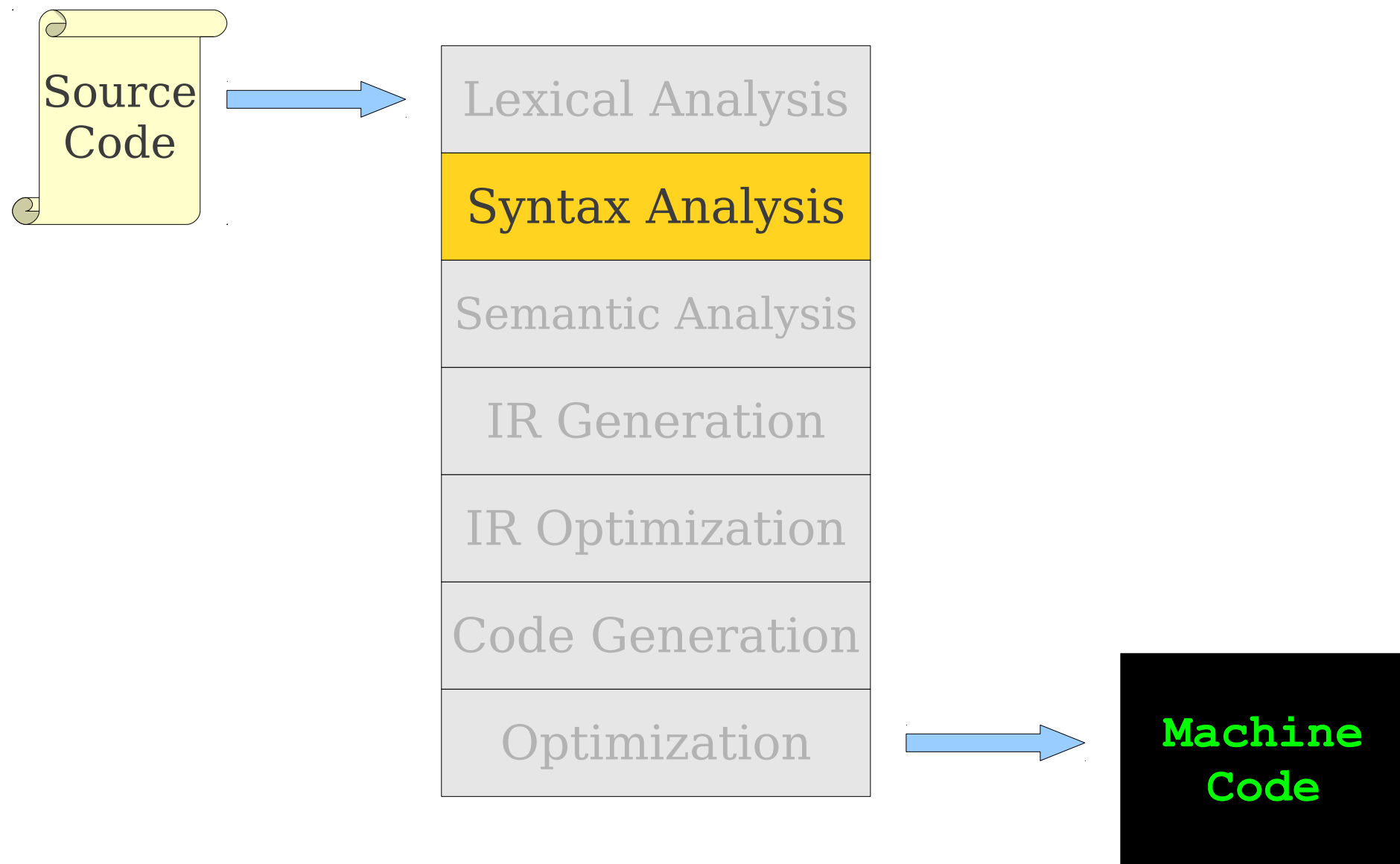


Syntax Analysis

- Introduction to parsing

Where We Are

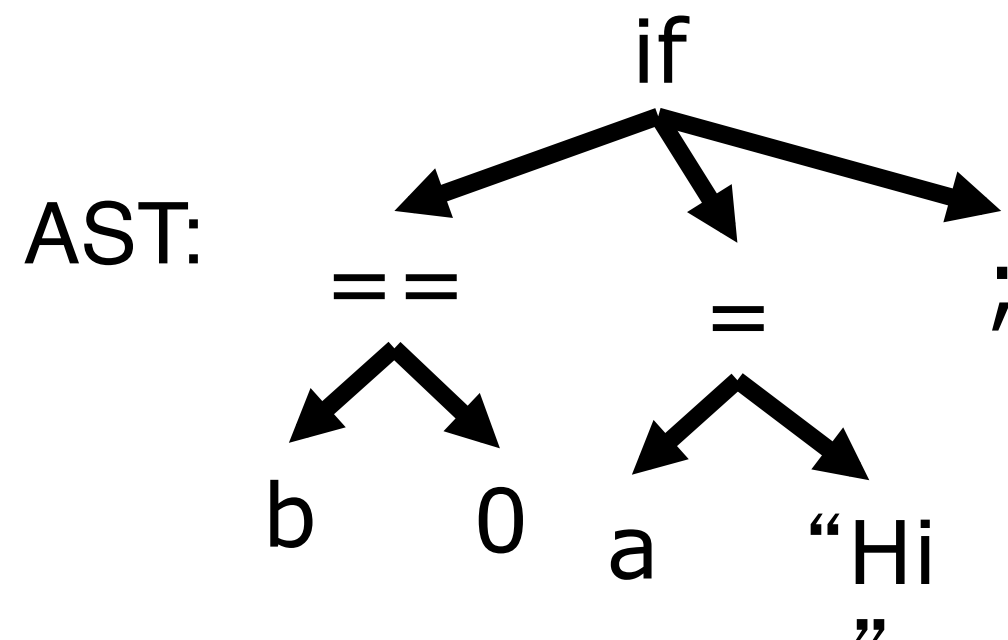


Overview of Syntax Analysis

- Input: stream of tokens from the lexer
- Output: Abstract Syntax Tree (AST)

Source code: `if (b==0) a = "Hi";`

- Report errors if the tokens do not properly encode a structure



What Parsing Doesn't Do

- Doesn't check: type agreement, variable declaration, variables initialization, etc.
- `int x = true;`
- `int y;`
- `z = f(y);`
- Deferred until semantic analysis

Outline

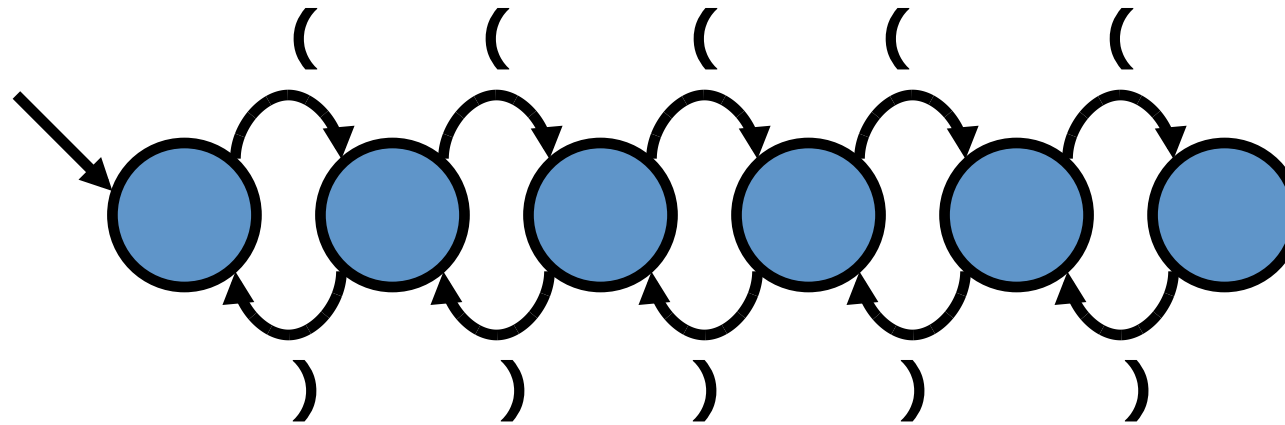
- Today: Formalism for syntax analysis
 - Grammars
 - Derivation
 - Ambiguity

Specify Language Syntax

- First problem: how to describe language syntax ?
 - Lexer: can describe tokens using ____ ?
 - Regular expressions: easy to implement, efficient (DFA)
 - Can we use regular expressions to specify programming language syntax?

To answer this...

- Consider: language of all strings that contain balanced parentheses
 - $() \quad (()) \quad (())() \quad (())()((()))$
 - $((\))(\ ()) \quad (())$
- Construct a Finite Automaton for this...?



- Limits of regular language: DFA has only finite number of states; cannot perform unbounded counting
- Need a More Powerful Representation

Context Free Grammar (CFG)

- Example: A specification of the balanced-parenthesis language:
 - $S \rightarrow (S) S$
 - $S \rightarrow \varepsilon$
- The definition is recursive
- If a grammar accepts a string, there is a **derivation** of that string using the **productions** of the grammar
 - $S \Rightarrow (S) \varepsilon \Rightarrow ((S) S) \varepsilon \Rightarrow ((\varepsilon) \varepsilon) \varepsilon \Rightarrow (())$

CFG Terminology

- **Terminals**

- Token or ε

- **Non-terminals**

- variables

- **Start symbol**

- Begins the derivation

- **Productions**

- **replacement rules** : Specify how non-terminals may be expanded to form strings
- LHS: single non-terminal, RHS: string of terminals (including ε) or non-terminals

- $S \rightarrow (S) S$
- $S \rightarrow \varepsilon$

Another Example...

Arithmetic Expressions

- Suppose we want to describe all legal arithmetic expressions using addition, subtraction, multiplication, and division.
- Here is one possible CFG:

E → **int**

E → **E Op E**

E → (**E**)

Op → +

Op → -

Op → *

Op → /

Non-terminal Symbols

Terminal Symbols

Production Rules

Start Symbols

A Notational Shorthand

E → **int**

E → **E Op E**

E → **(E)**

Op → **+**

Op → **-**

Op → *****

Op → **/**

E → **int** | **E Op E** | **(E)**

Op → **+** | **-** | ***** | **/**

- Vertical bar | is shorthand for multiple productions

CFGs for Programming Languages

BLOCK	→	STMT { STMTS }
STMTS	→	ϵ STMT STMTS
STMT	→	EXPR; if (EXPR) BLOCK while (EXPR) BLOCK do BLOCK while (EXPR); BLOCK ...
EXPR	→	identifier constant EXPR + EXPR EXPR - EXPR EXPR * EXPR ...

Scanner vs. Parser

Language is a set of **strings**

- each **string** is a finite sequence of symbols taken from a finite **alphabet**

Scanning:

- the **strings** are ___?
 - source programs
- the **alphabet** is ___?
 - the ASCII
- Formal Language is ___?
 - Regular expression
- Machine to recognize the language?
 - Finite Automata

Parsing:

- The **strings** are___?
 - Sequence of token
- the **alphabet** is ___?
 - set of token-types returned by the lexical analyzer
- Formal Language is ___?
 - Context Free Grammar
- Machine to recognize the language?
 - Pushdown automata => parsing algorithms for approximation

Some CFG Notation

- Capital letters at the beginning of the alphabet will represent nonterminals.
 - i.e. **A**, **B**, **C**, **D**
- Lowercase letters at the end of the alphabet will represent terminals.
 - i.e. **t**, **u**, **v**, **w**
- Lowercase Greek letters will represent arbitrary strings of terminals and nonterminals.
 - i.e. **α** , **γ** , **ω**

Examples

- We might write an arbitrary production as

$$\mathbf{A} \rightarrow \omega$$

- We might write a string of a nonterminal followed by a terminal as

$$\mathbf{A}t$$

- We might write an arbitrary production containing a nonterminal followed by a terminal as

$$\mathbf{B} \rightarrow \alpha \mathbf{A}t\omega$$

Derivation

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$
 $\Rightarrow E * (E \text{ Op } E)$
 $\Rightarrow \text{int} * (E \text{ Op } E)$
 $\Rightarrow \text{int} * (\text{int} \text{ Op } E)$
 $\Rightarrow \text{int} * (\text{int} \text{ Op } \text{int})$
 $\Rightarrow \text{int} * (\text{int} + \text{int})$

- This sequence of steps is called a **derivation**.
- A string $\alpha A \omega$ **yields** string $\alpha \gamma \omega$ iff $A \rightarrow \gamma$ is a production.
- If α yields β , we write $\alpha \Rightarrow \beta$.
- We say that α **derives** β iff there is a sequence of strings where
$$\alpha \Rightarrow \alpha_1 \Rightarrow \alpha_2 \Rightarrow \dots \Rightarrow \beta$$
- If α derives β , we write $\alpha \Rightarrow^* \beta$.

- Terminals: no replacement rules for them
- Terminals: tokens from the lexer

- Which of the strings are in the language of the given CFG?

- abcba
- acca
- aba
- abcbcba

- $S \rightarrow aXa$

- $X \rightarrow \varepsilon \mid bY$

- $Y \rightarrow \varepsilon \mid cXc$

Leftmost Derivations

STMTS → ϵ
| **STMT STMTS**

STMT → **EXPR**;
| **if** (**EXPR**) **BLOCK**
| **while** (**EXPR**) **BLOCK**
| **do** **BLOCK** **while** (**EXPR**);
| **BLOCK**
| ...

EXPR → **identifier**
| **constant**
| **EXPR + EXPR**
| **EXPR - EXPR**
| **EXPR * EXPR**
| **EXPR = EXPR**
| ...

Productions

Leftmost Derivations

- A **leftmost derivation** is a derivation in which each step expands the leftmost nonterminal.
- A **rightmost derivation** is a derivation in which each step expands the rightmost nonterminal.

$$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$$

$$\text{Op} \rightarrow + \mid - \mid * \mid /$$

Related Derivations

$$\begin{aligned} & E \\ \Rightarrow & E \text{ Op } E \\ \Rightarrow & \text{int Op } E \\ \Rightarrow & \text{int} * E \\ \Rightarrow & \text{int} * (E) \\ \Rightarrow & \text{int} * (E \text{ Op } E) \\ \Rightarrow & \text{int} * (\text{int Op } E) \\ \Rightarrow & \text{int} * (\text{int} + E) \\ \Rightarrow & \text{int} * (\text{int} + \text{int}) \end{aligned}$$

$$\begin{aligned} & E \\ \Rightarrow & E \text{ Op } E \\ \Rightarrow & E \text{ Op } (E) \\ \Rightarrow & E \text{ Op } (E \text{ Op } E) \\ \Rightarrow & E \text{ Op } (E \text{ Op } \text{int}) \\ \Rightarrow & E \text{ Op } (E + \text{int}) \\ \Rightarrow & E \text{ Op } (\text{int} + \text{int}) \\ \Rightarrow & E * (\text{int} + \text{int}) \\ \Rightarrow & \text{int} * (\text{int} + \text{int}) \end{aligned}$$

Derivations Revisited

- A derivation encodes two pieces of information:
 - What productions were applied produce the resulting string from the start symbol?
 - In what order were they applied?
- Multiple derivations might use the same productions, but apply them in a different order.

- Derivation: also a process of constructing a parse tree

Parse Trees

E \rightarrow **int** | **E Op E** | **(E)**

Op \rightarrow **+** | **-** | ***** | **/**

E

Parse Trees

E \rightarrow **int** | **E Op E** | (**E**)

Op \rightarrow **+** | **-** | ***** | **/**

E

E

Parse Trees

E

E
 \Rightarrow **E Op E**

E \rightarrow **int** | **E Op E** | **(E)**

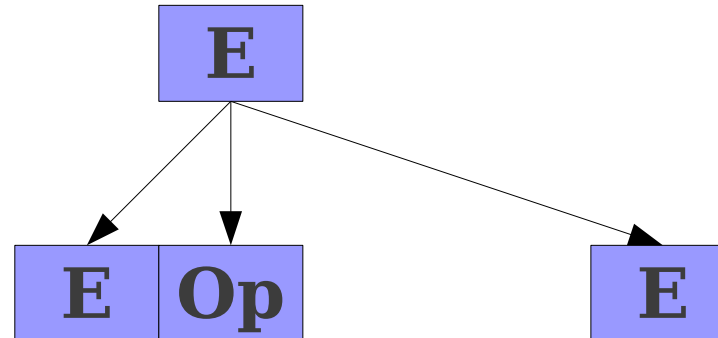
Op \rightarrow **+** | **-** | ***** | **/**

Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$

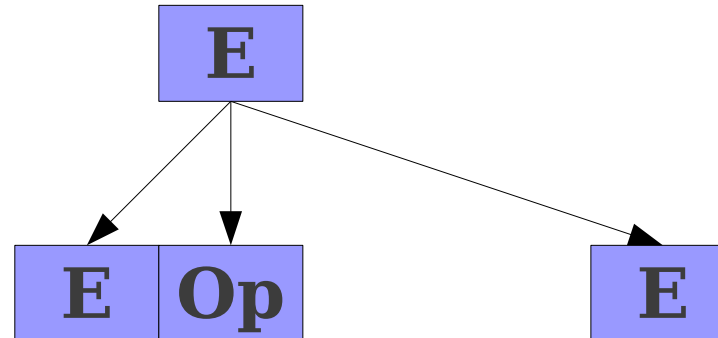


Parse Trees

E \rightarrow **int** | **E Op E** | (**E**)

Op \rightarrow **+** | **-** | ***** | **/**

E
 \Rightarrow **E Op E**
 \Rightarrow **int Op E**

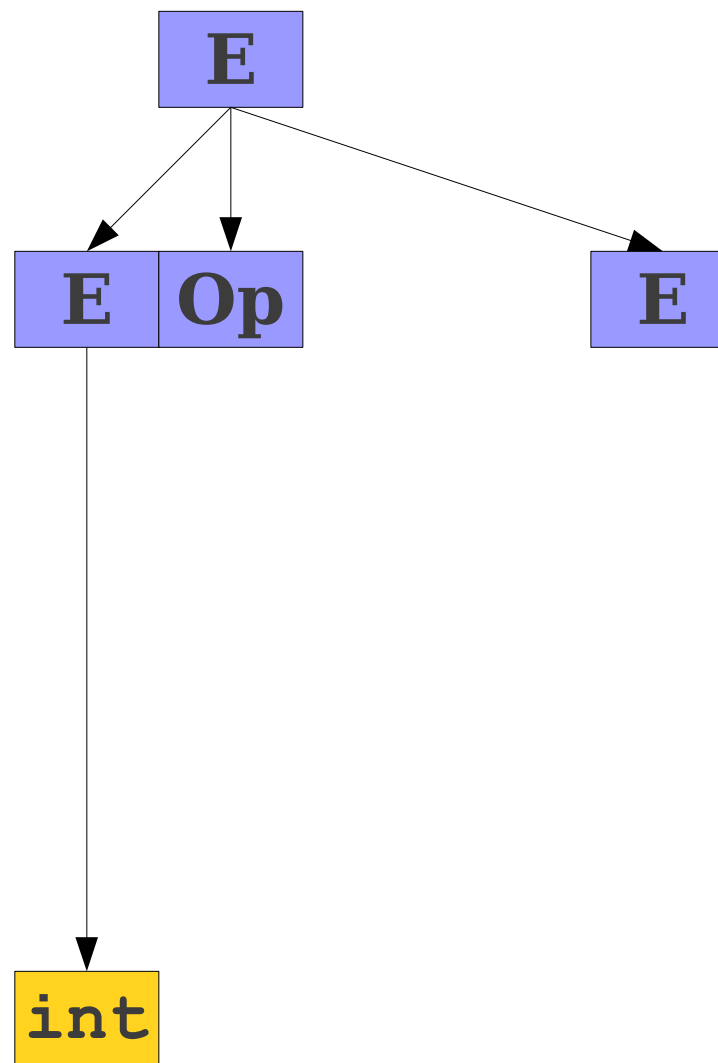


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow \text{int Op } E$

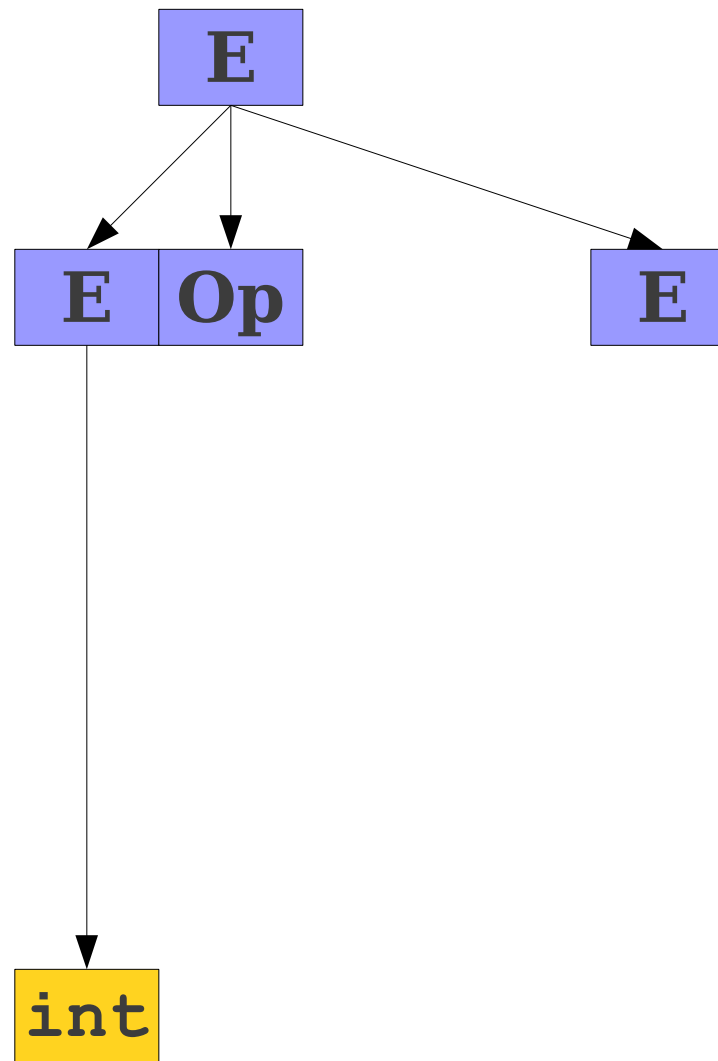


E \rightarrow **int** | **E Op E** | **(E)**

Op \rightarrow **+** | **-** | ***** | **/**

Parse Trees

E
 \Rightarrow **E Op E**
 \Rightarrow **int Op E**
 \Rightarrow **int * E**

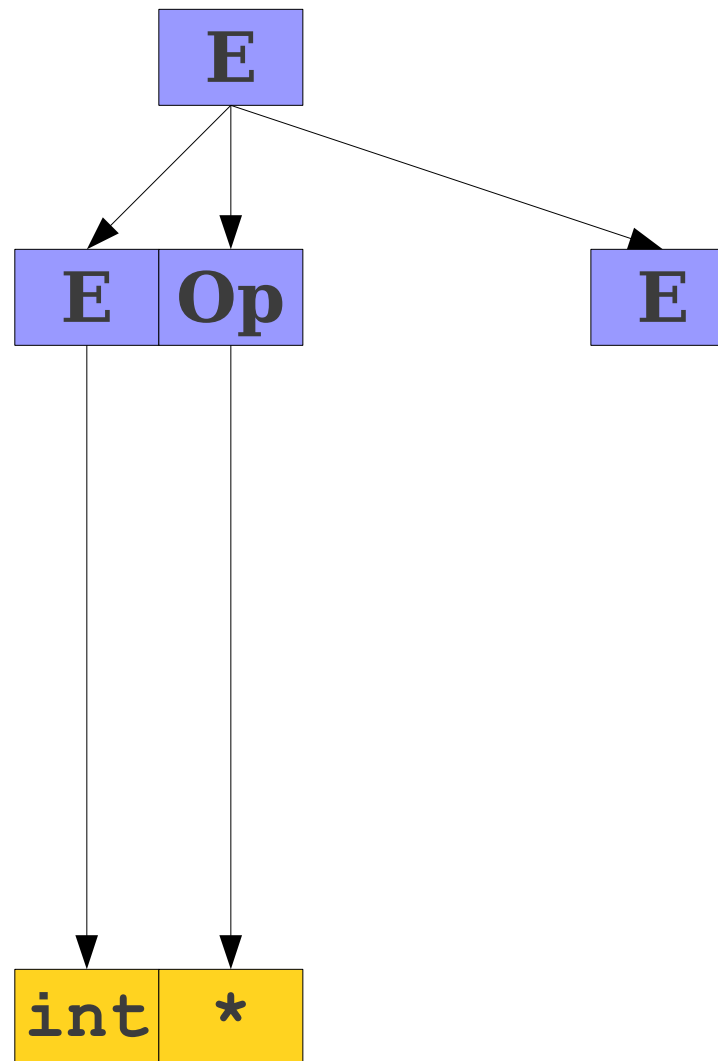


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow \text{int Op } E$
 $\Rightarrow \text{int } * E$

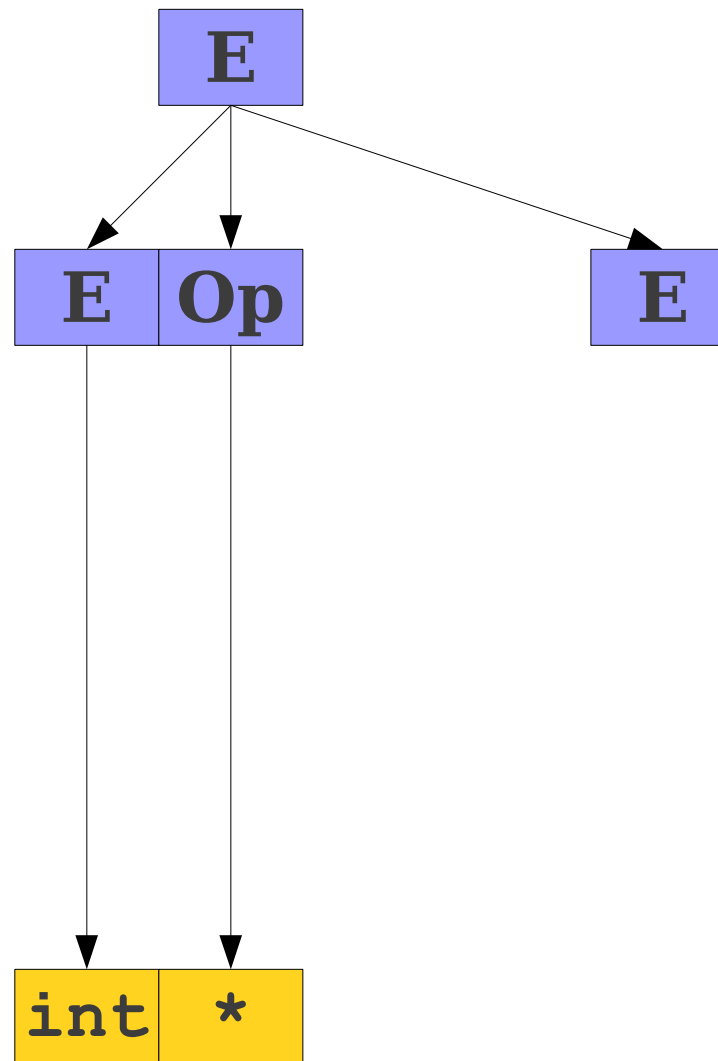


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow \text{int Op } E$
 $\Rightarrow \text{int } * E$
 $\Rightarrow \text{int } * (E)$

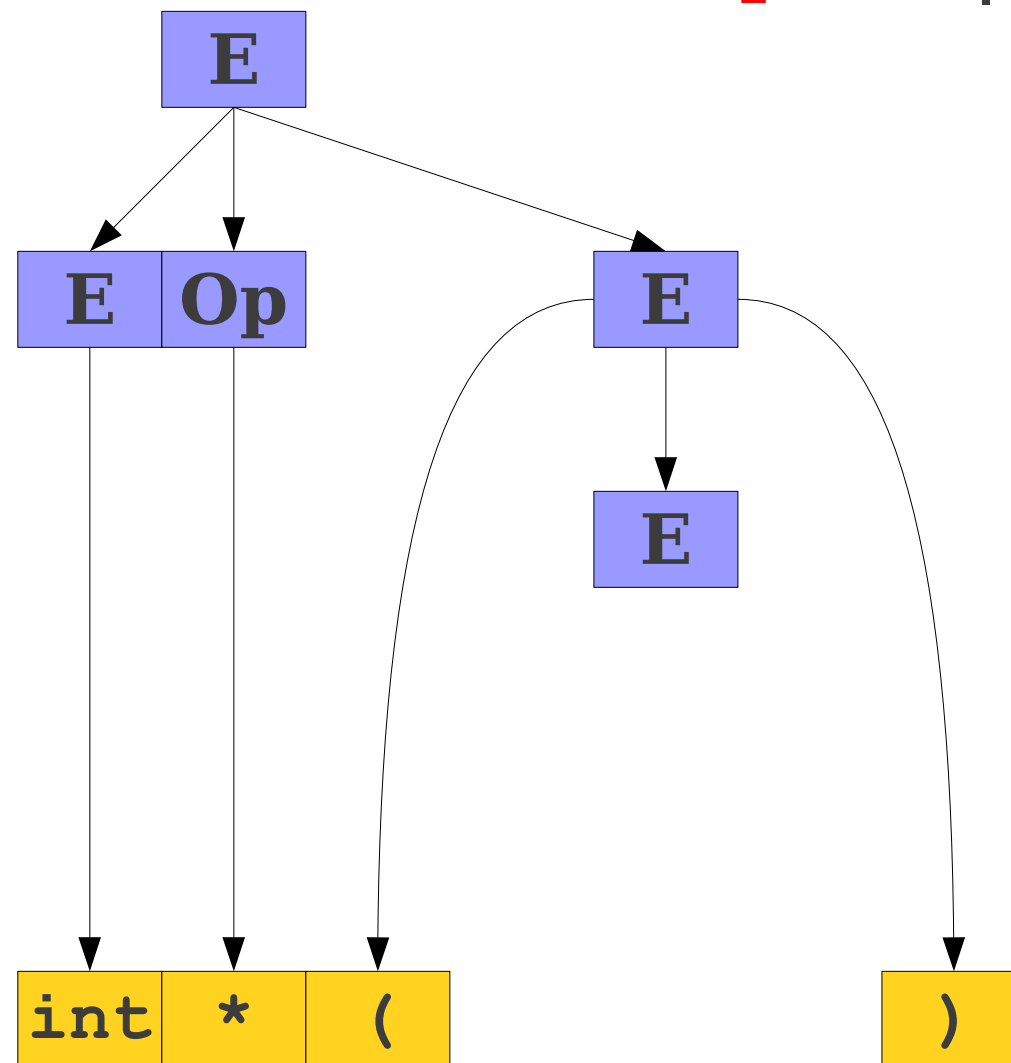


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow \text{int Op } E$
 $\Rightarrow \text{int } * E$
 $\Rightarrow \text{int } * (E)$

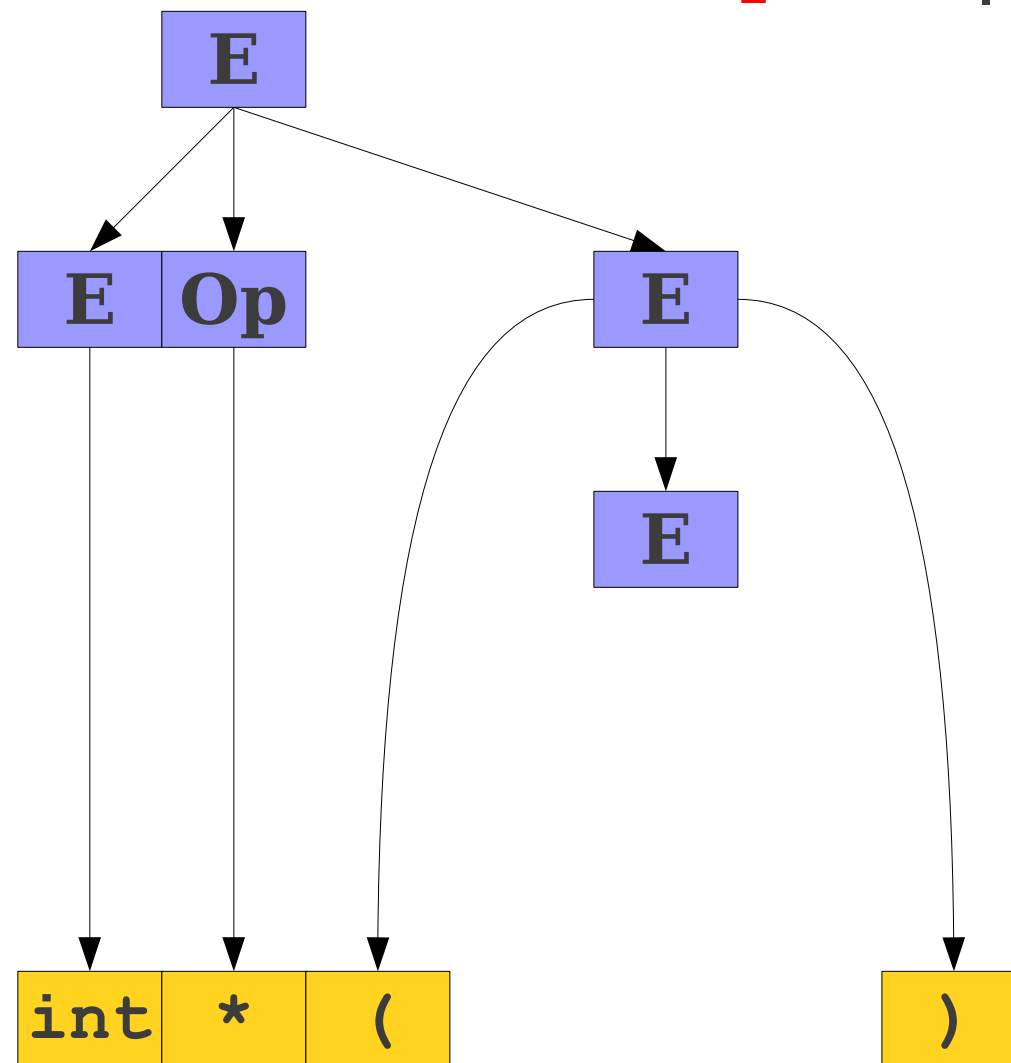


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow \text{int Op } E$
 $\Rightarrow \text{int } * E$
 $\Rightarrow \text{int } * (E)$
 $\Rightarrow \text{int } * (E \text{ Op } E)$

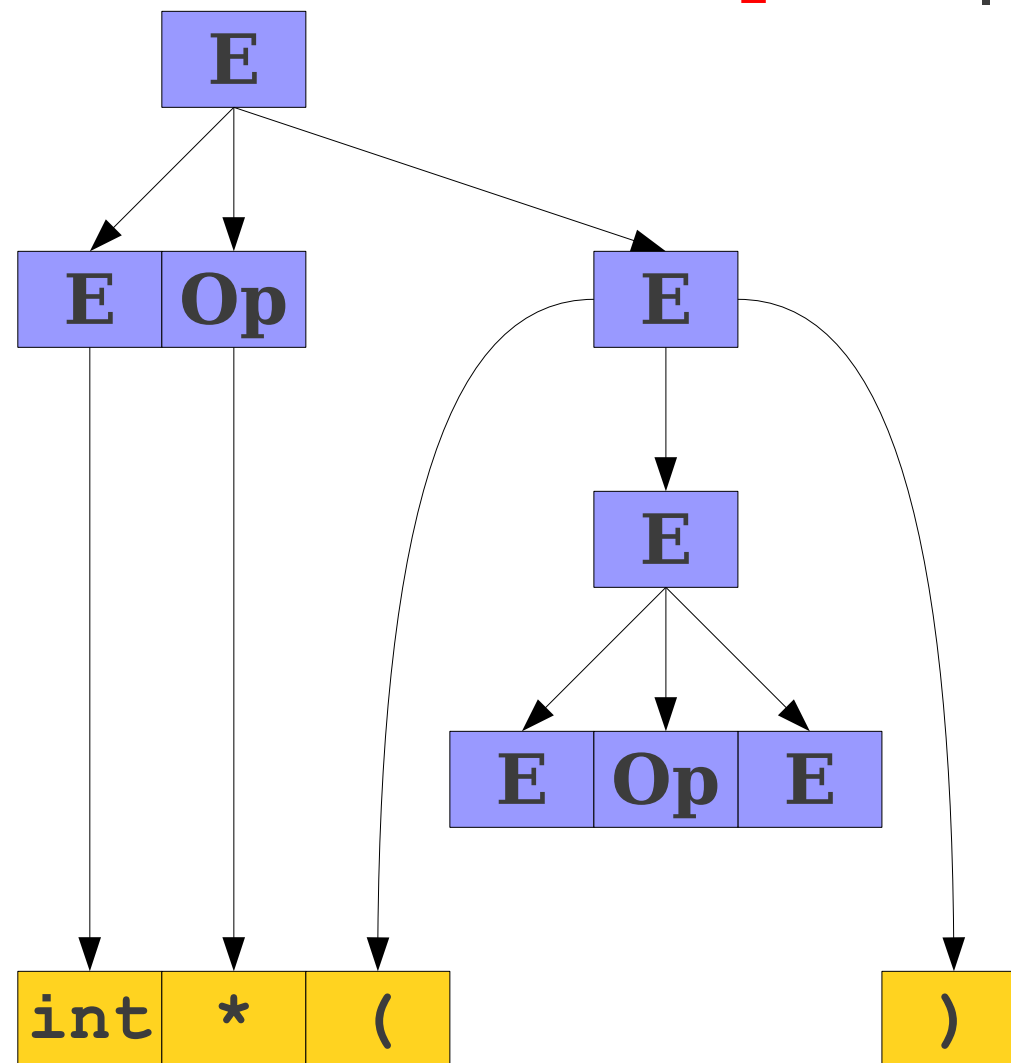


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow \text{int Op } E$
 $\Rightarrow \text{int } * E$
 $\Rightarrow \text{int } * (E)$
 $\Rightarrow \text{int } * (E \text{ Op } E)$



Parse Trees

$$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$$

Op \rightarrow + | - | * | /

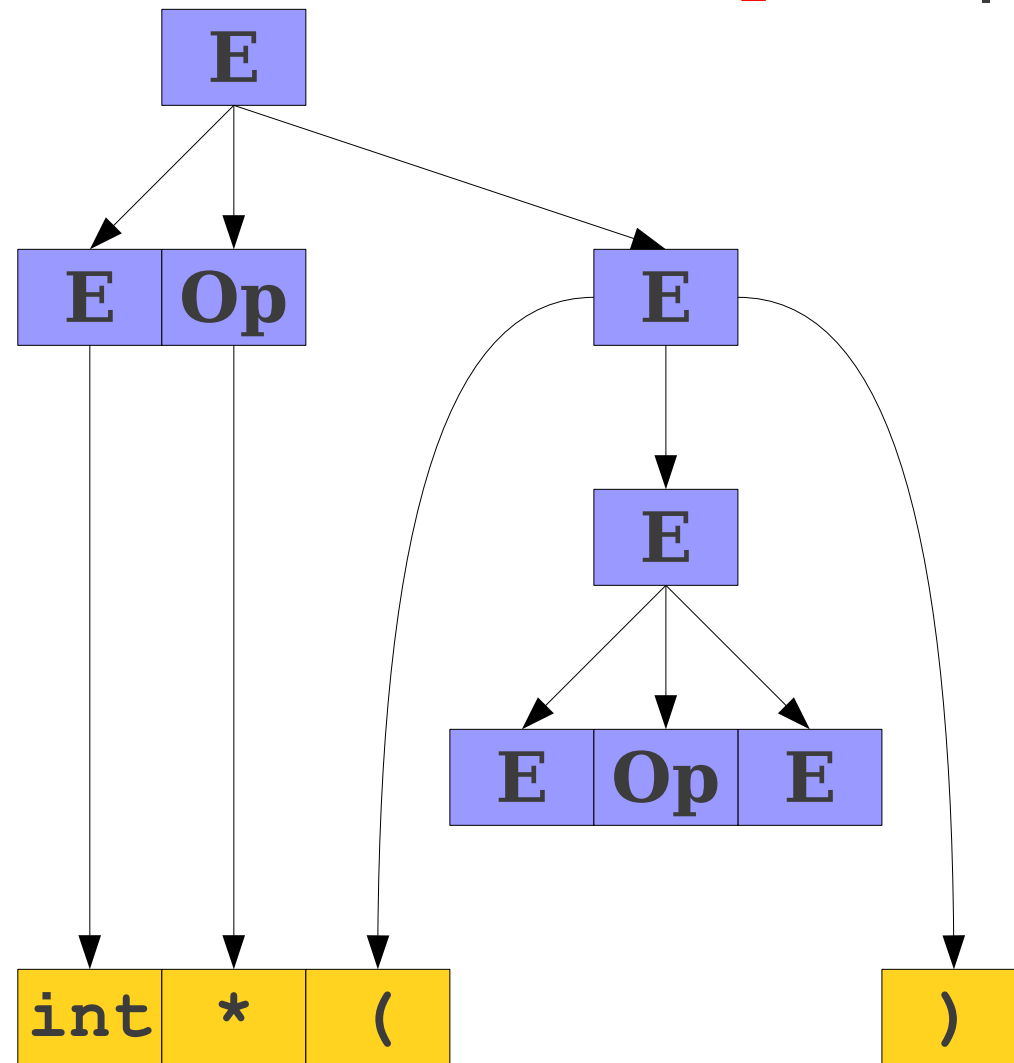
E

\Rightarrow **E Op E**

⇒ **int Op E**

⇒ int * **E**

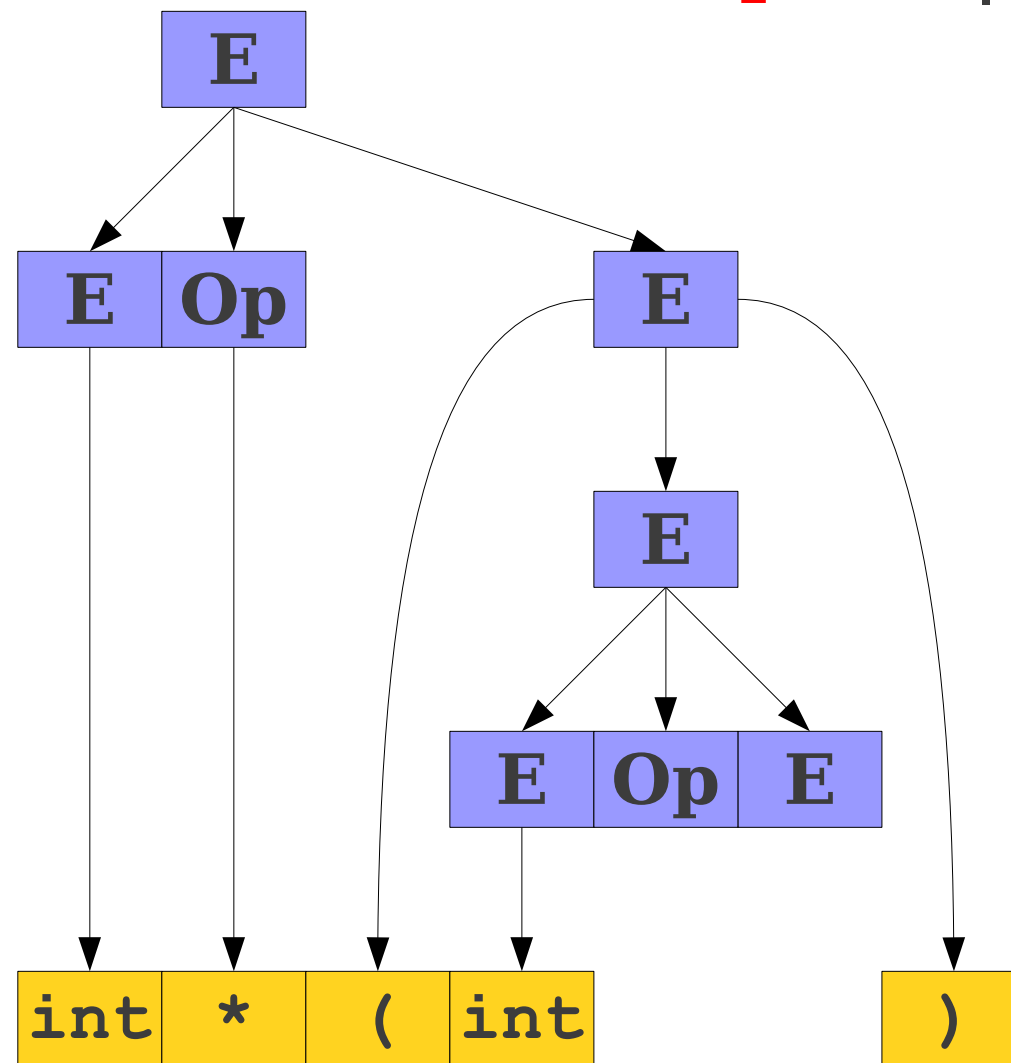
⇒ int * (E)

$$\Rightarrow \text{int} * (\text{E Op E})$$
$$\Rightarrow \text{int} * (\text{int Op E})$$


Parse Trees

$$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$$

Op \rightarrow + | - | * | /

$$\begin{aligned} & \mathbf{E} \\ \Rightarrow & \mathbf{E \ Op \ E} \\ \Rightarrow & \mathbf{int \ Op \ E} \\ \Rightarrow & \mathbf{int \ * \ E} \\ \Rightarrow & \mathbf{int \ * \ (E)} \\ \Rightarrow & \mathbf{int \ * \ (E \ Op \ E)} \\ \Rightarrow & \mathbf{int \ * \ (int \ Op \ E)} \end{aligned}$$


Parse Trees

$$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$$

Op \rightarrow + | - | * | /

E

⇒ **E Op E**

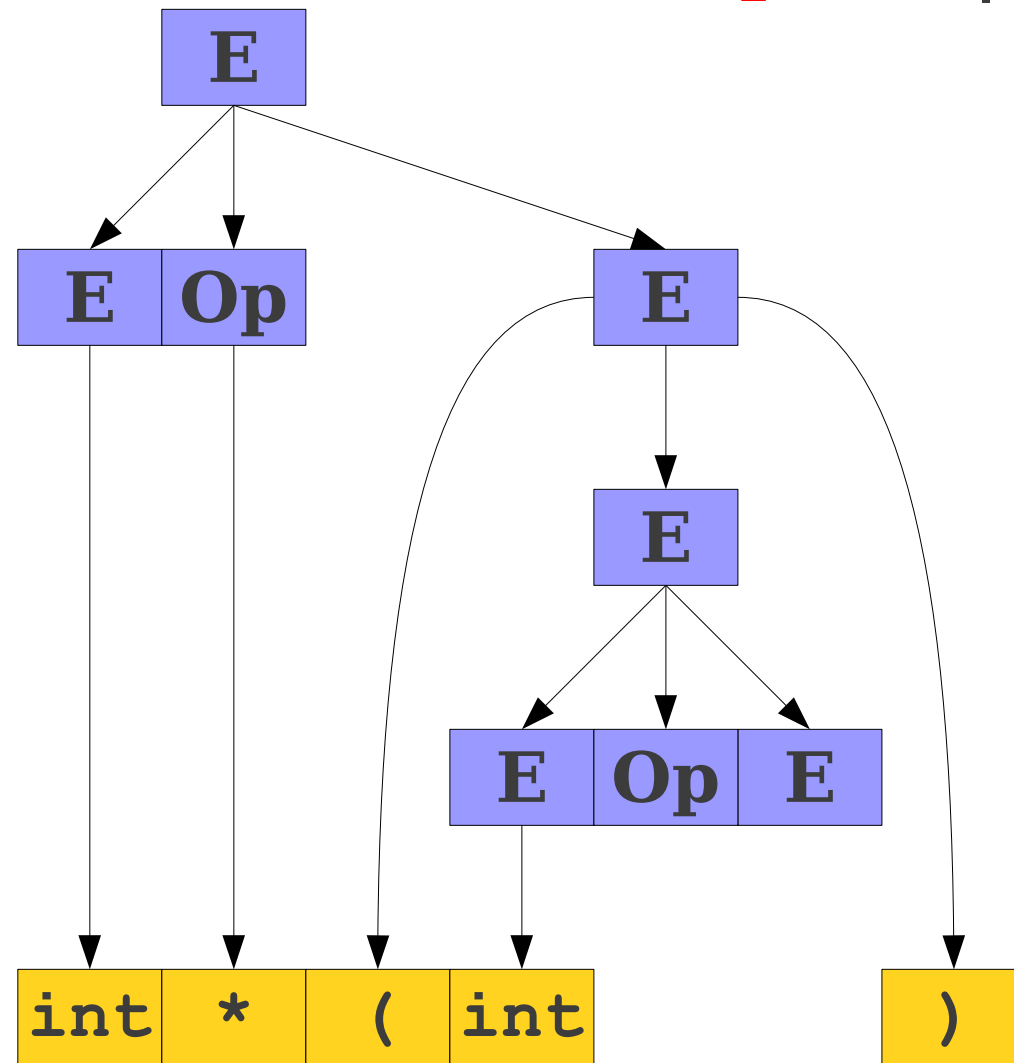
⇒ **int Op E**

⇒ int * **E**

⇒ int * (E)

$$\Rightarrow \text{int} * (\text{E Op E})$$
$$\Rightarrow \text{int} * (\text{int Op E})$$

⇒ int * (int + E)



Parse Trees

$$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$$

Op \rightarrow + | - | * | /

E

\Rightarrow **E Op E**

\Rightarrow **int Op E**

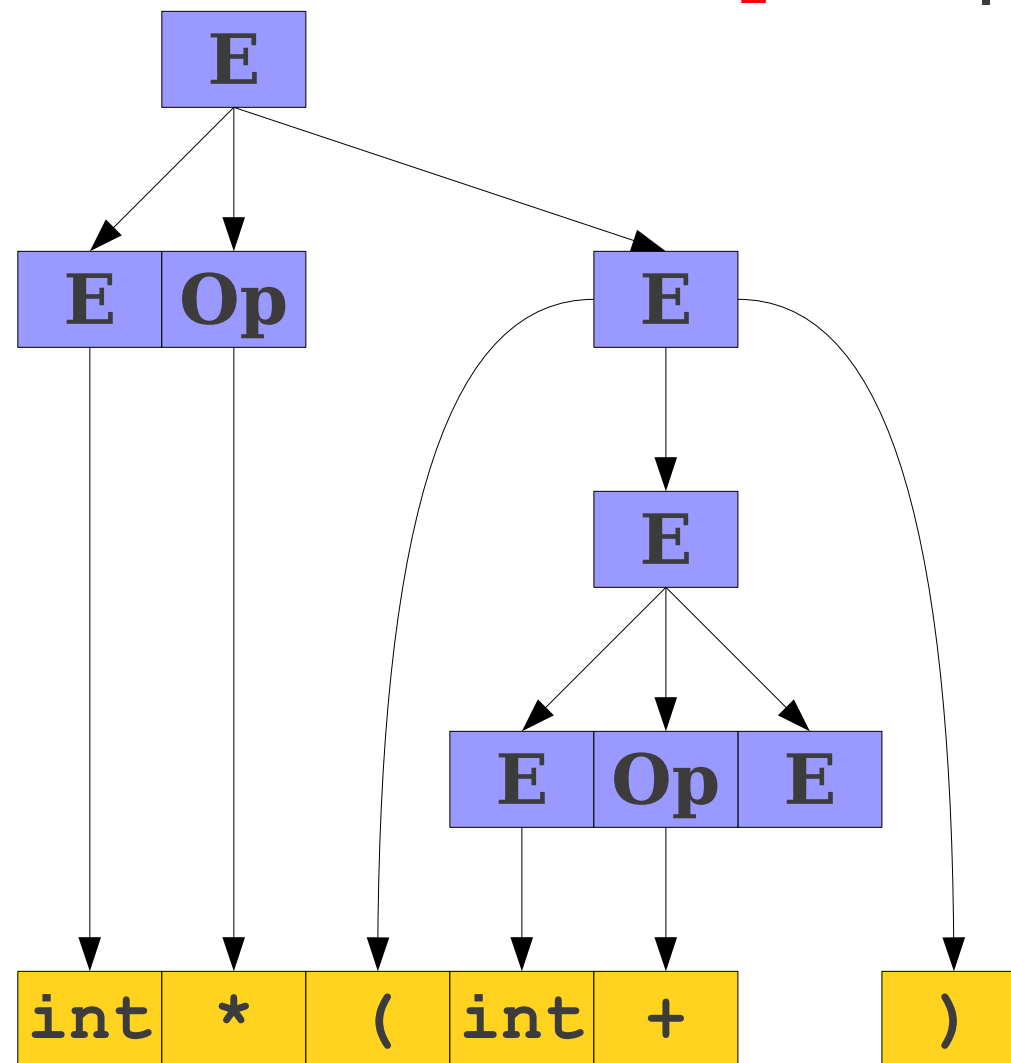
\Rightarrow **int * E**

\Rightarrow **int * (E)**

\Rightarrow **int * (E Op E)**

\Rightarrow **int * (int Op E)**

\Rightarrow **int * (int + E)**



Parse Trees

$$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$$

Op \rightarrow + | - | * | /

E

⇒ **E Op E**

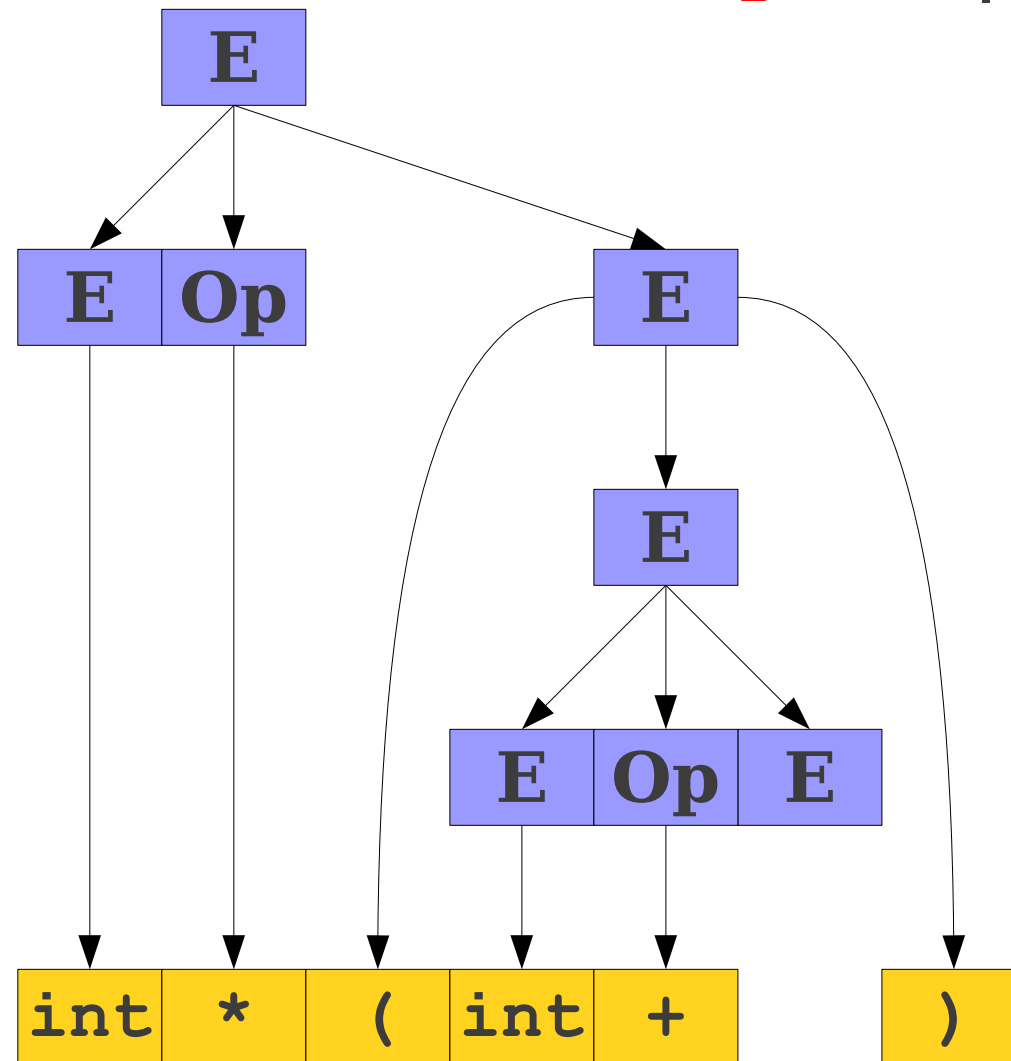
⇒ **int Op E**

⇒ int * E

⇒ int * (E)

$$\Rightarrow \text{int} * (\text{E Op E})$$
$$\Rightarrow \text{int} * (\text{int Op E})$$
$$\Rightarrow \text{int} * (\text{int} + \mathbf{E})$$

⇒ `int * (int + int)`



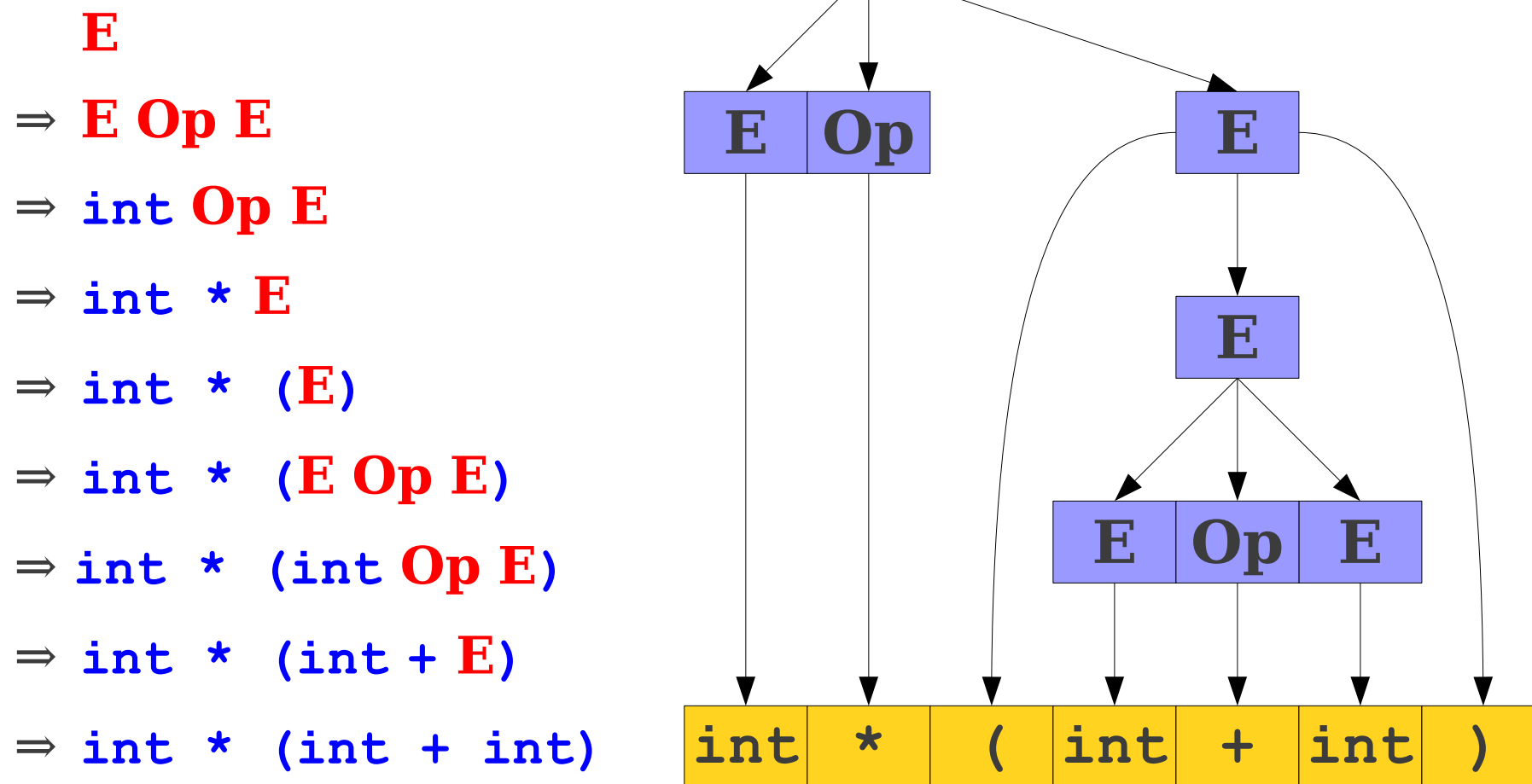
Parse Trees

$$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$$

$$\text{Op} \rightarrow + \mid - \mid * \mid /$$

$$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$$

Op \rightarrow + | - | * | /



Start symbol is the root

Non-leaf nodes are non-terminals

Leaf nodes are terminals

Inorder walk of the leaves is the generated string

Parse Trees

E \rightarrow **int** | **E Op E** | (**E**)

Op \rightarrow **+** | **-** | ***** | **/**

E

Parse Trees

E \rightarrow **int** | **E Op E** | (**E**)

Op \rightarrow **+** | **-** | ***** | **/**

E

E

Parse Trees

E

E
 \Rightarrow **E Op E**

E \rightarrow **int** | **E Op E** | (**E**)

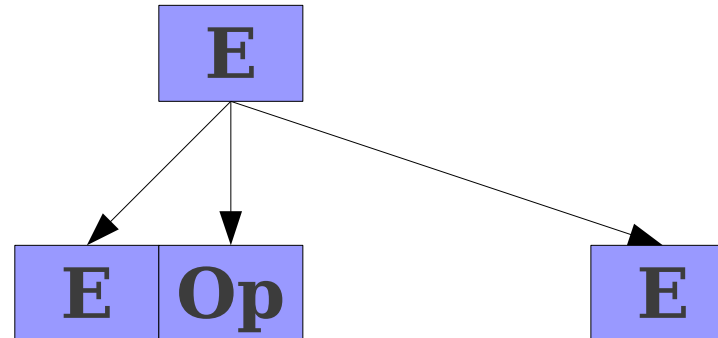
Op \rightarrow **+** | **-** | ***** | **/**

Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$

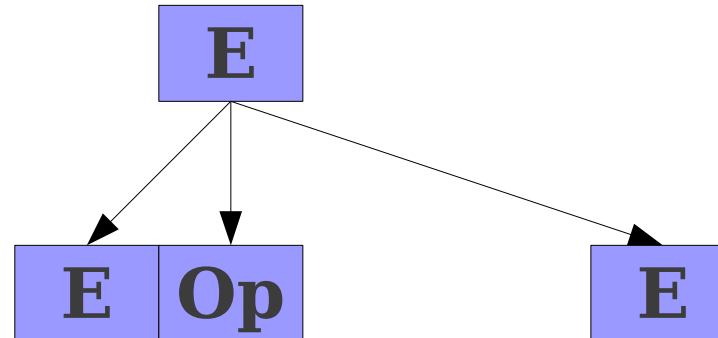


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$

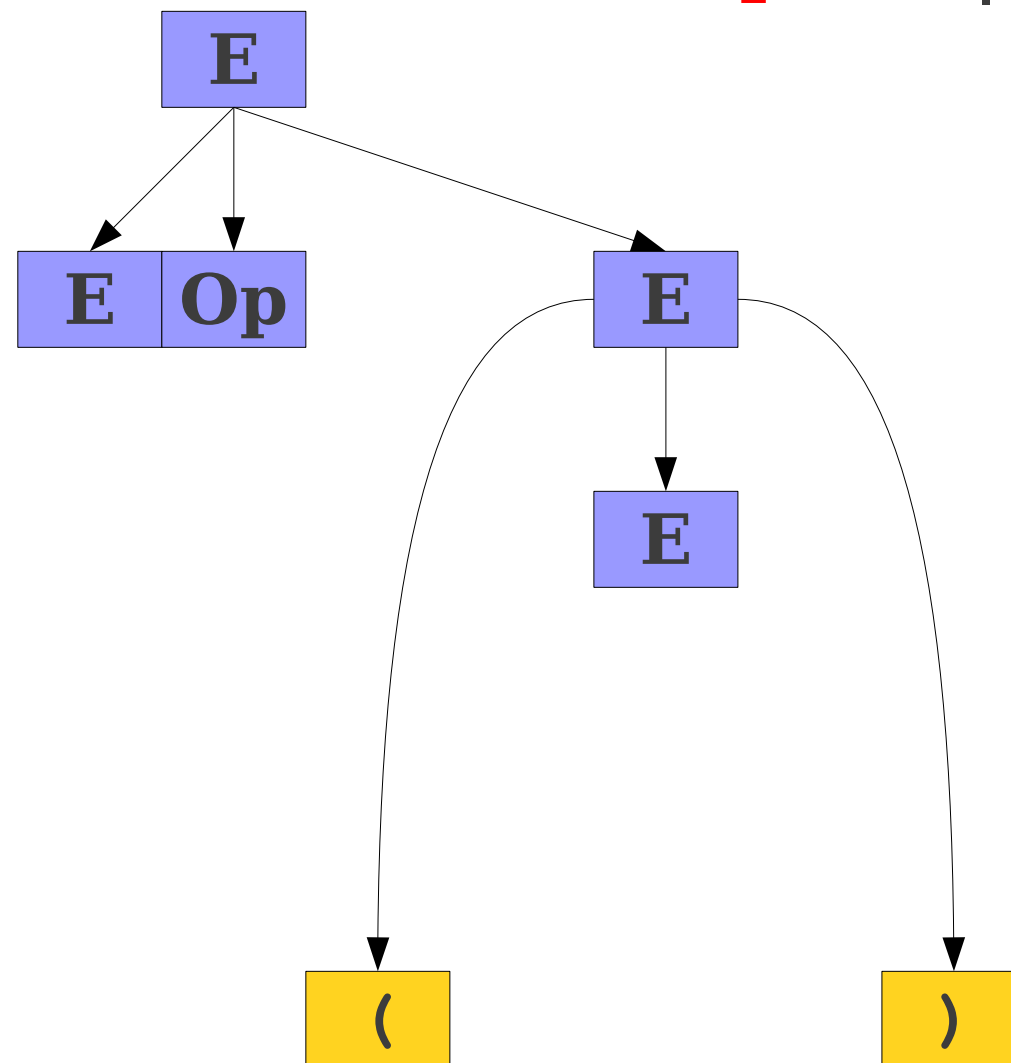


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$

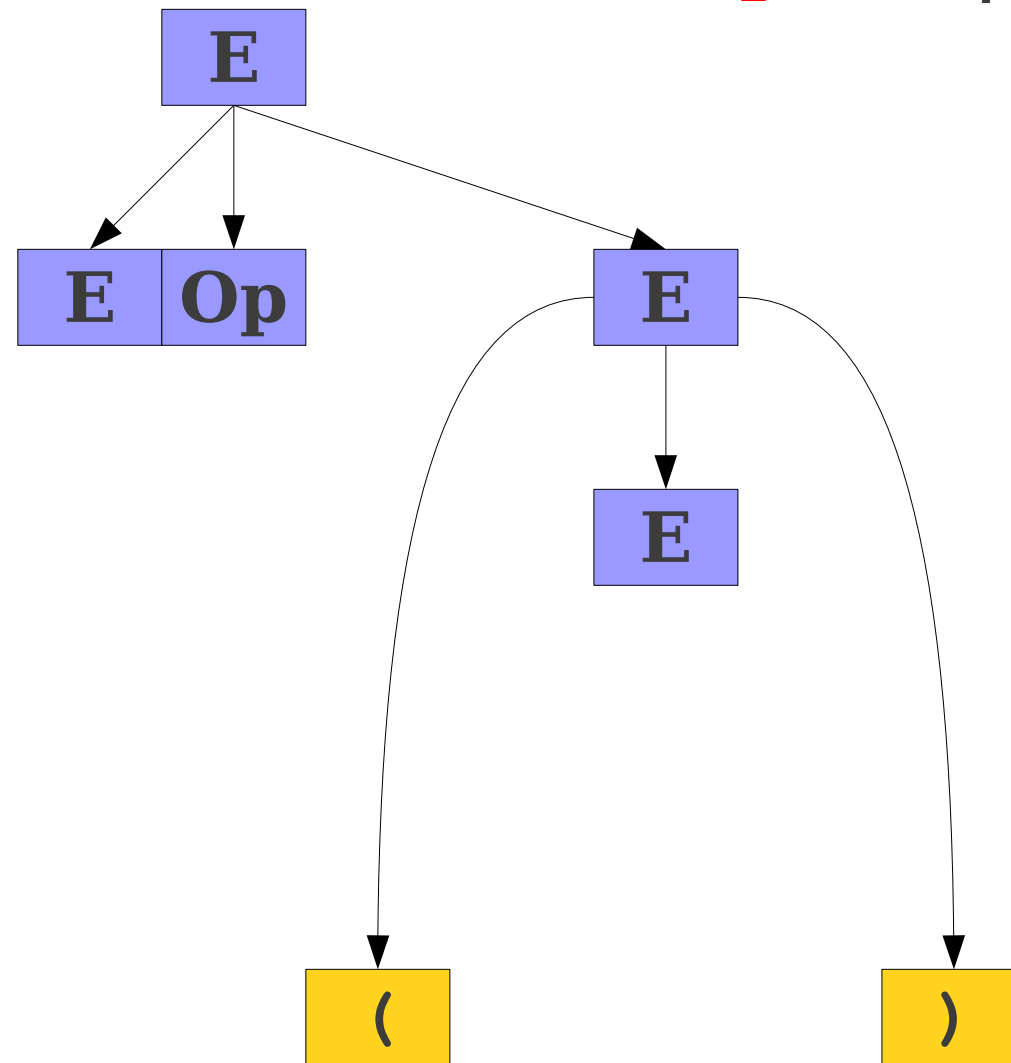


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$

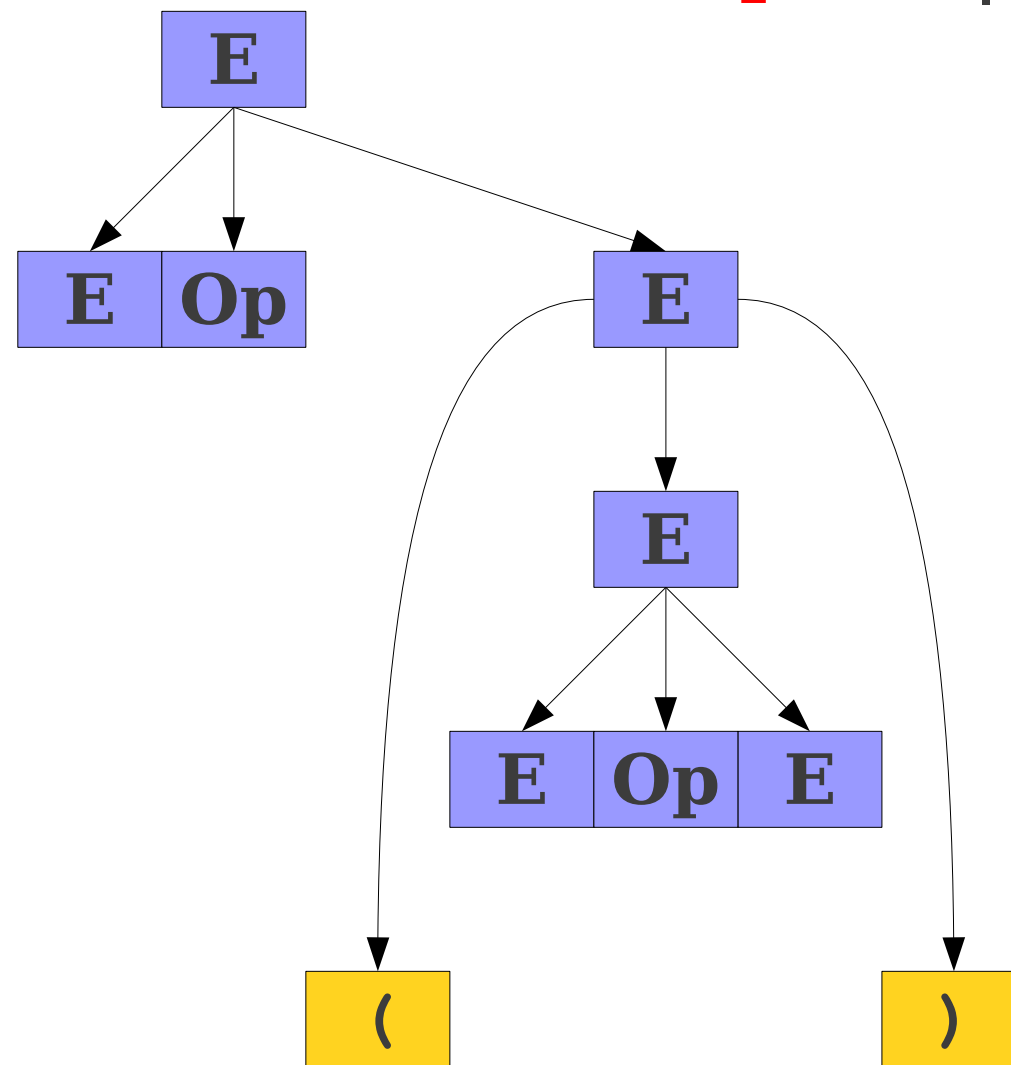


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

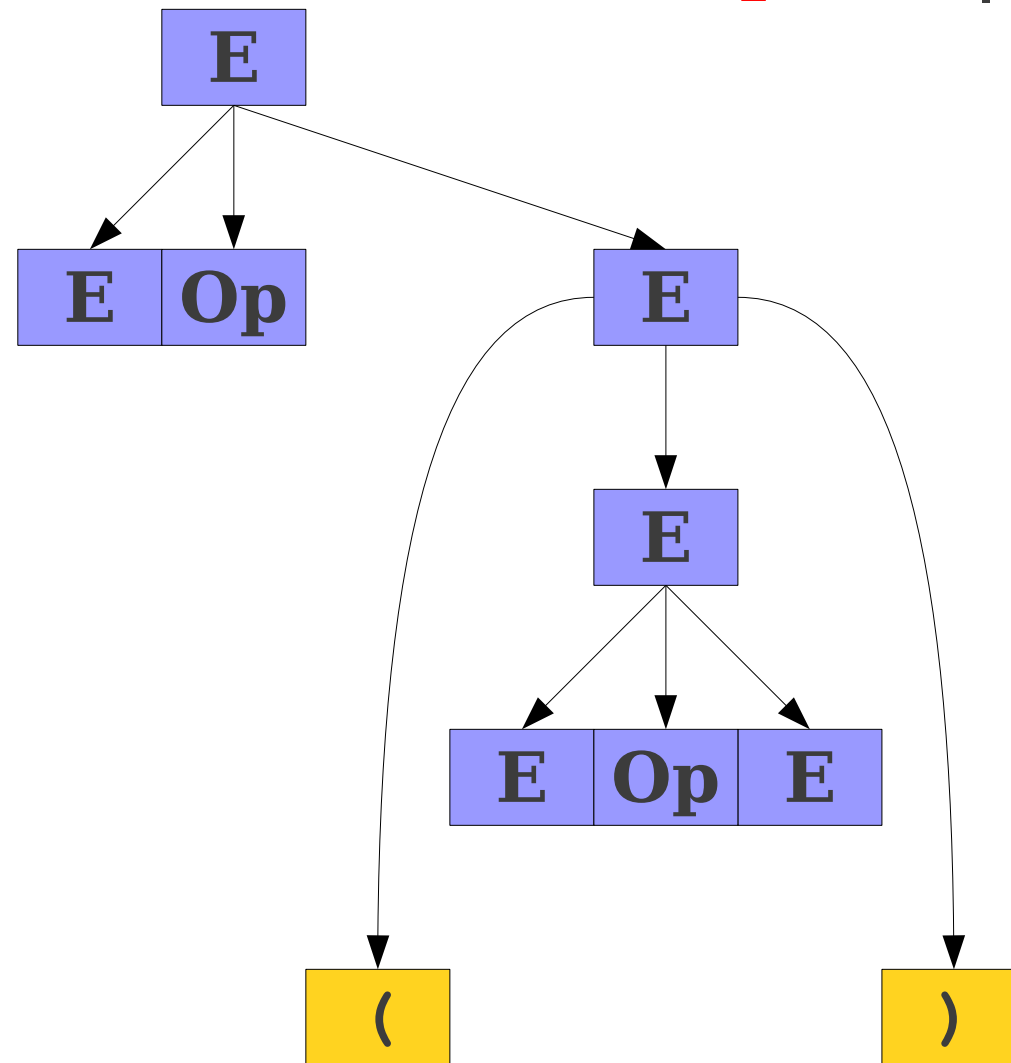
E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$



Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$
 $\text{Op} \rightarrow + \mid - \mid * \mid /$

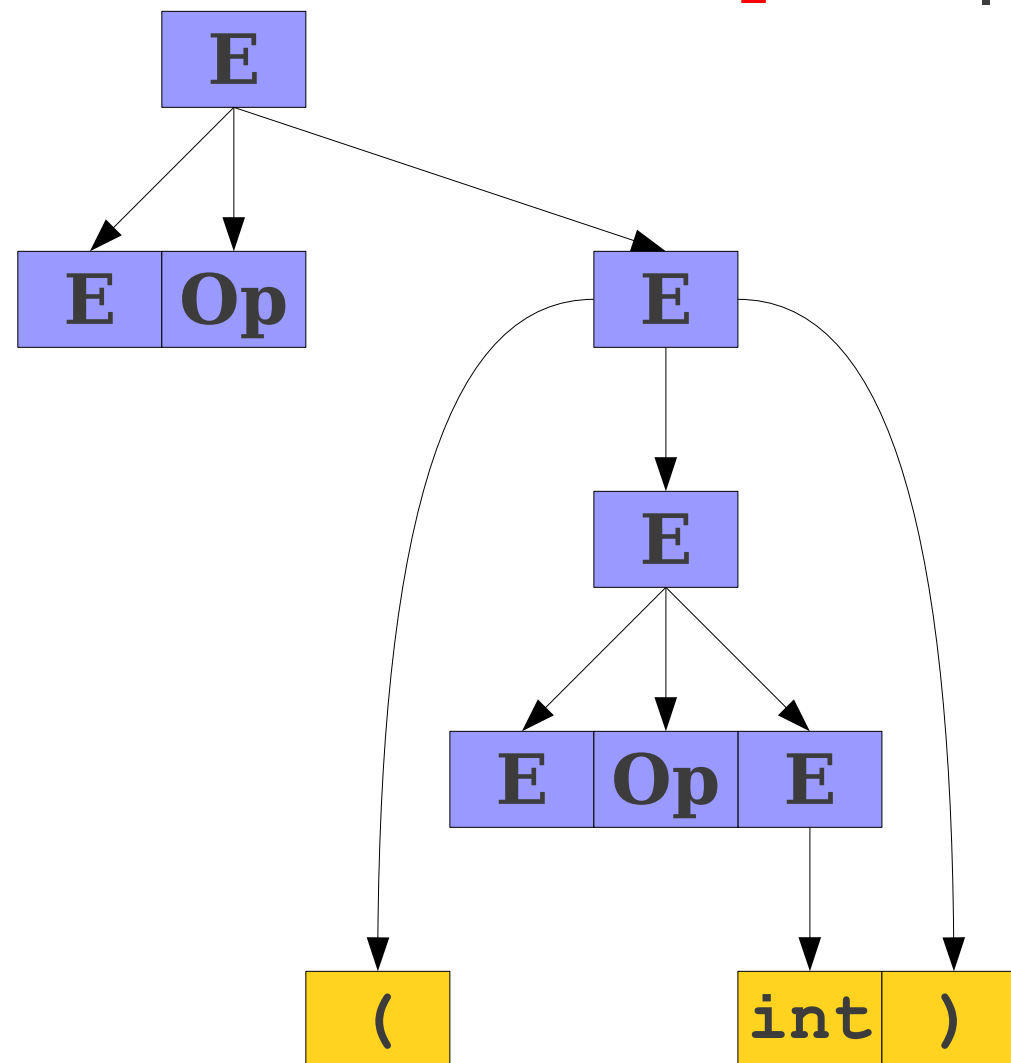
E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } \text{int})$



Parse Trees

$$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$$

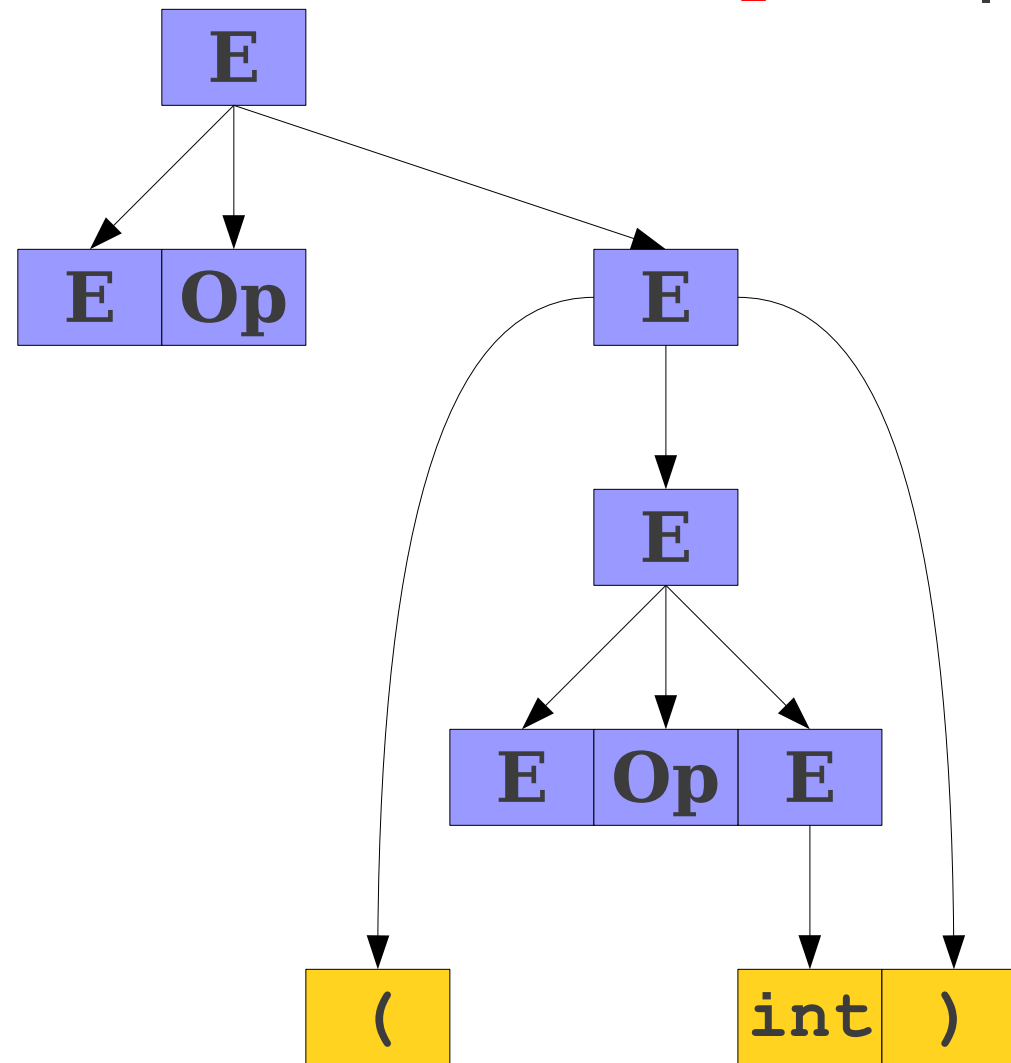
Op \rightarrow + | - | * | /

$$\begin{aligned} & \mathbf{E} \\ \Rightarrow & \mathbf{E \ Op \ E} \\ \Rightarrow & \mathbf{E \ Op \ (E)} \\ \Rightarrow & \mathbf{E \ Op \ (E \ Op \ E)} \\ \Rightarrow & \mathbf{E \ Op \ (E \ Op \ int)} \end{aligned}$$


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$
 $\text{Op} \rightarrow + \mid - \mid * \mid /$

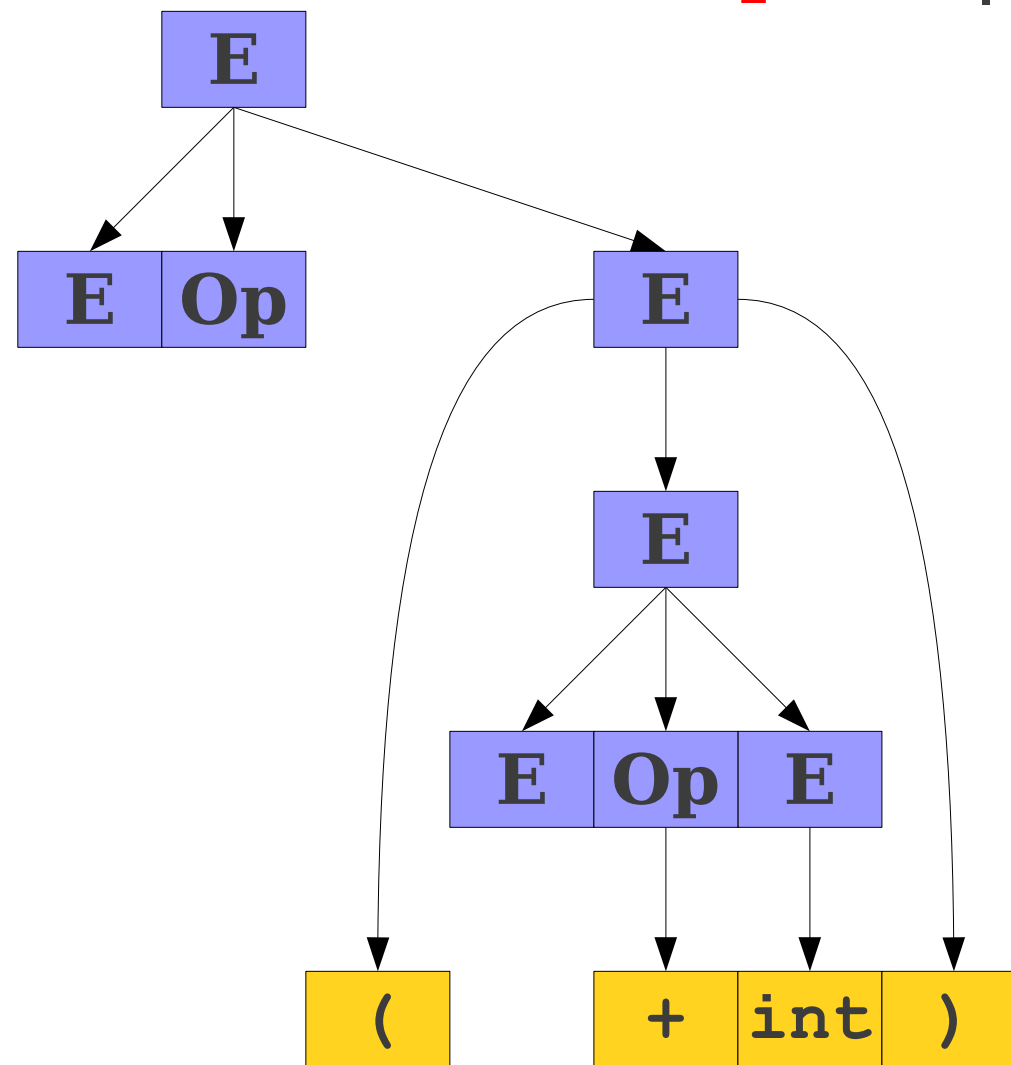
E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } \text{int})$
 $\Rightarrow E \text{ Op } (E + \text{int})$



Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$
 $\text{Op} \rightarrow + \mid - \mid * \mid /$

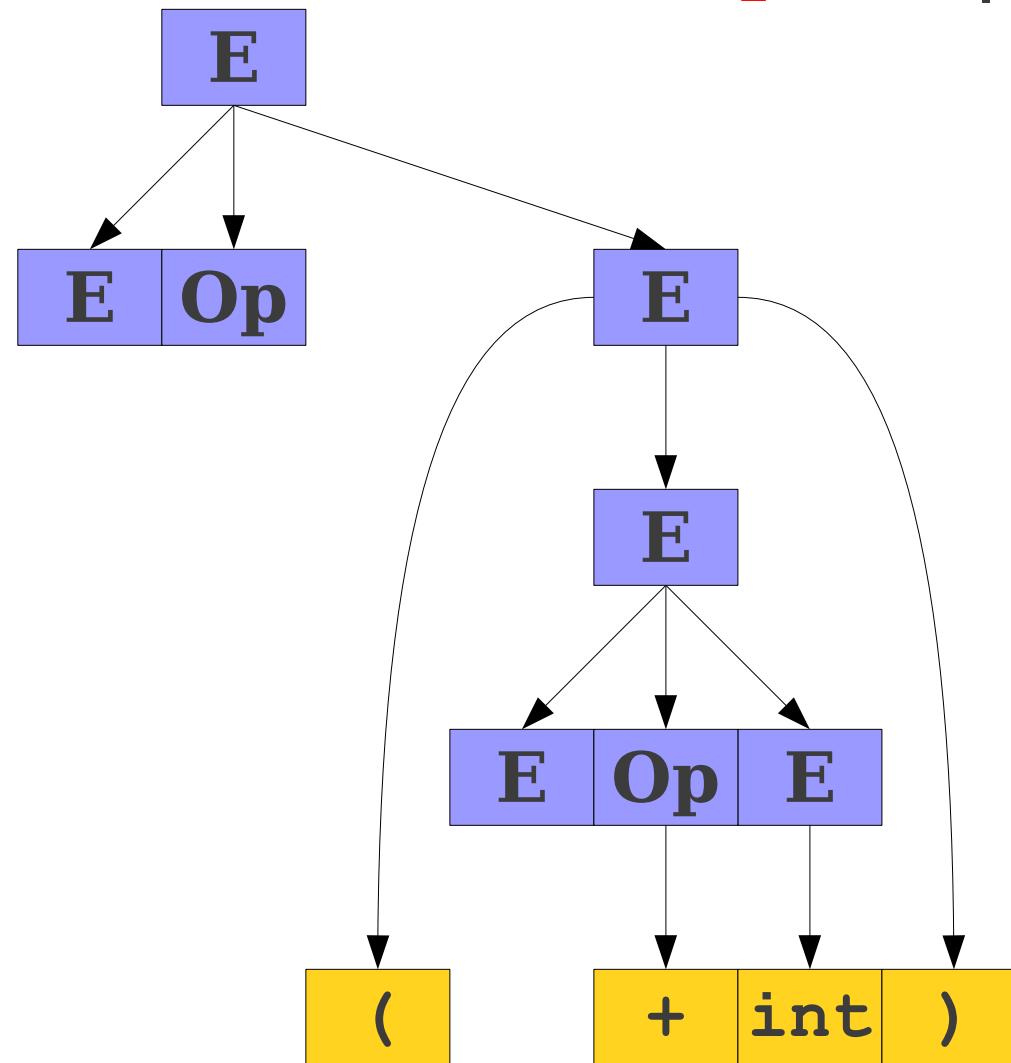
E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } \text{int})$
 $\Rightarrow E \text{ Op } (E + \text{int})$



Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$
 $\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } \text{int})$
 $\Rightarrow E \text{ Op } (E + \text{int})$
 $\Rightarrow E \text{ Op } (\text{int} + \text{int})$

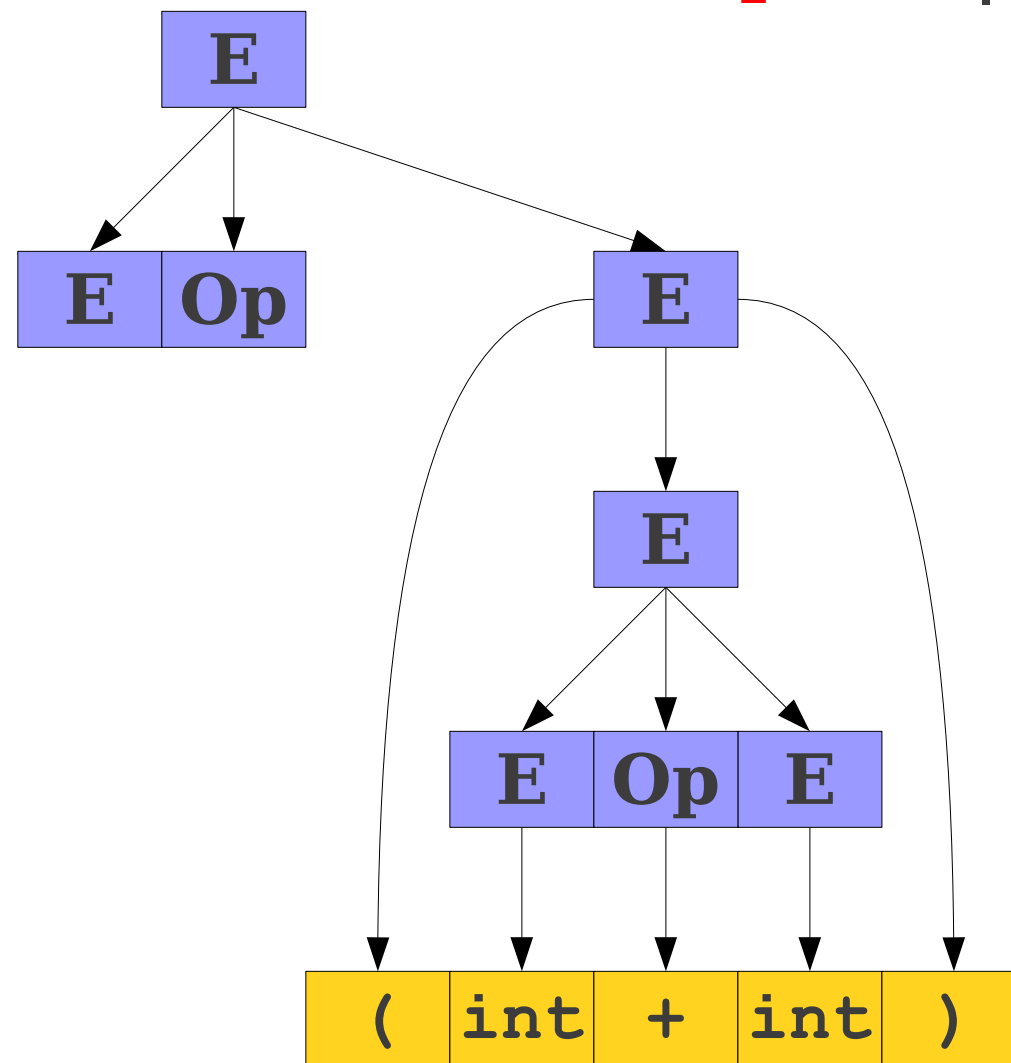


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } \text{int})$
 $\Rightarrow E \text{ Op } (E + \text{int})$
 $\Rightarrow E \text{ Op } (\text{int} + \text{int})$

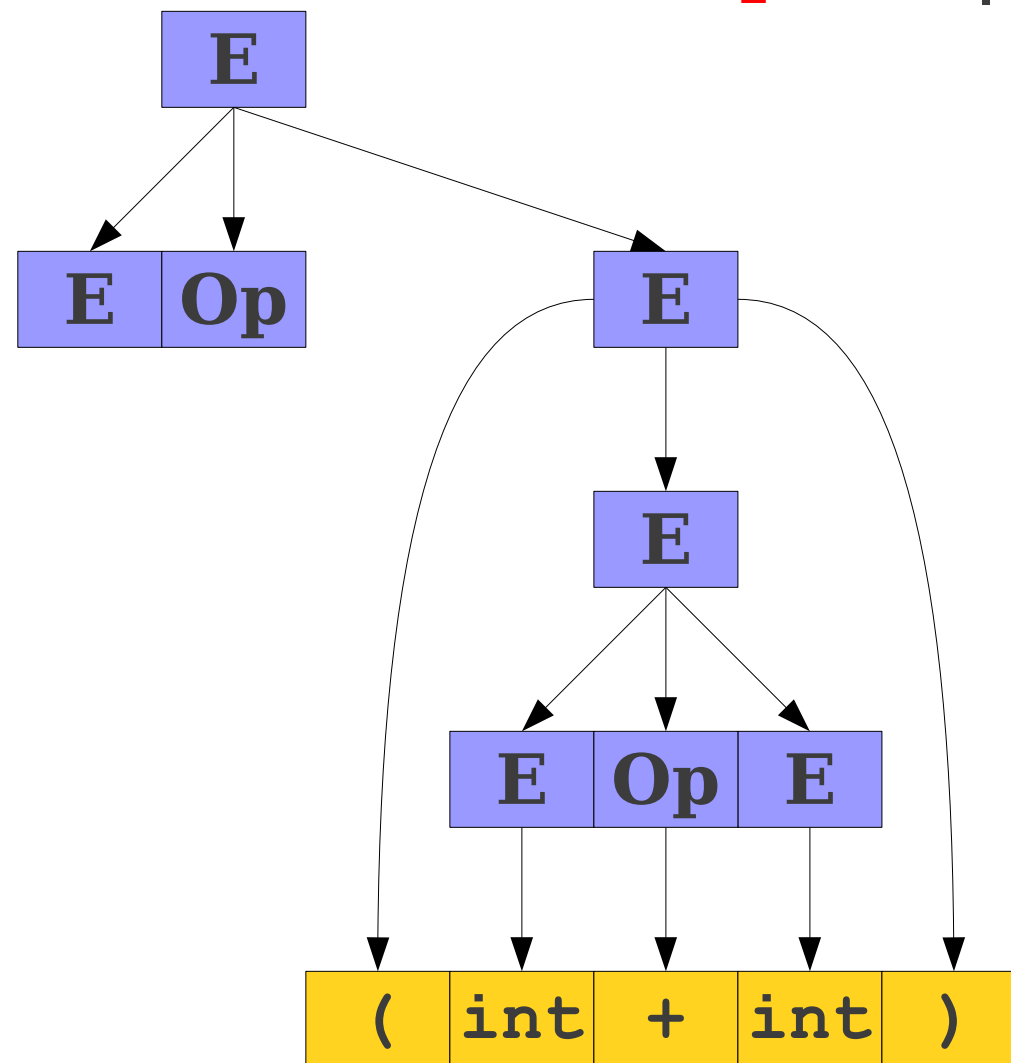


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } \text{int})$
 $\Rightarrow E \text{ Op } (E + \text{int})$
 $\Rightarrow E \text{ Op } (\text{int} + \text{int})$
 $\Rightarrow E * (\text{int} + \text{int})$

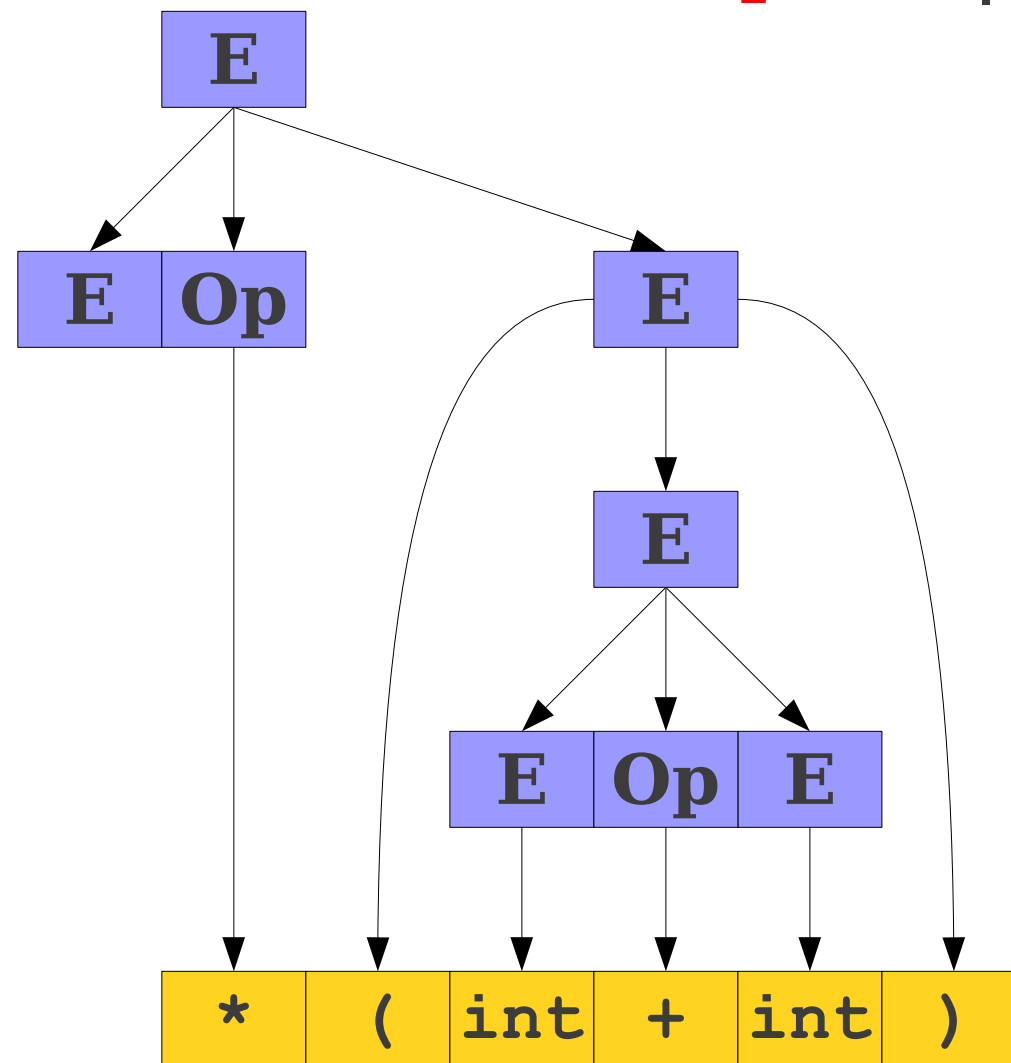


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } \text{int})$
 $\Rightarrow E \text{ Op } (E + \text{int})$
 $\Rightarrow E \text{ Op } (\text{int} + \text{int})$
 $\Rightarrow E * (\text{int} + \text{int})$

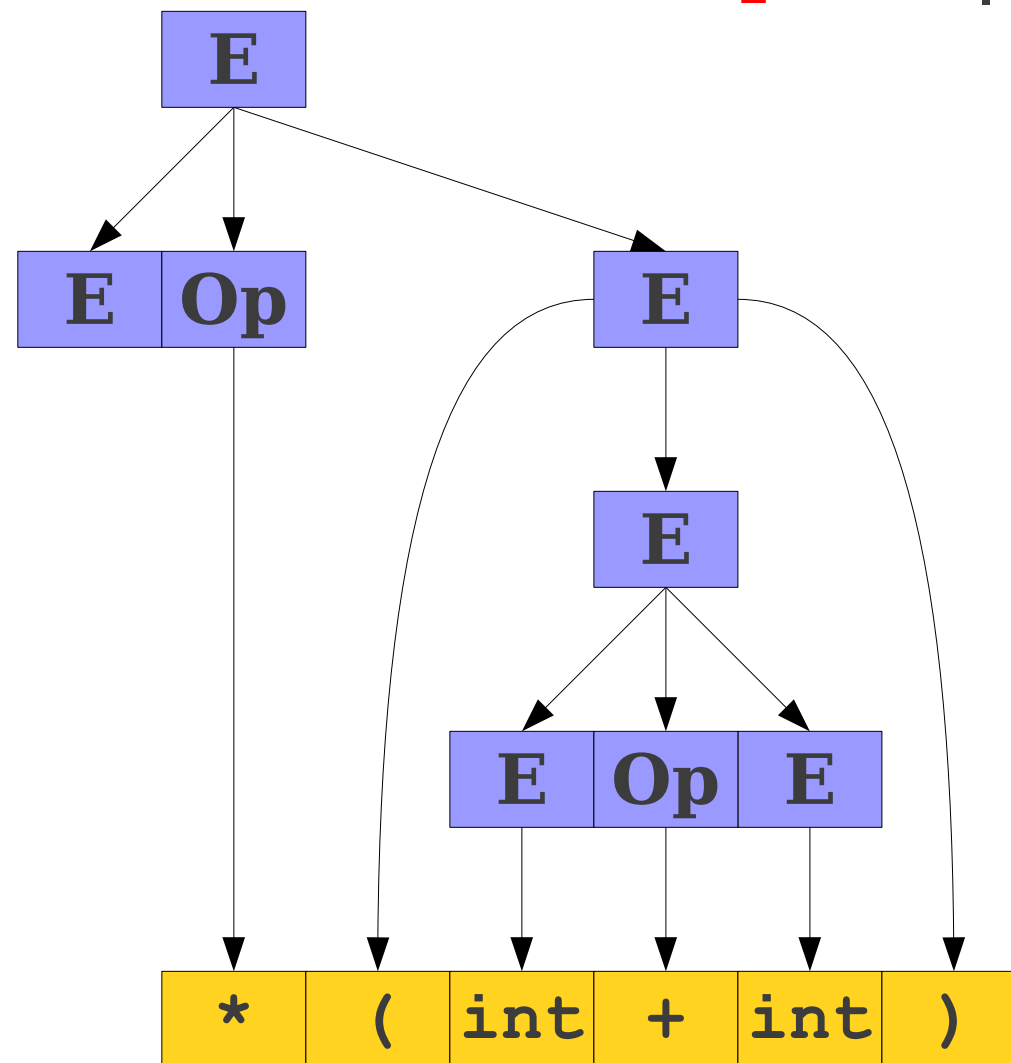


Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$

$\text{Op} \rightarrow + \mid - \mid * \mid /$

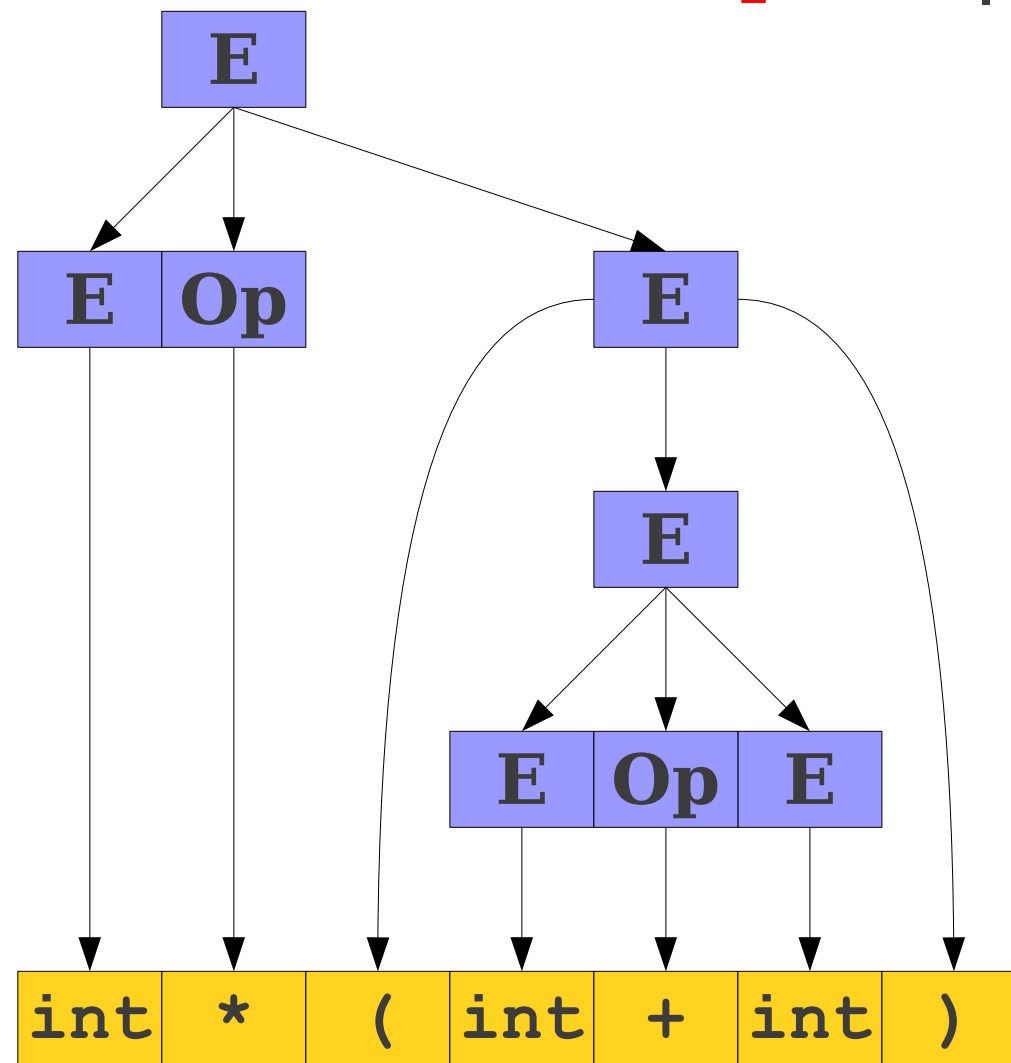
E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } \text{int})$
 $\Rightarrow E \text{ Op } (E + \text{int})$
 $\Rightarrow E \text{ Op } (\text{int} + \text{int})$
 $\Rightarrow E * (\text{int} + \text{int})$
 $\Rightarrow \text{int} * (\text{int} + \text{int})$



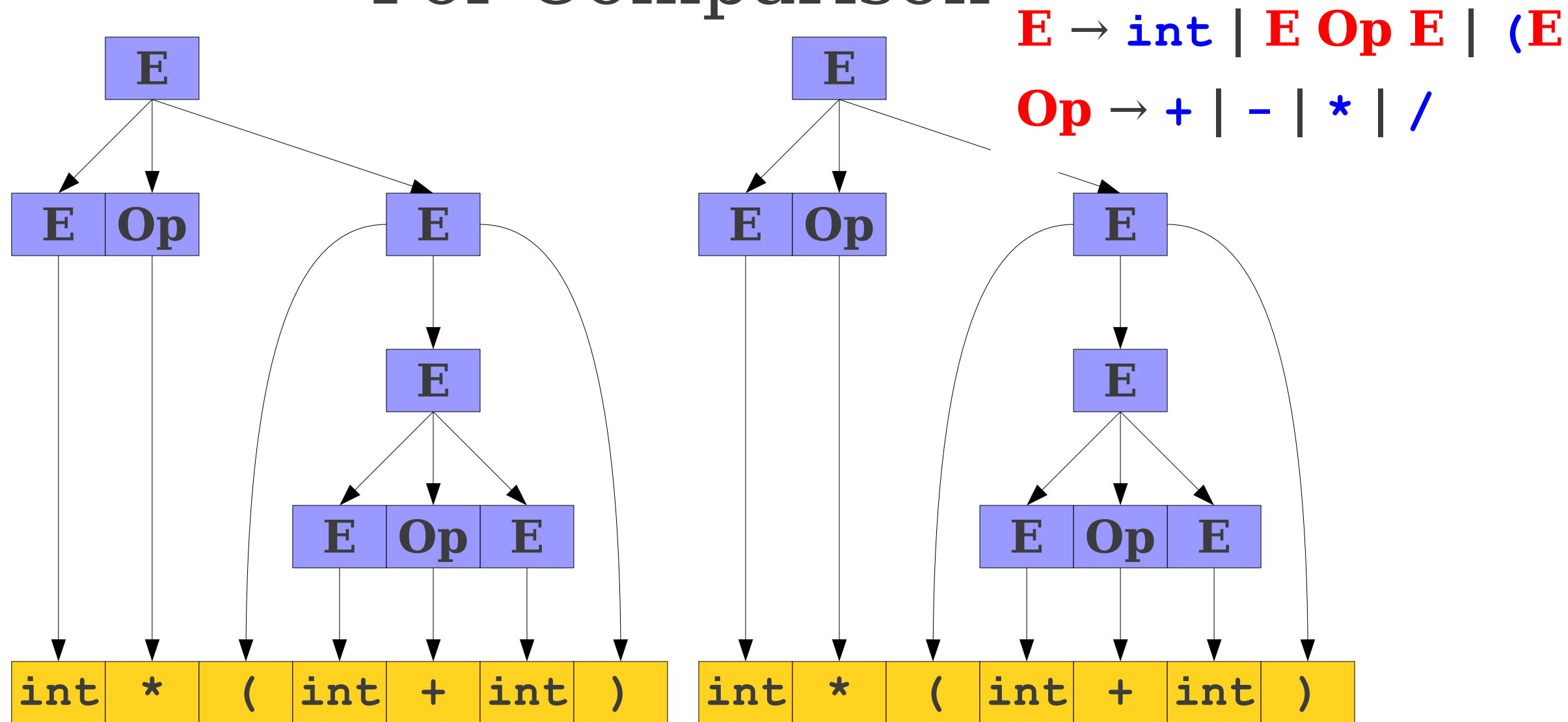
Parse Trees

$E \rightarrow \text{int} \mid E \text{ Op } E \mid (E)$
 $\text{Op} \rightarrow + \mid - \mid * \mid /$

E
 $\Rightarrow E \text{ Op } E$
 $\Rightarrow E \text{ Op } (E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } E)$
 $\Rightarrow E \text{ Op } (E \text{ Op } \text{int})$
 $\Rightarrow E \text{ Op } (E + \text{int})$
 $\Rightarrow E \text{ Op } (\text{int} + \text{int})$
 $\Rightarrow E * (\text{int} + \text{int})$
 $\Rightarrow \text{int} * (\text{int} + \text{int})$



For Comparison



Left-most derivation and right-most derivation generate the same parse tree!

But the order of the construction is different

Parse Trees

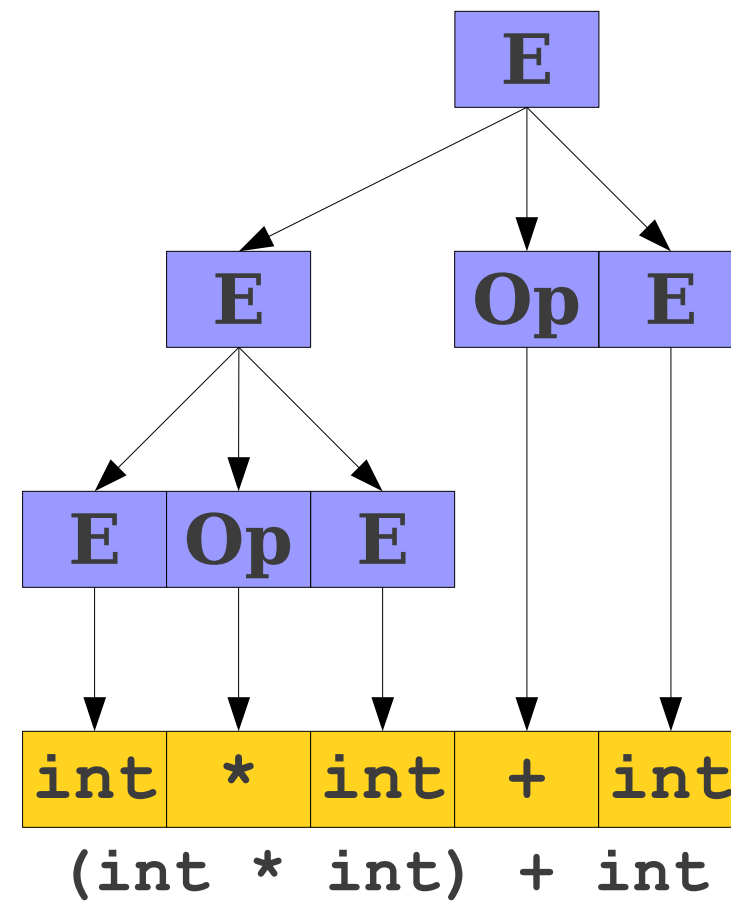
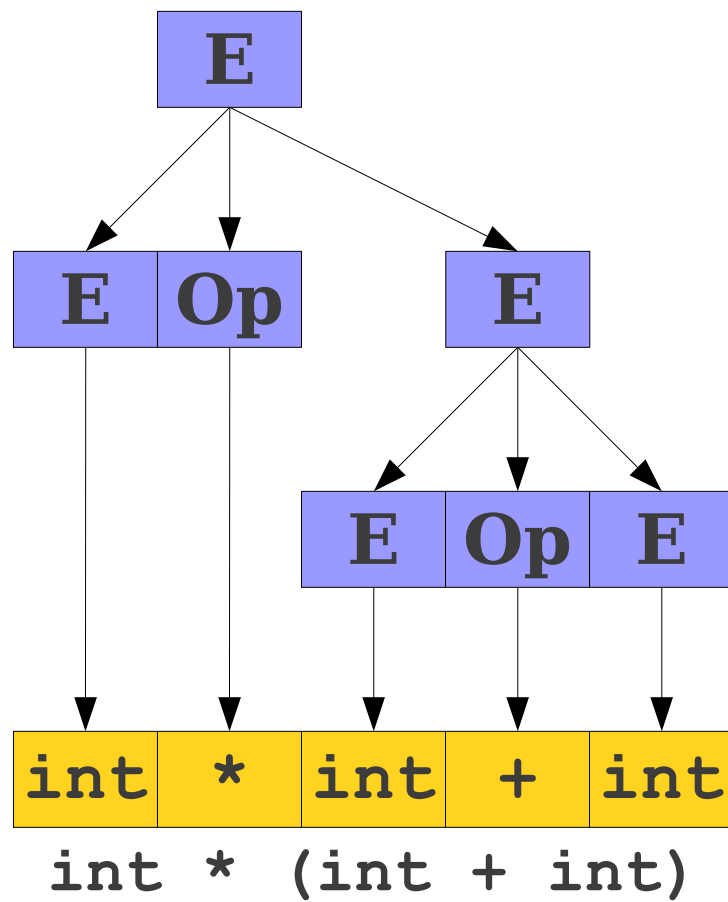
- A **parse tree** is a tree encoding the steps in a derivation.
- Internal nodes represent nonterminal symbols used in the production.
- Inorder walk of the leaves contains the generated string.
- Encodes what productions are used, not the order in which those productions are applied.

The Goal of Parsing

- Goal of syntax analysis: Recover the **structure** described by a series of tokens.
- If language is described as a CFG, goal is to recover a parse tree for the the input string.
 - Usually we do some simplifications on the tree; more on that later.
- **We will discuss how to do this more next class ...**

Challenges in Parsing

A Serious Problem



Ambiguity

- A CFG is said to be **ambiguous** if there is at least one string with two or more parse trees.
- Note that ambiguity is a property of *grammars*, not *languages*.

Is Ambiguity a Problem?

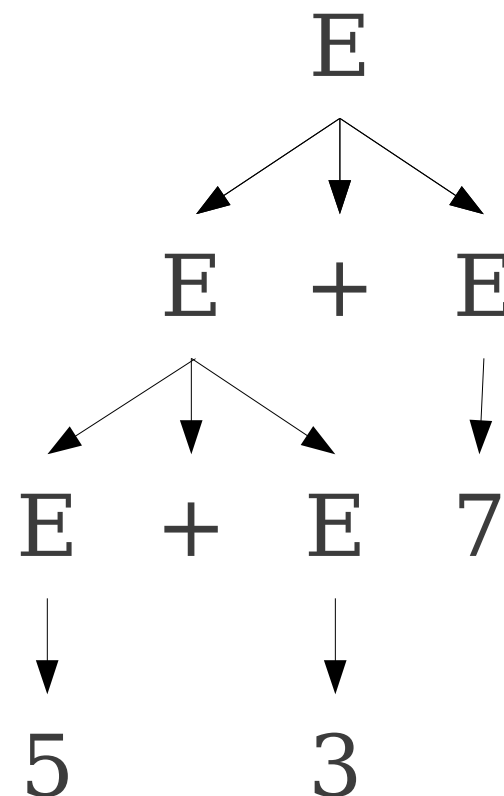
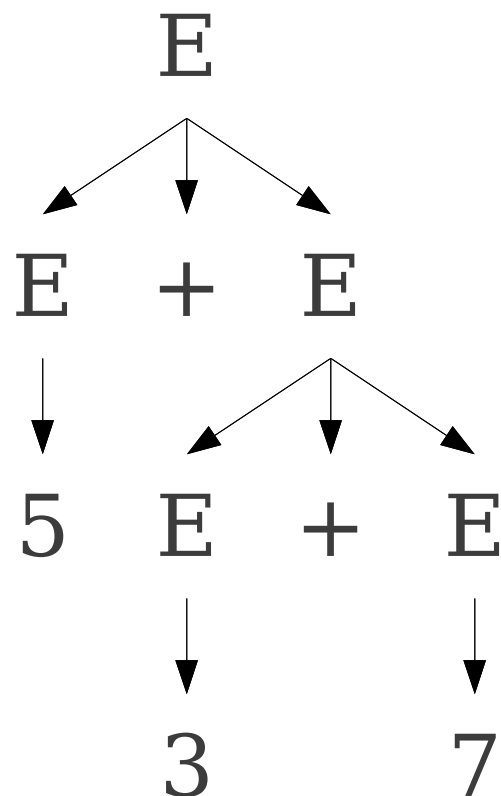
- Depends on **semantics**.

$$\mathbf{E} \rightarrow \mathbf{int} \mid \mathbf{E} + \mathbf{E}$$

Is Ambiguity a Problem?

- Depends on **semantics**.

E \rightarrow **int** | **E** + **E**



Is Ambiguity a Problem?

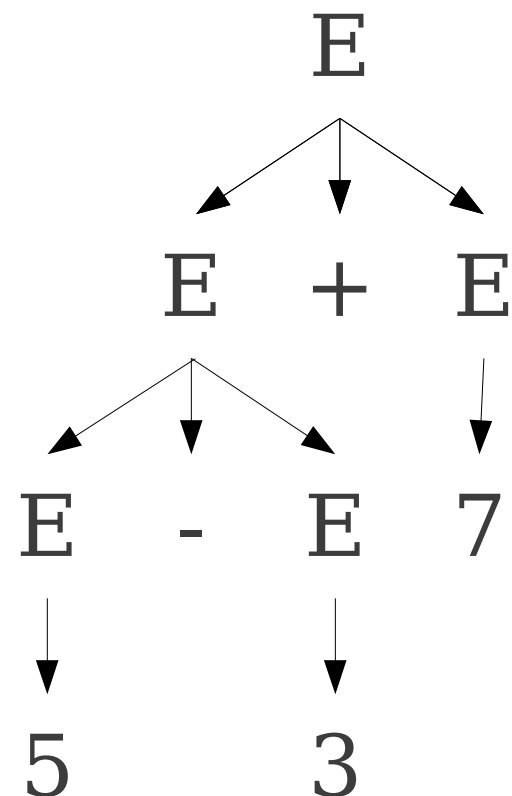
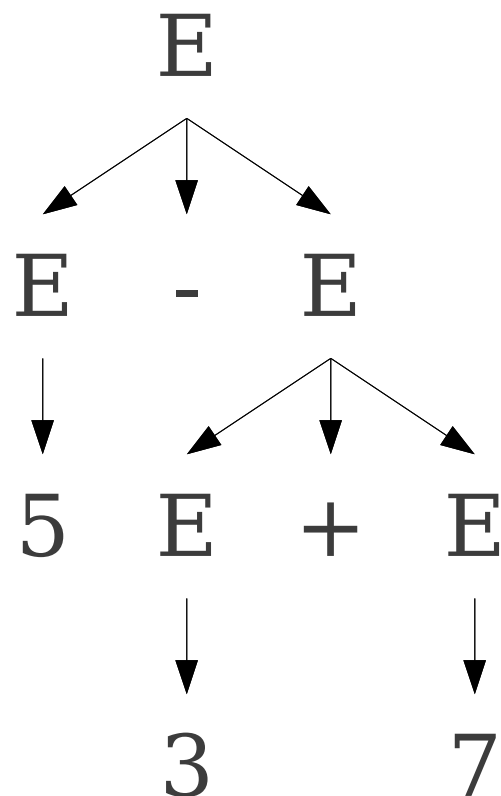
- Depends on **semantics**.

$$\mathbf{E} \rightarrow \mathbf{int} \mid \mathbf{E} + \mathbf{E} \mid \mathbf{E} - \mathbf{E}$$

Is Ambiguity a Problem?

- Depends on **semantics**.

E \rightarrow **int** | **E + E** | **E - E**



Different Parse Trees

$S \rightarrow S + S \mid S * S \mid \text{number}$

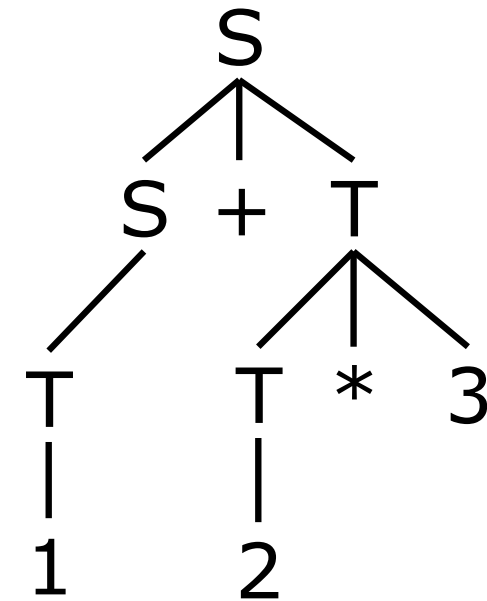
- Consider expression $1 + 2 * 3$
- Derivation 1: $S \rightarrow S + S \rightarrow 1 + S \rightarrow 1 + S * S \rightarrow 1 + 2 * S \rightarrow 1 + 2 * 3$
- Derivation 2: $S \rightarrow S * S \rightarrow S * 3 \rightarrow S + S * 3 \rightarrow S + 2 * 3 \rightarrow 1 + 2 * 3$



Eliminating Ambiguity



- Often can eliminate ambiguity by adding non-terminals & allowing recursion only on right or left
- $S \rightarrow S + T \mid T$
- $T \rightarrow T * \text{num} \mid \text{num}$
- T non-terminal enforces precedence
- Left-recursion : left-associativity



If-Then-Else

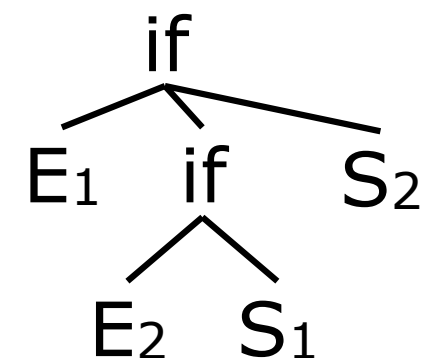
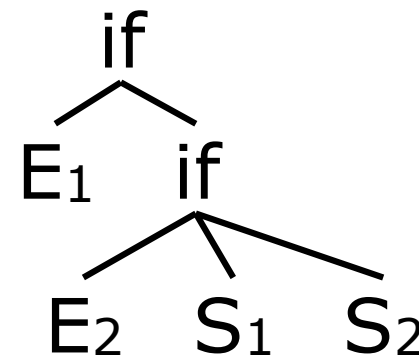
- How do we write a grammar for `if` statements?
- $S \rightarrow \text{if } (E) S$
- $S \rightarrow \text{if } (E) S \text{ else } S$
- $S \rightarrow X = E$
- Is this grammar OK?

No! Ambiguous

- $\text{if } (E_1) \text{ if } (E_2) S_1 \text{ else } S_2$

$S \rightarrow \text{if } (E) S$
$S \rightarrow \text{if } (E) S \text{ else } S$
$S \rightarrow \text{other}$

- $S \rightarrow \text{if } (E) S$
- $\rightarrow \text{if } (E) \text{ if } (E) S \text{ else } S$
- $S \rightarrow \text{if } (E) S \text{ else } S$
- $\rightarrow \text{if } (E) \text{ if } (E) S \text{ else } S$



- *Which “if” is the “else” attached to?*

Grammar for closest-if rule

- Want to rule out: if (E) if (E) S else S
- Problem: unmatched “if” may not occur as the “then” (consequent) clause of a containing “if”

statement \rightarrow matched | unmatched

matched \rightarrow if (E) matched else matched | other

unmatched \rightarrow if (E) statement |

if (E) matched else unmatched

Another example:

Context-Free Grammars

- A regular expression can be
 - Any letter
 - ε
 - The concatenation of regular expressions.
 - The union of regular expressions.
 - The Kleene closure of a regular expression.
 - A parenthesized regular expression.

Context-Free Grammars

- This gives us the following CFG:

$$\mathbf{R} \rightarrow \mathbf{a} \mid \mathbf{b} \mid \mathbf{c} \mid \dots$$

$$\mathbf{R} \rightarrow \text{“}\epsilon\text{”}$$

$$\mathbf{R} \rightarrow \mathbf{RR}$$

$$\mathbf{R} \rightarrow \mathbf{R} \text{ “} \mid \text{” } \mathbf{R}$$

$$\mathbf{R} \rightarrow \mathbf{R}^*$$

$$\mathbf{R} \rightarrow (\mathbf{R})$$

An Ambiguous Grammar

R \rightarrow **a** | **b** | **c** | ...

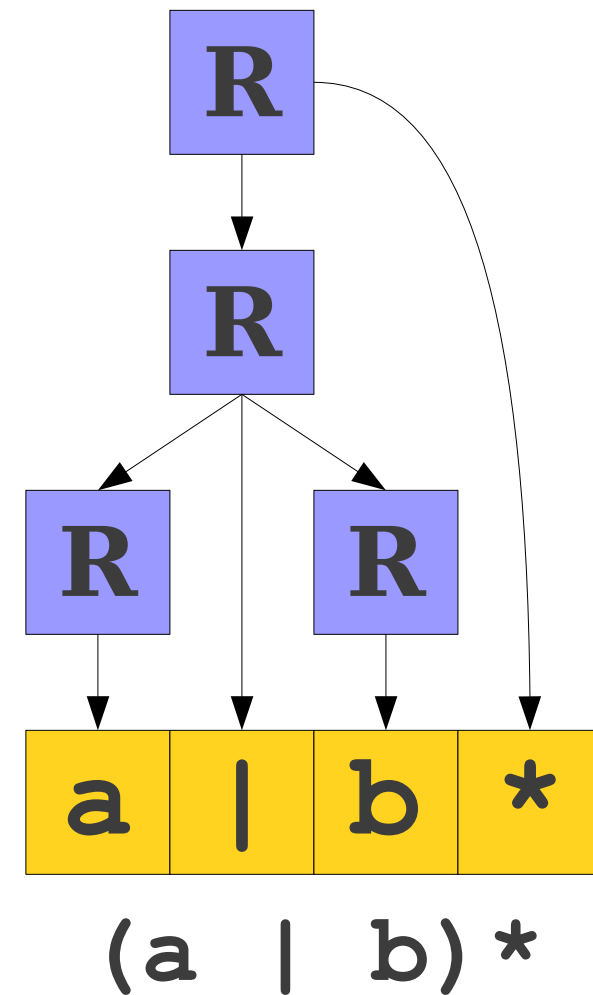
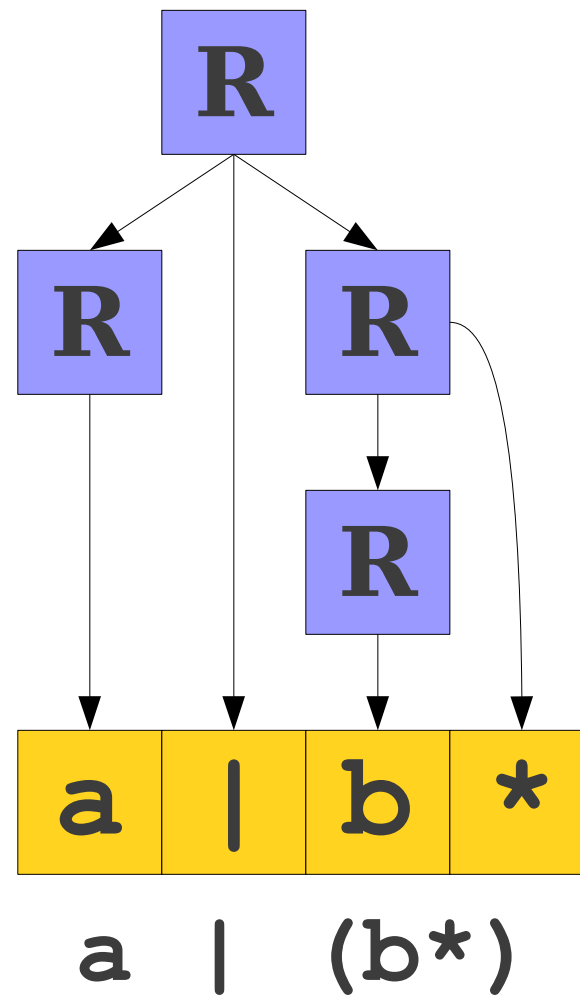
R \rightarrow " **ϵ** "

R \rightarrow **RR**

R \rightarrow **R** "**|**" **R**

R \rightarrow **R***

R \rightarrow (**R**)



Resolving Ambiguity

- We can try to resolve the ambiguity via layering:

$$\mathbf{R} \rightarrow \mathbf{a} \mid \mathbf{b} \mid \mathbf{c} \mid \dots$$

$$\mathbf{R} \rightarrow \text{"}\epsilon\text{"}$$

$$\mathbf{R} \rightarrow \mathbf{RR}$$

$$\mathbf{R} \rightarrow \mathbf{R} \text{"}\mid\text{"} \mathbf{R}$$

$$\mathbf{R} \rightarrow \mathbf{R}^*$$

$$\mathbf{R} \rightarrow (\mathbf{R})$$

$$\mathbf{R} \rightarrow \mathbf{S} \mid \mathbf{R} \text{"}\mid\text{"} \mathbf{S}$$

$$\mathbf{S} \rightarrow \mathbf{T} \mid \mathbf{ST}$$

$$\mathbf{T} \rightarrow \mathbf{U} \mid \mathbf{T}^*$$

$$\mathbf{U} \rightarrow \mathbf{a} \mid \mathbf{b} \mid \mathbf{c} \mid \dots$$

$$\mathbf{U} \rightarrow \text{"}\epsilon\text{"}$$

$$\mathbf{U} \rightarrow (\mathbf{R})$$

Why is this unambiguous?

$R \rightarrow S \mid R \text{ “|” } S$

$S \rightarrow T \mid ST$

$T \rightarrow U \mid T^*$

$U \rightarrow a \mid b \mid \dots$

$U \rightarrow \text{“}\epsilon\text{”}$

$U \rightarrow (R)$

Only generates
“atomic” expressions

Why is this unambiguous?

$R \rightarrow S \mid R \text{ “|” } S$

$S \rightarrow T \mid ST$

$T \rightarrow U \mid T^*$

$U \rightarrow a \mid b \mid \dots$

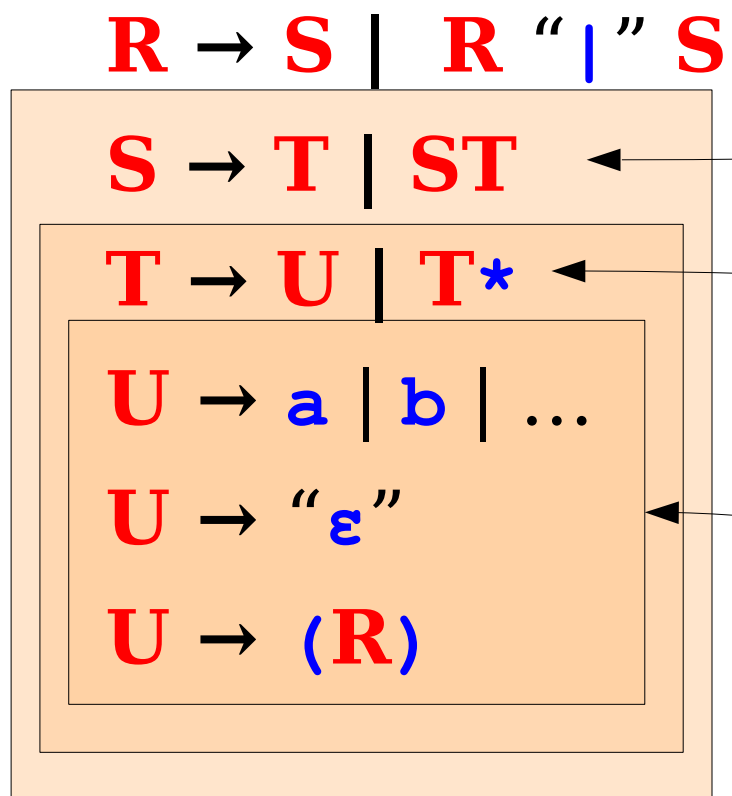
$U \rightarrow \epsilon$

$U \rightarrow (R)$

Puts stars onto
atomic expressions

Only generates
“atomic” expressions

Why is this unambiguous?

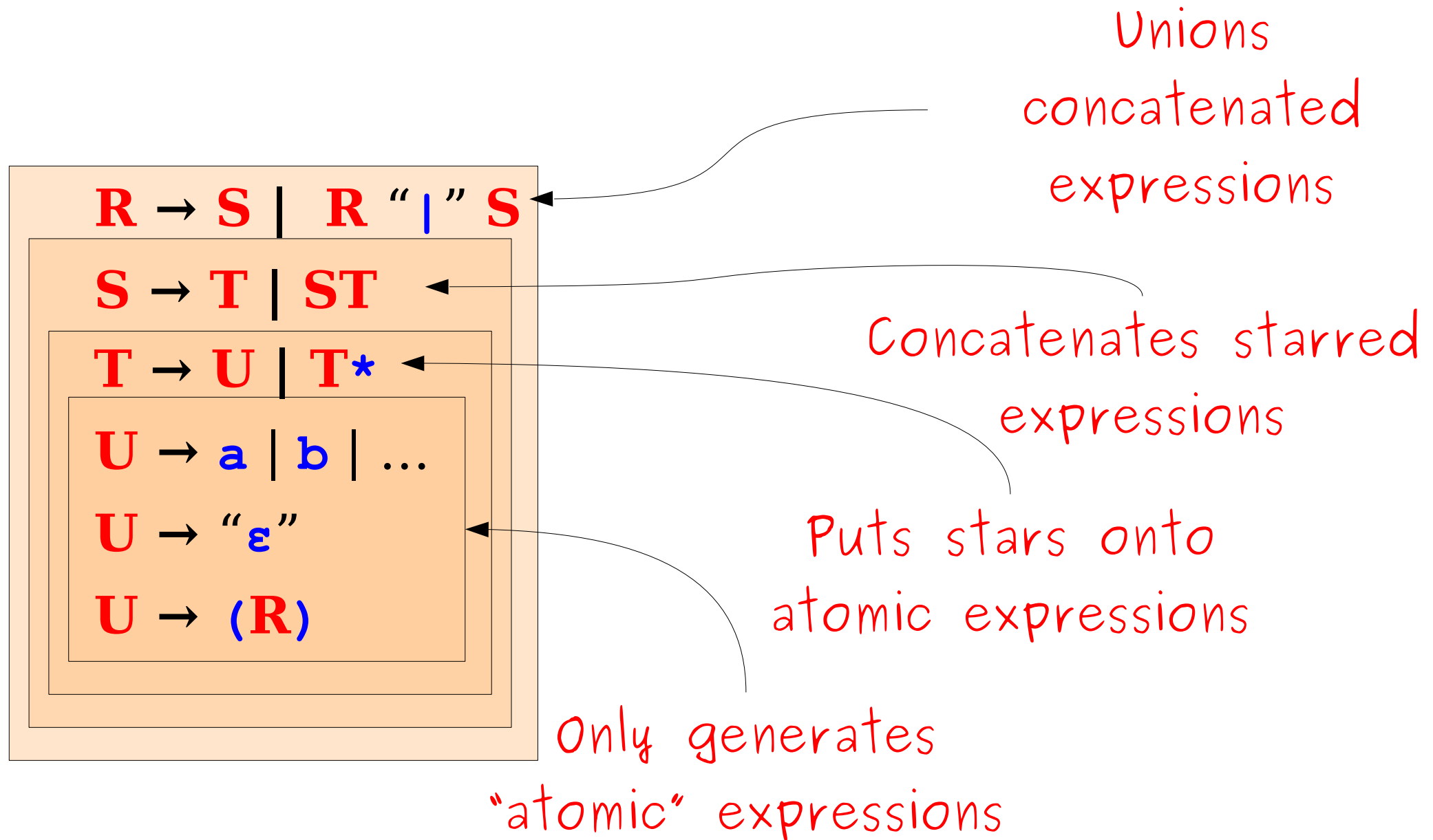


Concatenates starred
expressions

Puts stars onto
atomic expressions

Only generates
"atomic" expressions

Why is this unambiguous?



R

R \rightarrow **S** | **R** “|” **S**

S \rightarrow **T** | **ST**

T \rightarrow **U** | **T***

U \rightarrow **a** | **b** | **c** | ...

U \rightarrow “ **ϵ** ”

U \rightarrow (**R**)

a	b		c		a	*
---	---	--	---	--	---	---

R \rightarrow **S** | **R** “|” **S**

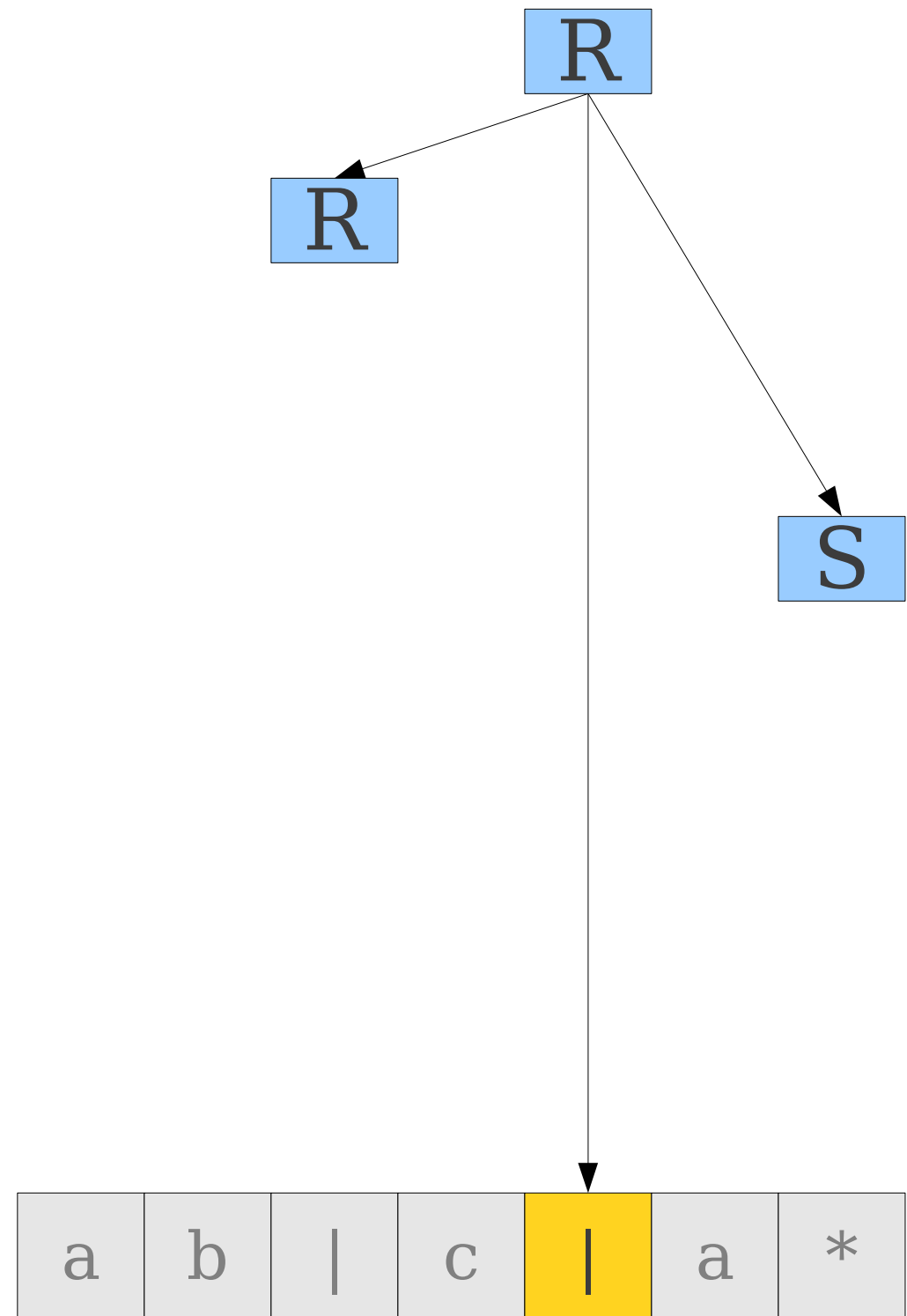
S \rightarrow **T** | **ST**

T \rightarrow **U** | **T***

U \rightarrow **a** | **b** | **c** | ...

U \rightarrow “ ϵ ”

U \rightarrow (**R**)



R \rightarrow **S** | **R** “|” **S**

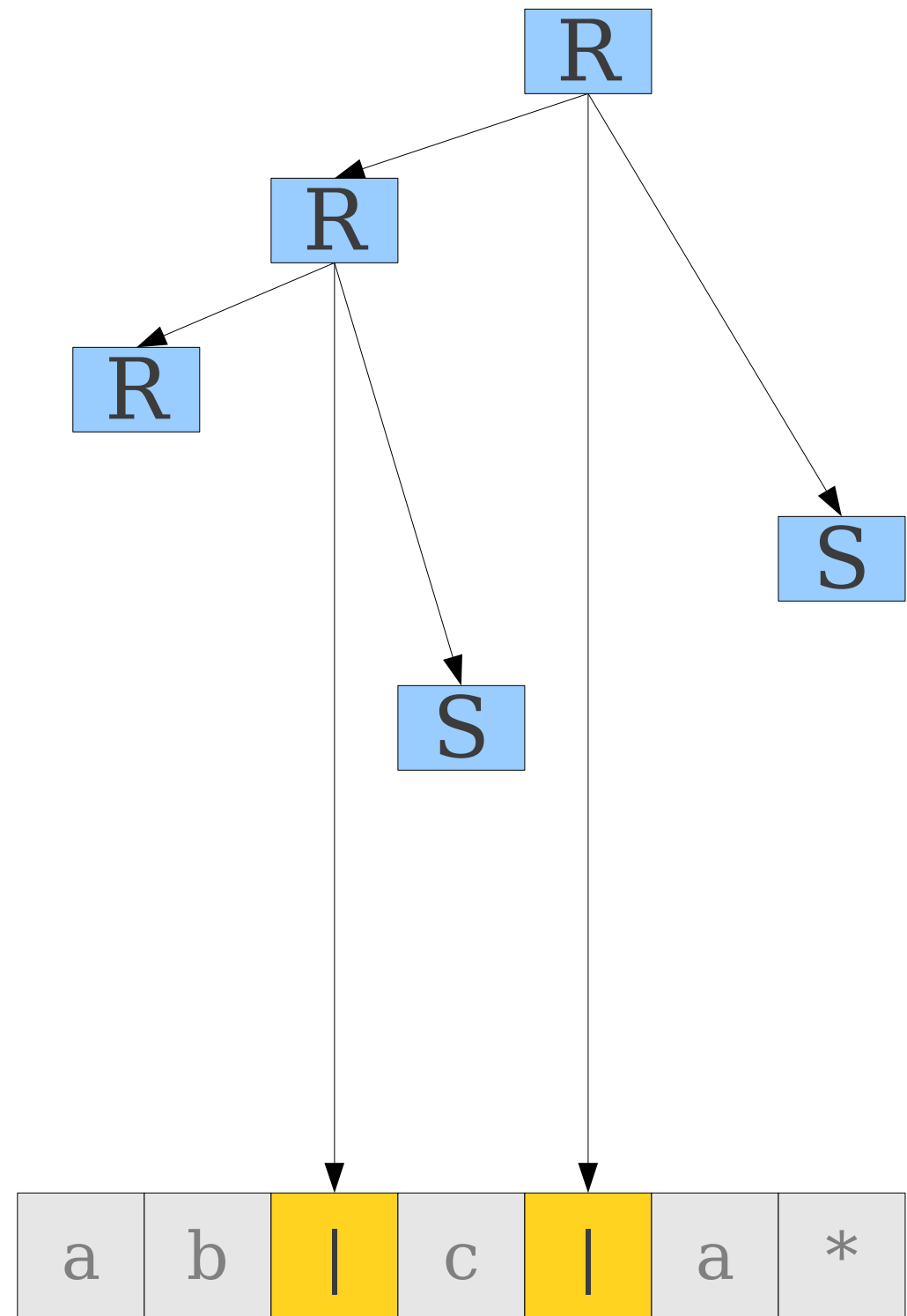
S → T | ST

T \rightarrow **U** | **T***

U \rightarrow **a** | **b** | **c** | ...

U \rightarrow "ε"

U \rightarrow **(R)**



R \rightarrow **S** | **R** “ | ” **S**

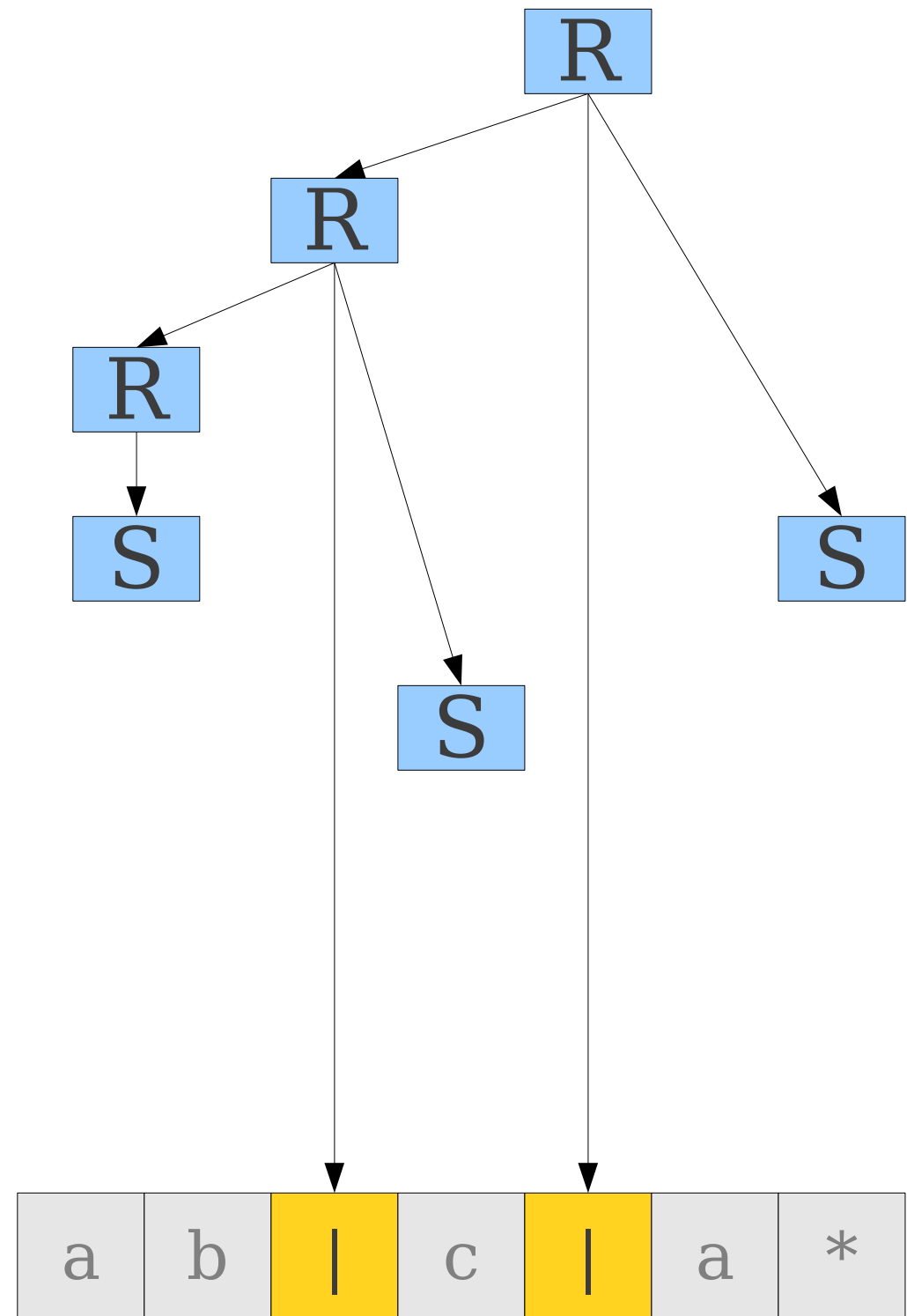
S \rightarrow **T** | **ST**

T \rightarrow **U** | **T***

U \rightarrow **a** | **b** | **c** | ...

U \rightarrow "ε"

U \rightarrow **(R)**



R \rightarrow **S** | **R** “ | ” **S**

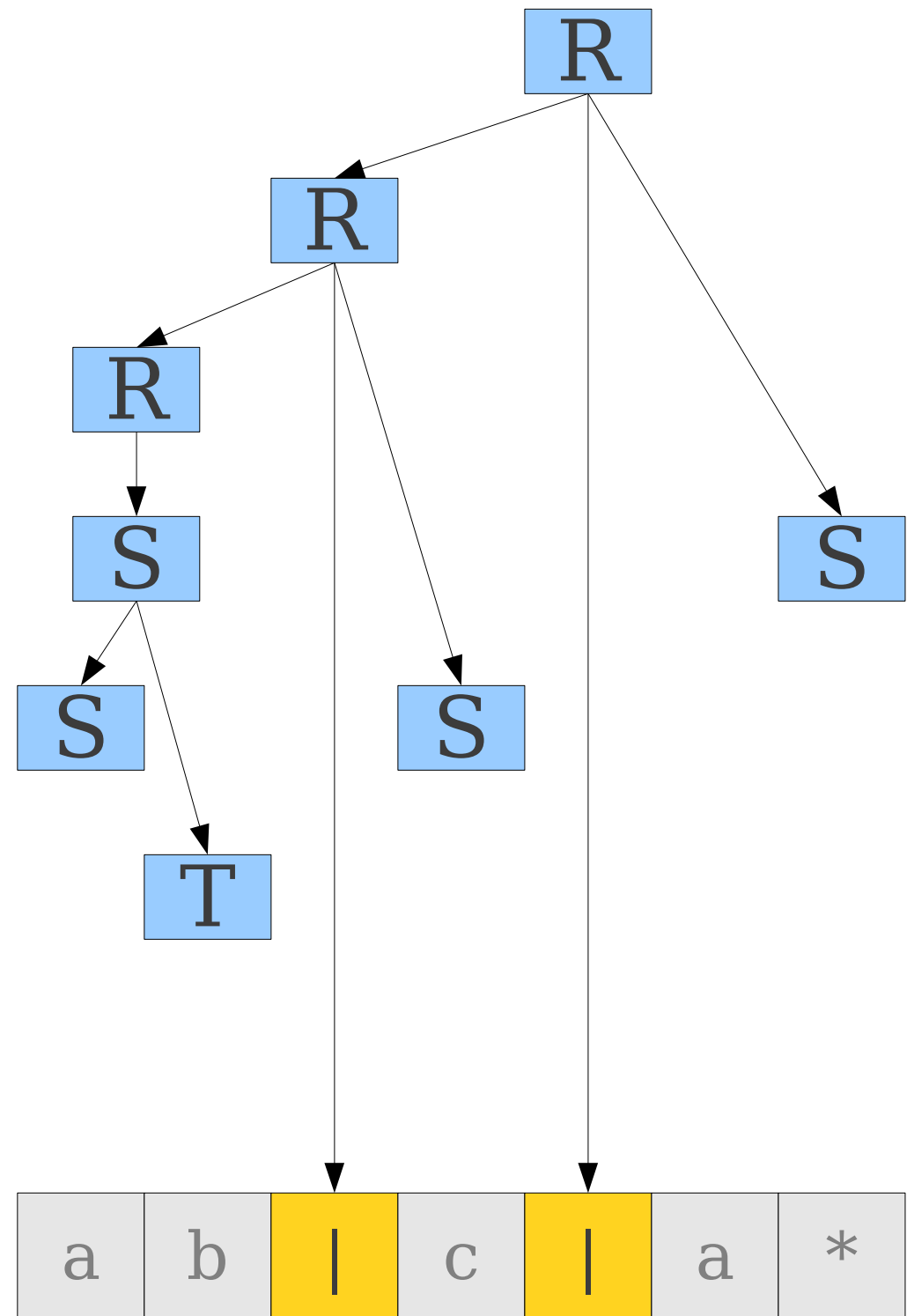
S → T | ST

T \rightarrow **U** | **T***

U \rightarrow **a** | **b** | **c** | ...

U \rightarrow “ **ϵ** ”

U \rightarrow **(R)**



$R \rightarrow S \mid R \mid S$

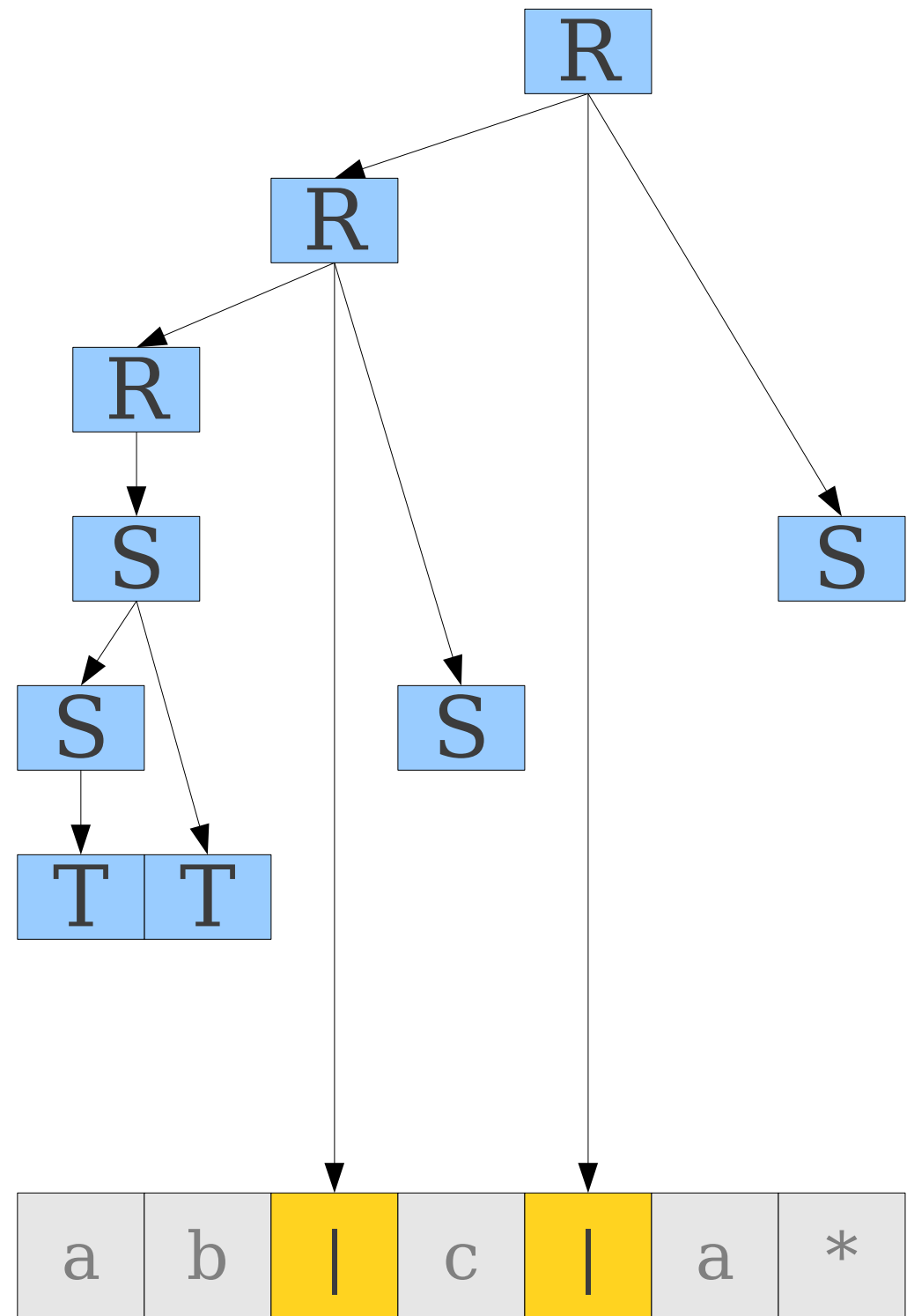
$S \rightarrow T \mid ST$

$T \rightarrow U \mid T^*$

$U \rightarrow a \mid b \mid c \mid \dots$

$U \rightarrow \epsilon$

$U \rightarrow (R)$



R \rightarrow **S** | **R** “|” **S**

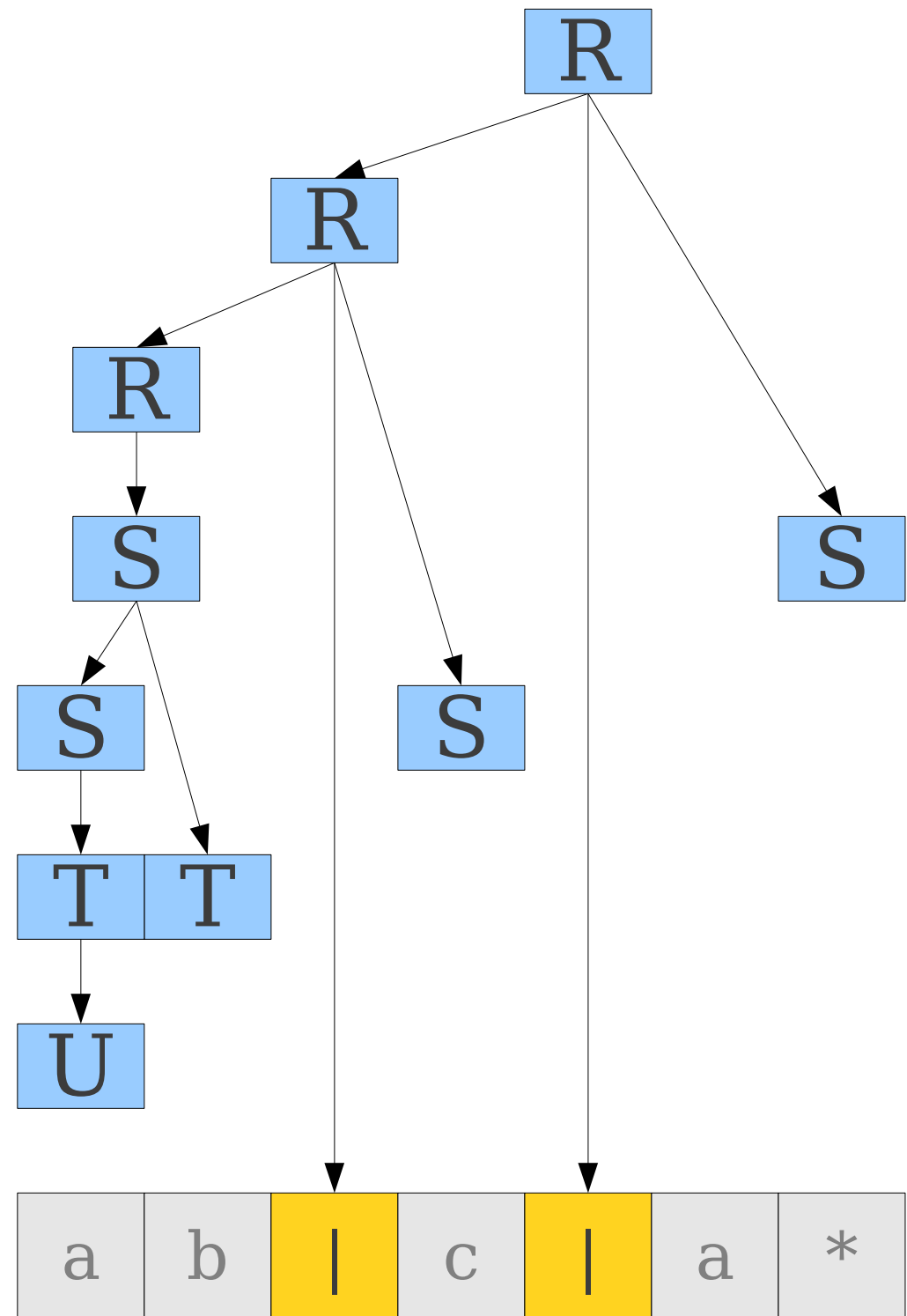
S \rightarrow **T** | **ST**

T \rightarrow **U** | **T***

U \rightarrow **a** | **b** | **c** | ...

U \rightarrow “ ϵ ”

U \rightarrow (**R**)



R \rightarrow **S** | **R** “|” **S**

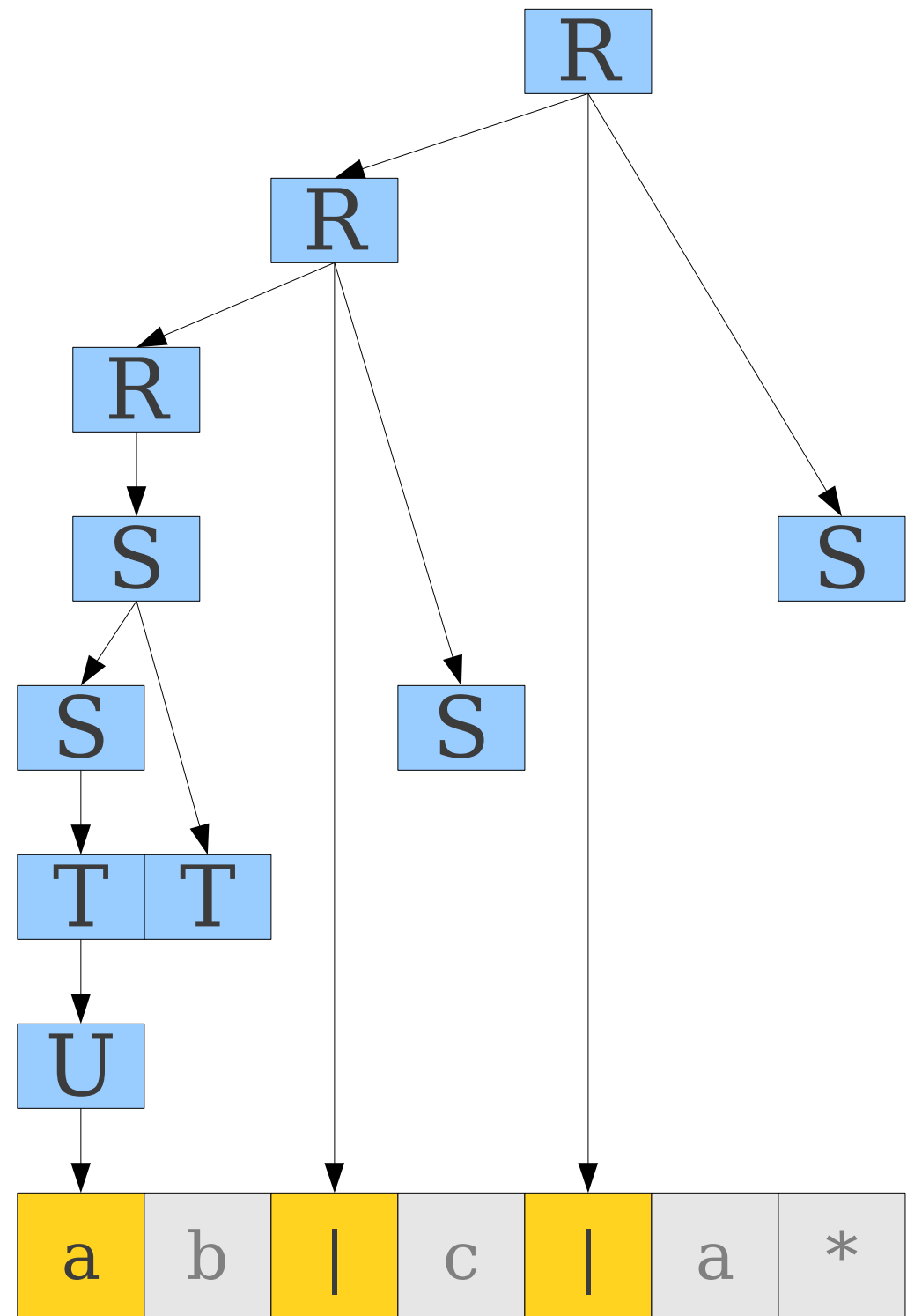
S \rightarrow **T** | **ST**

T \rightarrow **U** | **T***

U \rightarrow **a** | **b** | **c** | ...

U \rightarrow “ ϵ ”

U \rightarrow (**R**)



R → **S** | **R** “|” **S**

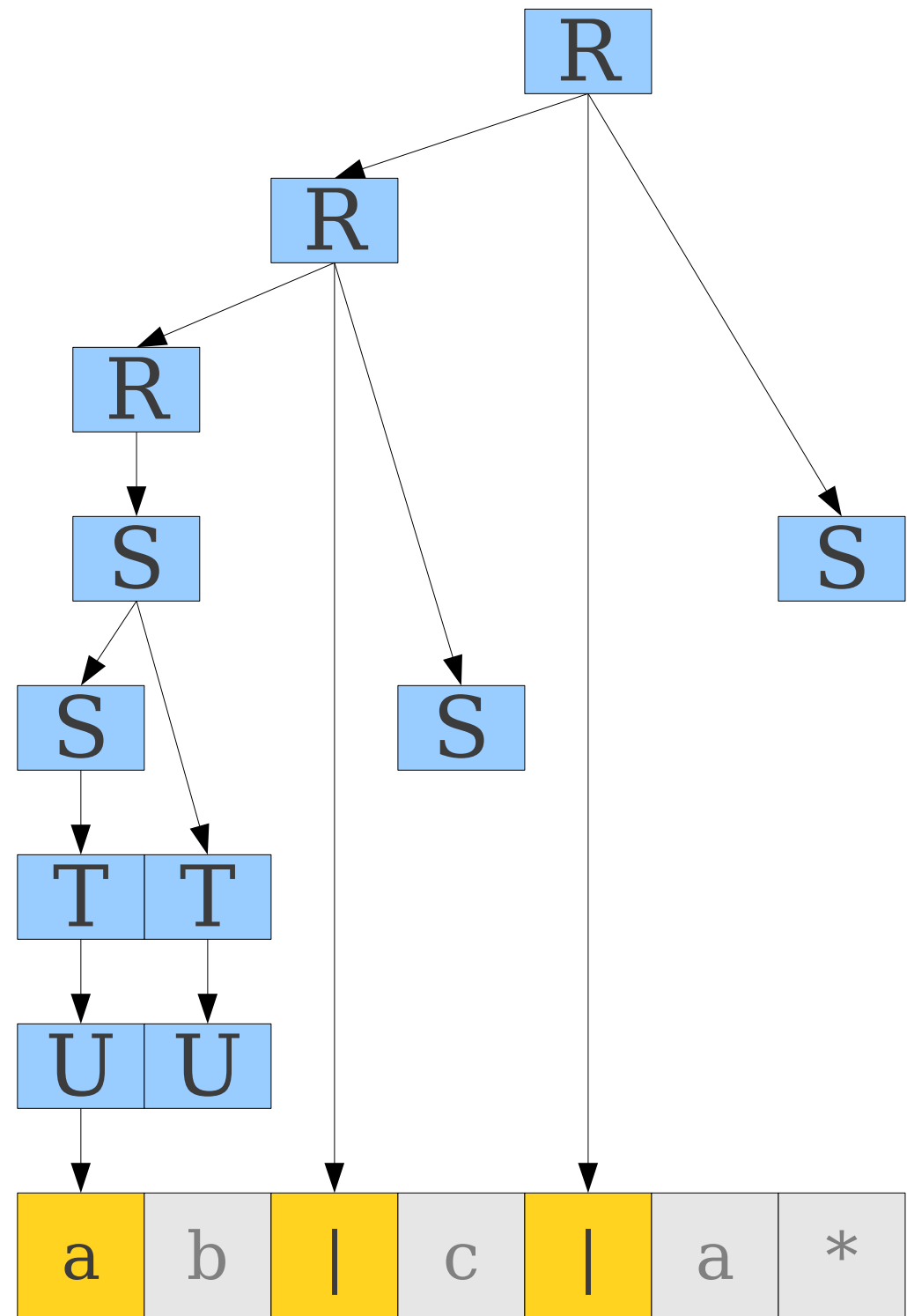
S → **T** | **ST**

T → **U** | **T***

U → **a** | **b** | **c** | ...

U → “**ε**”

U → (**R**)



R \rightarrow **S** | **R** “|” **S**

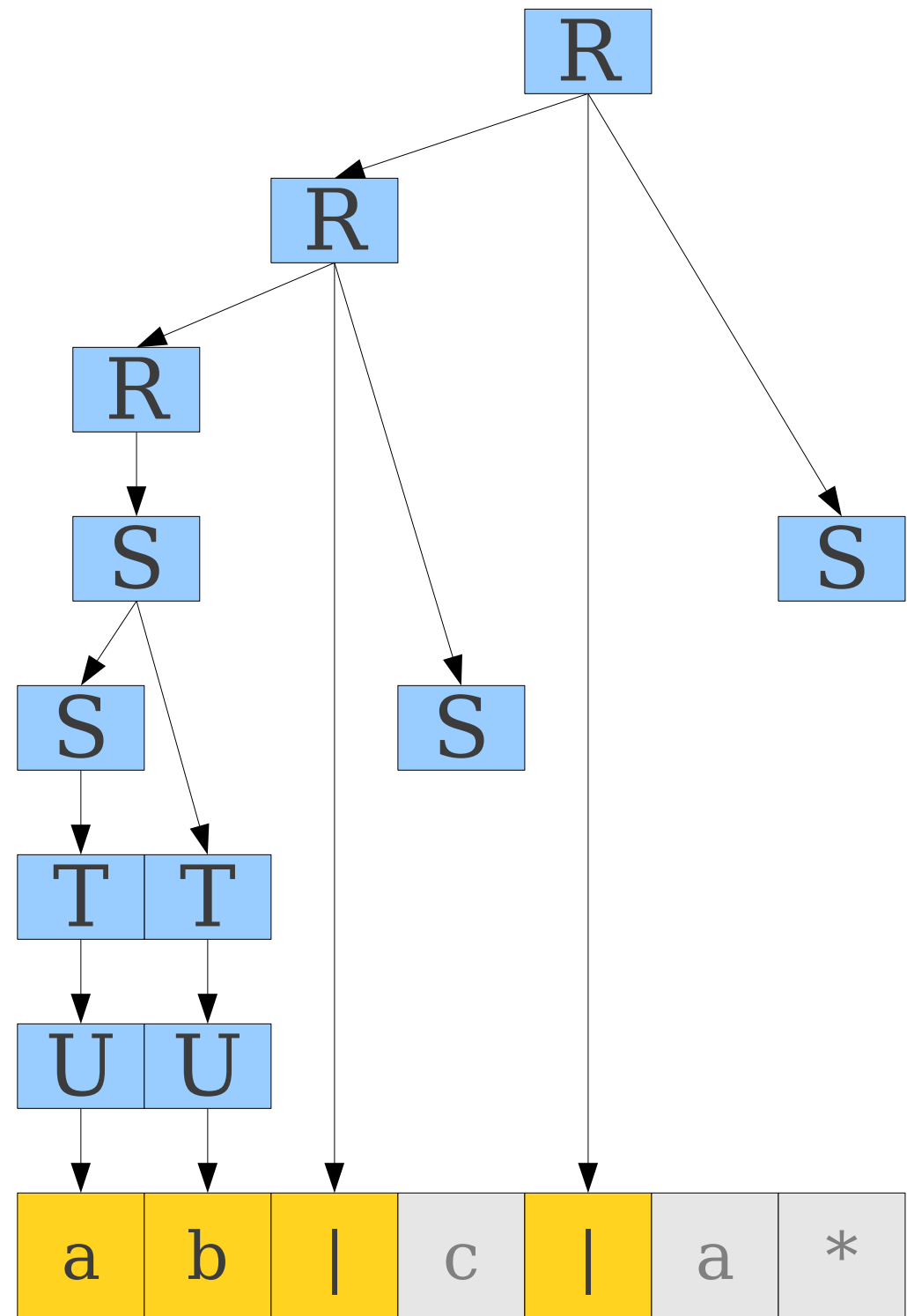
S → T | ST

T \rightarrow **U** | **T***

U \rightarrow **a** | **b** | **c** | ...

U \rightarrow "ε"

U \rightarrow **(R)**



R → **S** | **R** “|” **S**

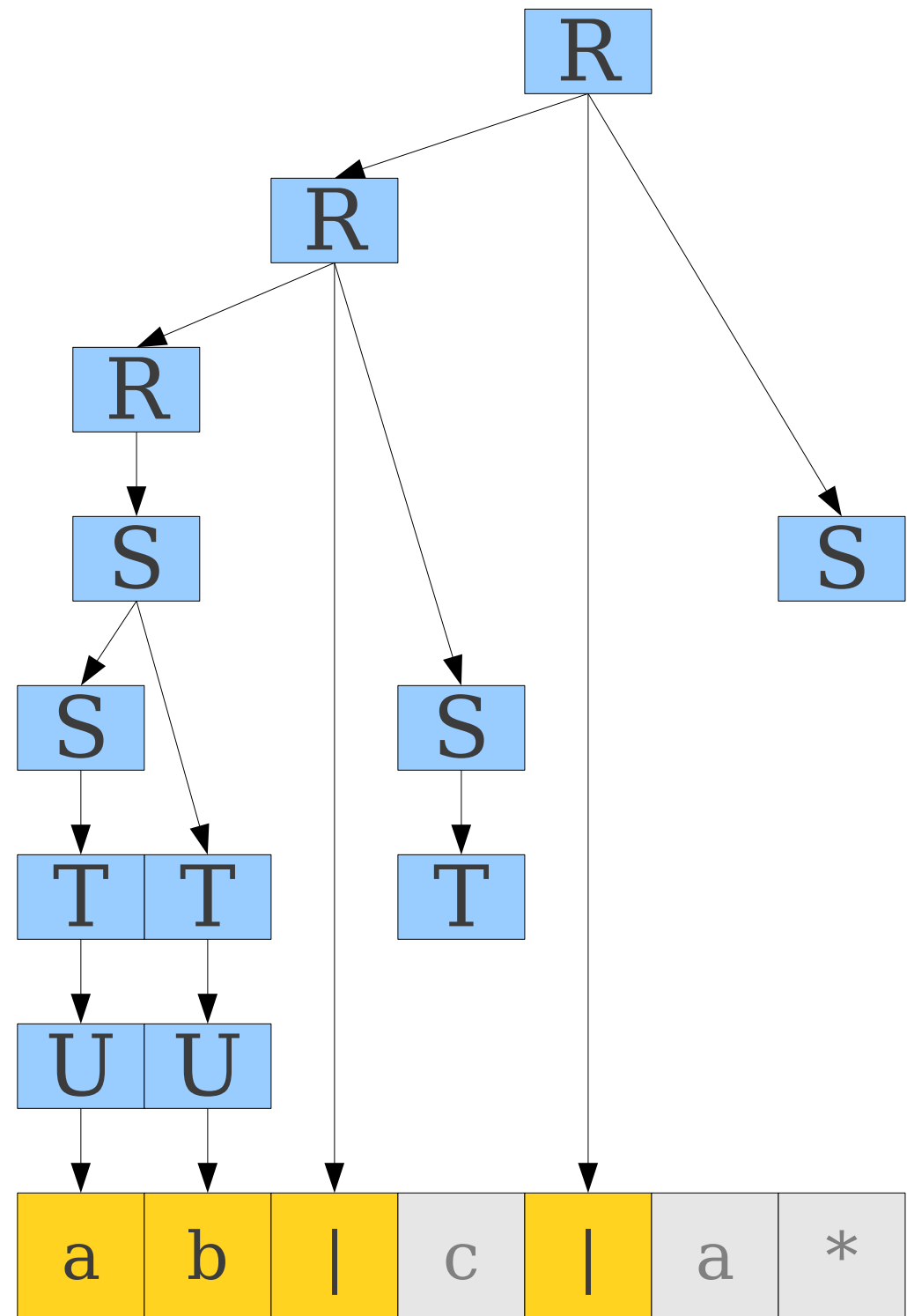
S → **T** | **ST**

T → **U** | **T***

U → **a** | **b** | **c** | ...

U → “ε”

U → (**R**)



R → **S** | **R** “|” **S**

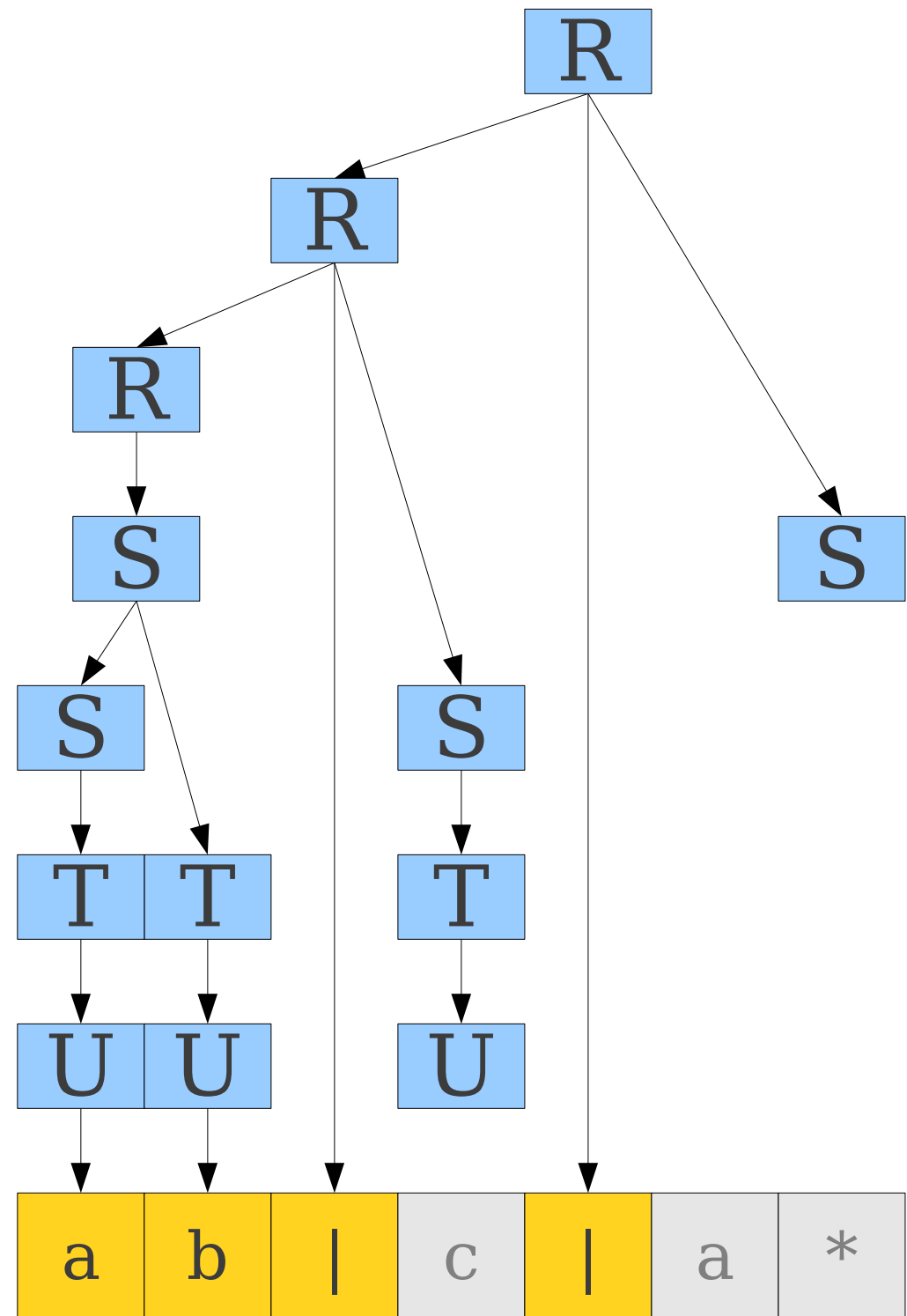
S → **T** | **ST**

T → **U** | **T***

U → **a** | **b** | **c** | ...

U → “ε”

U → (**R**)



$R \rightarrow S \mid R \mid S$

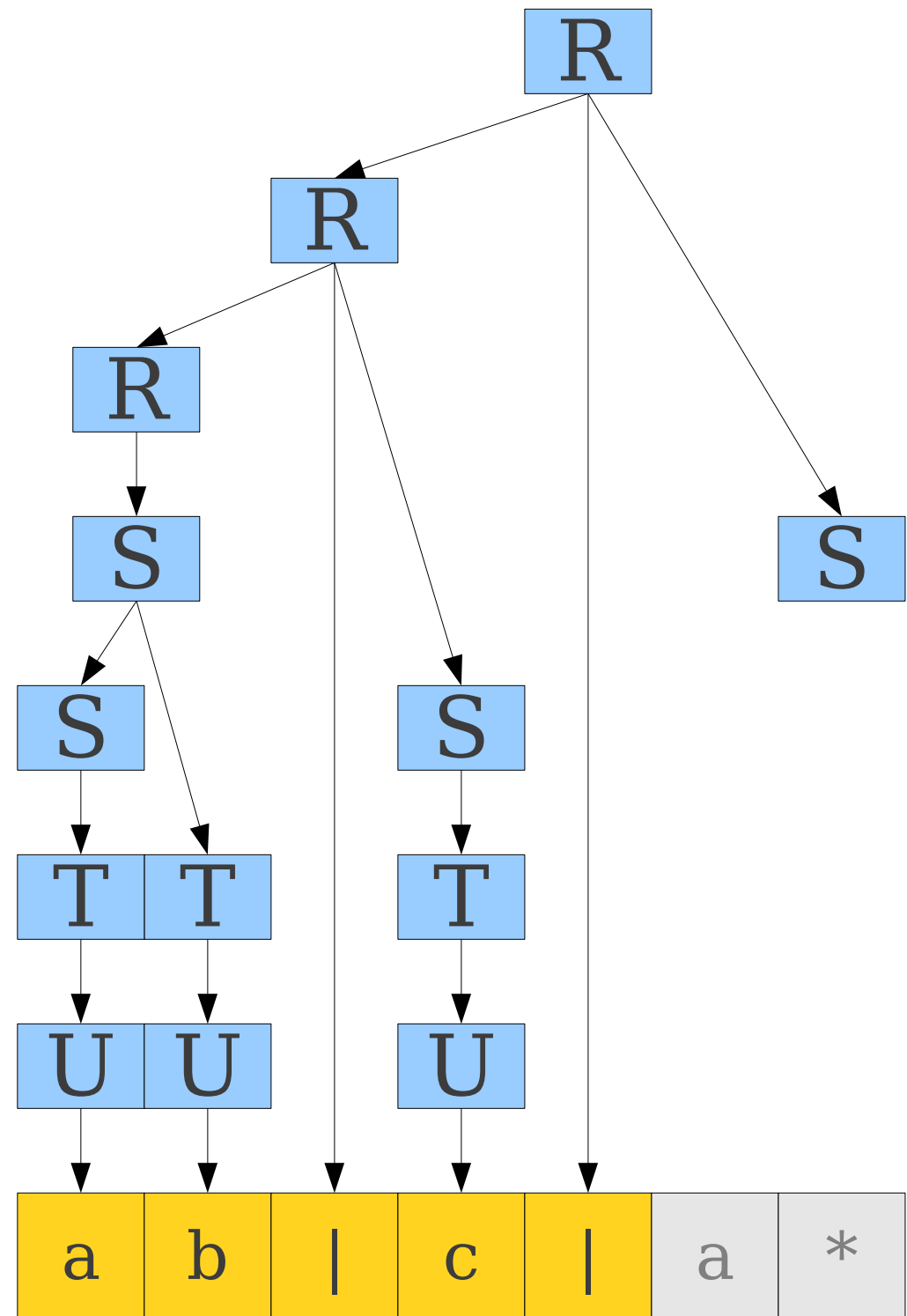
$S \rightarrow T \mid ST$

$T \rightarrow U \mid T^*$

$U \rightarrow a \mid b \mid c \mid \dots$

$U \rightarrow \epsilon$

$U \rightarrow (R)$



R \rightarrow **S** | **R** “|” **S**

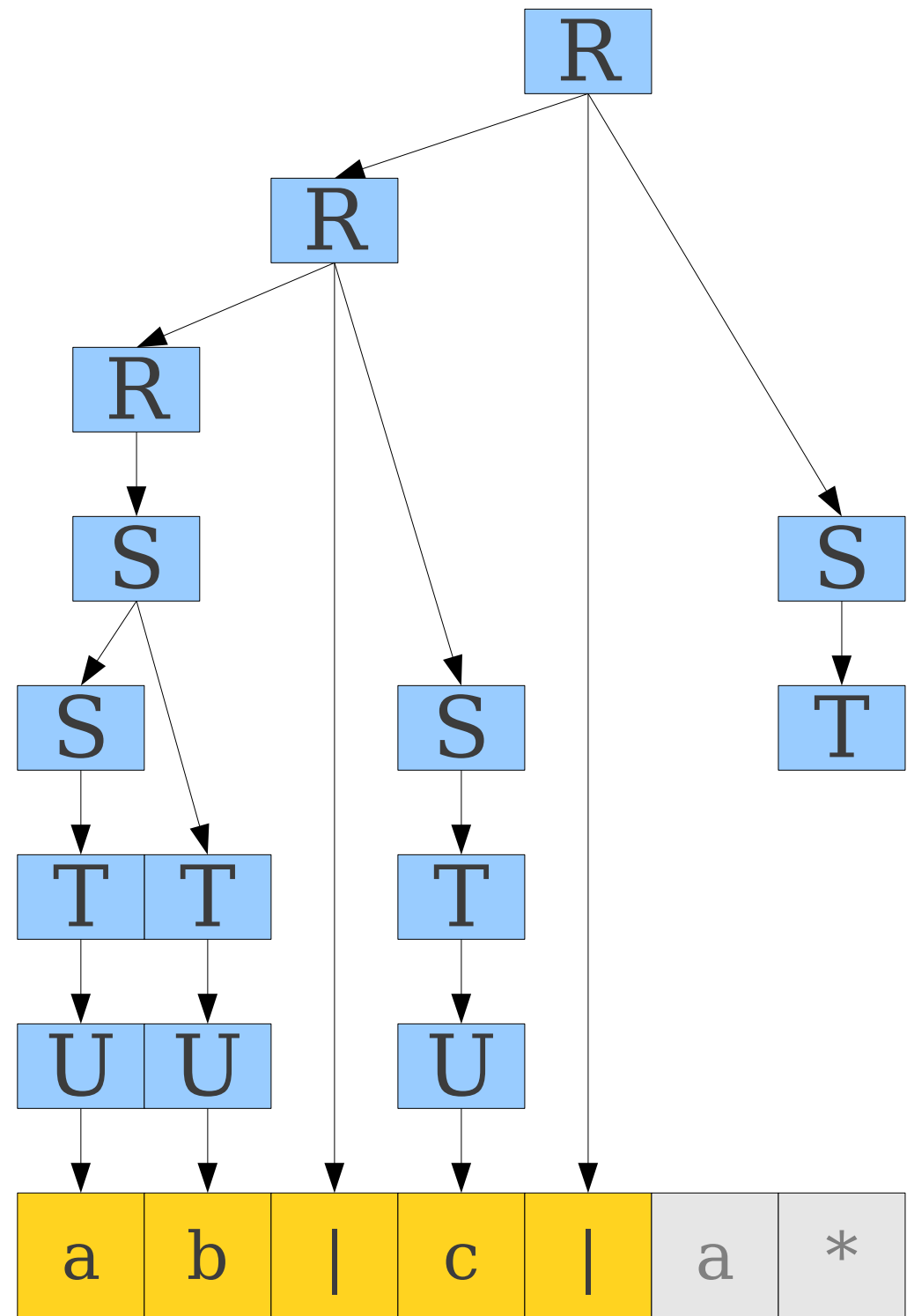
S \rightarrow **T** | **ST**

T \rightarrow **U** | **T***

U \rightarrow **a** | **b** | **c** | ...

U \rightarrow “ ϵ ”

U \rightarrow (**R**)



R \rightarrow **S** | **R** “|” **S**

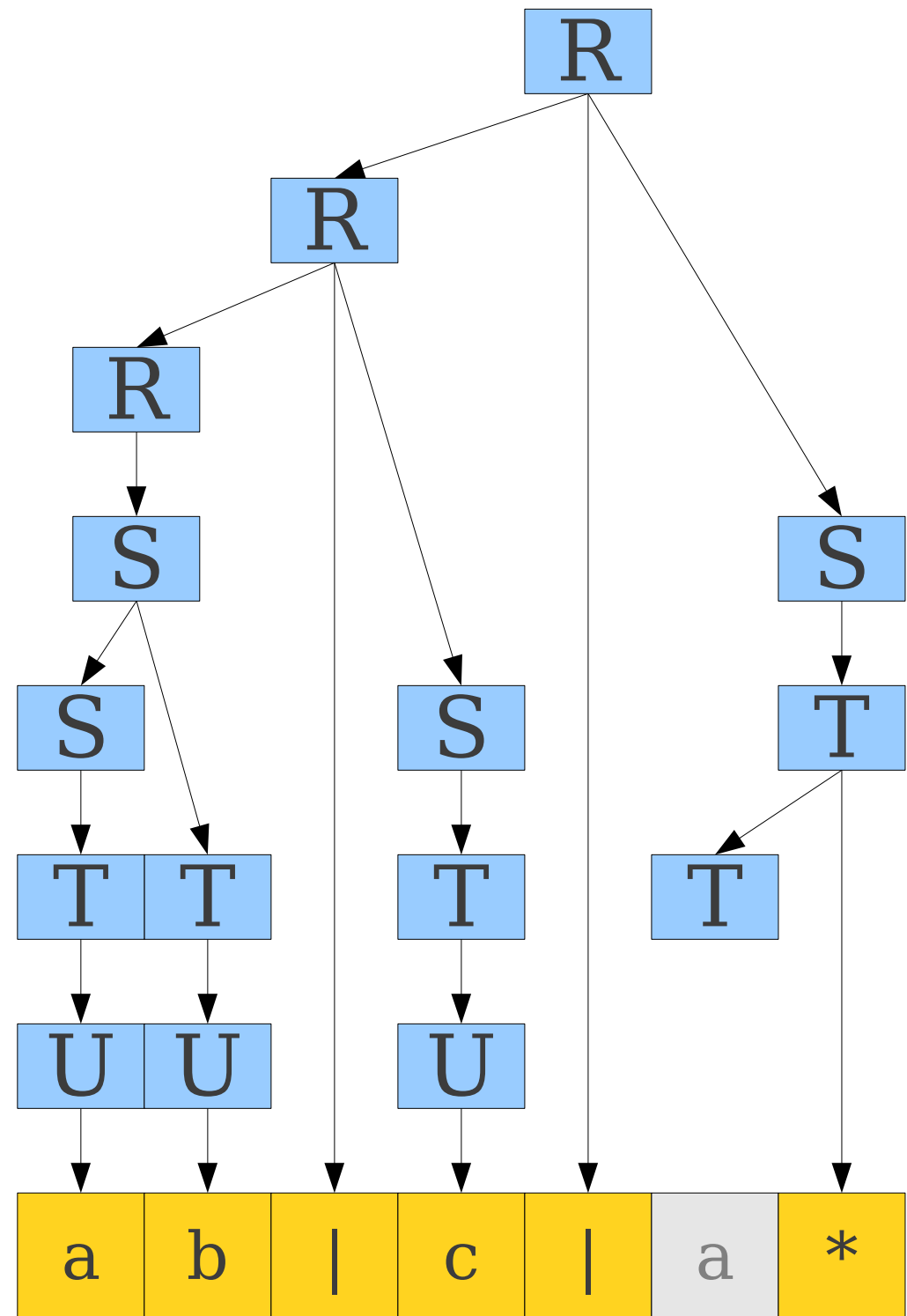
S → T | ST

T \rightarrow **U** | **T***

U \rightarrow **a** | **b** | **c** | ...

U \rightarrow “ **ϵ** ”

U \rightarrow **(R)**



R → **S** | **R** “|” **S**

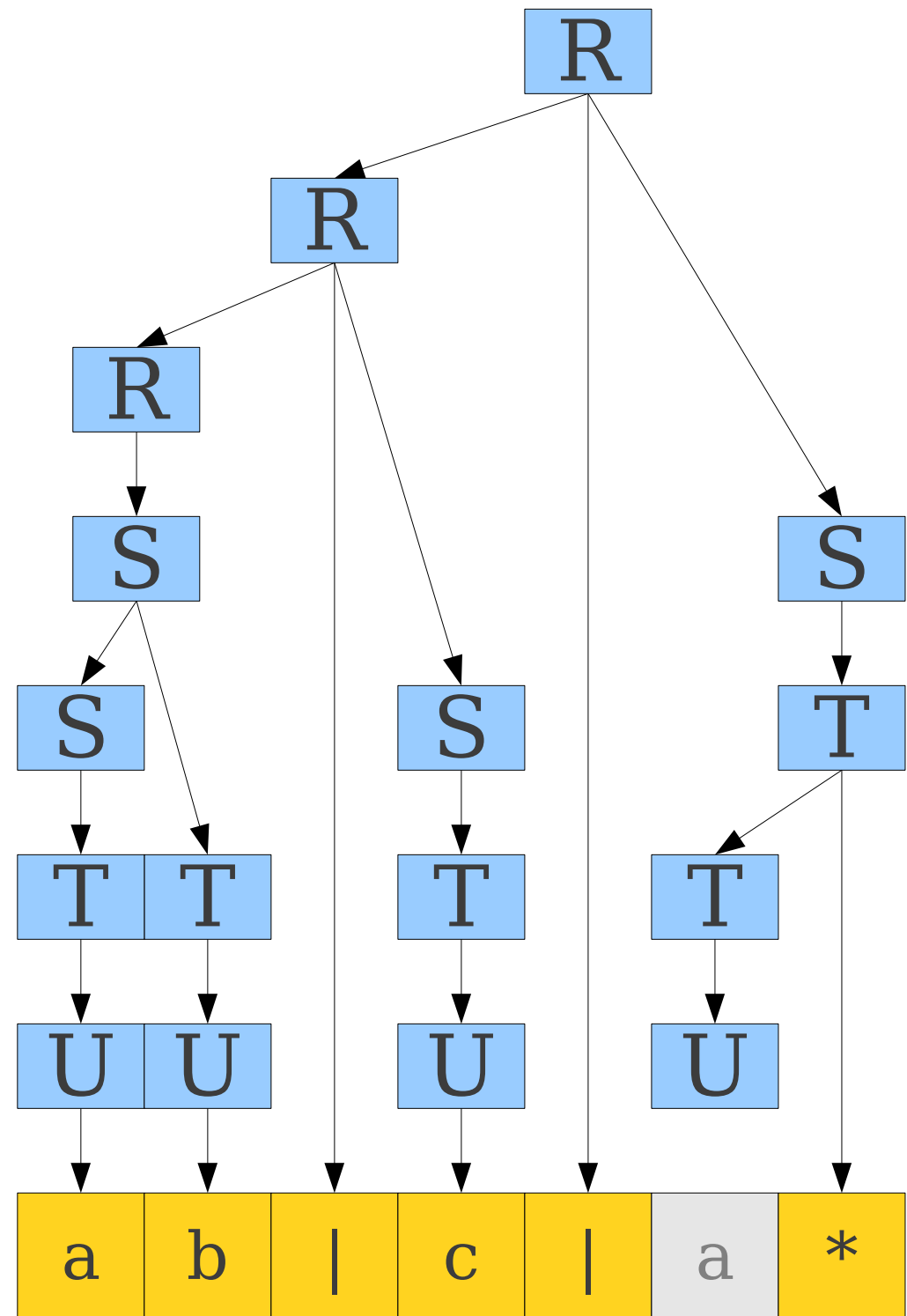
S → **T** | **ST**

T → **U** | **T***

U → **a** | **b** | **c** | ...

U → “ε”

U → (**R**)



R → **S** | **R** “|” **S**

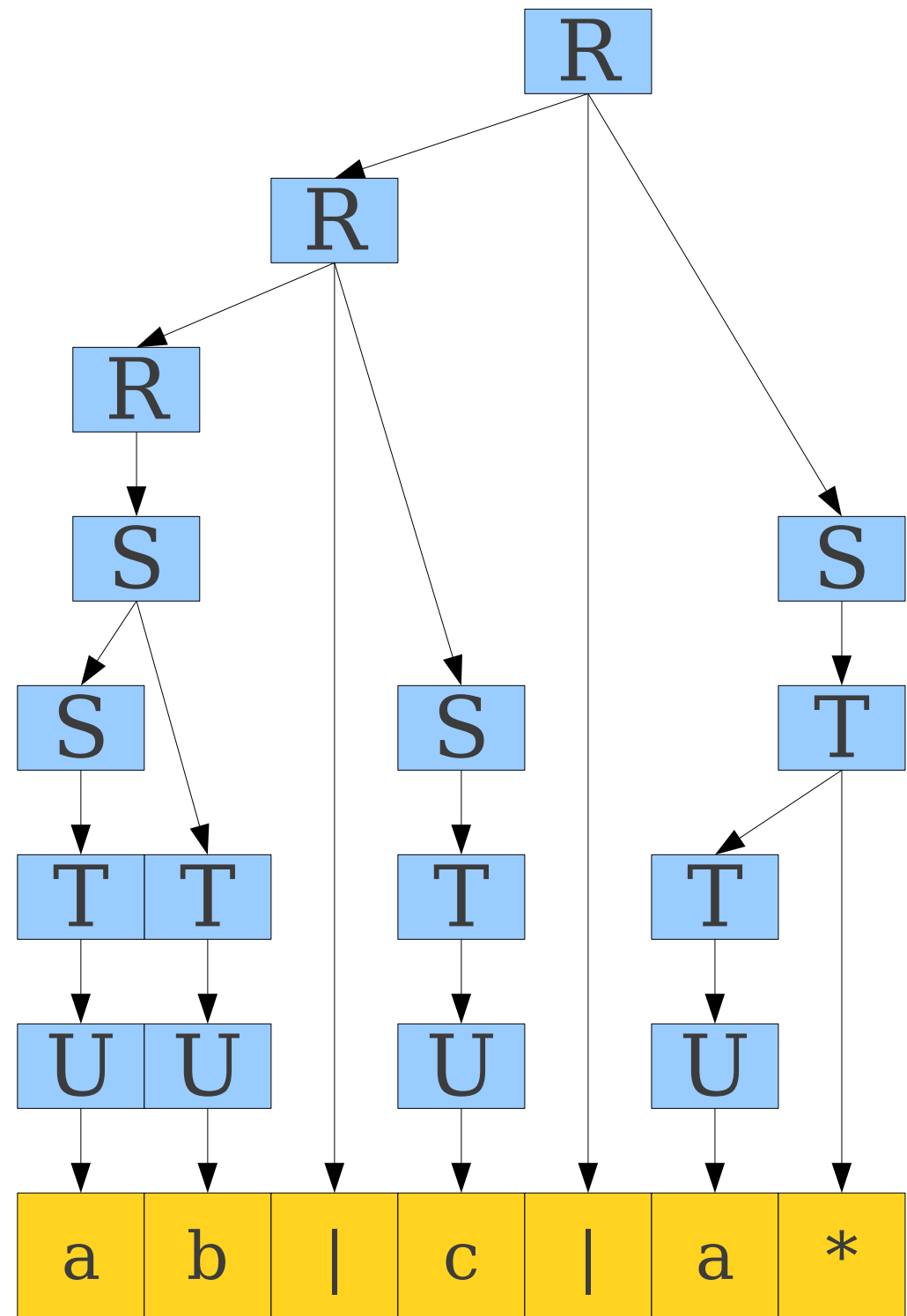
S → **T** | **ST**

T → **U** | **T***

U → **a** | **b** | **c** | ...

U → “**ε**”

U → (**R**)



Precedence Declarations

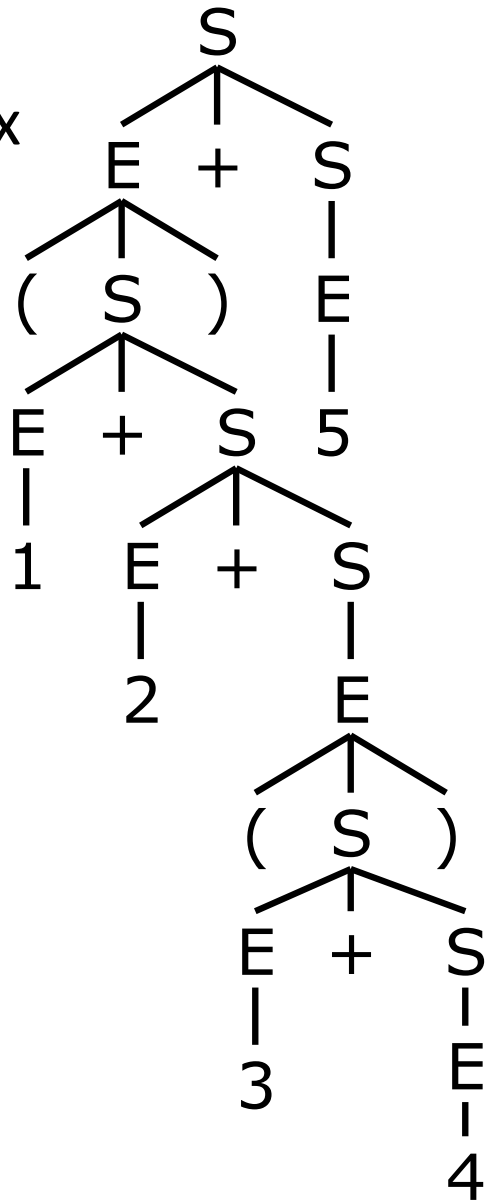
- If we leave the world of pure CFGs, we can often resolve ambiguities through **precedence declarations**.
 - e.g. multiplication has higher precedence than addition, but lower precedence than exponentiation.
- Allows for unambiguous parsing of ambiguous grammars.
- We'll see how this is implemented later on.

Abstract Syntax Trees (ASTs)

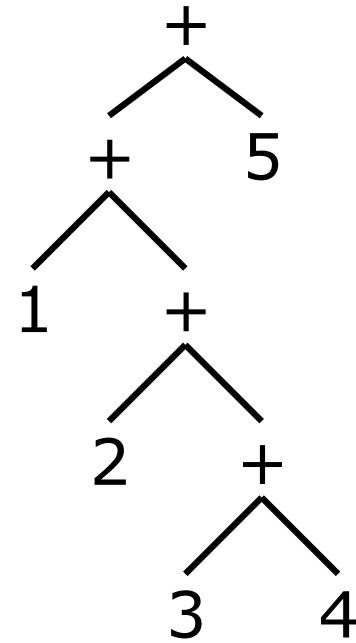
- A parse tree is a **concrete syntax tree**; it shows exactly how the text was derived.
- A more useful structure is an **abstract syntax tree**, which retains only the essential structure of the input.

Parse Tree vs. AST

- Parse Tree, aka concrete syntax



Abstract Syntax Tree



Discards/abstracts unnneeded information

The Structure of a Parse Tree

$R \rightarrow S \mid R \mid S$

$S \rightarrow T \mid ST$

$T \rightarrow U \mid T^*$

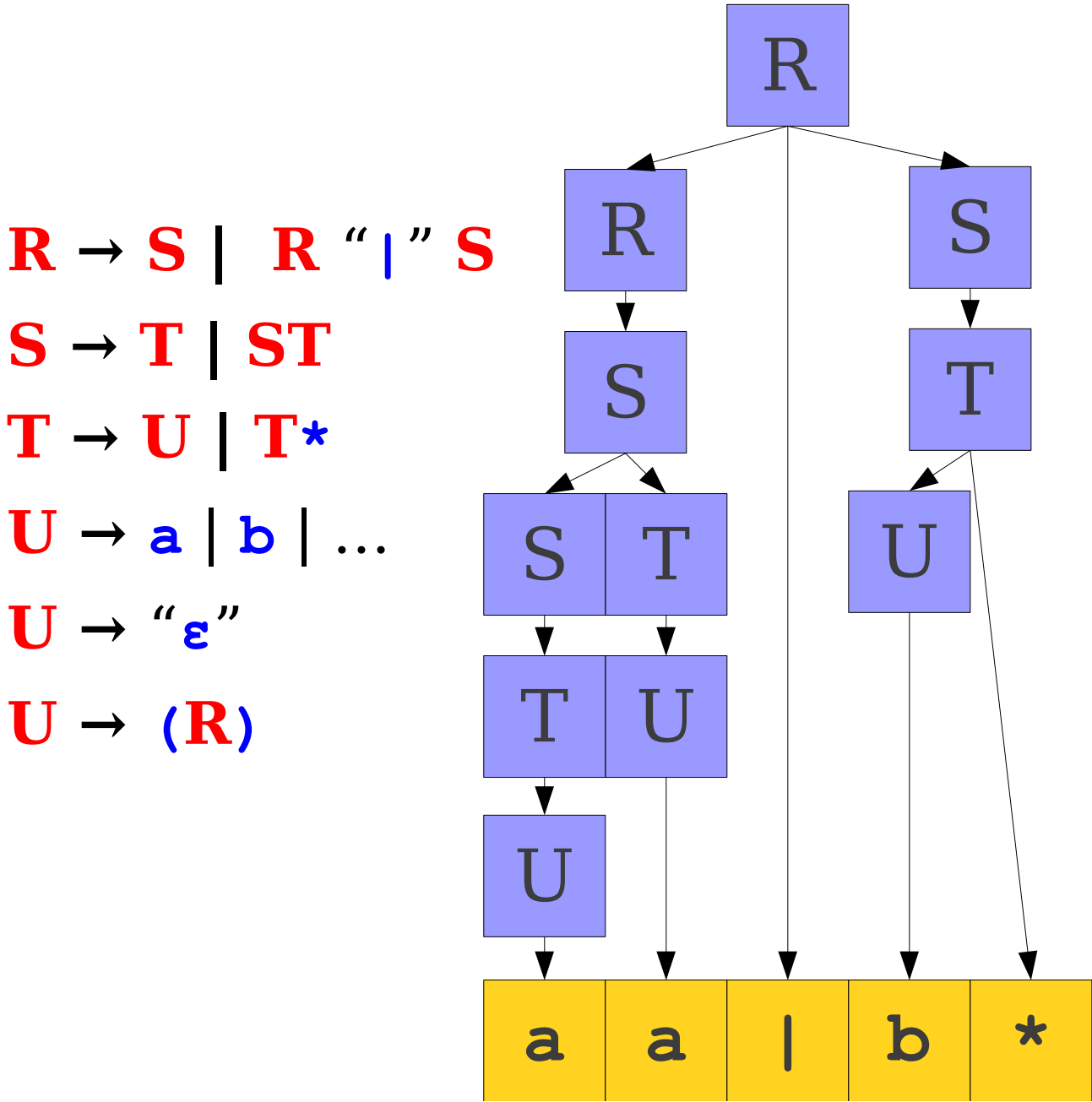
$U \rightarrow a \mid b \mid \dots$

$U \rightarrow \epsilon$

$U \rightarrow (R)$

a	a		b	*
---	---	--	---	---

The Structure of a Parse Tree



The Structure of a Parse Tree

$R \rightarrow S \mid R \mid S$

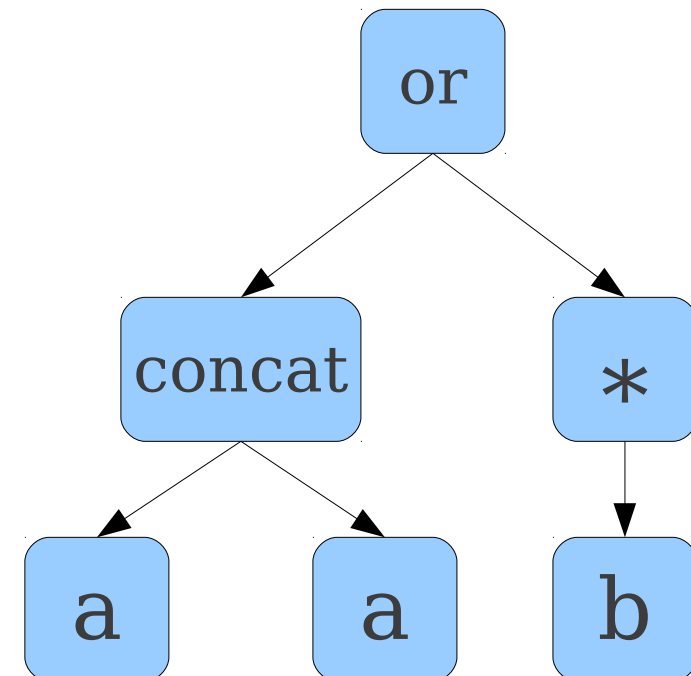
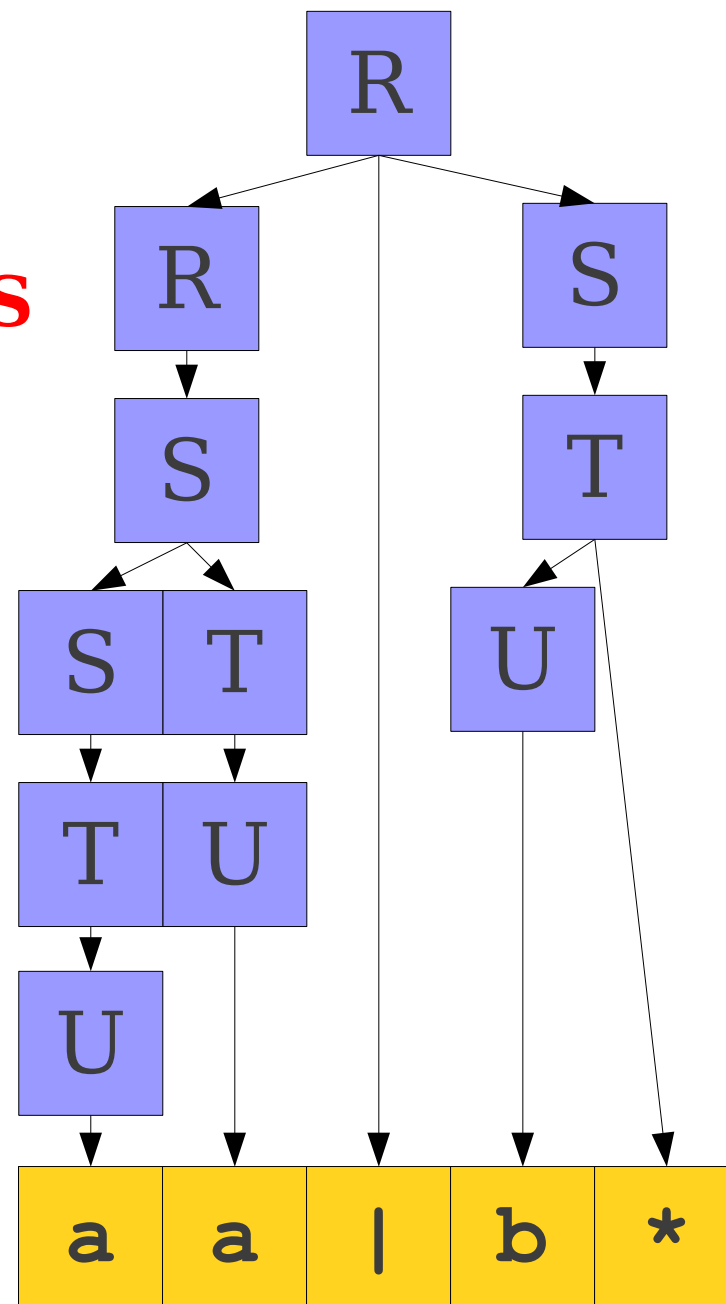
$S \rightarrow T \mid ST$

$T \rightarrow U \mid T^*$

$U \rightarrow a \mid b \mid \dots$

$U \rightarrow \epsilon$

$U \rightarrow (R)$



$R \rightarrow S \mid R \mid S$

$S \rightarrow T \mid ST$

$T \rightarrow U \mid T^*$

$U \rightarrow a \mid b \mid \dots$

$U \rightarrow \epsilon$

$U \rightarrow (R)$

a	(b		c)
---	---	---	--	---	---

$R \rightarrow S \mid R \mid S$

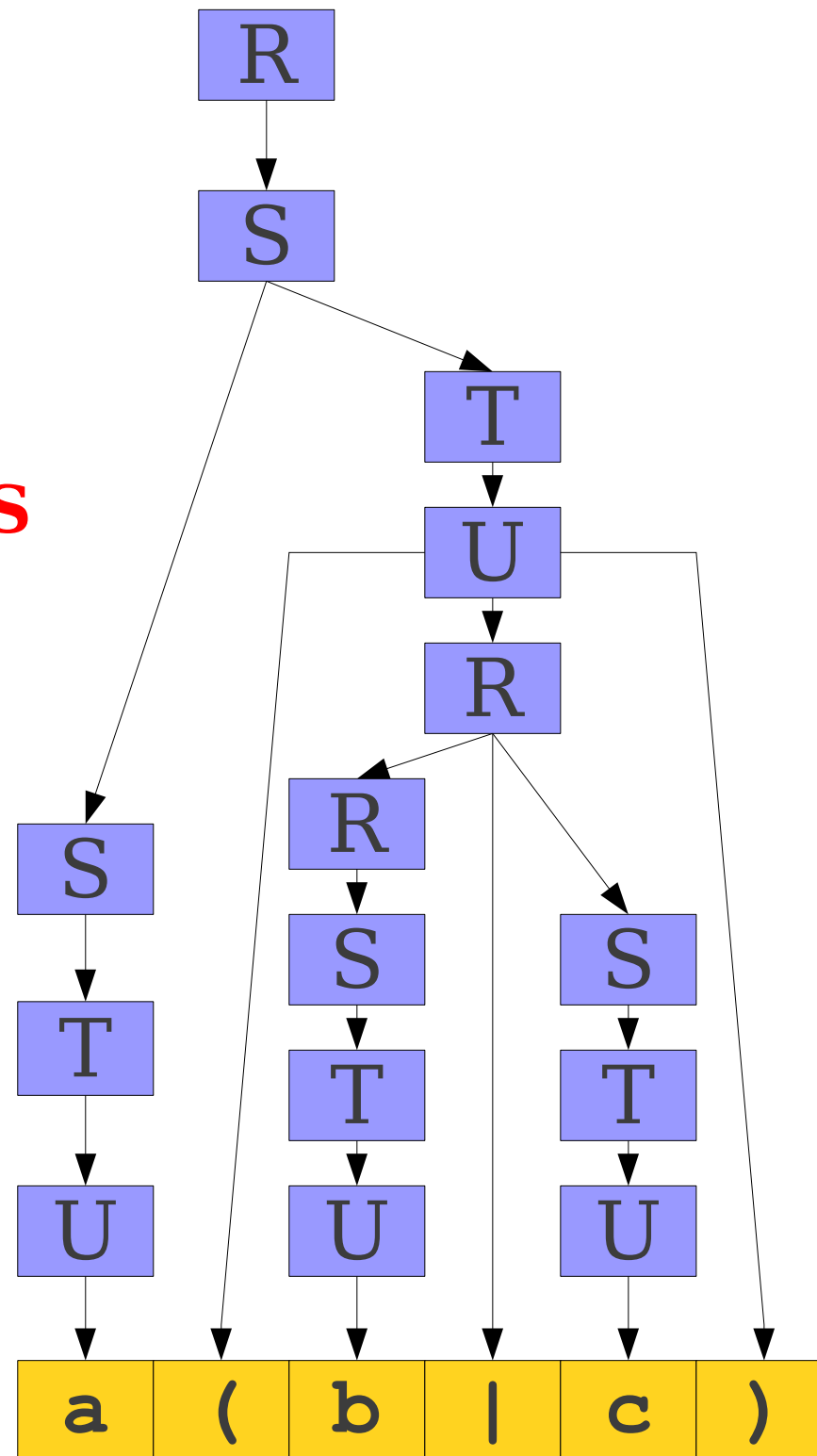
$S \rightarrow T \mid ST$

$T \rightarrow U \mid T^*$

$U \rightarrow a \mid b \mid \dots$

$U \rightarrow \epsilon$

$U \rightarrow (R)$



$R \rightarrow S \mid R \mid S$

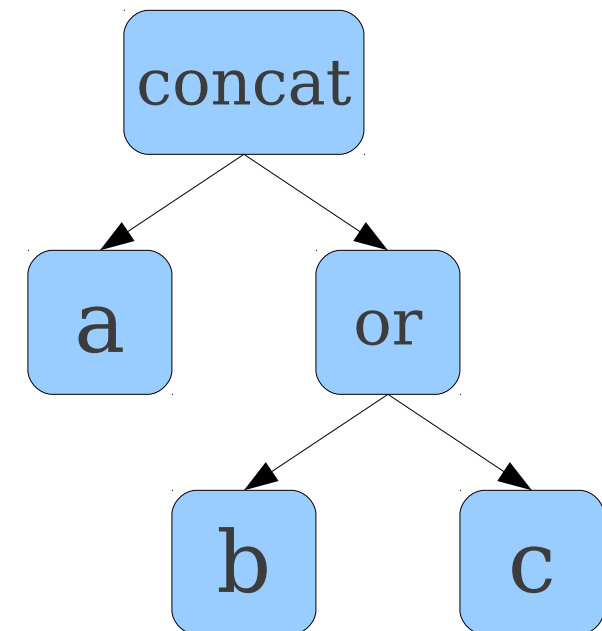
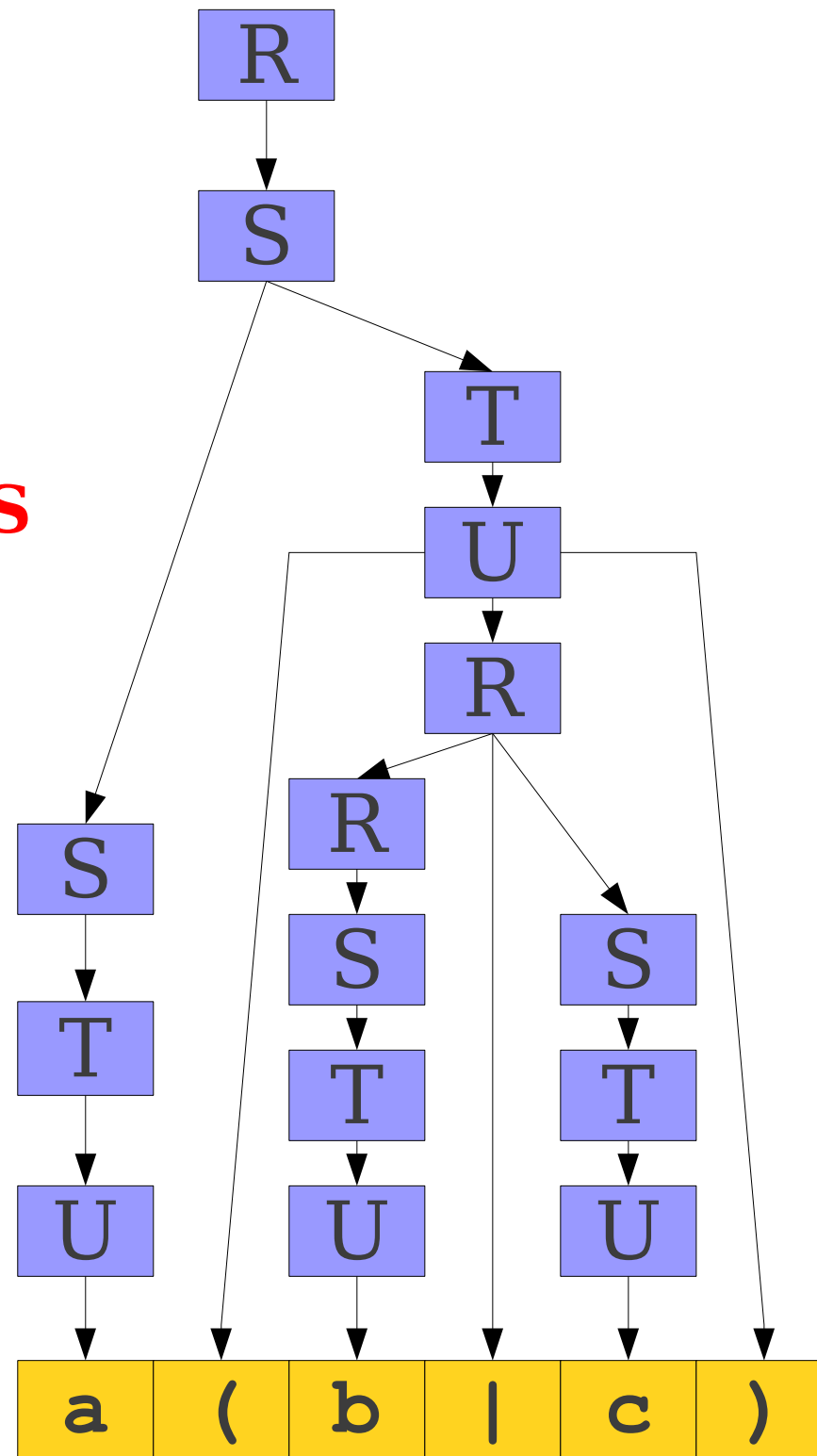
$S \rightarrow T \mid ST$

$T \rightarrow U \mid T^*$

$U \rightarrow a \mid b \mid \dots$

$U \rightarrow \epsilon$

$U \rightarrow (R)$



Summary

- Syntax analysis (**parsing**) extracts the structure from the tokens produced by the scanner.
- Languages are usually specified by **context-free grammars (CFGs)**.
- A **parse tree** shows how a string can be **derived** from a grammar.
- A grammar is **ambiguous** if it can derive the same string multiple ways.
- There is no algorithm for eliminating ambiguity; it must be done by hand.
- **Abstract syntax trees (ASTs)** contain an abstract representation of a program's syntax.
- **Semantic actions** associated with productions can be used to build ASTs.