Lecture 16

EECS 483: COMPILER CONSTRUCTION

Announcements

- Midterm
  - Grades will be released after we review the results on Wednesday's class (3/20).
- HW4: OAT v.1.0
  - Parsing & translation to LLVM IR
  - Helps to start early!
  - **Due: Tuesday, March 26th**

Personal Announcement

- My wife and I are having a baby
  - Due date: March 30, but could be any day now
  - We will have guest lectures (by our GSI Eric or a guest professor) for at least 4 lectures after the baby comes.
  - I will have my office hours remotely, will not be available for scheduled office hours.

Implementing First-Class Functions

- First attempt: Functions as Code
  - Represent a function value as its code
  - What about *local function definitions?*
    - `let f = fun x -> fun y -> x + y`
  - Every time we call `f 0, f 5, f 256`, we get a *different* function
  - In an substitution-based interpreter, we substitute a value in and get a different term each time we call the function.
  - Compilation
    - Infeasible to implement all of these possible functions statically in memory, impossible if the domain of the function is infinite!
    - Requires *runtime code generation*, which itself has high runtime overhead.
    - Usually not used for arbitrary first class functions, only in specialized situations

Implementing First-Class Functions

- Closures
  - Consider
    - `let f = fun x -> fun y -> x + y`
  - Each function `f 0, f 5, f 256,`… is implemented by substitution:
    - `(fun y -> x + y){ 0 / x }`
    - `(fun y -> x + y){ 5 / x }`
    - `(fun y -> 256 + y){ 256 / x }`
  - Idea: represent a first-class function as a ***pair*** of
    - A piece of code with **free** variables
    - An **environment** that provides all of the values of the free variables
  - In compilation
    - The code with free variables can be a code pointer to code that takes the environment as an argument
      - `fun (env, y) -> env.x + y`
    - The environment can be implemented in multiple ways
      - Array: fast access in the function
      - Linked list: more sharing between different closures

See fun.ml, cc.ml from lec15.zip

CLOSURES AND CLOSURE CONVERSION

Closure Conversion Summary

- A ***closure*** is a pair of an environment and a code pointer
  - the environment is a map data structure binding variables to values
  - environment could just be a list of the values (with known indices)

- Building a closure value:
  - code pointer is a function that takes an extra argument for the environment: A → B becomes (Env * A → B)
  - body of the closure "projects out" then variables from the environment
  - creates the environment map by bundling the free variables

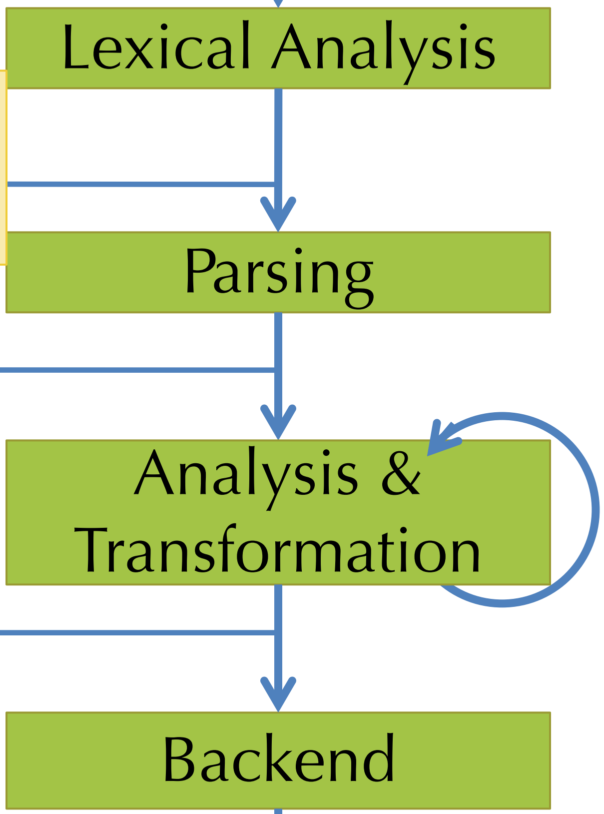
- Applying a closure:
  - project out the environment, invoke the function (pointer) with the environment and its "real" argument

- Hoisting:
  - Once closure converted, all functions can be lifted to the top level

Scope, Types, and Context

SEMANTIC ANALYSIS

Compilation in a Nutshell



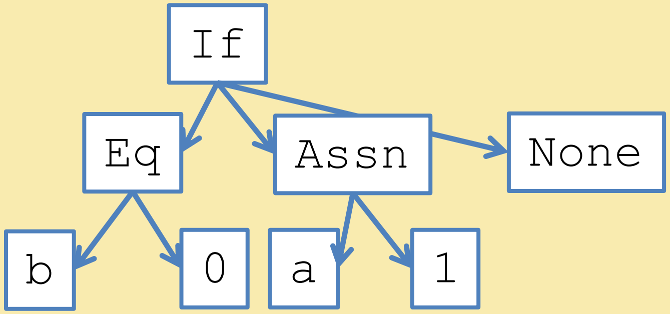
Source Code
(Character stream)

```
if (b == 0) { a = 1; }
```

Token stream:

if	(	b	==	0	)	{	a	=	0	;	}

Abstract Syntax Tree:



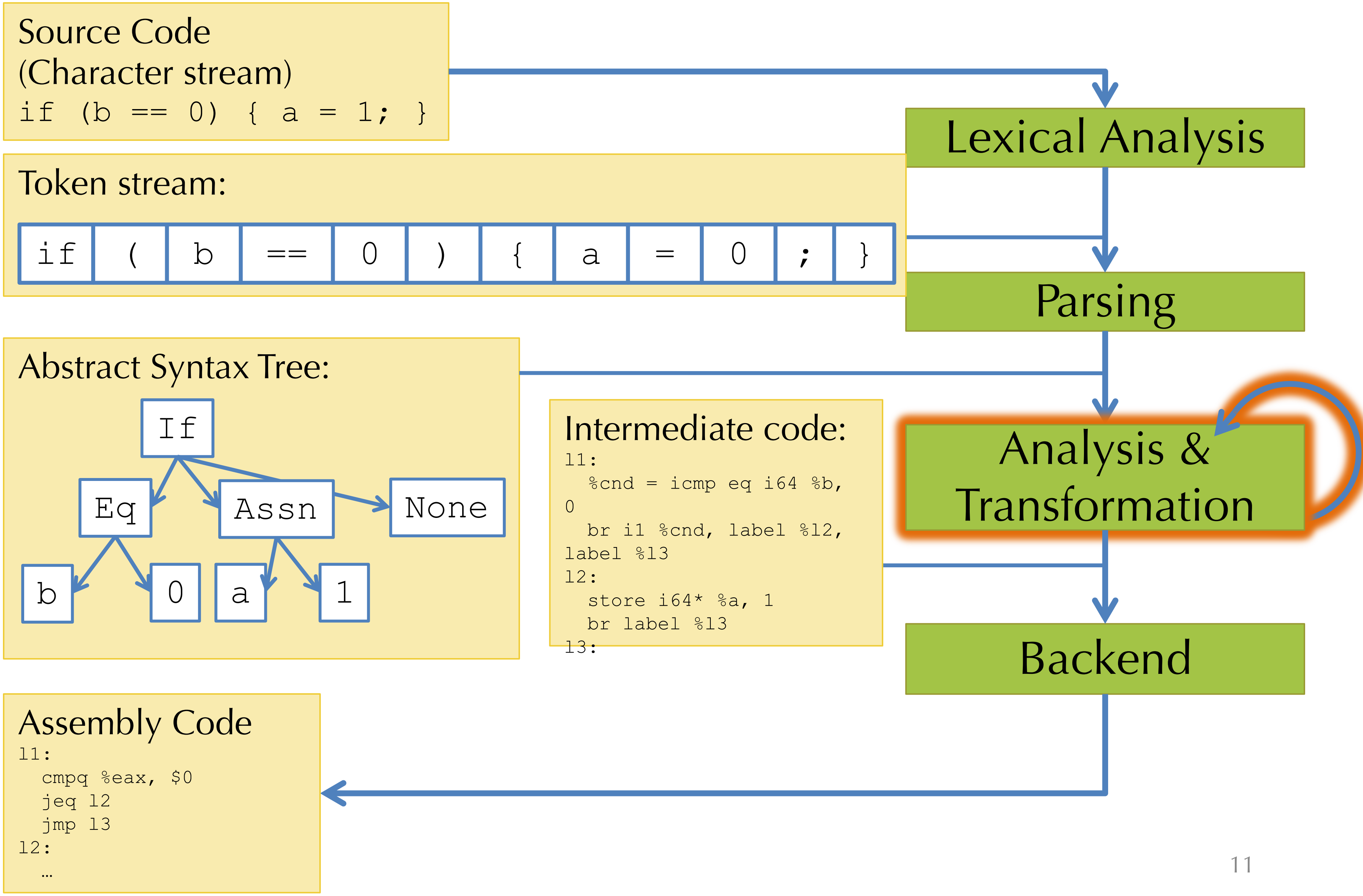
Intermediate code:

```
l1:
  %cnd = icmp eq i64 %b,
0
  br i1 %cnd, label %l2,
label %l3
l2:
  store i64* %a, 1
  br label %l3
l3:
```

Assembly Code

```
l1:
  cmpq %eax, $0
  jeq l2
  jmp l3
l2:
  …
```

Most of the Remainder of the Course



Source Code
(Character stream)

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if (b == 0) { a = 1; }
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Token stream:

if	(	b	==	0	)	{	a	=	0	;	}

Abstract Syntax Tree:

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  br label %l3
l3:
```

Assembly Code

```
l1:
  cmpq %eax, $0
  jeq l2
  jmp l3
l2:
  …
```

Static Program Analysis

- Static program analysis is analysis of a program at compile-time
- Used for two main purposes in the compiler:
  - Last stage of the frontend: “Type checking” or “Semantic Analysis”
    - Not every program that passes parsing is valid
      - `int main() { return x; }`
      - `int main() { return “hello world”; }`
    - If the type checker fails, the program is rejected, like a parse error
    - After the program passes the frontend, we consider it well-formed and will compile it.
  - During optimization: “static analysis”
    - We can do more optimizations if we know more about the program
    - Are these equivalent programs?
      - `int main() { int y = f(); return 0; }`
      - `int main() { return 0; }`
    - We can optimize the first to the second if we establish that f is side-effect free.
    - Since they take place after the frontend, the analysis never rejects the program
- Next few weeks: type checking, after that optimization and analyis

Type Checking as Grammar

	Specification	Implementation
Lexing	Regular Expressions	DFA
Parsing	CFG LL(1) grammars LR(1) grammars	Pushdown automata Recursive descent Shift/reduce parser
Type checking	Inference rules	Manual recursive descent

Variable Scoping

- Consider the problem of determining whether a programmer-declared variable is in scope.
- Issues:
  - Which variables are available at a given point in the program?
  - Shadowing – is it permissible to re-use the same identifier, or is it an error?

- Example: The following program is syntactically correct but not well-formed. (y and q are used without being defined anywhere)

```
int fact(int x) {
 var acc = 1;
 while (x > 0) {
  acc = acc * y;
  x = q  -  1;
   }
 return acc;
}
```

Q: Can we solve this problem by changing the parser to rule out such programs?

Inference Rules

- We can read a judgment $G \vdash e$ as
  "the expression e is well scoped and has free variables in G"
- For any environment G, expression e, and statements s_1, s_2.

$$G \vdash \text{if } (e)\ s_1 \text{ else } s_2$$

  holds if $G \vdash e$ and $G \vdash s_1$ and $G \vdash s_2$ all hold.
- More succinctly: we summarize these constraints as an ***inference rule***:

Premises

Conclusion

$$\frac{G \vdash e \qquad G \vdash s_1 \qquad G \vdash s_2}{G \vdash \text{if } (e)\ s_1 \text{ else } s_2}$$

- Such a rule can be used for *any* substitution of the syntactic metavariables G, e, s_1 and s_2.

Judgments

- A ***judgment*** is a (meta-syntactic) notation that *names* a relation among one or more sets.
  - The sets are usually built from object-language syntax elements and other “math” sets (*e.g.*, integers, natural numbers, *etc.*)
  - We usually describe them using metavariables that range over the sets.
  - Often use domain-specific notation to ease reading.
  - The meaning of judgments, *i.e.*, which sets they represent, is defined by (collections of) inference rules

- Example: When we say “G ⊢ e is a judgment where G is a context of variables and e is a term, defined by these […] inference rules” that is shorthand for this “math speak”:

  - Let Var be the set of all (syntactic) variables
  - Let Exp be the set {e | e is a term of the untyped lambda calculus}
  - Let $\mathcal{P}$(Var) be the (finite) powerset of variables (set of all finite sets)
  - Define *well-scoped* ⊆ ($\mathcal{P}$(Var), Exp) to be a relation satisfying the properties defined by the associated inference rules […]
  - Then “G ⊢ e” is notation that means that (G, e) ∈ *well-scoped*

Scope-Checking Lambda Calculus

- Consider how to identify "well-scoped" lambda calculus terms
  - Given: G, a *set* of variable identifiers, e, a term of the lambda calculus
  - *Judgment*: $G \vdash e$ "the free variables of e are included in G"

$$\frac{x \in G}{G \vdash x}$$

"the variable x is free, but in scope"

$$\frac{G \vdash e_1 \qquad G \vdash e_2}{G \vdash e_1\ e_2}$$

"G contains the free variables of e_1 and e_2"

$$\frac{G \cup \{x\} \vdash e}{G \vdash \text{fun } x \rightarrow e}$$

"x is available in the function body e"

Scope-checking Code

- Compare the OCaml code to the inference rules:
  - structural recursion over syntax
  - the check either “succeeds” or ”fails"

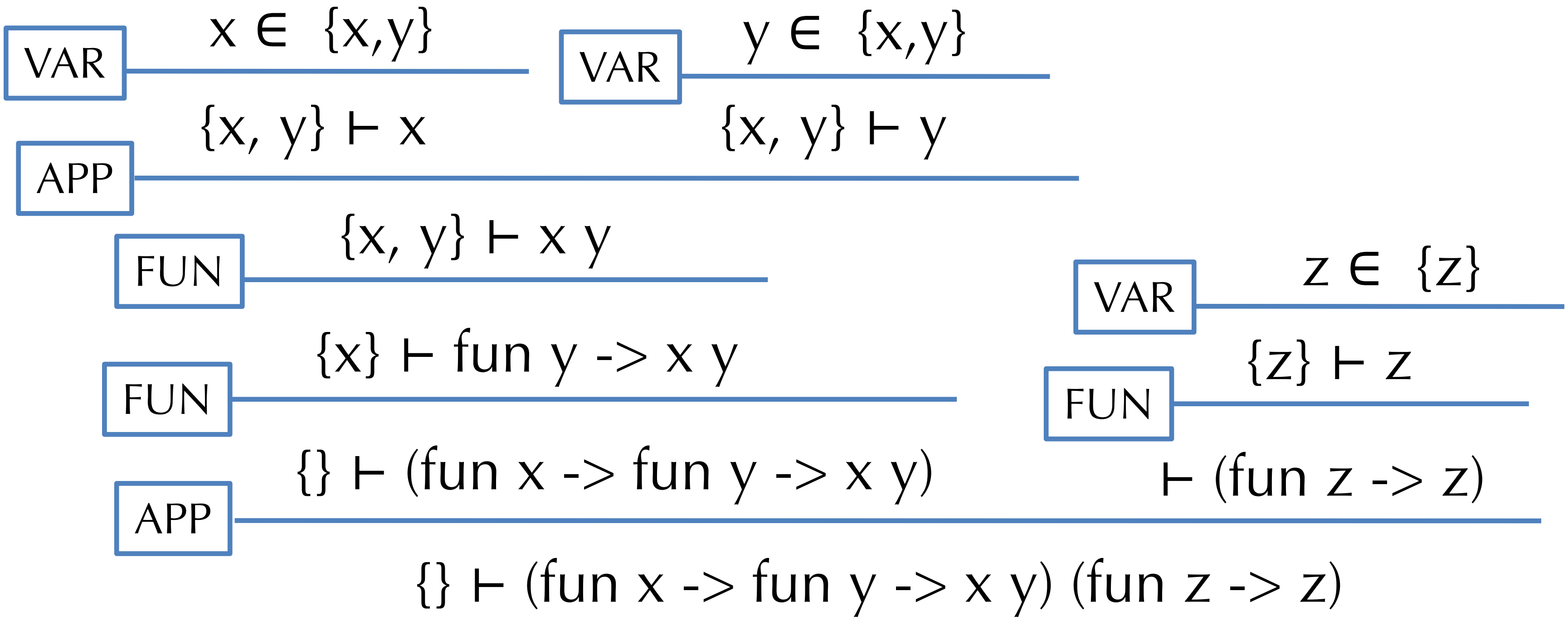
```
let rec scope_check (g:VarSet.t) (e:exp) : unit =
  begin match e with
  | Var x -> if VarSet.member x g then () else failwith (x ^ "not in scope")
  | App(e1, e2) -> ignore (scope_check g e1); scope_check g e2
  | Fun(x, e)   -> scope_check (VarSet.union g (VarSet.singleton x)) e
  end
```

$$\frac{x \in G}{G \vdash x} \qquad \frac{G \vdash e_1 \qquad G \vdash e_2}{G \vdash e_1\ e_2} \qquad \frac{G \cup \{x\} \vdash e}{G \vdash \text{fun } x \to e}$$

$$\boxed{\text{VAR}} \qquad \boxed{\text{APP}} \qquad \boxed{\text{FUN}}$$

- The inference rules are a *specification* of the intended behavior of this scope checking code.
  - they don’t specify the order in which the premises are checked

Example Derivation Tree



VAR

x ∈ {x,y}

VAR

y ∈ {x,y}

{x, y} ⊢ x

{x, y} ⊢ y

APP

{x, y} ⊢ x y

FUN

VAR

z ∈ {z}

{x} ⊢ fun y -> x y

FUN

{z} ⊢ z

FUN

{} ⊢ (fun x -> fun y -> x y)

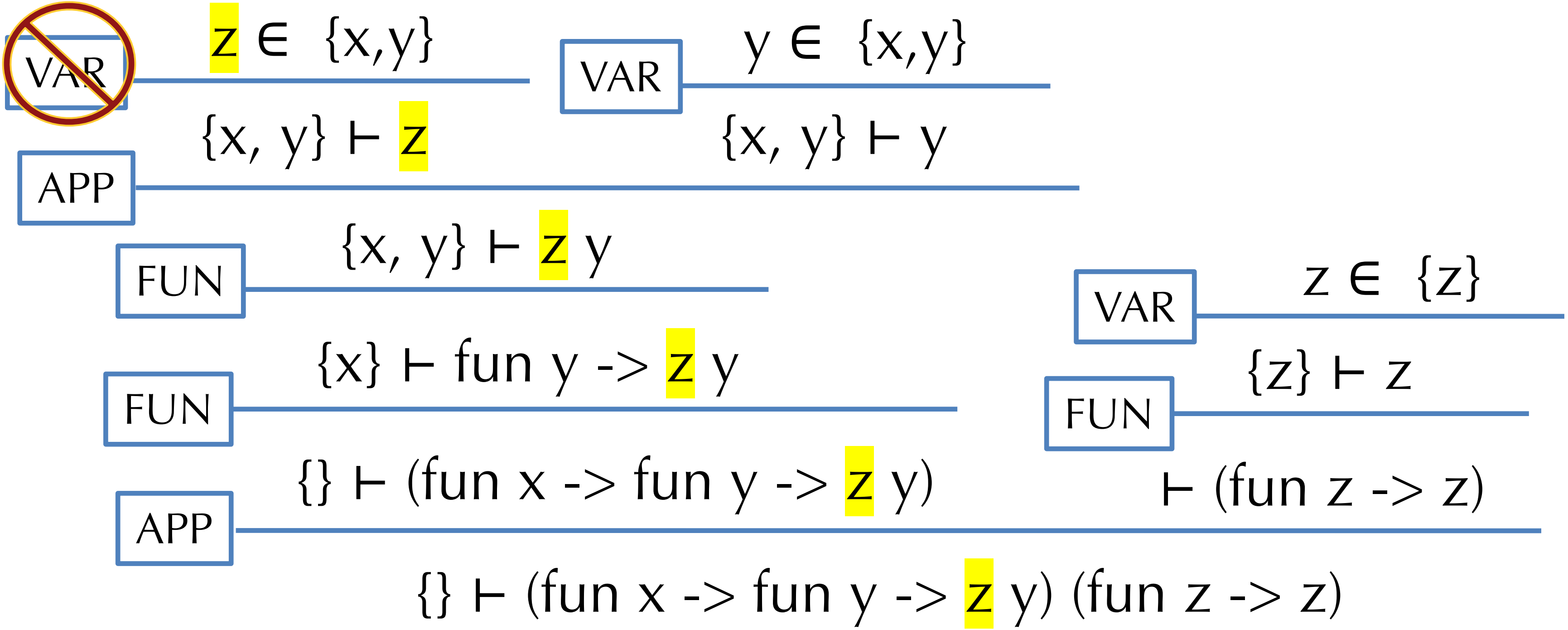
⊢ (fun z -> z)

APP

{} ⊢ (fun x -> fun y -> x y) (fun z -> z)

- Note: the OCaml function **scope_check** verifies the existence of this tree. The structure of the recursive calls when running **scope_check** is the same shape as this tree!
- Note that x ∈ E is implemented by the function **VarSet.mem**

Example Failed Derivation



- This program is *not* well scoped
  - The variable z is not bound in the body of the left function.
  - The typing derivation fails because the VAR rule cannot succeed
  - (The other parts of the derivation are OK, though!)

Uses of the inference rules

- We can do proofs by induction on the structure of the derivation.
- For example:

Lemma: If $G \vdash e$ then $fv(e) \subseteq G$.

Proof.

By induction on the derivation that $G \vdash e$.

$$\frac{x \in G}{G \vdash x}$$

- case: VAR then we have $e = x$ (for some variable x) and $x \in G$. But $fv(e) = fv(x) = \{x\}$, but then $\{x\} \subseteq G$.

$$\frac{G \vdash e_1 \quad G \vdash e_2}{G \vdash e_1\ e_2}$$

- case: APP then we have $e = e_1\ e_2$ (for some $e_1\ e_2$) and, by induction, we have $fv(e_1) \subseteq G$ and $fv(e_2) \subseteq G$, so $fv(e_1\ e_2) = fv(e_1) \cup fv(e_2) \subseteq G$

$$\frac{G \cup \{x\} \vdash e_1}{G \vdash \text{fun } x \to e_1}$$

- case: FUN then we have $e = (\text{fun } x \text{ -> } e_1)$ for some x, e_1 and, by induction, we have $fv(e_1) \subseteq G \cup \{x\}$, but then we also have $fv(\text{fun } x \text{ -> } e_1) = fv(e_1) \setminus \{x\} \subseteq ((G \cup \{x\}) \setminus \{x\}) \subseteq G$

$fv(x) = \{x\}$

$fv(\text{fun } x \to exp) = fv(exp) \setminus \{x\}$ *('x' is a bound in exp)*

$fv(exp_1\ exp_2) = fv(exp_1) \cup fv(exp_2)$

See tc.ml

STATICALLY RULING OUT PARTIALITY: TYPE CHECKING

Adding Integers to Lambda Calculus

exp ::=
- | …
- | n — *constant integers*
- | $exp_1 + exp_2$ — *binary arithmetic operation*

val ::=
- | fun x -> exp — *functions are values*
- | n — *integers are values*

$n\{v/x\}$	$= n$	*constants have no free vars.*
$(e_1 + e_2)\{v/x\}$	$= (e_1\{v/x\} + e_2\{v/x\})$	*substitute everywhere*

$$\frac{exp_1 \Downarrow n_1 \quad exp2 \Downarrow n_2}{exp_1 + exp_2 \;\Downarrow (n1 \; [\![+]\!] \; n2)}$$

Object-level '+'

Meta-level '+'

NOTE: there are no rules for the case where exp1 or exp2 evaluate to functions! The semantics is *undefined* in those cases.

Type Checking / Static Analysis

- Recall the interpreter from the Eval3 module:

```
let rec eval env e =
 match e with
 | …
 | Add (e1, e2) ->
     (match (eval env e1, eval env e2) with
       | (IntV i1, IntV i2) -> IntV (i1 + i2)
       | _ -> failwith "tried to add non-integers")
 | …
```

- The interpreter might fail at runtime.
  - Not all operations are defined for all values (*e.g.*, 3/0, 3 + true, …)
- A compiler can’t generate sensible code for this case.
  - A naïve implementation might “add” an integer and a function pointer

Type Judgments

- In the judgment: $E \vdash e : t$
  - E is a ***typing environment*** or a ***type context***
  - E maps variables to types. It is just a set of bindings of the form: $x_1 : t_1, x_2 : t_2, \ldots, x_n : t_n$
- For example: $x : \text{int}, b : \text{bool} \vdash \text{if (b) 3 else x} : \text{int}$
- What do we need to know to decide whether "if (b) 3 else x" has type int in the environment x : int, b : bool?
  - b must be a bool i.e. $x : \text{int}, b : \text{bool} \vdash b : \text{bool}$
  - 3 must be an int i.e. $x : \text{int}, b : \text{bool} \vdash 3 : \text{int}$
  - x must be an int i.e. $x : \text{int}, b : \text{bool} \vdash x : \text{int}$

Simply-typed Lambda Calculus

- For the language in “tc.ml” we have five inference rules:

INT

$$\frac{}{E \vdash i : \text{int}}$$

VAR

$$\frac{x : T \in E}{E \vdash x : T}$$

ADD

$$\frac{E \vdash e_1 : \text{int} \qquad E \vdash e_2 : \text{int}}{E \vdash e_1 + e_2 : \text{int}}$$

FUN

$$\frac{E, x : T \vdash e : S}{E \vdash \text{fun } (x{:}T) \text{ -> } e \ : T \text{ -> } S}$$

APP

$$\frac{E \vdash e_1 : T \text{ -> } S \qquad E \vdash e_2 : T}{E \vdash e_1\ e_2 : S}$$

- Note how these rules correspond to the code.

Type Checking Derivations

- A *derivation* or *proof tree* has (instances of) judgments as its nodes and edges that connect premises to a conclusion according to an inference rule.
- Leaves of the tree are *axioms* (i.e. rules with no premises)
  - Example: the INT rule is an axiom
- Goal of the typechecker: verify that such a tree exists.
- Example: Find a tree for the following program using the inference rules on the previous slide:

⊢ (fun (x:int) -> x + 3) 5 : int

Example Derivation Tree

$$\text{APP}\ \dfrac{\text{FUN}\ \dfrac{\text{ADD}\ \dfrac{\text{VAR}\ \dfrac{x : \text{int} \in x : \text{int}}{x : \text{int} \vdash x : \text{int}} \qquad \text{INT}\ \dfrac{}{x : \text{int} \vdash 3 : \text{int}}}{x : \text{int} \vdash x + 3 : \text{int}}}{\vdash (\text{fun } (x{:}\text{int}) \text{ -> } x + 3) : \text{int -> int}} \qquad \text{INT}\ \dfrac{}{\vdash 5 : \text{int}}}{\vdash (\text{fun } (x{:}\text{int}) \text{ -> } x + 3)\ 5 : \text{int}}$$

- Note: the OCaml function **typecheck** verifies the existence of this tree. The structure of the recursive calls when running **typecheck** is the same shape as this tree!
- Note that x : int ∈ E is implemented by the function **lookup**

Notes about this Typechecker

- The interpreter evaluates the body of a function only when it's applied.
- The typechecker always checks the body of the function
  - even if it's never applied
  - We *assume* the input has some type (say t_1) and reflect this in the type of the function (t_1 -> t_2).

- Dually, at a call site (e_1 e_2), we don't know what *closure* we're going to get.
  - But we can calculate e_1's type, check that e_2 is an argument of the right type, and determine what type e_1 will return.

- Question: Why is this an approximation?
- Question: What if well_typed always returns false?

oat.pdf

TYPECHECKING OAT

Checking Derivations

- A *derivation* or *proof tree* has (instances of) judgments as its nodes and edges that connect premises to a conclusion according to an inference rule.
- Leaves of the tree are *axioms* (i.e. rules with no premises)
  - Example: the INT rule is an axiom
- Goal of the type checker: verify that such a tree exists.
- Example1: Find a tree for the following program using the inference rules in oat.pdf:

```
var x1 = 0;
var x2 = x1 + x1;
x1 = x1 – x2;
return(x1);
```

Example2: There is no tree for this ill-scoped program:

```
var x2 = x1 + x1;
return(x2);
```

Example Derivation

```
var x1 = 0;
var x2 = x1 + x1;
x1 = x1 – x2;
return(x1);
```

$$\dfrac{\dfrac{\mathcal{D}_1 \quad \mathcal{D}_2 \quad \mathcal{D}_3 \quad \mathcal{D}_4}{G_0\,;\,\cdot\,;\texttt{int} \vdash \texttt{var}\ x_1 = 0;\ \texttt{var}\ x_2 = x_1 + x_1\,;\, x_1 = x_1 - x_2\,;\,\texttt{return}\ x_1\,;\ \Rightarrow\ \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}\,[\textsc{stmts}]}{\vdash \texttt{var}\ x_1 = 0;\ \texttt{var}\ x_2 = x_1 + x_1\,;\, x_1 = x_1 - x_2\,;\,\texttt{return}\ x_1\,;}\,[\textsc{prog}]$$

Example Derivation

$$\mathcal{D}_1 \quad = \quad \dfrac{\dfrac{\dfrac{\dfrac{}{G_0\,;\cdot \vdash 0 : \texttt{int}}\;[\textsc{INT}]}{G_0\,;\cdot \vdash 0 : \texttt{int}}\;[\textsc{CONST}]}{G_0\,;\cdot \vdash \texttt{var}\ x_1 = 0 \Rightarrow \cdot, x_1 : \texttt{int}}\;[\textsc{DECL}]}{G_0\,;\,\cdot\,;\texttt{int} \vdash \texttt{var}\ x_1 = 0;\ \Rightarrow \cdot, x_1 : \texttt{int}}\;[\textsc{SDECL}]$$

$$\mathcal{D}_2 \quad = \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\vdash + : (\texttt{int},\texttt{int}) \rightarrow \texttt{int}}\;[\textsc{ADD}] \quad \dfrac{x_1 : \texttt{int} \in \cdot, x_1 : \texttt{int}}{G_0\,;\cdot, x_1 : \texttt{int} \vdash x_1 : \texttt{int}}\;[\textsc{VAR}] \quad \dfrac{x_1 : \texttt{int} \in \cdot, x_1 : \texttt{int}}{G_0\,;\cdot, x_1 : \texttt{int} \vdash x_1 : \texttt{int}}\;[\textsc{VAR}]}{G_0\,;\cdot, x_1 : \texttt{int} \vdash x_1 + x_1 : \texttt{int}}\;[\textsc{BOP}]}{G_0\,;\cdot, x_1 : \texttt{int}\,;\texttt{int} \vdash \texttt{var}\ x_2 = x_1 + x_1;\ \Rightarrow \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}\;[\textsc{DECL}]}{G_0\,;\cdot, x_1 : \texttt{int}\,;\texttt{int} \vdash \texttt{var}\ x_2 = x_1 + x_1;\ \Rightarrow \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}\;[\textsc{SDECL}]$$

Example Derivation

$$x_1 : \mathtt{int} \in \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int}\ ;$$

$$\mathcal{D}_3 \quad \dfrac{\dfrac{\dfrac{}{\vdash \mathtt{-} : (\mathtt{int}, \mathtt{int}) \rightarrow \mathtt{int}}\,[\textsc{Add}] \quad \dfrac{x_1 : \mathtt{int} \in \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int}}{G_0\,; \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int} \vdash x_1 : \mathtt{int}}\,[\textsc{Var}] \quad \dfrac{x_2 : \mathtt{int} \in \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int}}{G_0\,; \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int} \vdash x_2 : \mathtt{int}}\,[\textsc{Var}]}{G_0\,; \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int} \vdash x_1 \mathtt{-} x_2 : \mathtt{int}}\,[\textsc{Bop}]}{G_0\,; \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int}\,; \mathtt{int} \vdash x_1 \mathtt{=} x_1 \mathtt{-} x_2\mathtt{;} \Rightarrow \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int}}\,[\textsc{Assn}]$$

$$\mathcal{D}_4 \quad = \quad \dfrac{\dfrac{x_1 : \mathtt{int} \in \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int}}{G_0\,; \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int} \vdash x_1 : \mathtt{int}}\,[\textsc{Var}]}{G_0\,; \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int}\,; \mathtt{int} \vdash \mathtt{return}\ x_1\mathtt{;} \Rightarrow \cdot, x_1 : \mathtt{int}, x_2 : \mathtt{int}}\,[\textsc{Ret}]$$

Type Safety

"Well typed programs do not go wrong."

– Robin Milner, 1978

Theorem: (simply typed lambda calculus with integers)

If $\vdash e : t$ then there exists a value v such that $e \Downarrow v$.

- Note: this is a *very* strong property.
  - Well-typed programs cannot "go wrong" by trying to execute undefined code (such as 3 + (fun x -> 2))
  - Simply-typed lambda calculus is guaranteed to terminate! (i.e. it isn't Turing complete)

Type Safety For General Languages

Theorem: (Type Safety)

If $\vdash P : t$ is a well-typed program, then either:

(a) the program terminates in a well-defined way, or

(b) the program continues computing forever

- Well-defined termination could include:
  - halting with a return value
  - raising an exception

- Type safety rules out undefined behaviors:
  - abusing "unsafe" casts: converting pointers to integers, etc.
  - treating non-code values as code (and vice-versa)
  - breaking the type abstractions of the language
- What is "defined" depends on the language semantics…

Why Inference Rules?

- They are a compact, precise way of specifying language properties.
  - E.g. ~20 pages for full Java vs. 100's of pages of prose Java Language Spec.

- Inference rules correspond closely to the recursive AST traversal that implements them

- Type checking (and type inference) is nothing more than attempting to prove a different judgment (E ⊢ e : t) by searching backwards through the rules.

- Compiling in a context is nothing more than a collection of inference rules specifying yet a different judgment (G ⊢ src ⇒ target)
  - Moreover, the compilation rules are very similar in structure to the typechecking rules

- Strong mathematical foundations
  - The "Curry-Howard-Lambek correspondence":
    - Programming Language : Logic : Category theory : Order Theory
    - Programs : Proof : Morphism : Inequality
    - Type : Proposition : Object : Element
    - See EECS 490, 590, my 598 if you're interested in type systems!

COMPILING

Compilation As Translating Judgments

- Consider the source typing judgment for source expressions:

$$C \vdash e : t$$

- How do we interpret this information in the target language?

$$[\![C \vdash e : t]\!] = \quad ?$$

- $[\![C]\!]$ translates contexts
- $[\![t]\!]$ is a target type
- $[\![e]\!]$ translates to a (potentially empty) stream of instructions, that, when run, computes the result into some operand

- INVARIANT: if $[\![C \vdash e : t\,]\!]$ = ty, operand , stream
  then the type (at the target level) of the operand is ty=$[\![t]\!]$

Example

- $C \vdash 341 + 5 : \text{int}$ what is $[\![C \vdash 341 + 5 : \text{int}]\!]$?

$$\dfrac{\dfrac{[\![\vdash 341 : \text{int}]\!] = \text{(i64, Const 341, [])}}{[\![C \vdash 341 : \text{int}]\!] = \text{(i64, Const 341, [])}} \qquad \dfrac{[\![\vdash 5 : \text{int}]\!] = \text{(i64, Const 5, [])}}{[\![C \vdash 5 : \text{int}]\!] = \text{(i64, Const 5, [])}}}{[\![C \vdash 341 + 5 : \text{int}]\!] = \text{(i64, \%tmp, [\%tmp = add i64 (Const 341) (Const 5)])}}$$

What about the Context?

- What is ⟦C⟧?
- Source level C has bindings like: x:int, y:bool
  - We think of it as a finite map from identifiers to types

- What is the interpretation of C at the target level?

- ⟦C⟧ maps source identifiers, “x” to source types and ⟦x⟧

- What is the interpretation of a variable ⟦x⟧ at the target level?
  - How are the variables used in the type system?

$$\frac{x{:}t \in L}{G;L \vdash x : t}\ \textbf{TYP\_VAR}$$

as expressions
(which denote values)

$$\frac{x{:}t \in L \quad G;L \vdash exp : t}{G;L;rt \vdash x = exp; \Rightarrow L}\ \textbf{TYP\_ASSN}$$

as addresses
(which can be assigned)

Interpretation of Contexts

- ⟦C⟧ = a map from source identifiers to types and target identifiers

- INVARIANT:
  x:t ∈ C means that

  (1) lookup ⟦C⟧ x = (t, %id_x)
  (2) the (target) type of %id_x is ⟦t⟧* (a pointer to ⟦t⟧)

Interpretation of Variables

- Establish invariant for expressions:

$$\left[\!\!\left[\frac{x : t \in L}{G ; L \vdash x : t} \;\; \textbf{TYP\_VAR} \right]\!\!\right] = \text{(\%tmp, [\%tmp = load i64* \%id\_x])}$$

as expressions
(which denote values)

where (i64, %id_x) = lookup ⟦L⟧ x

- What about statements?

$$\left[\!\!\left[\frac{x : t \in L \quad G ; L \vdash exp : t}{G ; L ; rt \vdash x = exp ; \Rightarrow L} \;\; \textbf{TYP\_ASSN} \right]\!\!\right] = \text{stream @}$$

[store ⟦t⟧ opn, ⟦t⟧* %id_x]

as addresses
(which can be assigned)

where (t, %id_x) = lookup ⟦L⟧ x
and ⟦G;L ⊢ exp : t⟧ = (⟦t⟧, opn, stream)

Other Judgments?

- Statement:
  ⟦C; rt ⊢ stmt ⇒ C′⟧ = ⟦C′⟧ , stream

- Declaration:
  ⟦G;L ⊢ t x = exp ⇒ G;L,x:t ⟧ = ⟦G;L,x:t⟧, stream

  INVARIANT: stream is of the form:
  stream' @
  [%id_x = alloca ⟦t⟧;
  store ⟦t⟧ opn, ⟦t⟧* %id_x]

  and ⟦G;L ⊢ exp : t ⟧ = (⟦t⟧, opn, stream′)

- Rest follow similarly

COMPILING CONTROL

Translating while

- Consider translating "while(e) s":
  - Test the conditional, if true jump to the body, else jump to the label after the body.

⟦C;rt ⊢ while(e) s ⇒ C'⟧ = ⟦C'⟧,

```
lpre:
        opn = ⟦C ⊢ e : bool⟧
        %test = icmp eq i1 opn, 0
        br %test, label %lpost, label %lbody
lbody:
    ⟦C;rt ⊢ s ⇒ C'⟧
    br %lpre
lpost:
```

- Note: writing opn = ⟦C ⊢ e : bool⟧ is pun
  - translating ⟦C ⊢ e : bool⟧ generates *code* that puts the result into opn
  - In this notation there is implicit collection of the code

Translating if-then-else

- Similar to while except that code is slightly more complicated because if-then-else must reach a merge and the else branch is optional.

$$[\![C;rt \vdash \text{if } (e_1)\ s_1 \text{ else } s_2 \Rightarrow C']\!] = [\![C']\!]$$

```
        opn = $[\![C \vdash e : bool]\!]$
        %test = icmp eq i1 opn, 0
        br %test, label %else, label %then
then:
    $[\![C;rt \vdash s_1 \Rightarrow C']\!]$
    br %merge
else:
        $[\![C; rt\ s_2 \Rightarrow C']\!]$
    br %merge
merge:
```

Connecting this to Code

- Instruction streams:
  - Must include labels, terminators, and “hoisted” global constants
- Must post-process the stream into a control-flow-graph
- See frontend.ml from HW4

OPTIMIZING CONTROL

Standard Evaluation

- Consider compiling the following program fragment:

```
if (x & !y | !w)
  z = 3;
else
  z = 4;
return z;
```

```
        %tmp1 = icmp Eq ⟦y⟧, 0      ; !y
        %tmp2 = and ⟦x⟧ ⟦tmp1⟧
        %tmp3 = icmp Eq ⟦w⟧, 0
        %tmp4 = or %tmp2, %tmp3
        %tmp5 = icmp Eq %tmp4, 0
        br %tmp4, label %else, label %then

then:
        store ⟦z⟧, 3
        br %merge

else:
        store ⟦z⟧, 4
        br %merge

merge:
        %tmp5 = load ⟦z⟧
        ret %tmp5
```

Observation

- Usually, we want the translation ⟦e⟧ to produce a value
  - $\llbracket C \vdash e : t \rrbracket = (ty, operand, stream)$
  - e.g. $\llbracket C \vdash e_1 + e_2 : \text{int} \rrbracket = (\text{i64}, \%\text{tmp}, [\%\text{tmp} = \text{add } \llbracket e_1 \rrbracket \ \llbracket e_2 \rrbracket])$

- But when the expression we're compiling appears in a test, the program jumps to one label or another after the comparison but otherwise never uses the value.

- In many cases, we can avoid "materializing" the value (i.e. storing it in a temporary) and thus produce better code.
  - This idea also lets us implement different functionality too: e.g. short-circuiting boolean expressions

Idea: Use a different translation for tests

Usual Expression translation:

$[\![C \vdash e : t]\!] = (ty, operand, stream)$

Conditional branch translation of booleans, without materializing the value:

$[\![C \vdash e : bool@]\!]$ ltrue lfalse = stream

$[\![C, rt \vdash \text{if } (e) \text{ then } s1 \text{ else } s2 \Rightarrow C']\!] = [\![C']\!],$

```
      insns3
then:
   [[s1]]
   br %merge
else:
      [[s2]]
   br %merge
merge:
```

Notes:

- takes two extra arguments: a "true" branch label and a "false" branch label.
- Doesn't "return a value"
- Aside: this is a form of continuation-passing translation…

where

$[\![C, rt \vdash s_1 \Rightarrow C']\!] = [\![C']\!], insns_1$

$[\![C, rt \vdash s_2 \Rightarrow C'']\!] = [\![C'']\!], insns_2$

$[\![C \vdash e : bool@]\!]$ then else = $insns_3$

Short Circuit Compilation: Expressions

- ⟦C ⊢ e : bool@⟧ ltrue lfalse = insns

$$\frac{}{\llbracket C \vdash \mathrm{false} : \mathrm{bool@} \rrbracket\ \mathrm{ltrue}\ \mathrm{lfalse} = [\mathtt{br\ \%lfalse}]}\ \mathrm{FALSE}$$

$$\frac{}{\llbracket C \vdash \mathrm{true} : \mathrm{bool@} \rrbracket\ \mathrm{ltrue}\ \mathrm{lfalse} = [\mathtt{br\ \%ltrue}]}\ \mathrm{TRUE}$$

$$\frac{\llbracket C \vdash e : \mathrm{bool@} \rrbracket\ \mathrm{lfalse}\ \mathrm{ltrue} = \mathrm{insns}}{\llbracket C \vdash !e : \mathrm{bool@} \rrbracket\ \mathrm{ltrue}\ \mathrm{lfalse} = \mathrm{insns}}\ \mathrm{NOT}$$

Short Circuit Evaluation

Idea: build the logic into the translation

$$\frac{[\![C \vdash \mathrm{e1} : \mathrm{bool@}]\!]\ \mathrm{ltrue}\ \mathrm{right} = \mathrm{insns}_1 \qquad [\![C \vdash \mathrm{e2} : \mathrm{bool@}]\!]\ \mathrm{ltrue}\ \mathrm{lfalse} = \mathrm{insns}_2}{[\![C \vdash \mathrm{e1|e2} : \mathrm{bool@}]\!]\ \mathrm{ltrue}\ \mathrm{lfalse} =}$$

insns_1
right:
insn_2

$$\frac{[\![C \vdash \mathrm{e1} : \mathrm{bool@}]\!]\ \mathrm{right}\ \mathrm{lfalse} = \mathrm{insns}_1 \qquad [\![C \vdash \mathrm{e2} : \mathrm{bool@}]\!]\ \mathrm{ltrue}\ \mathrm{lfalse} = \mathrm{insns}_2}{[\![C \vdash \mathrm{e1\&e2} : \mathrm{bool@}]\!]\ \mathrm{ltrue}\ \mathrm{lfalse} =}$$

insns_1
right:
insn_2

where **right** is a fresh label

Short-Circuit Evaluation

- Consider compiling the following program fragment:

```
if (x & !y | !w)
  z = 3;
else
  z = 4;
return z;
```

```
        %tmp1 = icmp Eq ⟦x⟧, 0
        br %tmp1, label %right2, label %right1

right1:
        %tmp2 = icmp Eq ⟦y⟧, 0
        br %tmp2, label %then, label %right2

right2:
        %tmp3 = icmp Eq ⟦w⟧, 0
        br %tmp3, label %then, label %else

then:
        store ⟦z⟧, 3
        br %merge

else:
        store ⟦z⟧, 4
        br %merge

merge:
        %tmp5 = load ⟦z⟧
        ret %tmp5
```

Beyond describing “structure”… describing “properties”

Types as sets

Subsumption

TYPES, MORE GENERALLY

Arrays

- Array constructs are not hard
- First: add a new type constructor: T[]

NEW

$$\frac{E \vdash e_1 : \text{int} \qquad E \vdash e_2 : T}{E \vdash \text{new } T[e_1](e_2) \ : T[\,]}$$

e_1 is the size of the newly allocated array. e_2 initializes the elements of the array.

INDEX

$$\frac{E \vdash e_1 : T[\,] \qquad E \vdash e_2 : \text{int}}{E \vdash e_1[e_2] \ : T}$$

UPDATE

$$\frac{E \vdash e_1 : T[\,] \qquad E \vdash e_2 : \text{int} \quad E \vdash e_3 : T}{E \vdash e_1[e_2] = e_3 \text{ ok}}$$

Note: These rules don't ensure that the array index is in bounds – that should be checked *dynamically*.

Tuples

- ML-style tuples with statically known number of products:
- First: add a new type constructor: $T_1 * \ldots * T_n$

TUPLE

$$\frac{E \vdash e_1 : T_1 \quad \ldots \quad E \vdash e_n : T_n}{E \vdash (e_1, \ldots, e_n) : T_1 * \ldots * T_n}$$

PROJ

$$\frac{E \vdash e : T_1 * \ldots * T_n \quad 1 \leq i \leq n}{E \vdash \#i\ e \ : \ T_i}$$

References

- ML-style references (note that ML uses only expressions)
- First, add a new type constructor: T ref

REF

$$\frac{E \vdash e : T}{E \vdash \text{ref } e : T \text{ ref}}$$

DEREF

$$\frac{E \vdash e : T \text{ ref}}{E \vdash \,!e \;: T}$$

ASSIGN

$$\frac{E \vdash e_1 : T \text{ ref} \qquad E \vdash e_2 : T}{E \vdash e_1 := e_2 \;: \text{unit}}$$

Note the similarity with the rules for arrays…