a}, h', e) \\ c^{-\sigma} m_1\left(h', h\right) &= \mu + W_h(\underline{a}, h', e). \end{aligned}$$

We already know that $m_1\left(h', h\right) > 0$ **iff** $\mu_t = 0$. We also know that $m_1\left(h', h\right) \geq 0$.It follows that if $\mu_t > 0$ then $m_1\left(h', h\right) = 0$. So from the second FOC:

$$\mu = -W_h(\underline{a}, h', e).$$

This means that the gradient of the value function with respect to human capital is negative, which is inconsistent with the economic logic of the problem. Hence, the constraint of human capital investment never binds, except for the trivial case when the household has zero resources. Intuitively, convex human capital investment function implies that it is always profitable for the agent to invest an incremental amount into human capital.

8. Update guesses. The final $a'(a, h, e)$ is the element-wise maximum of its unconstrained and liquidity-constrained counterparts. Replace the unconstrained $h'(a, h, e)$ with constrained one at the exact same points. Compute consumption from the budget constraint as

$$c(a, h, e) = (1+r)a + g(h, e, w) - m\left(h'(a, h, e), h\right) - a'(a, h, e)$$

Finally use the envelope conditions and to update the guesses

$$V_a(a, h, e) = (1+r)(c(a, h, e)^{-\sigma} - \min\{\bar{c} - c, 0\}^2)$$

$$V_h(a, h, e) = \left[g_1(h, e, w) - m_2\left(h'(a, h, e), h\right)\right] c(a, h, e)^{-\sigma} (c(a, h, e)^{-\sigma} - \min\{\bar{c} - c, 0\}^2)$$

Go back to step 2, repeat until convergence.

9.3 Dynamic transitions

To obtain the dynamic transition paths following the shock, I follow the MIT-shock method (c.f. Boppart et al. (2017)):

1. Choose a time T at which the economy has presumably reached a steady state.[36]
2. Solve for the stationary equilibrium at T.
3. Guess a transition path for all aggregate variables, and obtain the relevant prices given the aggregates.
4. Given the guess for the transition path, solve the policy functions backwards from $t = T - 1$.
5. Calculate the transition matrix for the joint state in every time period, using the policy functions obtained in step **4.** Iterate the joint distribution forward, starting with the initial stationary distribution.
6. Calculate the implied evolution of the aggregate states at each point in time.
7. Compare the path of the aggregates with the initial guess and update the guess until convergence is reached.

For the simulation of the joint distribution I use a histogram approach with linear interpolation between gridpoints (c.f. Angelopoulos et al. (2021)).

9.4 Calibration of the Covid-19 shock

I calibrate the impact of Covid-19 on exogenous human capital productivity and the corresponding additional benefit payments as follows:

1. I obtain the percentage reductions in gross earnings and post all taxes and transfers net labour incomes for the 10 deciles of the pre-Covid-19 gross earnings distribution from Figure 1 in Brewer & Tasseva (2021).
2. I assign every state combination of (a_t, h_t, e_t) to one gross earnings decile using the stationary distribution of households across states and the stationary wage rate.
3. I initiate the vector κ to a random guess.
4. I parameterize both e_t^{shock} and b_t^{shock} as polynomial functions of e_t :
$$e_t^{shock}(e_t) = \kappa_0 + \kappa_1 e_t^{0.5} + \kappa_2 e_t + \kappa_3 e_t^2 \tag{26}$$
$$b_t^{shock}(e_t) = \max(0, \kappa_4 + \kappa_5 e_t^{0.5} + \kappa_6 e_t + \kappa_7 e_t^2) \tag{27}$$
5. Given the current value of $e_t^{shock}(e_t)$ I calculate the aggregate labour supply, using the fact that the distribution of human capital is fixed at the arrival of the shock. Using the aggregate supply of capital which is also fixed at the beginning of the shock period, I calculate the wage rate $w_t = (1-\alpha)A_t \left(\frac{K_t}{L_t}\right)^{\alpha}$.

[36] For my Covid-19 application I set T= 500. The MIT shock dynamics use T=150, since the shock is relatively small.

6. Using w_t and $e_t^{shock}(e_t)$ I calculate the implied average gross income for each initial gross earnings decile. Then I use the current value of $b_t^{shock}(e_t)$ to calculate the corresponding net labour income values.

7. Evaluate the fit of the approximation, by comparing the sum of squared percentage deviations of the gross and net earnings losses implied by the model relative to the numbers obtained in step 1.

8. Return to step 3, update κ using a nonlinear solver, and repeat until the desired level of convergence is achieved.

10 Appendix C

10.1 Data

Understanding Society (UnSoc, ISER (2020)) is a comprehensive longitudinal survey of about 40,000 households in the UK. It investigates a broad spectrum of social, economic and behavioural aspects, making it pertinent to a variety of researchers and decision-makers. Data gathering for each wave spans 24 months, and the first wave ran from January 2009 to January 2011. Although the waves overlap, the same respondents are interviewed at approximately the same time every year; no one is interviewed twice within a wave or a calendar year (see Knies (2018)). The UnSoc data in this paper refer to the free "End User Licence" versions of the datasets, SN-6614. In this paper I use waves 1-9, which means that the last wave ends just before the Covid-19 pandemic.

10.2 Sample selection

My primary sample consists of the General Population Sample, including the Northern Ireland sample and the Ethnic Minority boost samples. I drop those respondents who completed proxy interviews and all those where relevant information is missing. I restrict the sample to those individuals who are the heads of their respective households and of prime working age (25 to 55), and in full- or part-time employment, i.e. (w_jbstat == 2 & w_jbsemp ==1).

10.3 Definitions of income

For pre-policy labour income, I use monthly gross labour income in the current job (fimnlabgrs_dv) and multiply by 12 to arrive at annual gross labour income. For post-policy income I use monthly net labour income in the current job (fimnlabnet_dv) as well as social benefit income (fimnsben_dv), again annualised. All values are deflated using the annual Consumer Price Deflator for the UK (2015 = 100). I also trim the top and bottom 1% of values of gross labour income in every wave.

10.4 Mincerian regression

In order to partial out the observable components, I run a mincer-type regression of the natural logarithm of gross labour income $\ln(\underline{w}_{i,t})$ on a number of demographic variables:

$$\ln(\underline{w}_{i,t}) = \beta_0 + \sum_{k=1}^{K} \beta_k D_{k,i,t} + \epsilon_{i,t}, \tag{28}$$

where $D_{K,i,t}$ contains demographic information about the household:

1. An indicator for the sex of the respondent household member (w_sex_dv).
2. A third-order polynomial of mean household age (calculated from w_age_dv).
3. A dummy for each of the 12 UK government office regions (w_gor_dv).
4. A dummy for the year of the interview (w_intdaty_dv).

I collect the residuals $\epsilon_{i,t}$ and generate a new variable which will serve as the proxy for the pre-policy distribution of labour income targeted by the model: $y_{i,t} = e^{\epsilon_{i,t}}$ which is normalised to have mean 1.

10.5 Human capital proxy

The third wave of Understanding Society was supported by the Economic and Social Research Council (ESRC) and the Department for Business, Innovation, and Skills' Large Facilities Capital Fund, resulting in the addition of a module that included questions about the cognitive and psychological characteristics of adults (16 years and older). I use the results from some of these survey questions to construct a measure of general intelligence, which I use as a proxy for human capital.

I follow Whitley et al. (2016) by constructing a composite measure of cognitive ability by choosing 5 exercises from the cognitive module; these were Numeric Ability, Subtraction, Number Sequence Completion, Word Recall, and Verbal Fluency. I also use a self-reported measure of general health. I standardise each of these measures individually to have mean 0 and standard deviation of 1 and then perform a Principal Component Analysis on the set of standardised measures.

I obtain the first principal component and normalise it to have positive support: h^{UnSoc}. I take the resulting measure to be a proxy for the distribution of human capital which is targeted in the SMM procedure.