Lecture 22

EECS 483: COMPILER CONSTRUCTION

Announcements

- HW5: OAT v. 2.0
  - Fully released now
  - Due on Friday, April 12
- HW6: Analysis and Optimization
  - Introduce in discussion
  - Due on Thursday, May 2
- Guest lectures
  - Lectures on Optimization and Dataflow analysis – Eric
  - Next lectures: Cyrus (4/15), Steven (4/17), Max (last two lectures)

AVAILABLE EXPRESSIONS

Available Expressions

- Idea: want to perform common subexpression elimination:
  - a = x + 1 → a = x + 1
    … → …
    b = x + 1 → b = a
- This transformation is safe if x+1 means computes the same value at both places (i.e. x hasn't been assigned).
  - "x+1" is an *available expression*
- Dataflow values:
  - in[n] = set of nodes whose values are available on entry to n
  - out[n] = set of nodes whose values are available on exit of n

Available Expressions Step 1

- Define the sets of values
- Define gen[n] and kill[n] as follows:
- Quadruple forms n:

Quadruple forms n	gen[n]	kill[n]
a = b op c	{n} - kill[n]	uses[a]
a = load b	{n} - kill[n]	uses[a]
store b, a	Ø	uses[[x]] (for all x that may equal a)
br L	Ø	Ø
br a L1 L2	Ø	Ø
$a = f(b_1,\ldots,b_n)$	Ø	uses[a]∪ uses[[x]] (for all x)
$f(b_1,\ldots,b_n)$	Ø	uses[[x]] (for all x)
return a	Ø	Ø

Note the need for “may alias” information…

Note that functions are assumed to be impure…

Available Expressions Step 2

- Define the constraints that an available expressions solution must satisfy.
- $out[n] \supseteq gen[n]$
  "The expressions made available by n that reach the end of the node"

- $in[n] \subseteq out[n']$ if n' is in $pred[n]$
  "The expressions available at the beginning of a node include those that reach the exit of *every* predecessor"

- $out[n] \cup kill[n] \supseteq in[n]$
  "The expressions available on entry either reach the end of the node or are killed by it."
  - Equivalently: $out[n] \supseteq in[n] - kill[n]$

Note similarities and differences with constraints for "reaching definitions".

Available Expressions Step 3

- Convert constraints to iterated update equations:
- $in[n] := \bigcap_{n' \in pred[n]} out[n']$
- $out[n] := gen[n] \cup (in[n] - kill[n])$

- Algorithm: initialize in[n] and out[n] to {set of all nodes}
  - Iterate the update equations until a fixed point is reached

- The algorithm terminates because in[n] and out[n] *decrease* only *monotonically*
  - At most to a minimum of the empty set
- The algorithm is precise because it finds the *largest* sets that satisfy the constraints.

GENERAL DATAFLOW ANALYSIS

Comparing Dataflow Analyses

- Look at the update equations in the inner loop of the analyses
- Liveness: (backward)
  - Let $gen[n] = use[n]$ and $kill[n] = def[n]$
  - $out[n] := = \bigcup_{n' \in succ[n]} in[n']$
  - $in[n] := gen[n] \cup (out[n] - kill[n])$

- Reaching Definitions: (forward)
  - $in[n] := \bigcup_{n' \in pred[n]} out[n']$
  - $out[n] := gen[n] \cup (in[n] - kill[n])$

- Available Expressions: (forward)
  - $in[n] := \bigcap_{n' \in pred[n]} out[n']$
  - $out[n] := gen[n] \cup (in[n] - kill[n])$

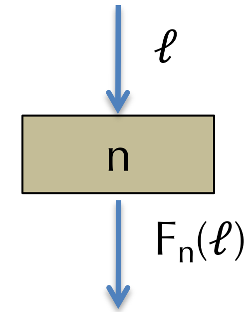
Common Features

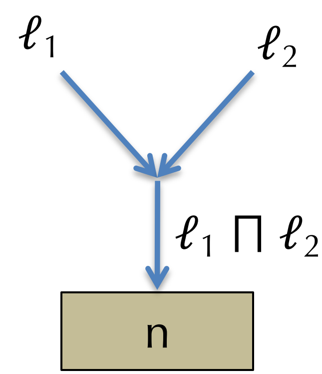
- All of these analyses have a *domain* over which they solve constraints.
  - Liveness, the domain is sets of variables
  - Reaching defns., Available exprs. the domain is sets of nodes
- Each analysis has a notion of gen[n] and kill[n]
  - Used to explain how information propagates across a node.
- Each analysis is propagates information either *forward* or *backward*
  - Forward: in[n] defined in terms of predecessor nodes' out[]
  - Backward: out[n] defined in terms of successor nodes' in[]
- Each analysis has a way of aggregating information
  - Liveness & reaching definitions take union (∪)
  - Available expressions uses intersection (∩)
  - Union expresses a property that holds for *some* path (existential)
  - Intersection expresses a property that holds for *all* paths (universal)

(Forward) Dataflow Analysis Framework

A forward dataflow analysis can be characterized by:

1. A domain of dataflow values $\mathcal{L}$
   - e.g. $\mathcal{L}$ = the powerset of all variables
   - Think of $\ell \in \mathcal{L}$ as a property, then "x $\in \ell$" means "x has the property"
2. For each node n, a flow function $F_n : \mathcal{L} \rightarrow \mathcal{L}$
   - So far we've seen $F_n(\ell) = \text{gen}[n] \cup (\ell - \text{kill}[n])$
   - So: $\text{out}[n] = F_n(\text{in}[n])$
   - "If ℓ is a property that holds before the node n, then $F_n(\ell)$ holds after n"
3. A combining operator $\sqcap$
   - "If we know *either* ℓ_1 *or* ℓ_2 holds on entry to node n, we know at most $\ell_1 \sqcap \ell_2$"
   - $\text{in}[n] := \bigsqcap_{n' \in \text{pred}[n]} \text{out}[n']$





Generic Iterative (Forward) Analysis

for all n, in[n] := $\top$, out[n] := $\top$
repeat until no change
    for all n

$$in[n] := \sqcap_{n' \in pred[n]} out[n']$$

    out[n] := $F_n(in[n])$
    end
end

- Here, $\top \in \mathcal{L}$ ("top") represents having the "maximum" amount of information.
  - Having "more" information enables more optimizations
  - "Maximum" amount could be inconsistent with the constraints.
  - Iteration refines the answer, eliminating inconsistencies

Structure of $\mathcal{L}$

- The domain has structure that reflects the "amount" of information contained in each dataflow value.
- Some dataflow values are more informative than others:
  - Write $\ell_1 \sqsubseteq \ell_2$ whenever ℓ_2 provides at least as much information as ℓ_1.
  - The dataflow value ℓ_2 is "better" for enabling optimizations.

- Example 1: for liveness analysis, *smaller* sets of variables are more informative.
  - Having smaller sets of variables live across an edge means that there are fewer conflicts for register allocation assignments.
  - So: $\ell_1 \sqsubseteq \ell_2$ if and only if $\ell_1 \supseteq \ell_2$

- Example 2: for available expressions analysis, larger sets of nodes are more informative.
  - Having a larger set of nodes (equivalently, expressions) available means that there is more opportunity for common subexpression elimination.
  - So: $\ell_1 \sqsubseteq \ell_2$ if and only if $\ell_1 \subseteq \ell_2$

$\mathcal{L}$ as a Partial Order

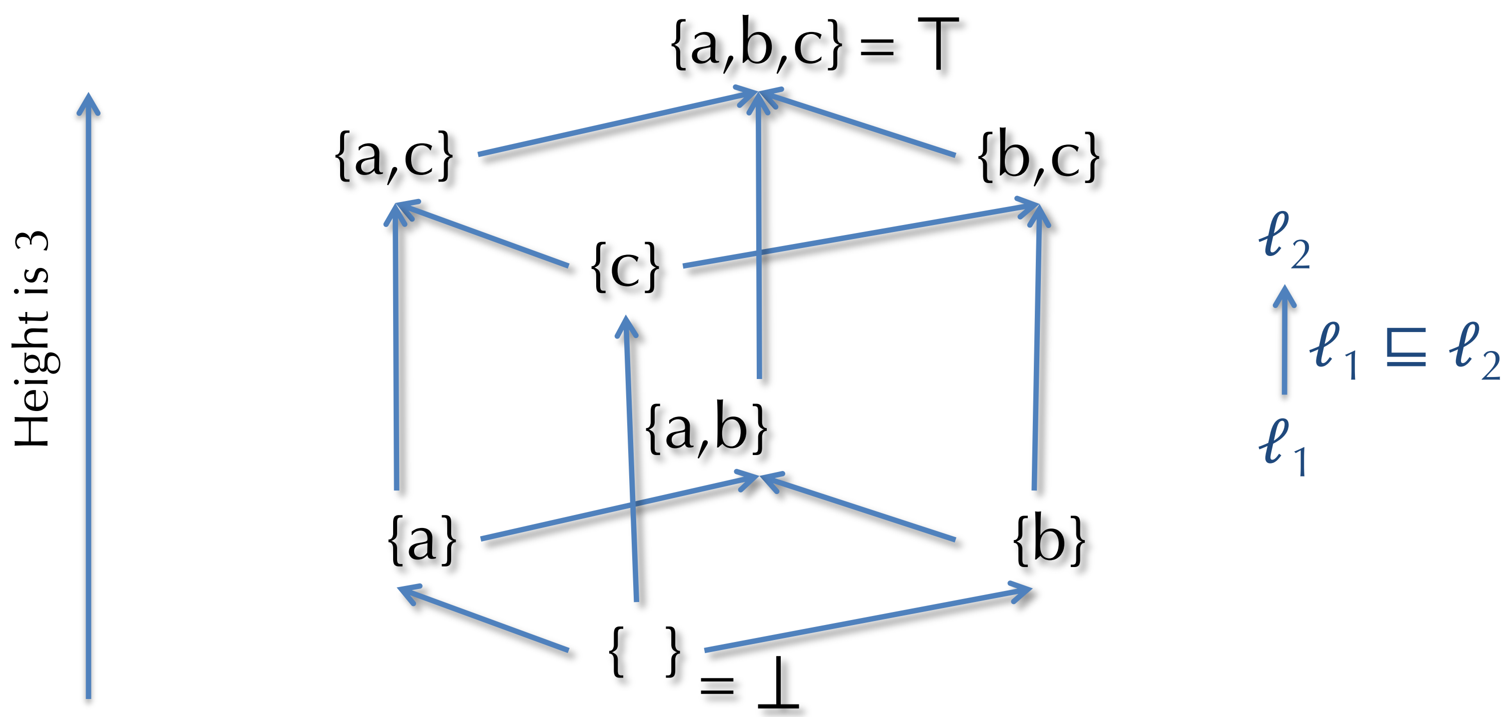
- $\mathcal{L}$ is a *partial order* defined by the ordering relation $\sqsubseteq$.
- A partial order is an ordered set.
- Some of the elements might be *incomparable*.
  - That is, there might be $\ell_1, \ell_2 \in \mathcal{L}$ such that neither $\ell_1 \sqsubseteq \ell_2$ nor $\ell_2 \sqsubseteq \ell_1$

- Properties of a partial order:
  - *Reflexivity:* $\ell \sqsubseteq \ell$
  - *Transitivity:* $\ell_1 \sqsubseteq \ell_2$ and $\ell_2 \sqsubseteq \ell_3$ implies $\ell_1 \sqsubseteq \ell_2$
  - *Anti-symmetry:* $\ell_1 \sqsubseteq \ell_2$ and $\ell_2 \sqsubseteq \ell_1$ implies $\ell_1 = \ell_2$

- Examples:
  - Integers ordered by $\leq$
  - Types ordered by <:
  - Sets ordered by $\subseteq$ or $\supseteq$

Subsets of {a,b,c} ordered by ⊆

Partial order presented as a Hasse diagram.



order $\sqsubseteq$ is $\subseteq$ meet $\sqcap$ is $\cap$ join $\sqcup$ is $\cup$

Meets and Joins

- The combining operator $\sqcap$ is called the "meet" operation.
- It constructs the *greatest lower bound*:
  - $\ell_1 \sqcap \ell_2 \sqsubseteq \ell_1$ and $\ell_1 \sqcap \ell_2 \sqsubseteq \ell_2$
    "the meet is a lower bound"
  - If $\ell \sqsubseteq \ell_1$ and $\ell \sqsubseteq \ell_2$ then $\ell \sqsubseteq \ell_1 \sqcap \ell_2$
    "there is no greater lower bound"

- Dually, the $\sqcup$ operator is called the "join" operation.
- It constructs the *least upper bound*:
  - $\ell_1 \sqsubseteq \ell_1 \sqcup \ell_2$ and $\ell_2 \sqsubseteq \ell_1 \sqcup \ell_2$
    "the join is an upper bound"
  - If $\ell_1 \sqsubseteq \ell$ and $\ell_2 \sqsubseteq \ell$ then $\ell_1 \sqcup \ell_2 \sqsubseteq \ell$
    "there is no smaller upper bound"

- A partial order that has all meets and joins is called a *lattice*.
  - If it has just meets, it's called a *meet semi-lattice*.

Another Way to Describe the Algorithm

- Algorithm repeatedly computes (for each node n):
- $\text{out}[n] := F_n(\text{in}[n])$

- Equivalently: $\text{out}[n] := F_n(\bigsqcap_{n' \in \text{pred}[n]} \text{out}[n'])$
  - By definition of in[n]
- We can write this as a simultaneous update of the vector of out[n] values:
  - let $x_n = \text{out}[n]$
  - Let $\mathbf{X} = (x_1, x_2, \ldots, x_n)$ it's a vector of points in $\mathcal{L}$
  - $\mathbf{F}(\mathbf{X}) = (F_1(\bigsqcap_{j \in \text{pred}[1]} \text{out}[j]), F_2(\bigsqcap_{j \in \text{pred}[2]} \text{out}[j]), \ldots, F_n(\bigsqcap_{j \in \text{pred}[n]} \text{out}[j]))$

- Any solution to the constraints is a *fixpoint* **X** of **F**
  - i.e. $\mathbf{F}(\mathbf{X}) = \mathbf{X}$

Iteration Computes Fixpoints

- Let $\mathbf{X}_0 = (\top, \top, \ldots, \top)$
- Each loop through the algorithm apply F to the old vector:
  $\mathbf{X}_1 = \mathbf{F}(\mathbf{X}_0)$
  $\mathbf{X}_2 = \mathbf{F}(\mathbf{X}_1)$
  …
- $\mathbf{F}^{k+1}(\mathbf{X}) = \mathbf{F}(\mathbf{F}^k(\mathbf{X}))$
- A fixpoint is reached when $\mathbf{F}^k(\mathbf{X}) = \mathbf{F}^{k+1}(\mathbf{X})$
  - That's when the algorithm stops.
- Wanted: a maximal fixpoint
  - Because that one is more informative/useful for performing optimizations

Monotonicity & Termination

- Each flow function F_n maps lattice elements to lattice elements; to be sensible is should be *monotonic*:
- $F : \mathcal{L} \rightarrow \mathcal{L}$ is *monotonic* iff:
  $\ell_1 \sqsubseteq \ell_2$ implies that $F(\ell_1) \sqsubseteq F(\ell_2)$
  - Intuitively: "If you have more information entering a node, then you have more information leaving the node."

- Monotonicity lifts point-wise to the function: $\mathbf{F} : \mathcal{L}^n \rightarrow \mathcal{L}^n$
  - vector $(x_1, x_2, \ldots, x_n) \sqsubseteq (y_1, y_2, \ldots, y_n)$ iff $x_i \sqsubseteq y_i$ for each i

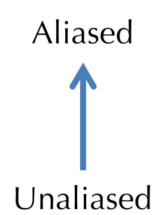
- Note that **F** is consistent: $\mathbf{F}(\mathbf{X}_0) \sqsubseteq \mathbf{X}_0$
  - So each iteration moves at least one step down the lattice (for some component of the vector)
  - $\ldots \sqsubseteq \mathbf{F}(\mathbf{F}(\mathbf{X}_0)) \sqsubseteq \mathbf{F}(\mathbf{X}_0) \sqsubseteq \mathbf{X}_0$

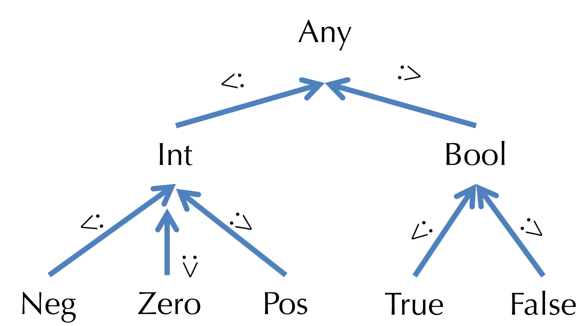
- Therefore, # steps needed to reach a fixpoint is at most the height H of $\mathcal{L}$ times the number of nodes: O(Hn)

Building Lattices?

- Information about individual nodes or variables can be lifted *pointwise:*
  - If $\mathcal{L}$ is a lattice, then so is $\{ f : X \to \mathcal{L} \}$ where $f \sqsubseteq g$ if and only if $f(x) \sqsubseteq g(x)$ for all $x \in X$.

- Like *types*, the dataflow lattices are *static approximations* to the dynamic behavior:
  - Could pick a lattice based on subtyping:
  - Or other information:





- Points in the lattice are sometimes called dataflow "facts"

“Classic” Constant Propagation

- Constant propagation can be formulated as a dataflow analysis.

- Idea: propagate and fold integer constants in one pass:

x = 1;
y = 5 + x;
z = y * y;
	⇒	x = 1;
		y = 6;
		z = 36;

- Information about a single variable:
  - Variable is never defined.
  - Variable has a single, constant value.
  - Variable is assigned multiple values.

Domains for Constant Propagation

- We can make a constant propagation lattice $\mathcal{L}$ for *one variable* like this:

$\top$ = multiple values

…, -3, -2, -1, 0, 1, 2, 3, …

$\bot$ = never defined

- To accommodate multiple variables, we take the product lattice, with one element per variable.
  - Assuming there are three variables, x, y, and z, the elements of the product lattice are of the form (ℓ_x, ℓ_y, ℓ_z).
  - Alternatively, think of the product domain as a context that maps variable names to their "*abstract interpretations*"
- What are "meet" and "join" in this product lattice?
- What is the height of the product lattice?

Flow Functions

- Consider the node $x = y \text{ op } z$
- $F(\ell_x, \ell_y, \ell_z) = ?$

- $F(\ell_x, \top, \ell_z) = (\top, \top, \ell_z)$
- $F(\ell_x, \ell_y, \top) = (\top, \ell_y, \top)$

"If either input might have multiple values the result of the operation might too."

- $F(\ell_x, \perp, \ell_z) = (\perp, \perp, \ell_z)$
- $F(\ell_x, \ell_y, \perp) = (\perp, \ell_y, \perp)$

"If either input is undefined the result of the operation is too."

- $F(\ell_x, i, j) = (i \text{ op } j, i, j)$

"If the inputs are known constants, calculate the output statically."

- Flow functions for the other nodes are easy…
- Monotonic?
- Distributes over meets?

Dataflow Analysis: Summary

- Many dataflow analyses fit into a common framework.
- Key idea: *Iterative solution* of a system of equations over a *lattice* of constraints.
  - Iteration terminates if flow functions are monotonic.

- Dataflow analyses as presented work for an "imperative" intermediate representation.
  - The values of temporary variables are updated ("mutated") during evaluation.
  - Such mutation complicates calculations
  - SSA = "Single Static Assignment" eliminates this problem, by introducing more temporaries – each one assigned to only once.
  - Next up: Converting to SSA, finding loops and dominators in CFGs