Lecture 10

CIS 341: COMPILERS

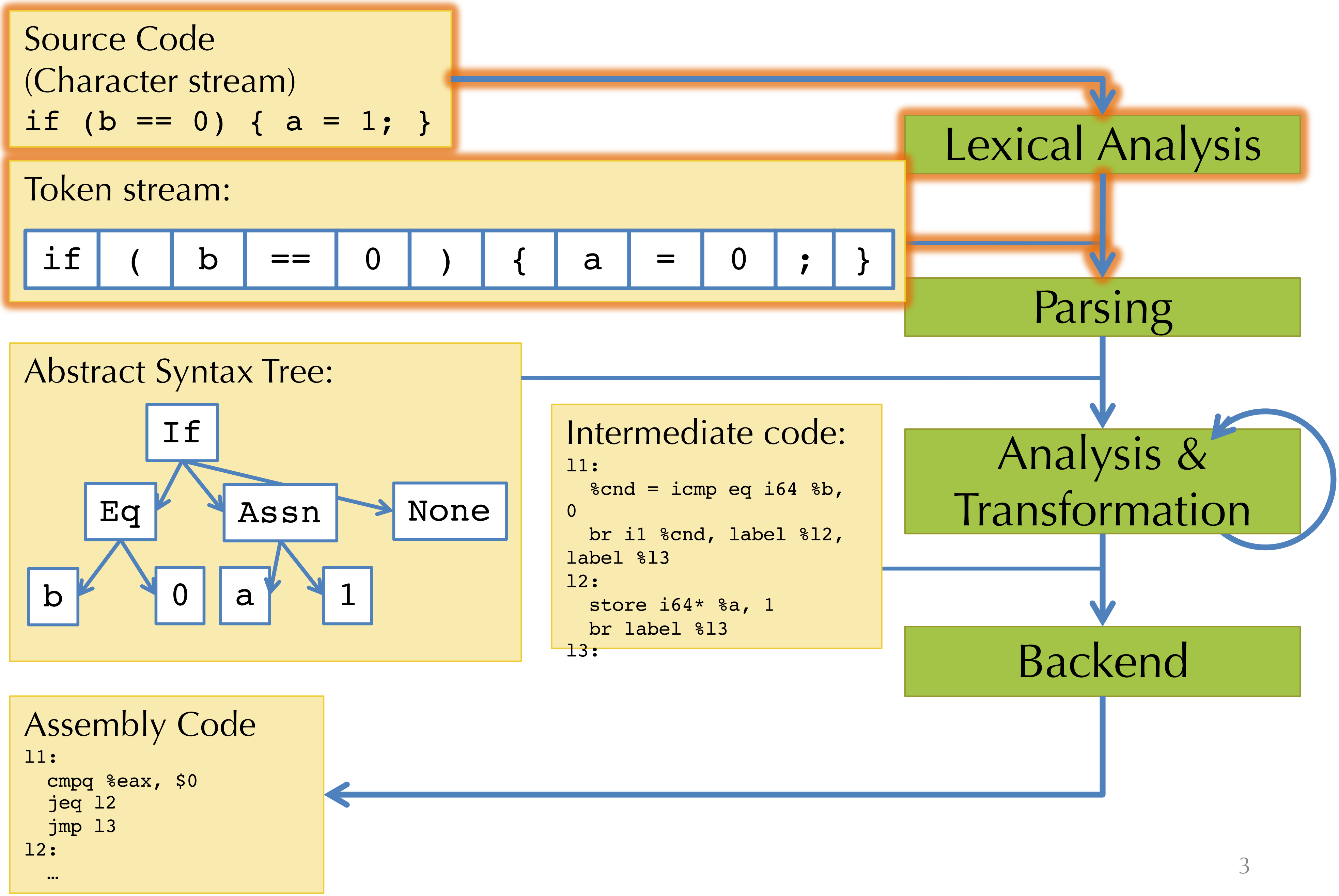
Announcements

- HW3: LLVM lite
  - Available on the course web pages.
  - Due: Weds., February 23rd at 11:59:59pm

START EARLY!!

- Midterm: March 3rd
  - In class
  - One-page, letter-sized, double-sided "cheat sheet" of notes permitted
  - See examples of previous exams on the web pages

Last Time: Lexing



Lexing By Hand

- How hard can it be?
  - Tedious and painful!

- Problems:
  - Precisely define tokens
  - Matching tokens simultaneously
  - Reading too much input (need look ahead)
  - Error handling
  - Hard to compose/interleave tokenizer code
  - Hard to maintain

PRINCIPLED SOLUTION TO LEXING

Regular Expressions

- Regular expressions precisely describe sets of strings.
- A regular expression R has one of the following forms:
  - ε — Epsilon stands for the empty string
  - `'a'` — An ordinary character stands for itself
  - $R_1 \mid R_2$ — Alternatives, stands for choice of R_1 or R_2
  - R_1R_2 — Concatenation, stands for R_1 followed by R_2
  - `R*` — Kleene star, stands for *zero or more* repetitions of `R`
- *Useful extensions:*
  - `"foo"` — Strings, equivalent to `'f''o''o'`
  - `R+` — One or more repetitions of `R`, equivalent to `RR*`
  - `R?` — Zero or one occurrences of `R`, equivalent to $(\varepsilon|R)$
  - `['a'-'z']` — One of `a` or `b` or `c` or … `z`, equivalent to `(a|b|…|z)`
  - `[^'0'-'9']` — Any character except `0` through `9`
  - `R as x` — Name the string matched by `R` as `x`

Example Regular Expressions

- Recognize the keyword "if": `"if"`
- Recognize a digit: `['0'-'9']`
- Recognize an integer literal: `'-'?['0'-'9']+`
- Recognize an identifier:
  `(['a'-'z']|['A'-'Z'])(['0'-'9']|'_'|['a'-'z']|['A'-'Z'])*`

- In practice, it's useful to be able to *name* regular expressions:

```
let lowercase = ['a'-'z']
let uppercase = ['A'-'Z']
let character = uppercase | lowercase
```

How to Match?

- Consider the input string: `ifx = 0`
  - Could lex as: | `if` | `x` | `=` | `0` | or as: | `ifx` | `=` | `0` |

- Regular expressions alone are ambiguous, need a rule for choosing between the options above
- Most languages choose "longest match"
  - So the 2^{nd} option above will be picked
  - Note that only the first option is "correct" for parsing purposes

- Conflicts: arise due to two tokens whose regular expressions have a shared prefix
  - Ties broken by giving some matches higher priority
  - Example: keywords have priority over identifiers
  - Usually specified by order the rules appear in the lex input file

Lexer Generators

- Reads a list of regular expressions: `R1,…,Rn` , one per token.
- Each token has an attached "action" `Ai` (just a piece of code to run when the regular expression is matched):

```
rule token = parse
| '-'?digit+                { Int (Int32.of_string (lexeme lexbuf)) }
| '+'                       { PLUS }
| 'if'                      { IF }
| character (digit|character|'_')*       { Ident (lexeme lexbuf) }
| whitespace+                            { token lexbuf }
```

token
regular expressions

actions

- Generates scanning code that:
  1. Decides whether the input is of the form `(R1|…|Rn)*`
  2. Whenever the scanner matches a (longest) token, it runs the associated action

lexlex.mll

DEMO: OCAMLLEX

Implementation Strategies

- Most Tools: lex, ocamllex, flex, etc.:
  - Table-based
  - Deterministic Finite Automata (DFA)
  - Goal: Efficient, compact representation, high performance

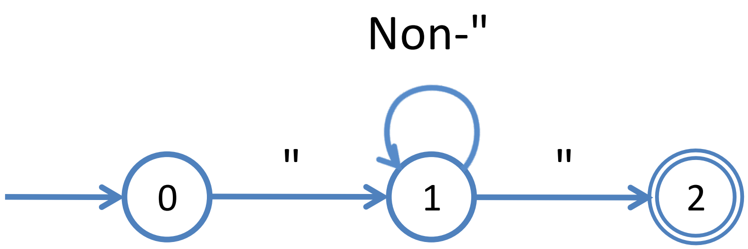
- Other approaches:
  - Brzozowski derivatives
  - Idea: directly manipulate the (abstract syntax of) the regular expression
  - Compute partial "derivatives"
    - Regular expression that is "left-over" after seeing the next character
  - Elegant, purely functional, implementation
  - (very cool!)

Finite Automata

- Consider the regular expression: ‘"’[^’"’]*’"’
- An automaton (DFA) can be represented as:
  - A transition table:

	"	Non-"
0	1	ERROR
1	2	1
2	ERROR	ERROR

  - A graph:

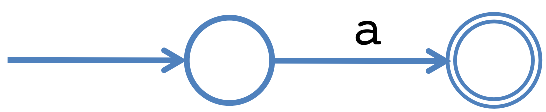


RE to Finite Automaton?

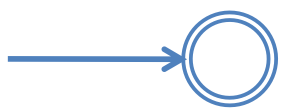
- Can we build a finite automaton for every regular expression?
  - Yes! Recall CIS 262 for the complete theory…

- Strategy: consider every possible regular expression (by induction on the structure of the regular expressions):

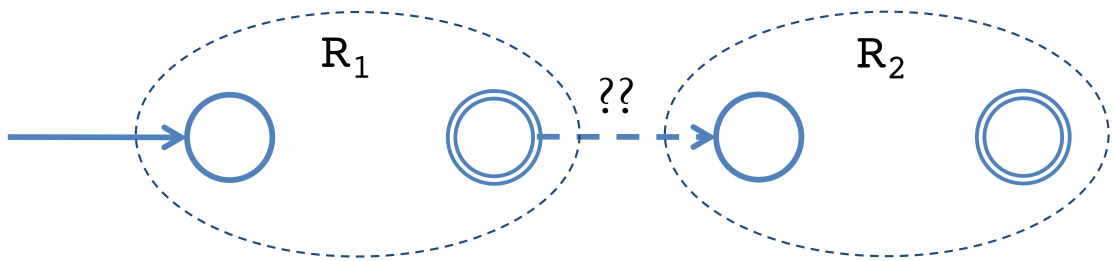
`'a'`



ε



R_1R_2

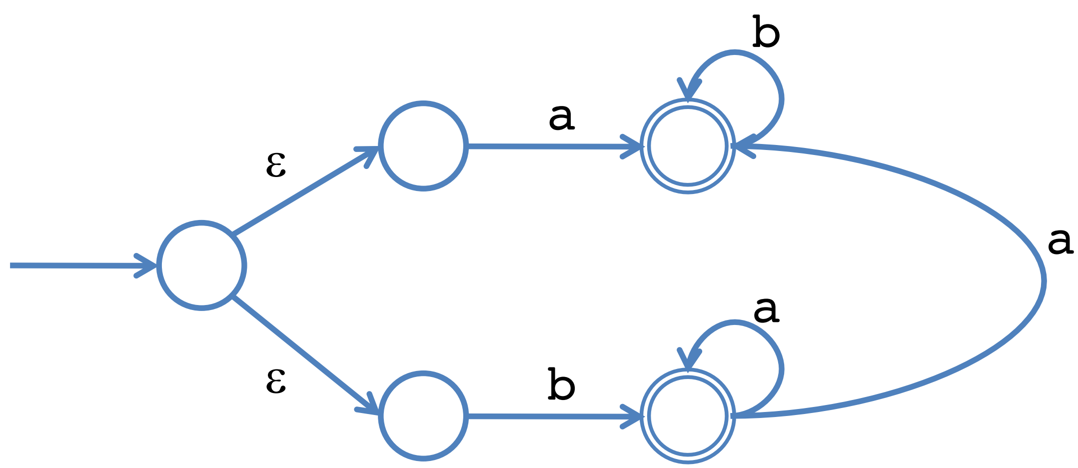


What about?

$R_1 \mid R_2$

Nondeterministic Finite Automata

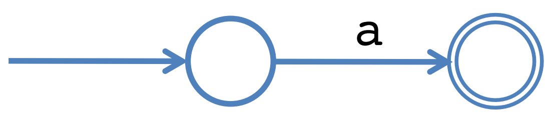
- A finite set of states, a start state, and accepting state(s)
- Transition arrows connecting states
  - Labeled by input symbols
  - Or ε (which does not consume input)
- *Nondeterministic*: two arrows leaving the same state may have the same label



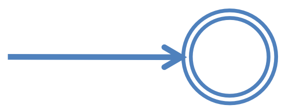
RE to NFA?

- Converting regular expressions to NFAs is easy.
- Assume each NFA has one start state, unique accept state

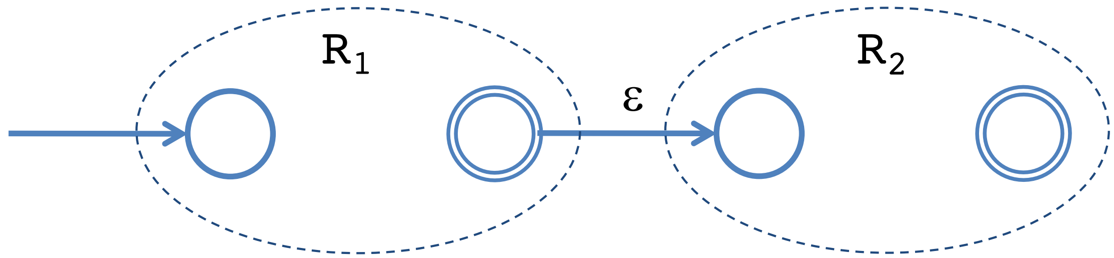
'a'



ε

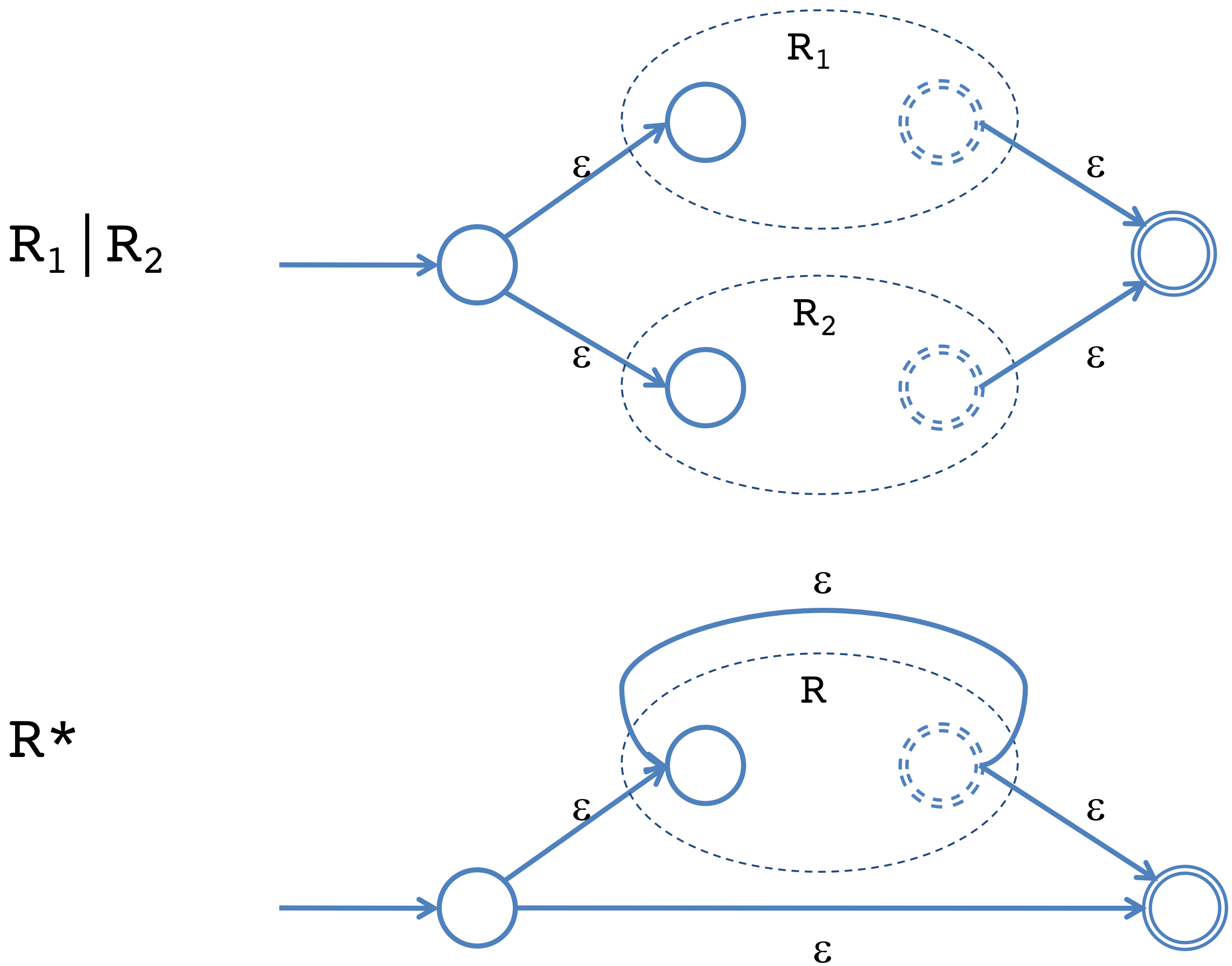


R_1R_2



RE to NFA (cont'd)

- Sums and Kleene star are easy with NFAs



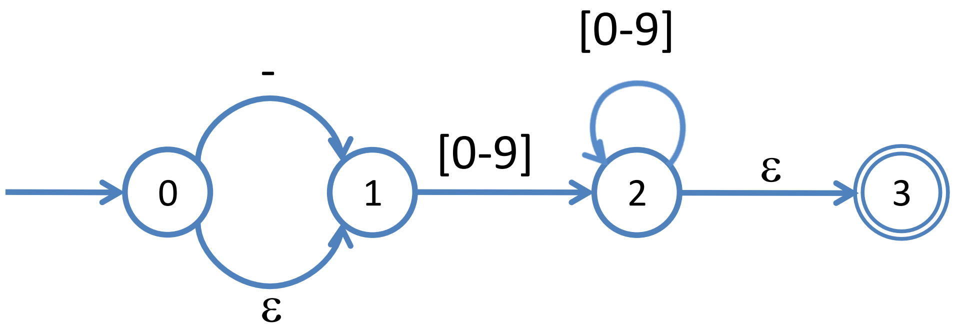
DFA versus NFA

- DFA:
  - Action of the automaton for each input is fully determined
  - Automaton accepts if the input is consumed upon reaching an accepting state
  - Obvious table-based implementation

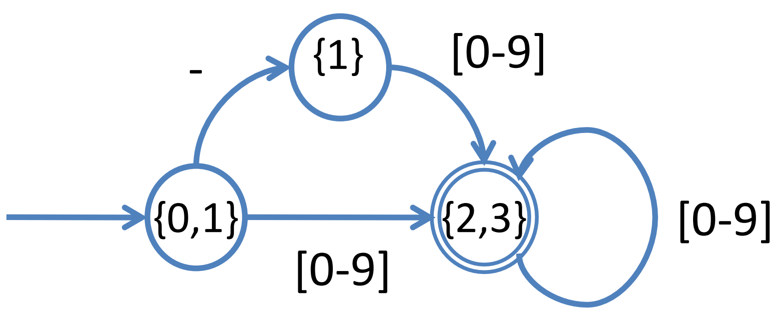
- NFA:
  - Automaton potentially has a choice at every step
  - Automaton accepts an input string if there *exists* a way to reach an accepting state
  - Less obvious how to implement efficiently

NFA to DFA conversion (Intuition)

- Idea: Run all possible executions of the NFA "in parallel"
- Keep track of a set of possible states: "finite fingers"
- Consider: `-?[0-9]+`
- NFA representation:



- DFA representation:



Summary of Lexer Generator Behavior

- Take each regular expression R_i and it's action A_i
- Compute the NFA formed by $(R_1 \mid R_2 \mid \ldots \mid R_n)$
  - Remember the actions associated with the accepting states of the R_i
- Compute the DFA for this big NFA
  - There may be multiple accept states (why?)
  - A single accept state may correspond to one or more actions (why?)
- Compute the minimal equivalent DFA
  - There is a standard algorithm due to Myhill & Nerode
- Produce the transition table
- Implement longest match:
  - Start from initial state
  - Follow transitions, remember last accept state entered (if any)
  - Accept input until no transition is possible (i.e. next state is "ERROR")
  - Perform the highest-priority action associated with the last accept state; if no accept state there is a lexing error

Lexer Generators in Practice

- Many existing implementations: lex, Flex, Jlex, ocamllex, …
  - For example ocamllex program
    - see lexlex.mll, olex.mll, piglatin.mll on course website
- Error reporting:
  - Associate line number/character position with tokens
  - Use a rule to recognize '\n' and increment the line number
  - The lexer generator itself usually provides character position info.
- Sometimes useful to treat comments specially
  - Nested comments: keep track of nesting depth

- Lexer generators are usually designed to work closely with parser generators…

lexlex.mll, olex.mll, piglatin.mll

DEMO: OCAMLLEX

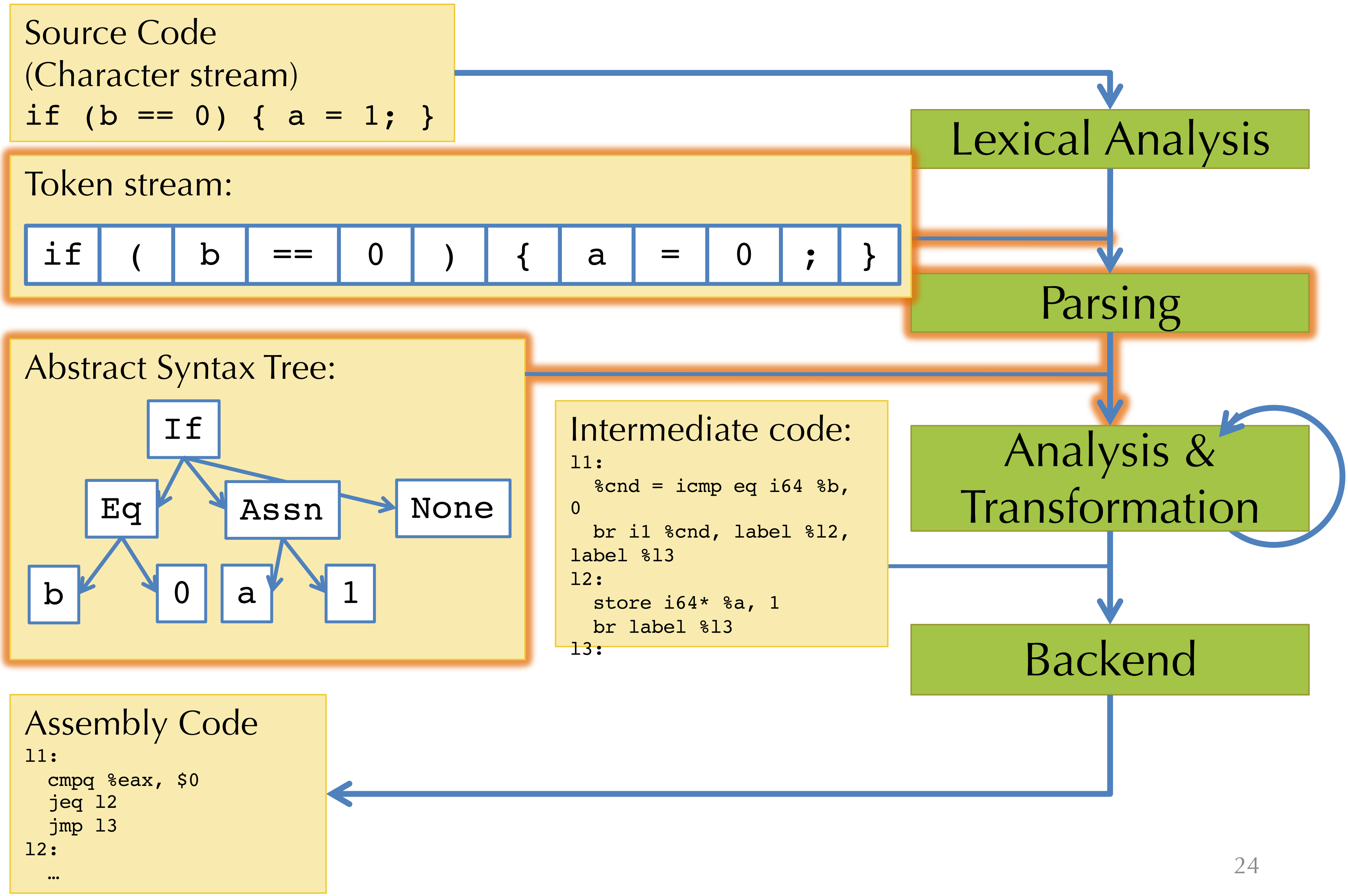
lexlex.mll, olex.mll, piglatin.mll

DEMO: OCAMLLEX

Creating an abstract representation of program syntax.

PARSING

Parsing

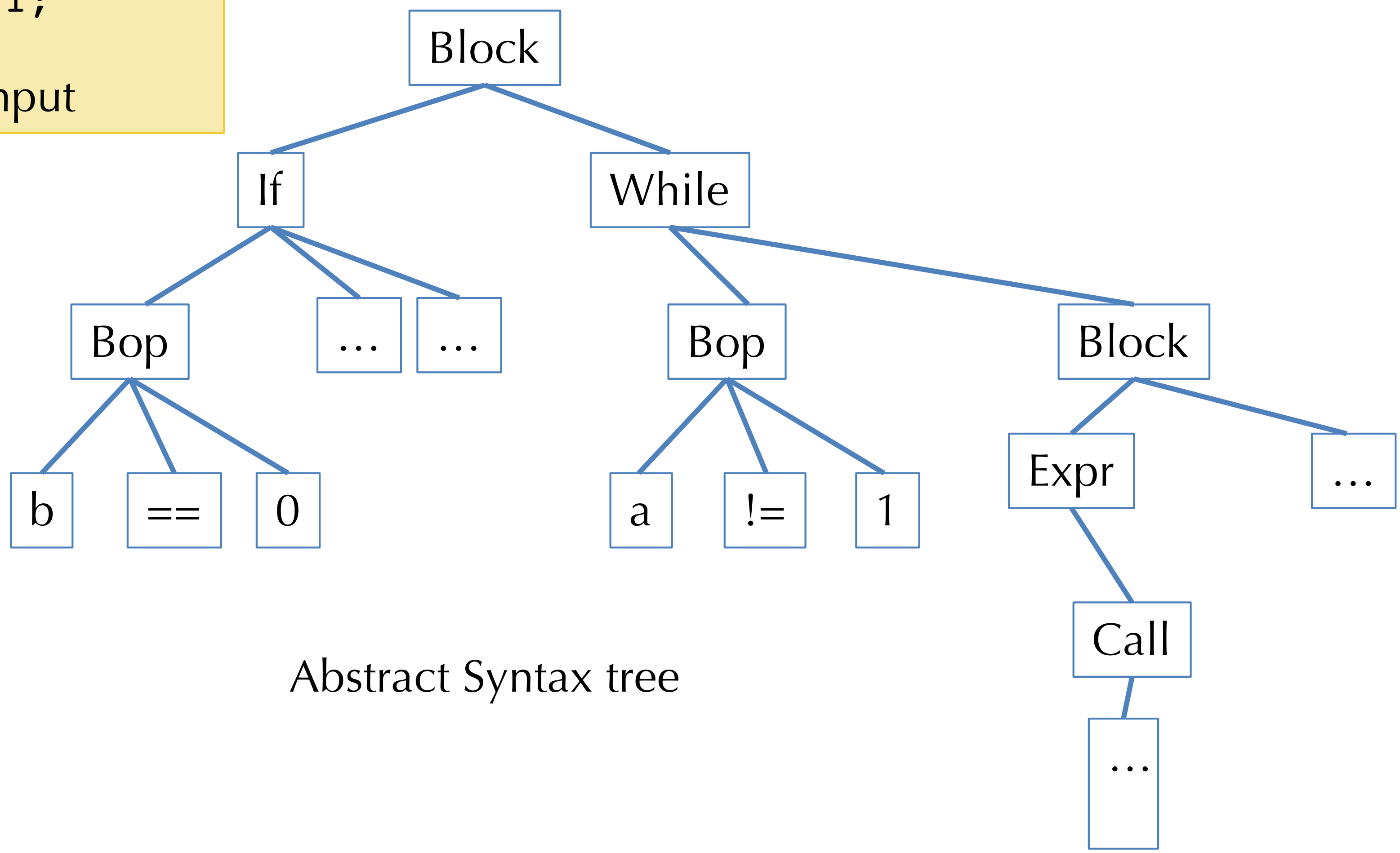


Source Code
(Character stream)
if (b == 0) { a = 1; }
Lexical Analysis
Token stream:
if (b == 0) { a = 0 ; }
Parsing
Abstract Syntax Tree:
If
Eq
Assn
None
b
0
a
1
Intermediate code:
l1:
%cnd = icmp eq i64 %b, 0
br i1 %cnd, label %l2, label %l3
l2:
store i64* %a, 1
br label %l3
l3:
Analysis & Transformation
Backend
Assembly Code
l1:
cmpq %eax, $0
jeq l2
jmp l3
l2:
…

Parsing: Finding Syntactic Structure

```
{
  if (b == 0) a = b;
  while (a != 1) {
    print_int(a);
    a = a – 1;
  }
}
```

Source input



Abstract Syntax tree

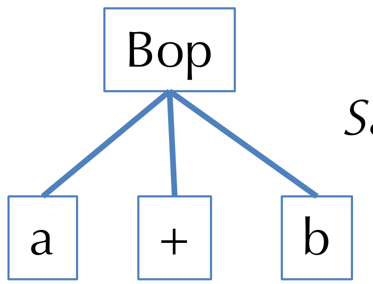
Syntactic Analysis (Parsing): Overview

- Input: stream of tokens (generated by lexer)
- Output: abstract syntax tree

- Strategy:
  – Parse the token stream to traverse the "concrete" syntax
  – During traversal, build a tree representing the "abstract" syntax

- Why abstract? Consider these three *different* concrete inputs:

```
a + b
(a + ((b)))
((a) + (b))
```



Same abstract syntax tree

- Note: parsing doesn't check many things:
  – Variable scoping, type agreement, initialization, …

Specifying Language Syntax

- First question: how to describe language syntax precisely and conveniently?
- Previously we described tokens using regular expressions
  - Easy to implement, efficient DFA representation
  - Why not use regular expressions on tokens to specify programming language syntax?

- Limits of regular expressions:
  - DFA's have only finite # of states
  - So… DFA's can't "count"
  - For example, consider the language of all strings that contain balanced parentheses – easier than most programming languages, but not regular.

- So: we need more expressive power than DFA's

CONTEXT FREE GRAMMARS

Context-free Grammars

- Here is a specification of the language of balanced parens:

$$S \longmapsto (S)S$$
$$S \longmapsto \varepsilon$$

Note: Once again we have to take care to distinguish meta-language elements (e.g. "S" and "$\longmapsto$") from object-language elements (e.g. "(").*

- The definition is *recursive* – S mentions itself.
- Idea: "derive" a string in the language by starting with S and rewriting according to the rules:
  - Example: $S \longmapsto (S)S \longmapsto ((S)S)S \longmapsto ((\varepsilon)S)S \longmapsto ((\varepsilon)S)\varepsilon \longmapsto ((\varepsilon)\varepsilon)\varepsilon = (())$
- You can replace the "nonterminal" S by one of its definitions anywhere
- A context-free grammar accepts a string iff there is a derivation from the start symbol

* And, since we're writing this description in English, we are careful distinguish the meta-meta-language (e.g. words) from the meta-language and object-language (e.g. symbols) by using quotes.

CFGs Mathematically

- A Context-free Grammar (CFG) consists of
  - A set of *terminals* (e.g., a lexical token or ε)
  - A set of *nonterminals* (e.g., S and other syntactic variables)
  - A designated nonterminal called the *start symbol*
  - A set of productions: LHS $\longmapsto$ RHS
    - LHS is a nonterminal
    - RHS is a *string* of terminals and nonterminals

- Example: The balanced parentheses language:

$$S \longmapsto (S)S$$

$$S \longmapsto \varepsilon$$

- How many terminals? How many nonterminals? Productions?

Another Example: Sum Grammar

- A grammar that accepts parenthesized sums of numbers:

$$
\begin{array}{lcll}
S & \mapsto & E + S & \mid \; E \\
E & \mapsto & \text{number} & \mid \; (\, S \,)
\end{array}
$$

e.g.: (1 + 2 + (3 + 4)) + 5

- Note the vertical bar '|' is shorthand for multiple productions:

$$
\left.
\begin{array}{l}
S \mapsto E + S \\
S \mapsto E \\
E \mapsto \text{number} \\
E \mapsto (S)
\end{array}
\right\}
$$

4 productions
2 nonterminals: S, E
4 terminals: (,), +, number
Start symbol: S

Derivations in CFGs

- Example: derive (1 + 2 + (3 + 4)) + 5
- $\underline{\mathbf{S}} \longmapsto \underline{\mathbf{E}} + S$

  $\longmapsto (\underline{\mathbf{S}}) + S$

  $\longmapsto (\underline{\mathbf{E}} + S) + S$

  $\longmapsto (1 + \underline{\mathbf{S}}) + S$

  $\longmapsto (1 + \underline{\mathbf{E}} + S) + S$

  $\longmapsto (1 + 2 + \underline{\mathbf{S}}) + S$

  $\longmapsto (1 + 2 + \underline{\mathbf{E}}) + S$

  $\longmapsto (1 + 2 + (\underline{\mathbf{S}})) + S$

  $\longmapsto (1 + 2 + (\underline{\mathbf{E}} + S)) + S$

  $\longmapsto (1 + 2 + (3 + \underline{\mathbf{S}})) + S$

  $\longmapsto (1 + 2 + (3 + \underline{\mathbf{E}})) + S$

  $\longmapsto (1 + 2 + (3 + 4)) + \underline{\mathbf{S}}$

  $\longmapsto (1 + 2 + (3 + 4)) + \underline{\mathbf{E}}$

  $\longmapsto (1 + 2 + (3 + 4)) + 5$

$S \longmapsto E + S \mid E$
$E \longmapsto \text{number} \mid (S)$

For arbitrary strings α, β, γ and production rule $A \longmapsto \beta$
a single step of the derivation is:

$$\alpha A \gamma \longmapsto \alpha \beta \gamma$$

(*substitute* β for an occurrence of A)

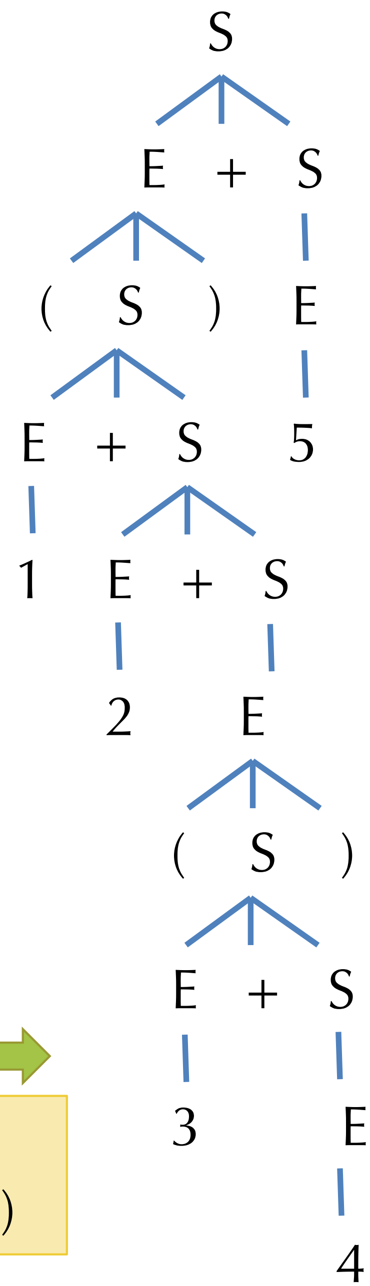
In general, there are many possible derivations for a given string

Note: Underline indicates symbol being expanded.

From Derivations to Parse Trees

- Tree representation of the derivation
- Leaves of the tree are terminals
  - In-order traversal yields the input sequence of tokens
- Internal nodes: nonterminals
- No information about the *order* of the derivation steps

- (1 + 2 + (3 + 4)) + 5



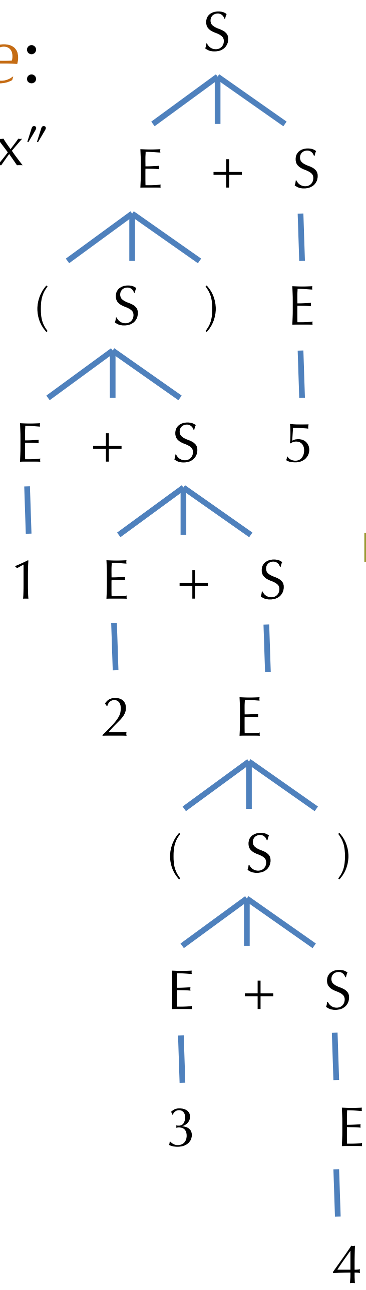
Parse Tree

$S \longmapsto E + S \mid E$
$E \longmapsto \text{number} \mid (S)$

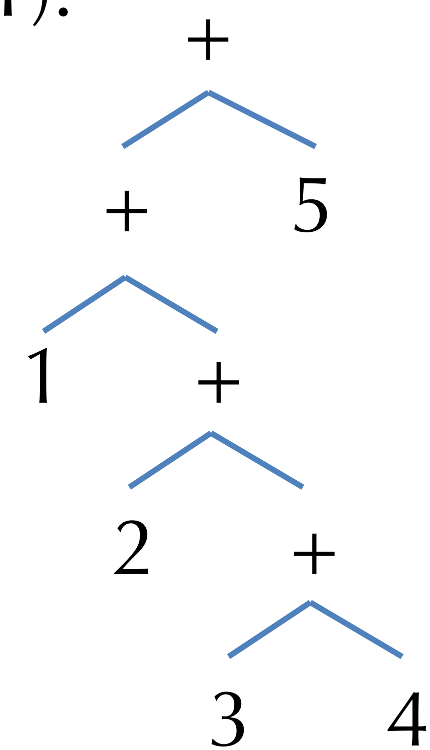
From Parse Trees to Abstract Syntax

- *Parse tree*:

"concrete syntax"



- *Abstract syntax tree* (AST):



- Hides, or *abstracts*, unneeded information.

Derivation Orders

- Productions of the grammar can be applied in any order.
- There are two standard orders:
  - *Leftmost derivation*: Find the left-most nonterminal and apply a production to it.
  - *Rightmost derivation*: Find the right-most nonterminal and apply a production there.

- Note that both strategies (and any other) yield the same parse tree!
  - Parse tree doesn't contain the information about what order the productions were applied.

Example: Left- and rightmost derivations

- Leftmost derivation:
- $\underline{\mathbf{S}} \longmapsto \underline{\mathbf{E}} + S$
  $\longmapsto (\underline{\mathbf{S}}) + S$
  $\longmapsto (\underline{\mathbf{E}} + S) + S$
  $\longmapsto (1 + \underline{\mathbf{S}}) + S$
  $\longmapsto (1 + \underline{\mathbf{E}} + S) + S$
  $\longmapsto (1 + 2 + \underline{\mathbf{S}}) + S$
  $\longmapsto (1 + 2 + \underline{\mathbf{E}}) + S$
  $\longmapsto (1 + 2 + (\underline{\mathbf{S}})) + S$
  $\longmapsto (1 + 2 + (\underline{\mathbf{E}} + S)) + S$
  $\longmapsto (1 + 2 + (3 + \underline{\mathbf{S}})) + S$
  $\longmapsto (1 + 2 + (3 + \underline{\mathbf{E}})) + S$
  $\longmapsto (1 + 2 + (3 + 4)) + \underline{\mathbf{S}}$
  $\longmapsto (1 + 2 + (3 + 4)) + \underline{\mathbf{E}}$
  $\longmapsto (1 + 2 + (3 + 4)) + 5$

Rightmost derivation:

$\underline{\mathbf{S}} \longmapsto E + \underline{\mathbf{S}}$
$\longmapsto E + \underline{\mathbf{E}}$
$\longmapsto \underline{\mathbf{E}} + 5$
$\longmapsto (\underline{\mathbf{S}}) + 5$
$\longmapsto (E + \underline{\mathbf{S}}) + 5$
$\longmapsto (E + E + \underline{\mathbf{S}}) + 5$
$\longmapsto (E + E + \underline{\mathbf{E}}) + 5$
$\longmapsto (E + E + (\underline{\mathbf{S}})) + 5$
$\longmapsto (E + E + (E + \underline{\mathbf{S}})) + 5$
$\longmapsto (E + E + (E + \underline{\mathbf{E}})) + 5$
$\longmapsto (E + E + (\underline{\mathbf{E}} + 4)) + 5$
$\longmapsto (E + \underline{\mathbf{E}} + (3 + 4)) + 5$
$\longmapsto (\underline{\mathbf{E}} + 2 + (3 + 4)) + 5$
$\longmapsto (1 + 2 + (3 + 4)) + 5$

$S \longmapsto E + S \mid E$
$E \longmapsto \text{number} \mid (S)$

Loops and Termination

- Some care is needed when defining CFGs
- Consider:

  $$\begin{array}{lll} S & \mapsto & E \\ E & \mapsto & S \end{array}$$

  – This grammar has nonterminal definitions that are "nonproductive". (i.e. they don't mention any terminal symbols)
  – There is no finite derivation starting from S, so the language is empty.

- Consider: $S \mapsto (S)$

  – This grammar is productive, but again there is no finite derivation starting from S, so the language is empty

- Easily generalize these examples to a "chain" of many nonterminals, which can be harder to find in a large grammar

- Upshot: be aware of "vacuously empty" CFG grammars.
  – Every nonterminal should eventually rewrite to an alternative that contains only terminal symbols.