

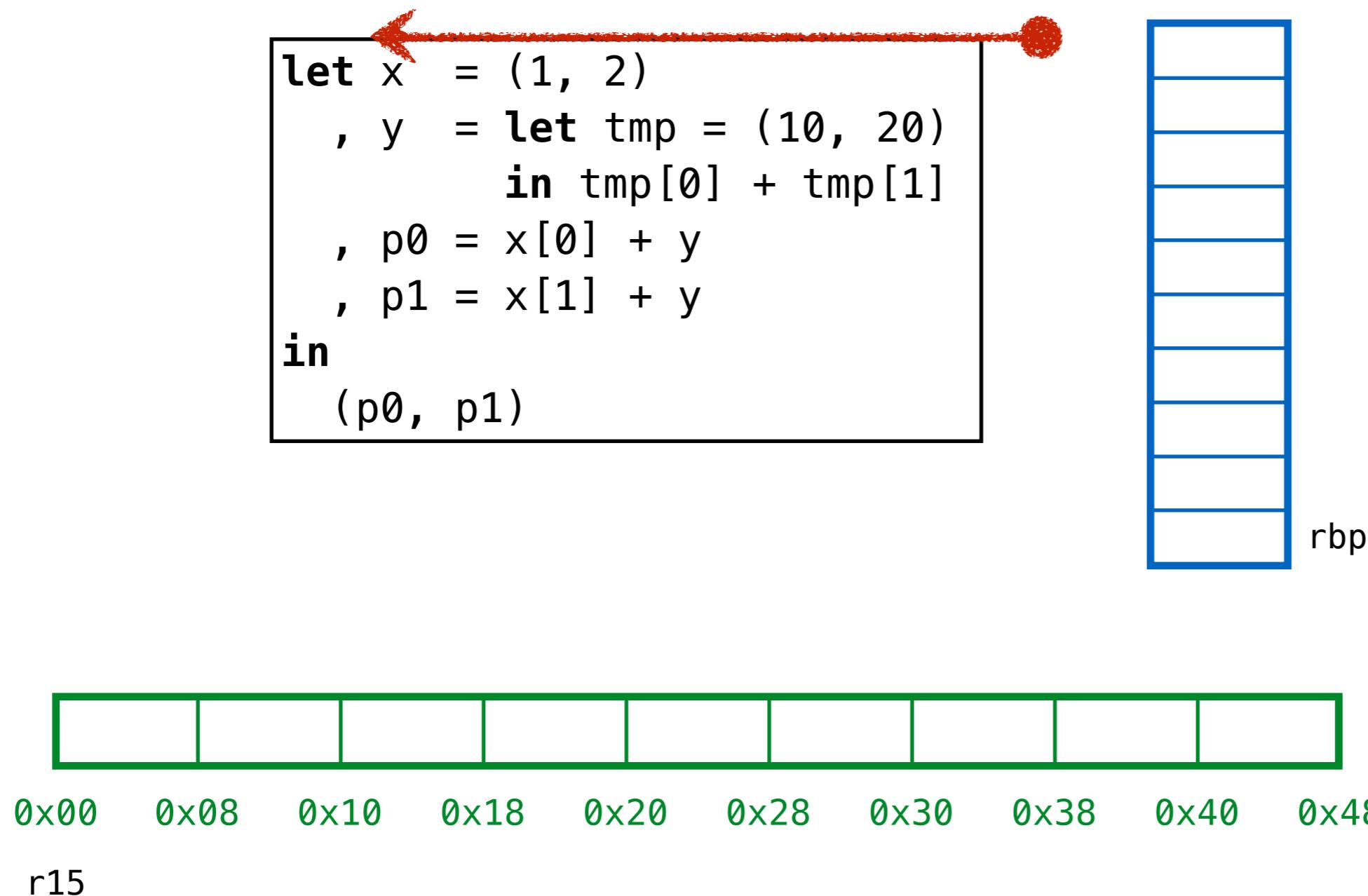
Garter

Garbage Collection

Garter / GC

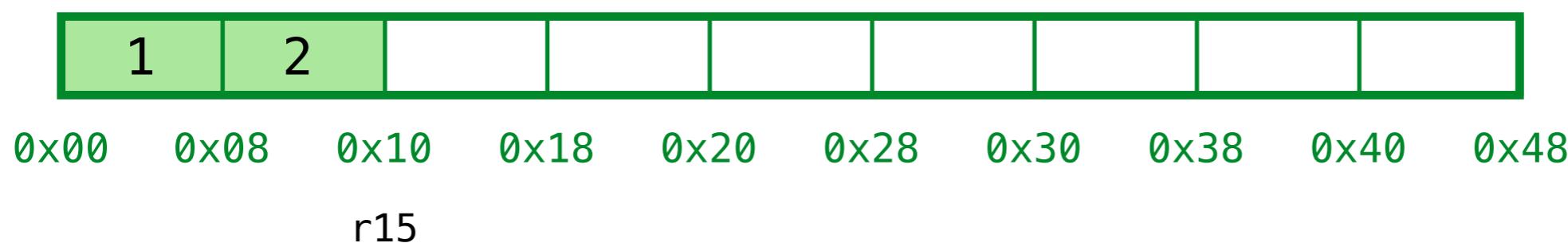
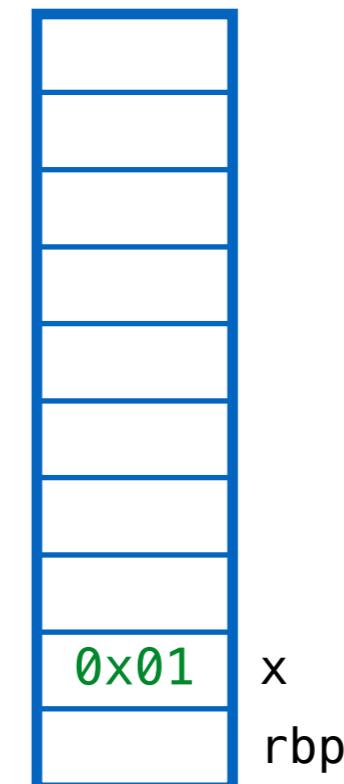
Example 1

ex1: garbage at end



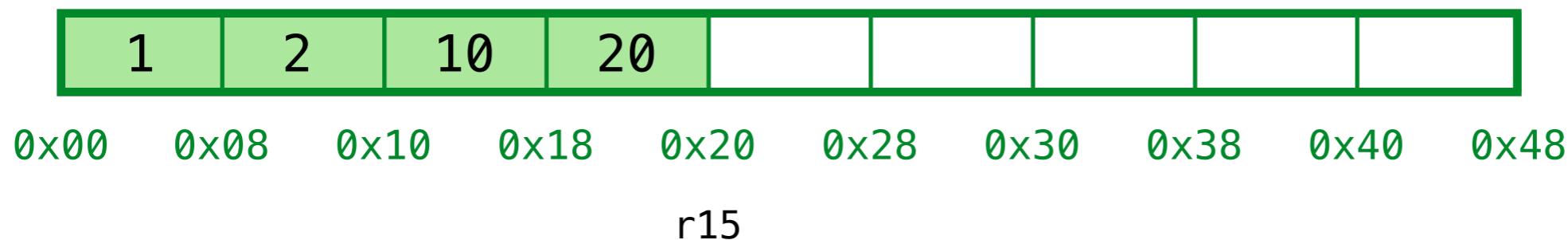
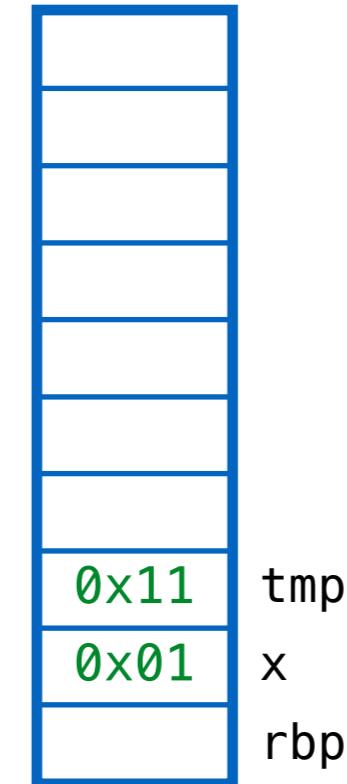
ex1: garbage at end

```
let x = (1, 2) ←  
      , y = let tmp = (10, 20)  
              in tmp[0] + tmp[1]  
      , p0 = x[0] + y  
      , p1 = x[1] + y  
in  
(p0, p1)
```



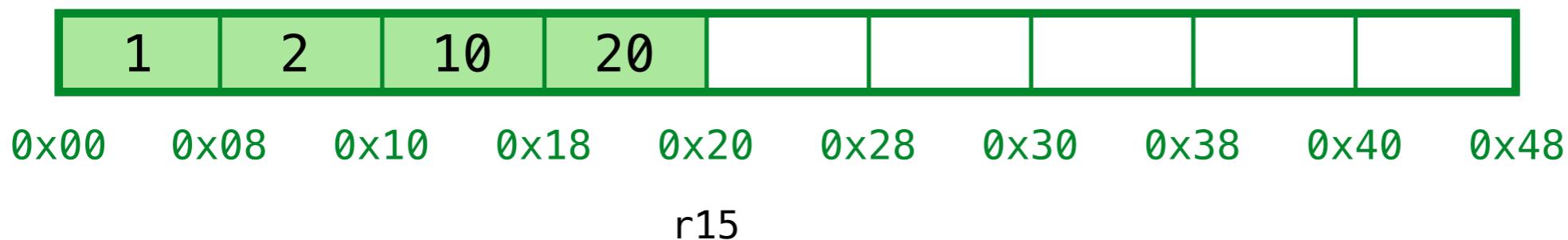
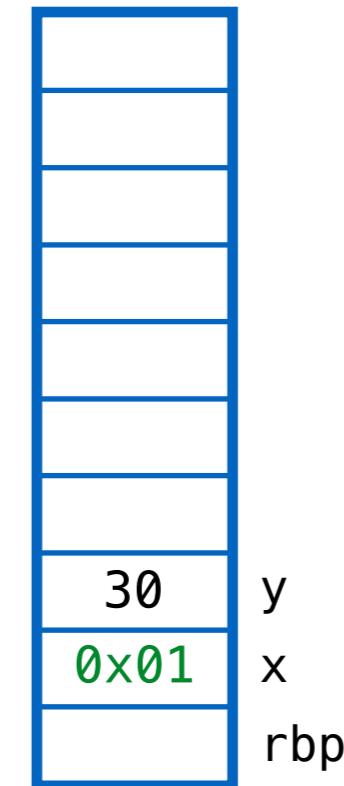
ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1)
```



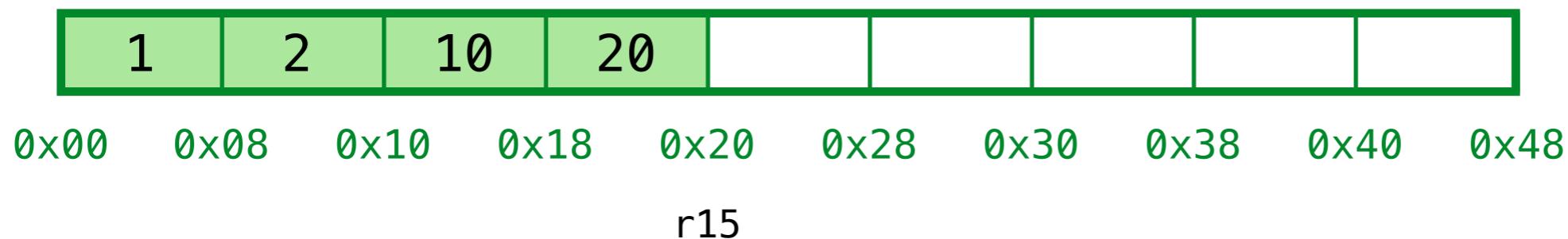
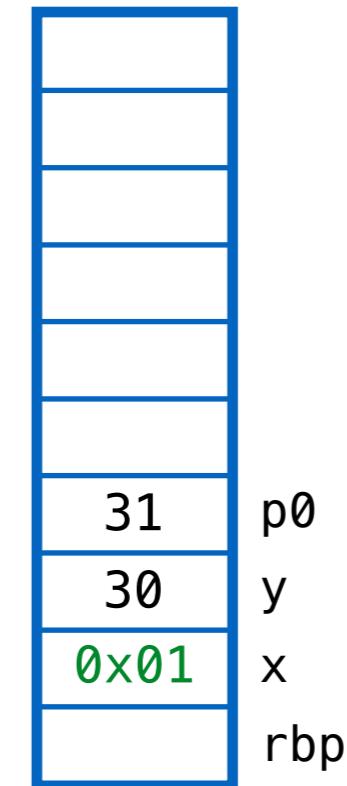
ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1)
```



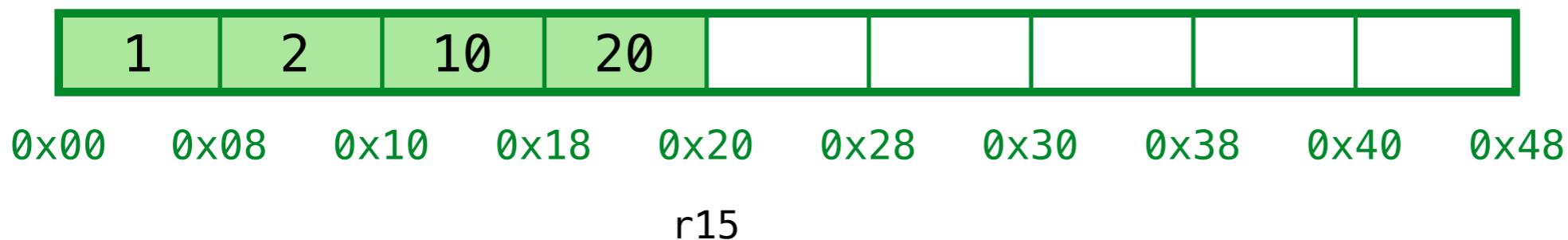
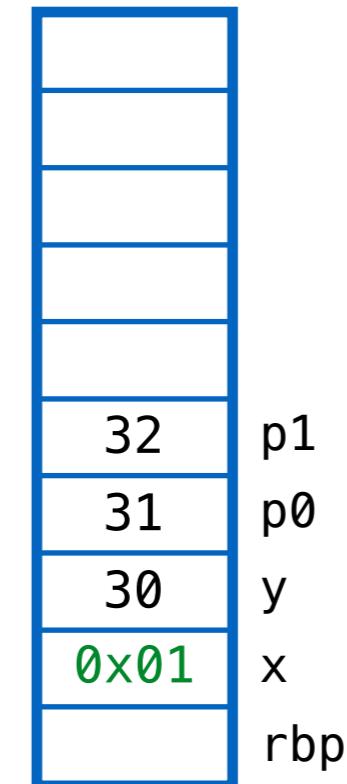
ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1)
```



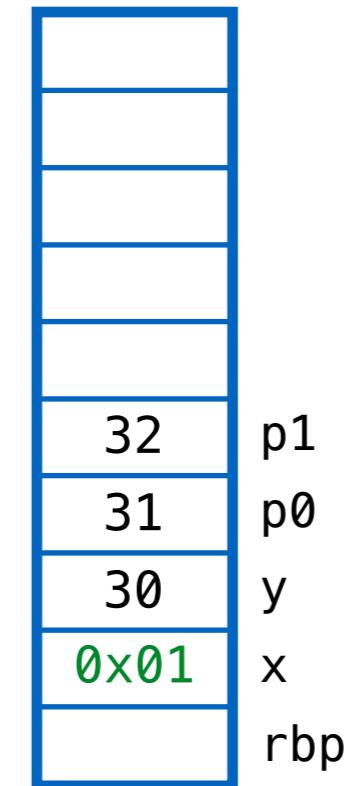
ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in (p0, p1)
```

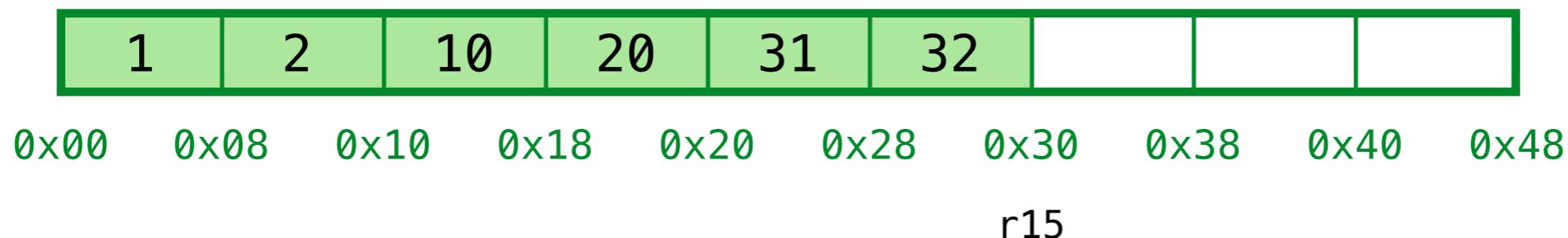


ex1: garbage at end

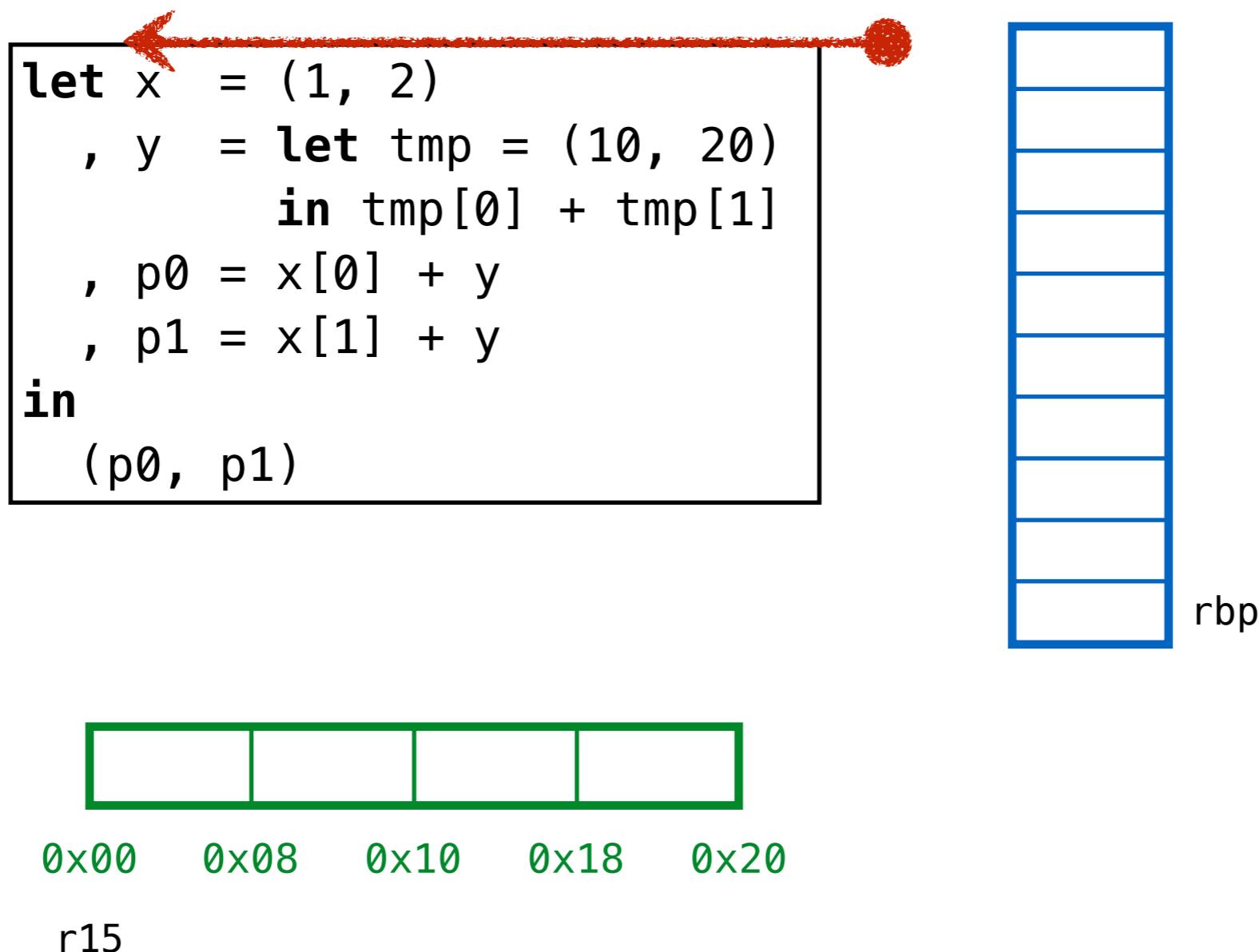
```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1) ←
```



Result (rax) = 0x21



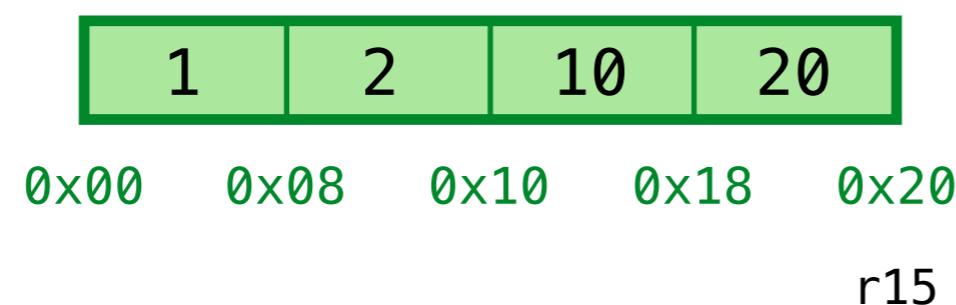
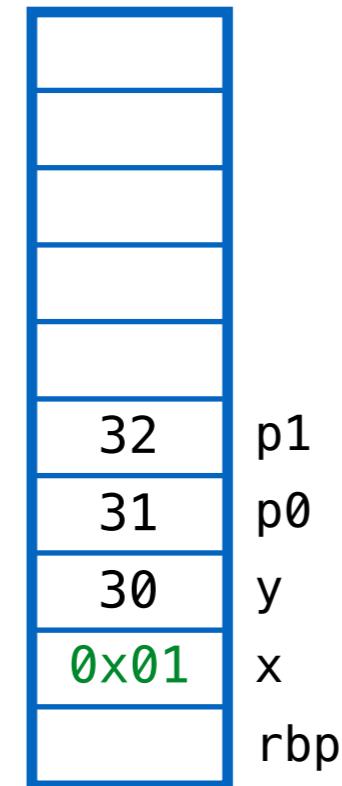
ex1: garbage at end



Suppose we had a smaller, 4-word heap

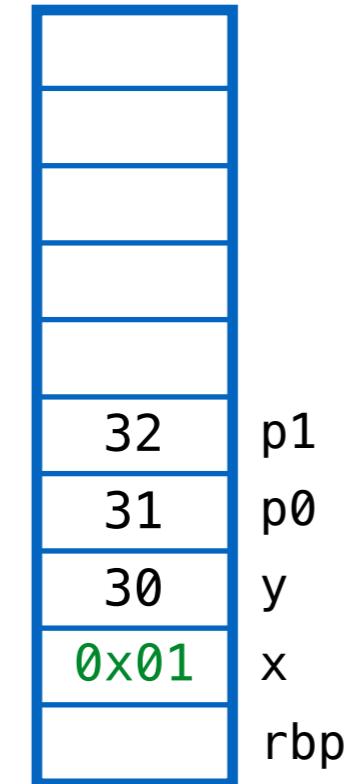
ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```

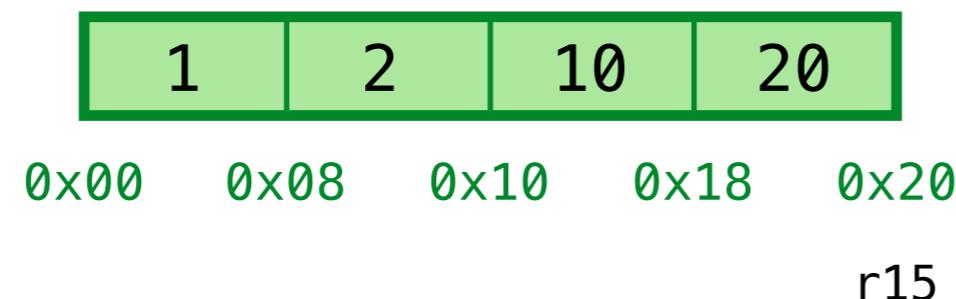


ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



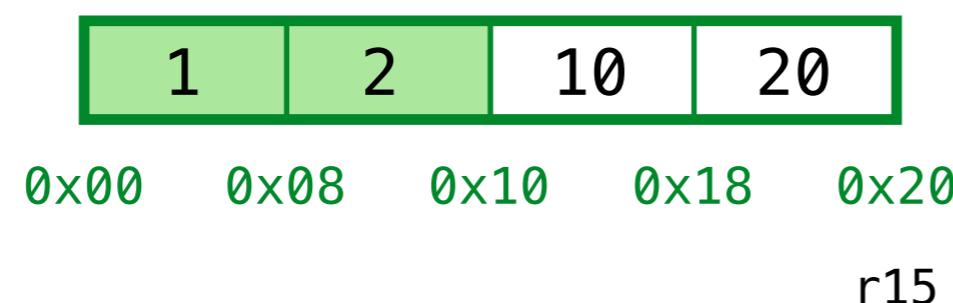
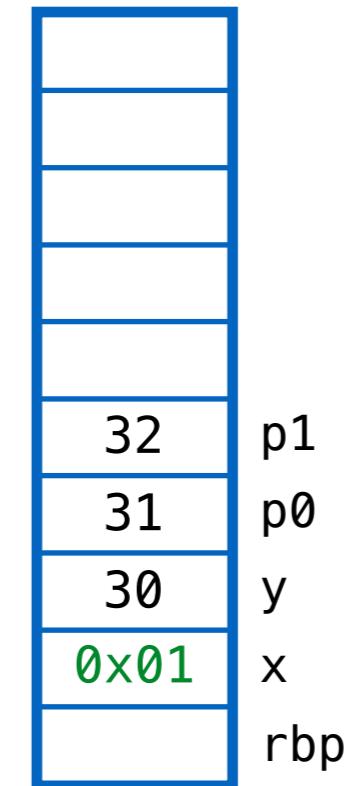
Out of memory!
Can't allocate (p0, p1)



ex1: garbage at end

```
let x  = (1, 2)
    , y  = let tmp = (10, 20)
            in tmp[0] + tmp[1]
    , p0 = x[0] + y
    , p1 = x[1] + y
in ←
    (p0, p1)
```

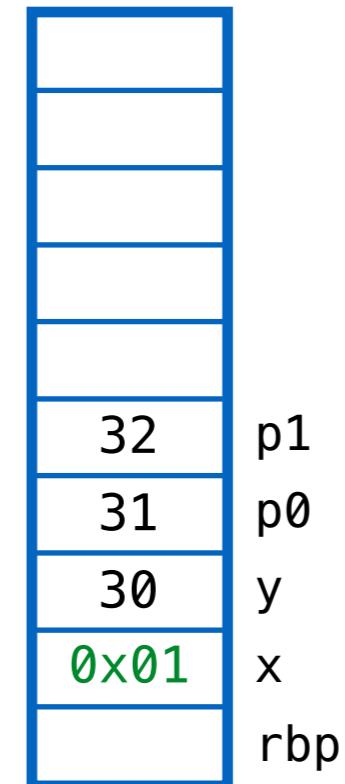
(10, 20) is “garbage”



Q: How to determine if cell is garbage?

ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in (p0, p1)
```



(10, 20) is “garbage”

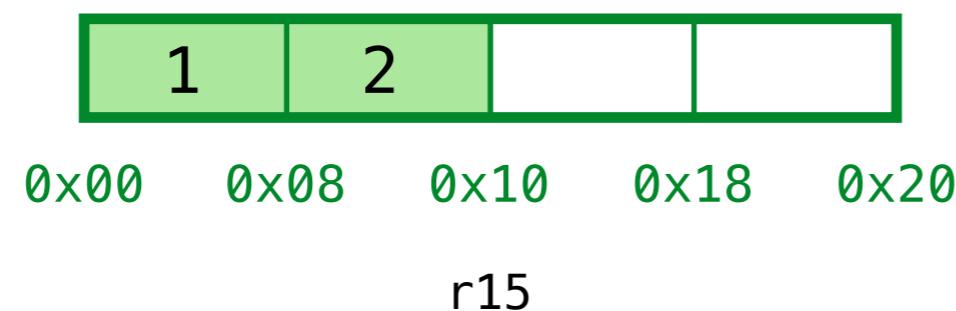
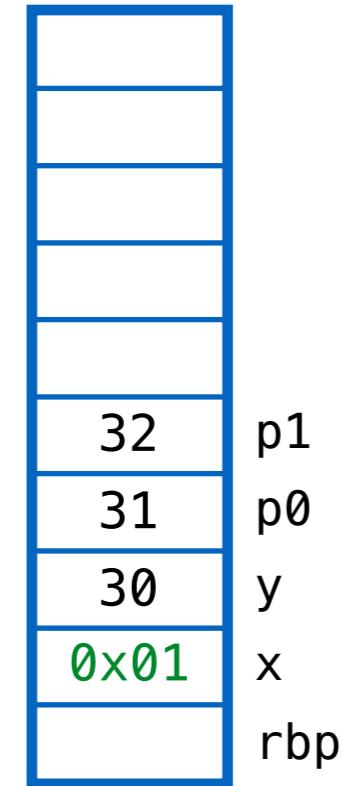


0x00 0x08 0x10 0x18 0x20

r15

ex1: garbage at end

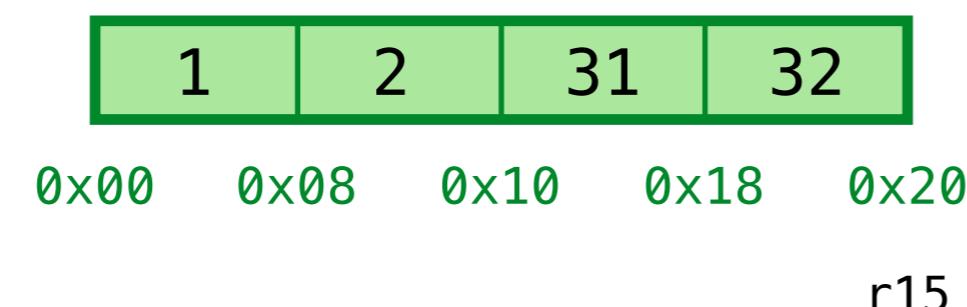
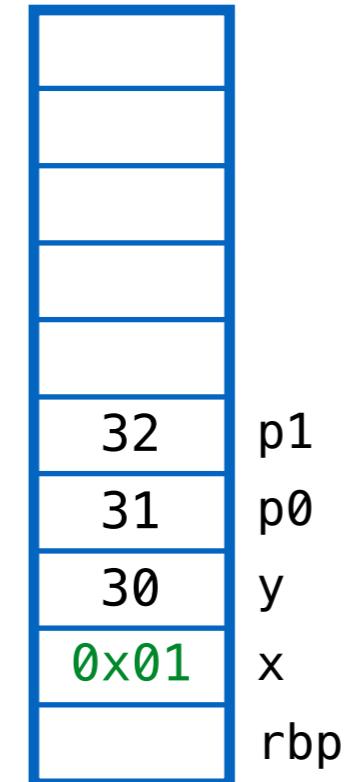
```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



ex1: garbage at end

```
let x = (1, 2)
, y = let tmp = (10, 20)
      in tmp[0] + tmp[1]
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1) ←
```

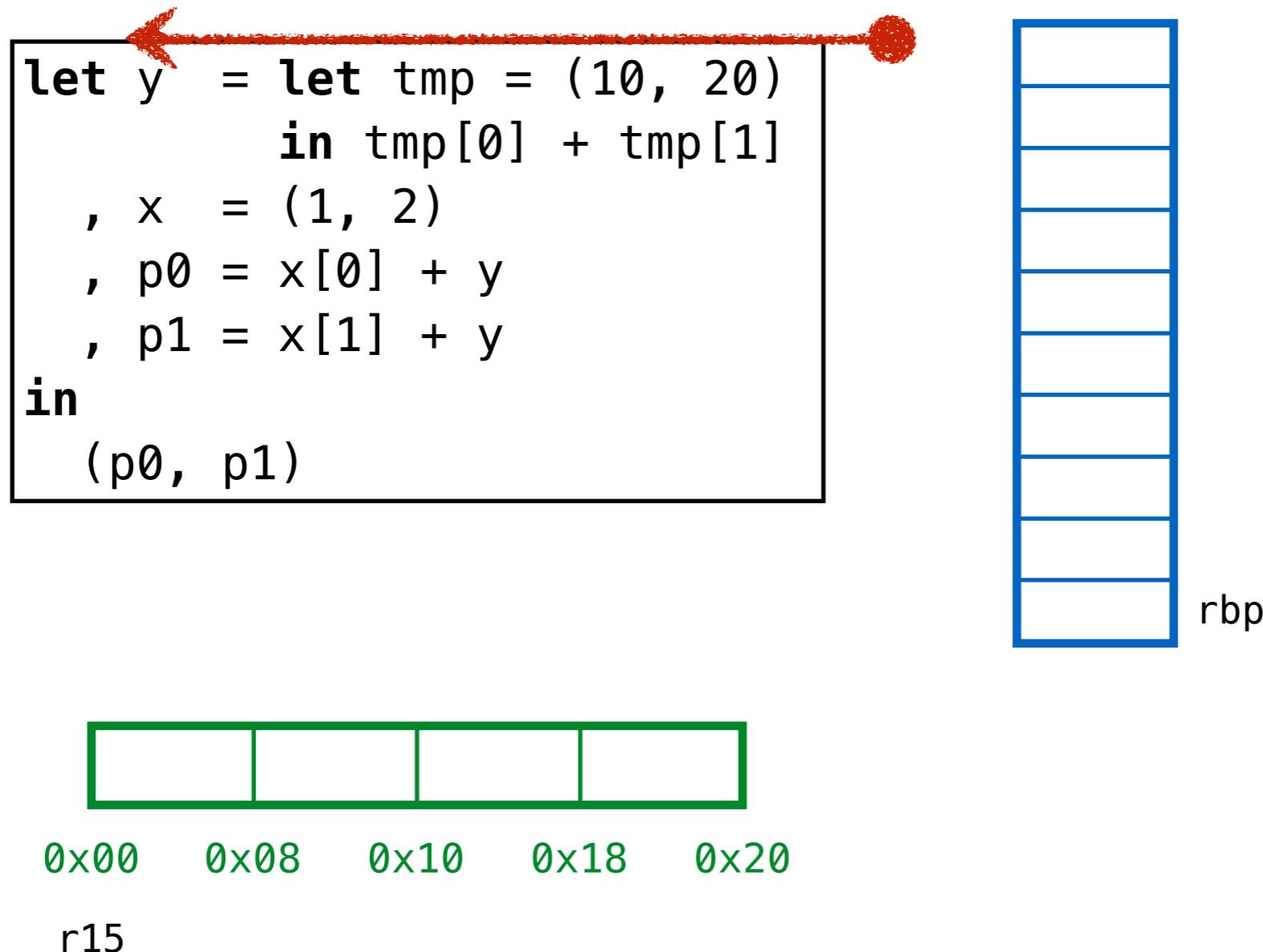
Result (rax) = 0x11



Garter / GC

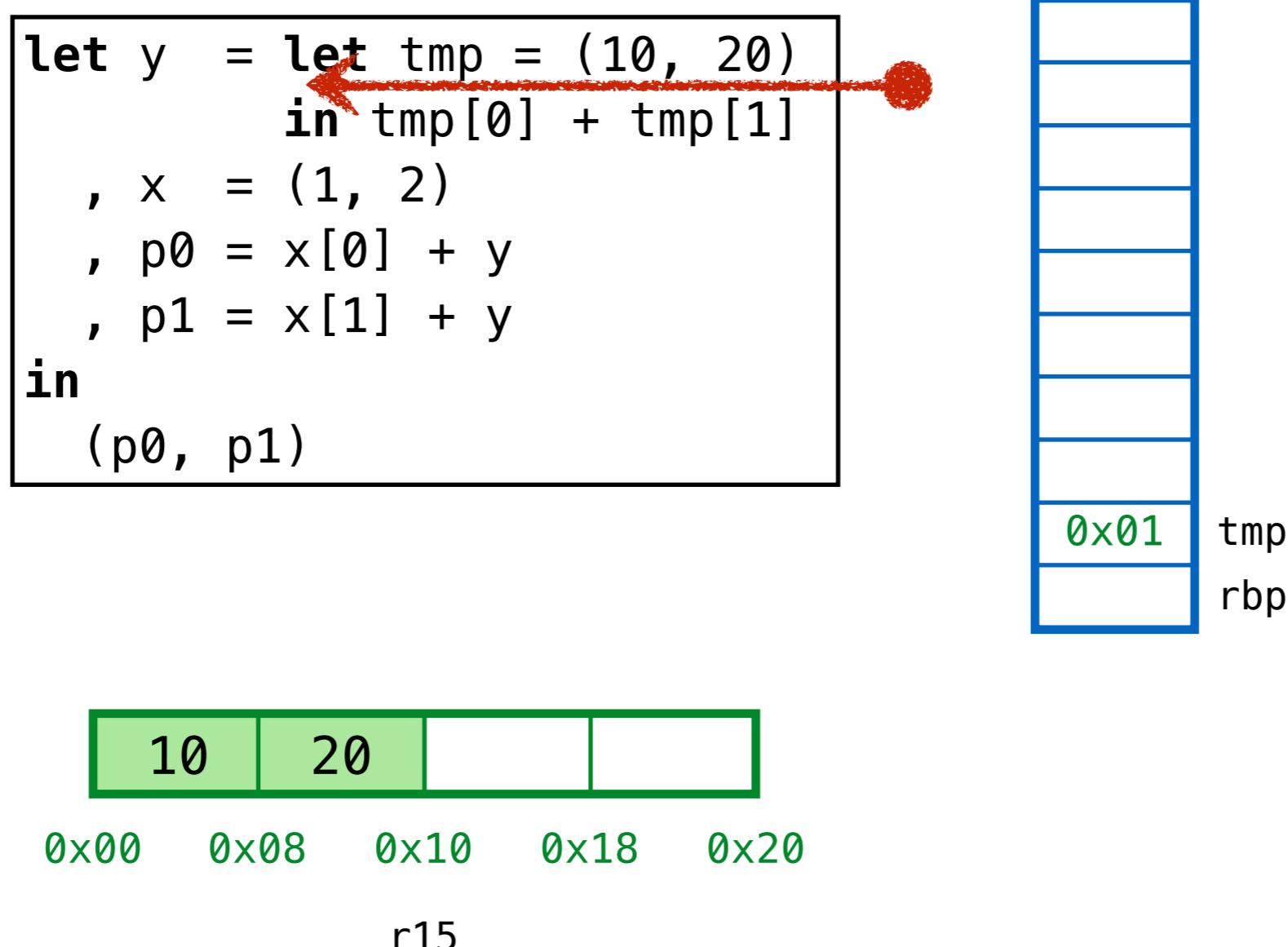
Example 2

ex2: garbage in the middle

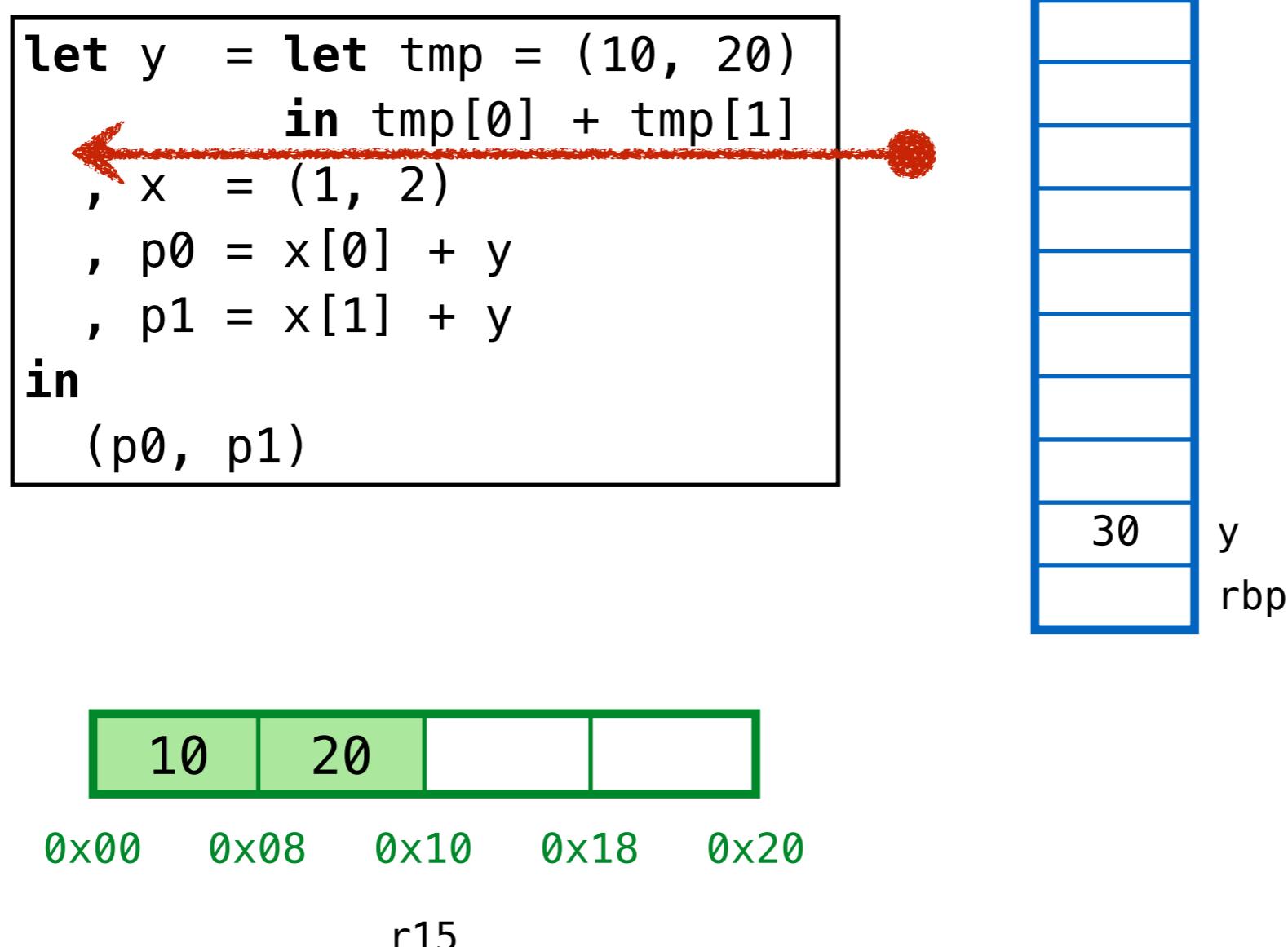


Start with a 4-word heap

ex2: garbage in the middle

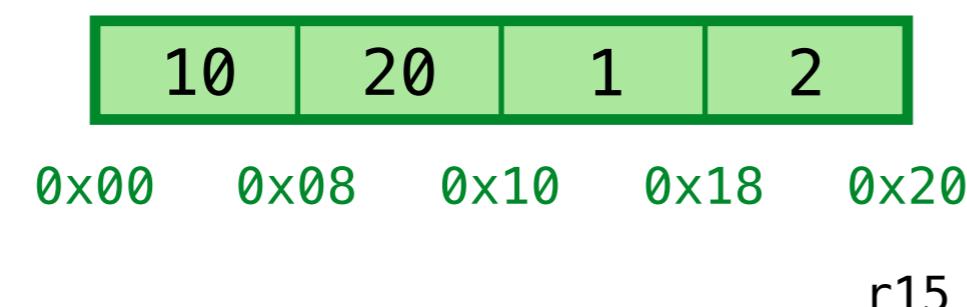
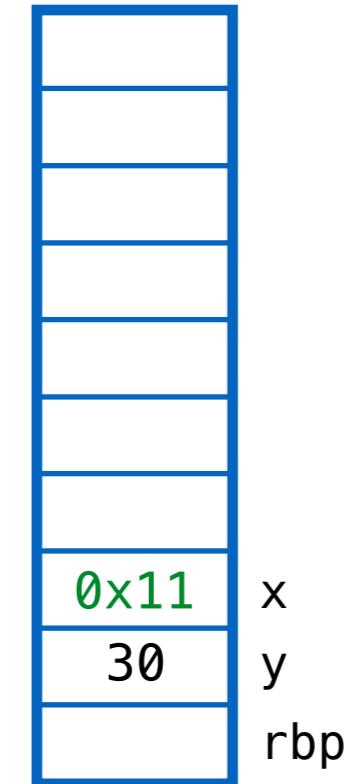


ex2: garbage in the middle



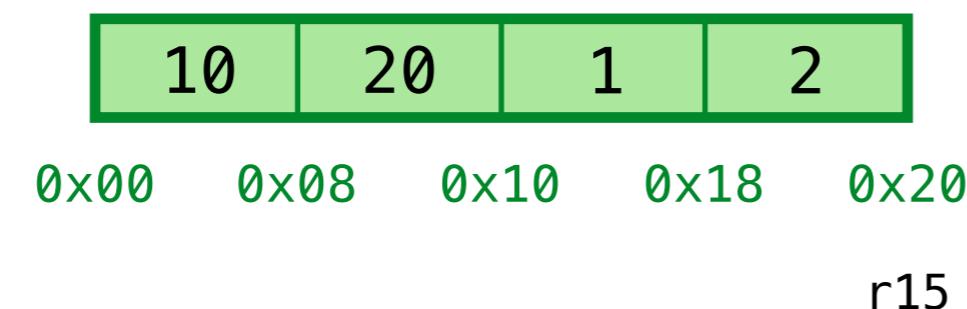
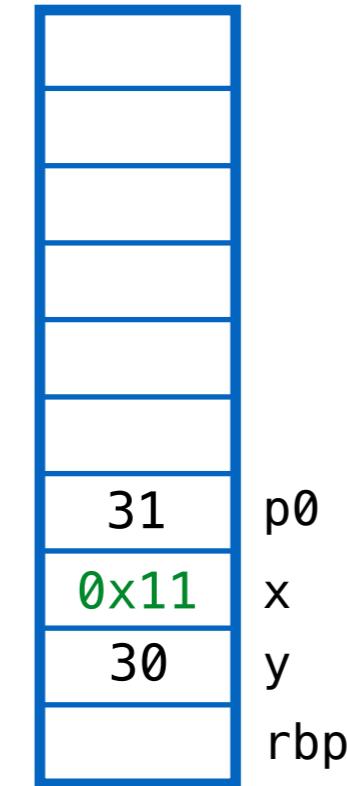
ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
        , x = (1, 2)
        , p0 = x[0] + y
        , p1 = x[1] + y
in
  (p0, p1)
```



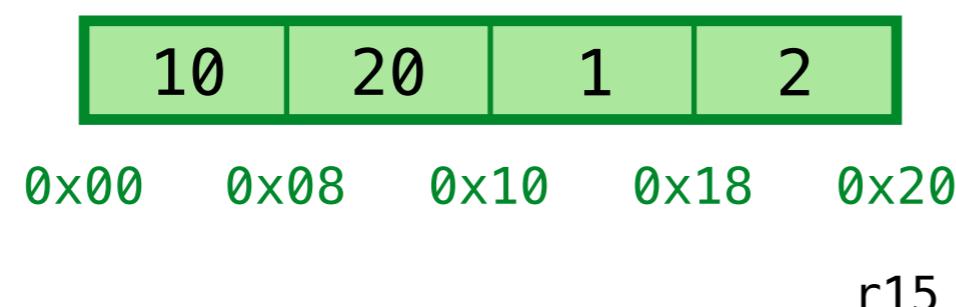
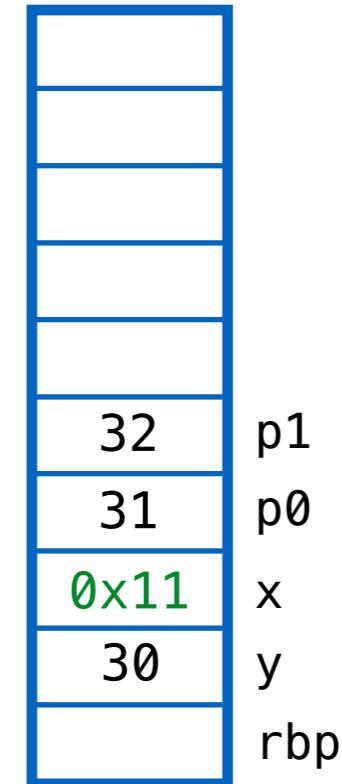
ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1)
```



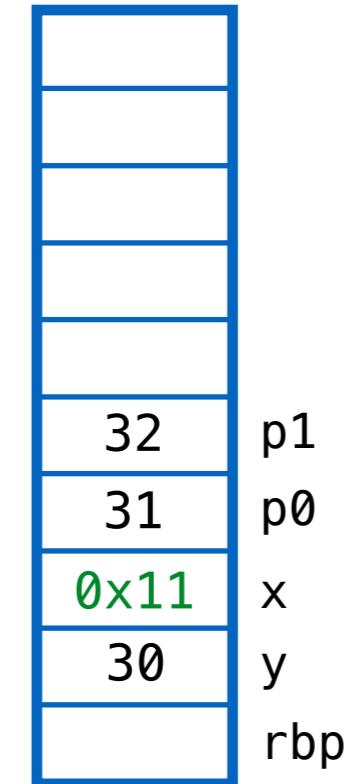
ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
      , x  = (1, 2)
      , p0 = x[0] + y
      , p1 = x[1] + y
in ←
  (p0, p1)
```



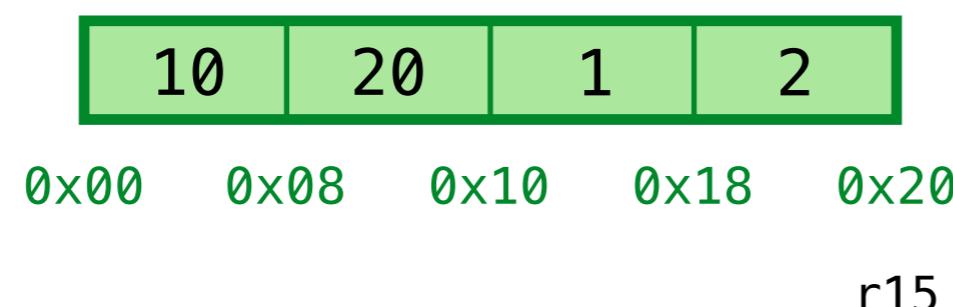
ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
      , x  = (1, 2)
      , p0 = x[0] + y
      , p1 = x[1] + y
in ←
  (p0, p1)
```



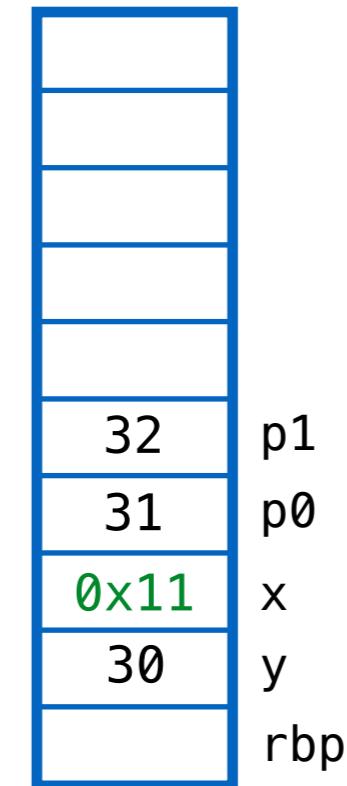
Out of memory!

Can't allocate (p0, p1)

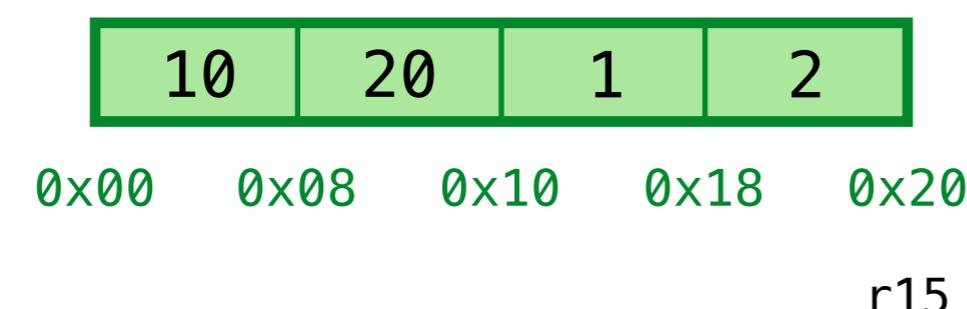


ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
      (p0, p1)
```

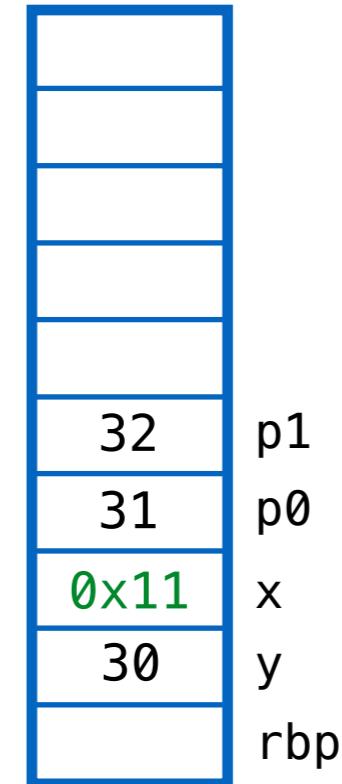


Lets reclaim & recycle garbage!

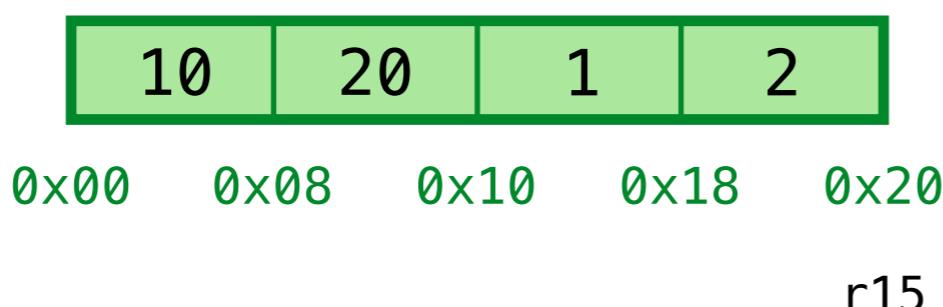


ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
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, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

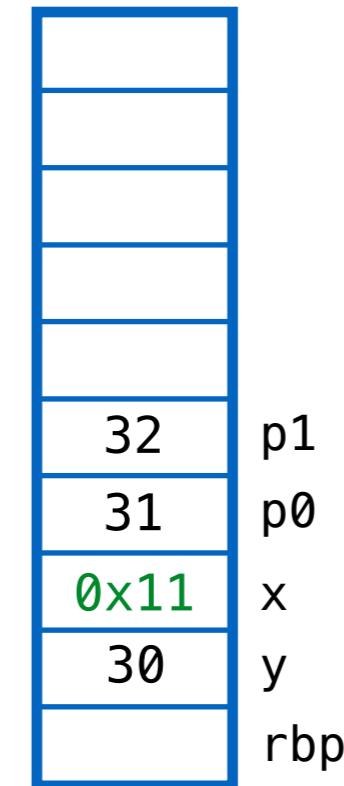


QUIZ: Which cells are garbage?

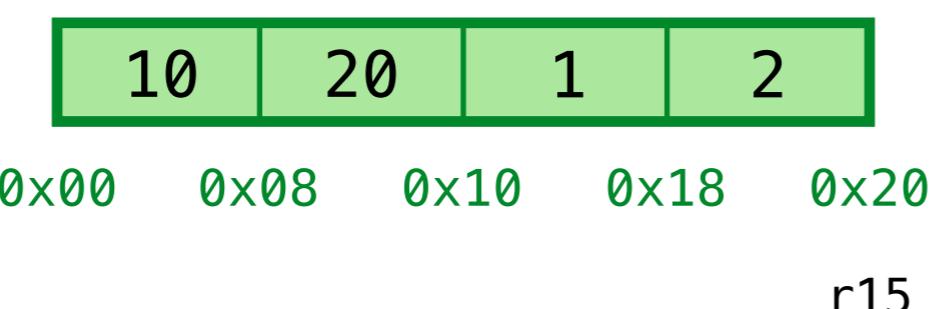
- (A) 0x00, 0x08 (B) 0x08, 0x10 (C) 0x18, 0x20 (D) None (E) All

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
      (p0, p1)
```



Lets reclaim & recycle garbage!



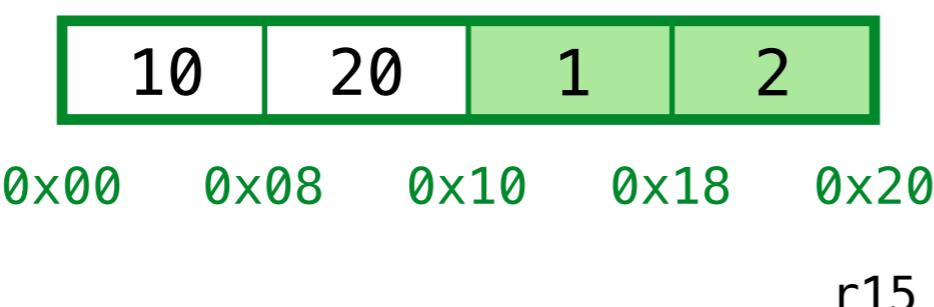
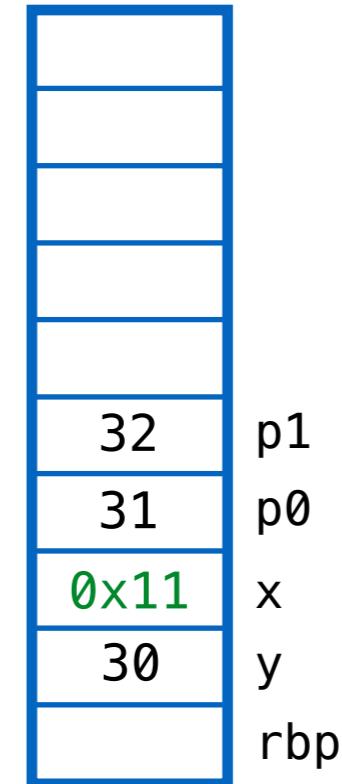
QUIZ: Which cells are garbage?

Those that are *not reachable from stack*

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
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in ←
(p0, p1)
```

Lets reclaim & recycle garbage!

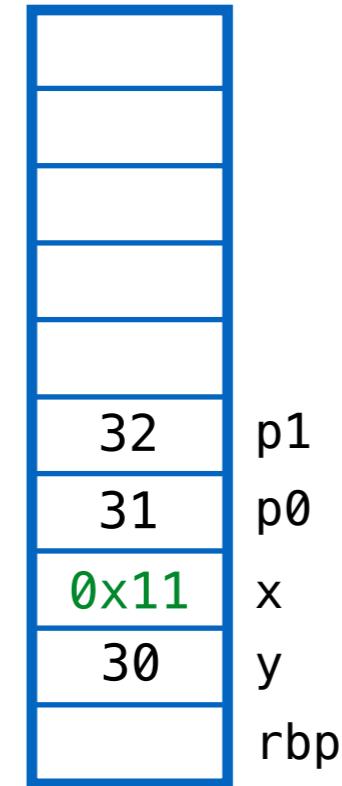


QUIZ: Which cells are garbage?

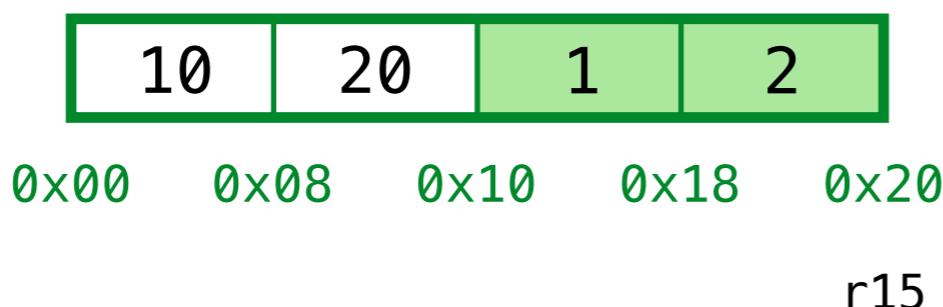
Those that are *not reachable from stack*

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

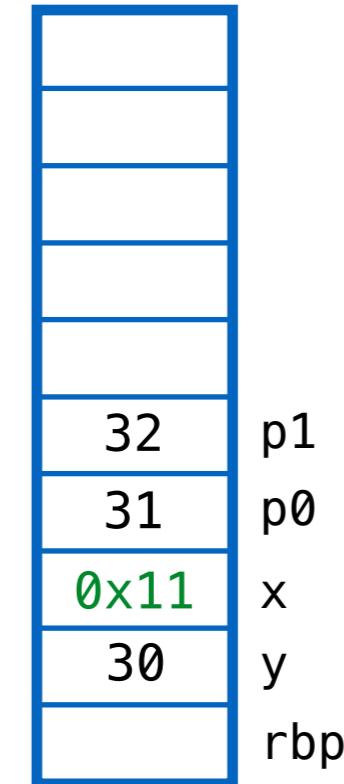


Q: How to reclaim space?

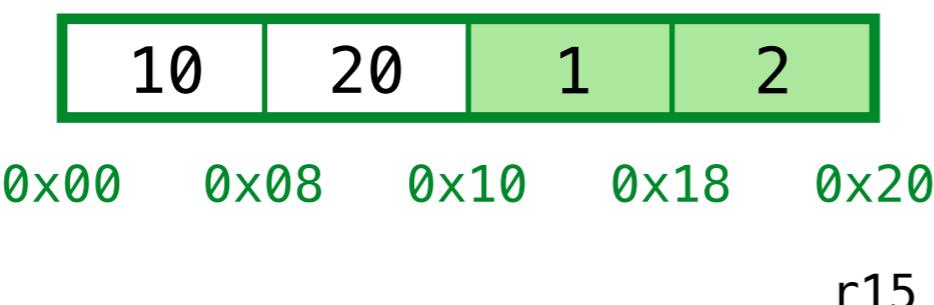
Why is it not enough to rewind r15?

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

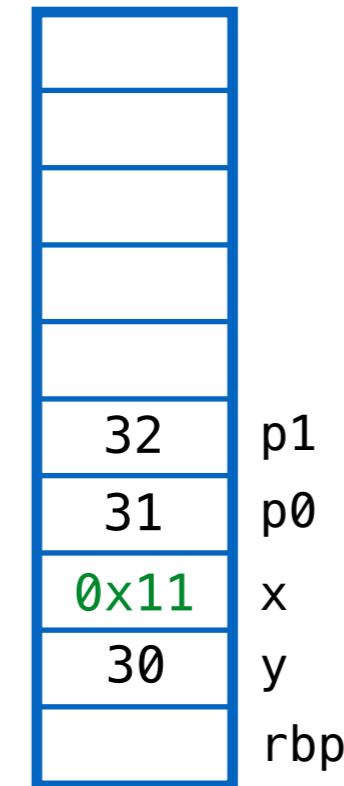


Why is it not enough to rewind r15?

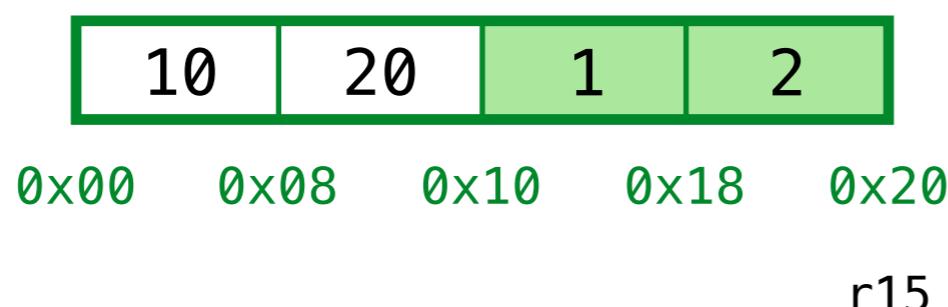
Want free space to be *contiguous* (i.e. go to end of heap)

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

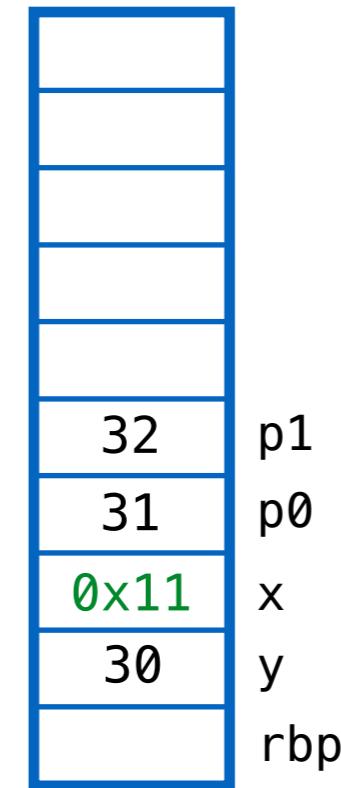


Solution: Compaction

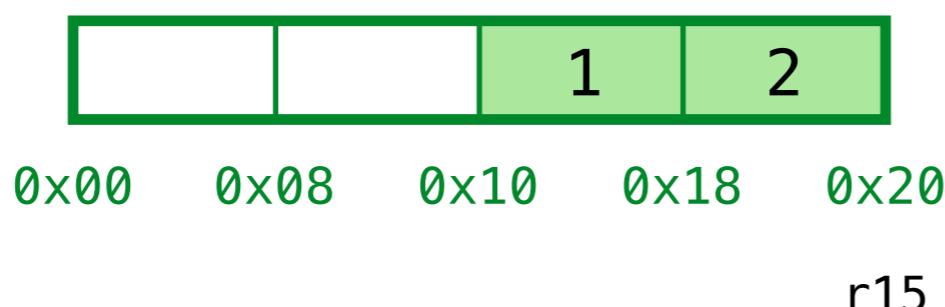
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
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(p0, p1)
```



Lets reclaim & recycle garbage!

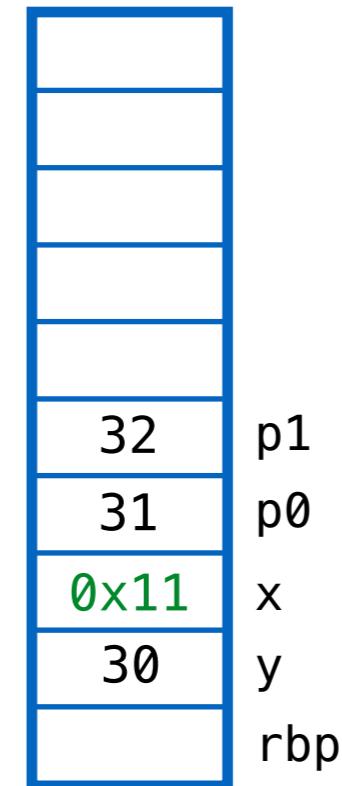


Solution: Compaction

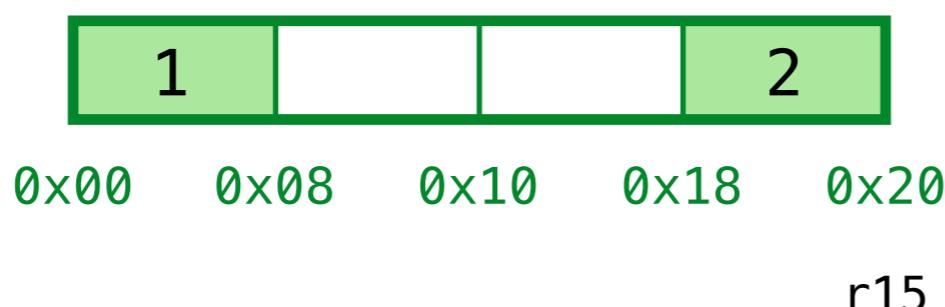
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
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in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

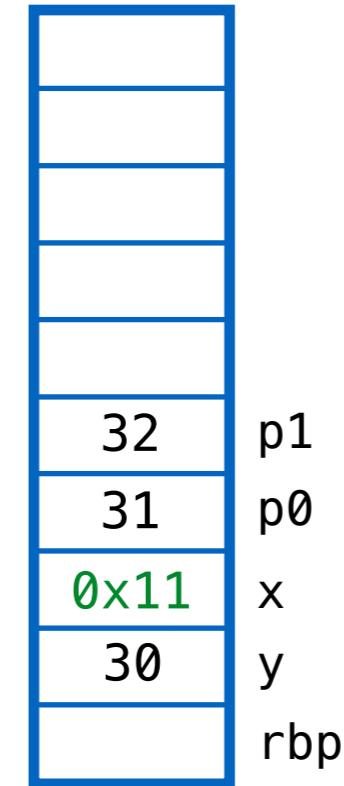


Solution: Compaction

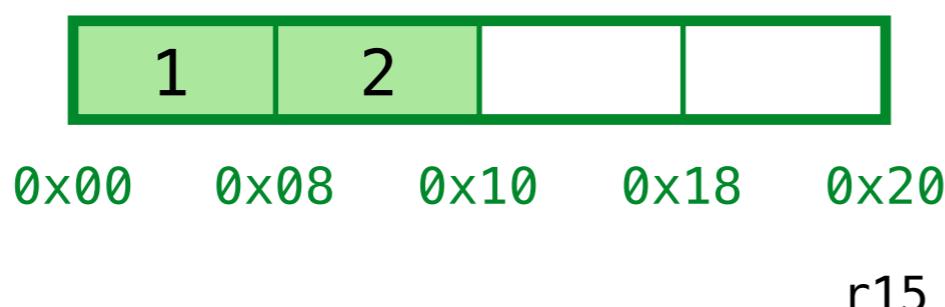
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

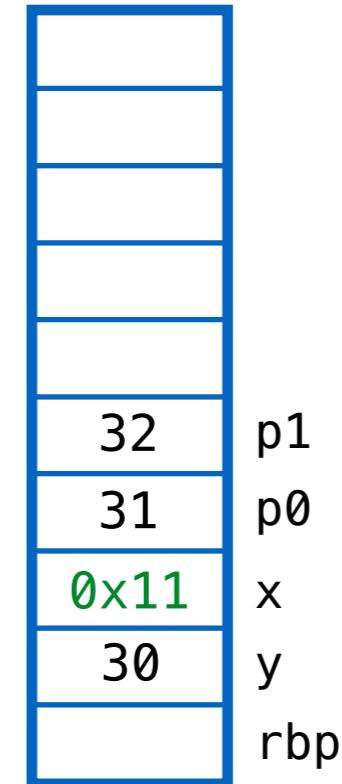


Solution: Compaction

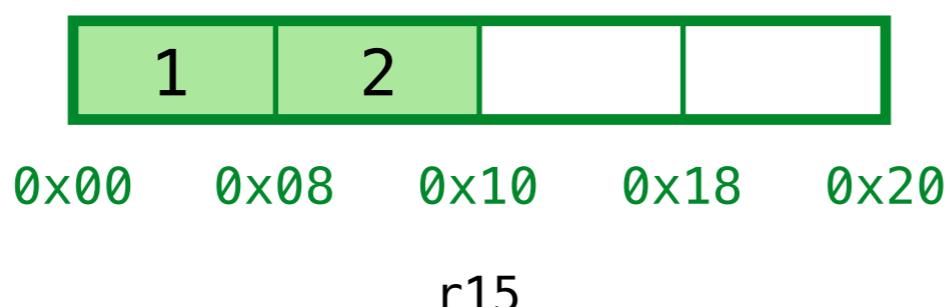
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in ←
(p0, p1)
```



Lets reclaim & recycle garbage!

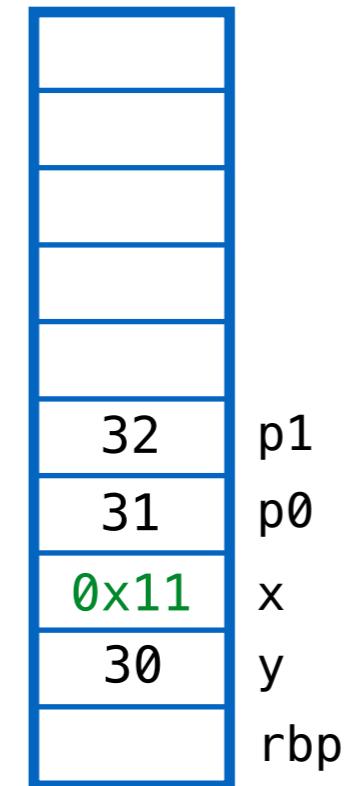


Solution: Compaction

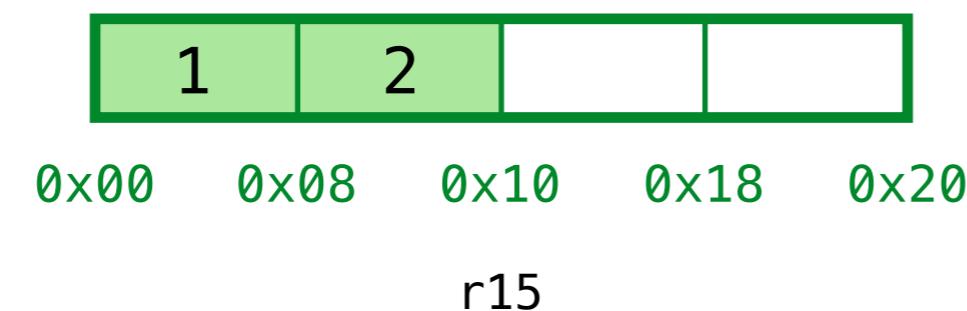
Copy “live” cells into “garbage” ... *and then* ... rewind r15!

ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
      , x  = (1, 2)
      , p0 = x[0] + y
      , p1 = x[1] + y
in ←
  (p0, p1)
```

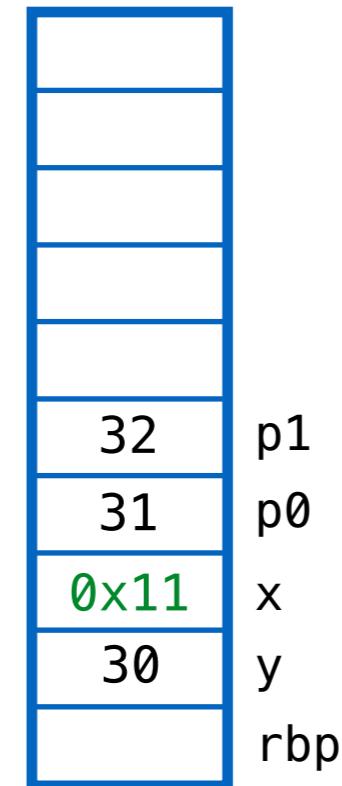


Yay! Have space for (p0, p1)

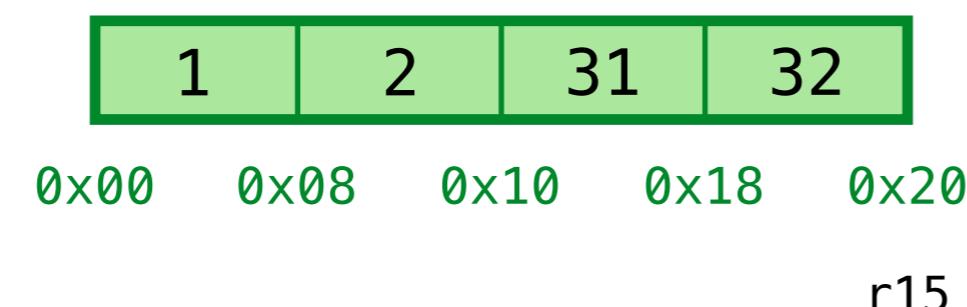


ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1) ←
```



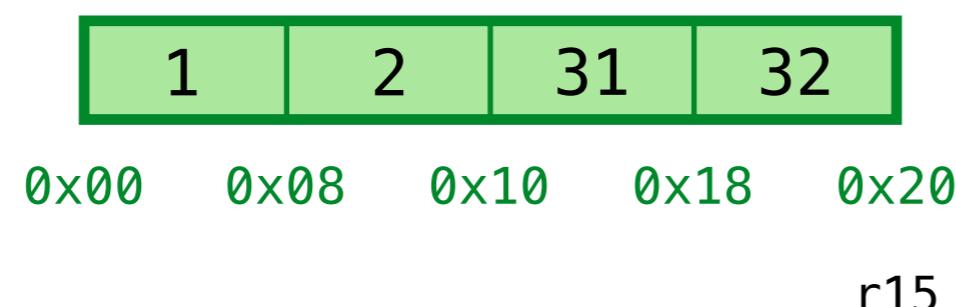
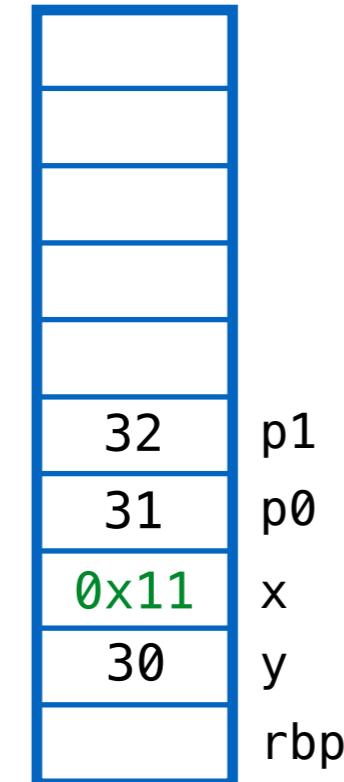
Yay! Have space for (p0, p1)



ex2: garbage in the middle

```
let y  = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x  = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1) ←
```

Result (rax) = 0x09

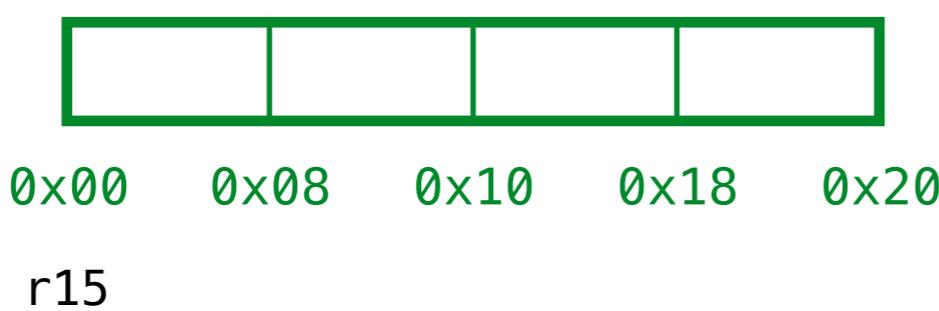
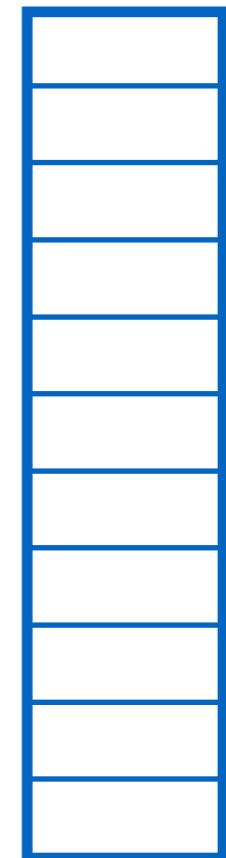


Garter / GC

Example 3

ex3: garbage in the middle (with stack)

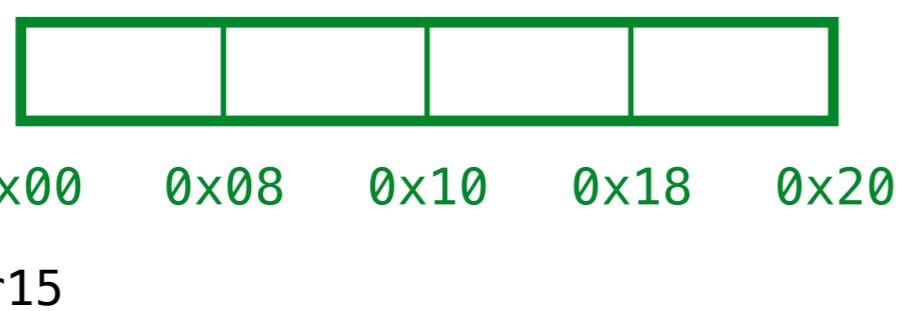
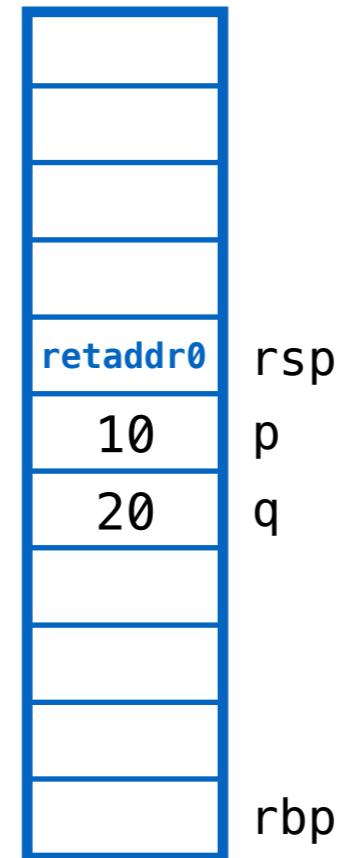
```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]
←
let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
x[0] + y + z
```



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

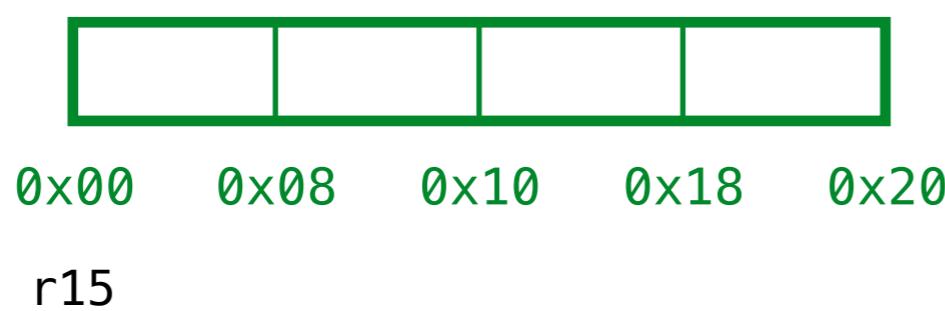
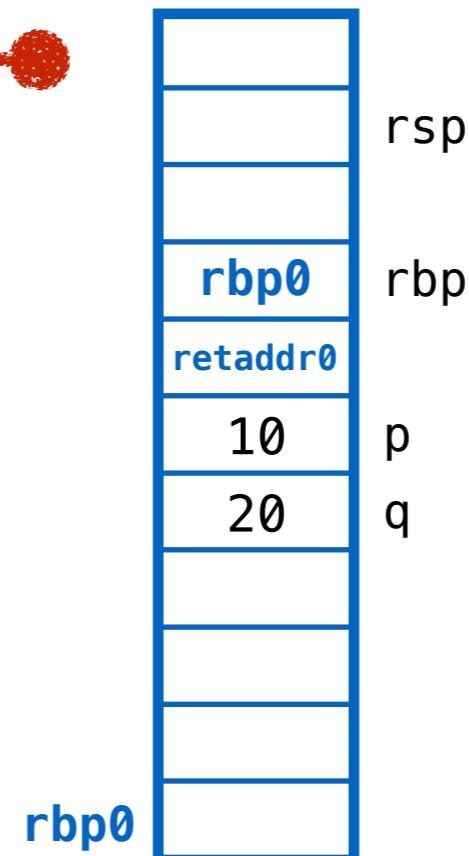
let y  = foo(10, 20) ←
, x  = (y, y + 1)
, z  = foo(100, 200)
in
x[0] + y + z
```



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

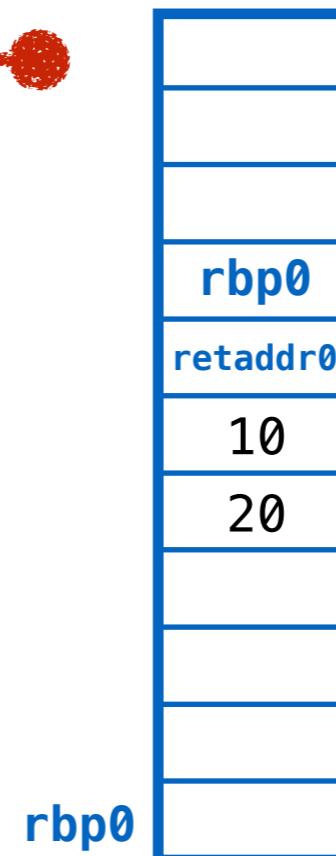
let y = foo(10, 20)
    , x = (y, y + 1)
    , z = foo(100, 200)
in
    x[0] + y + z
```



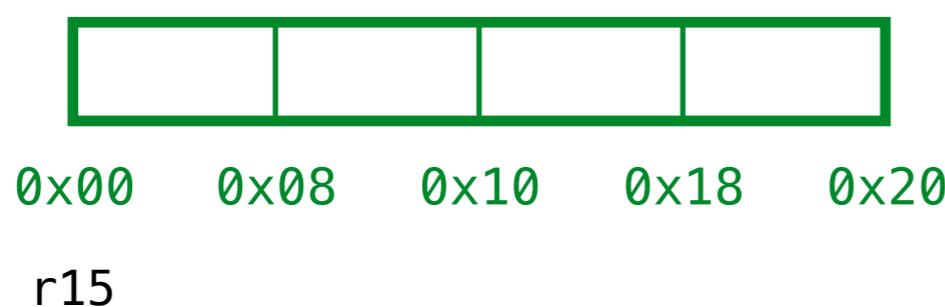
ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

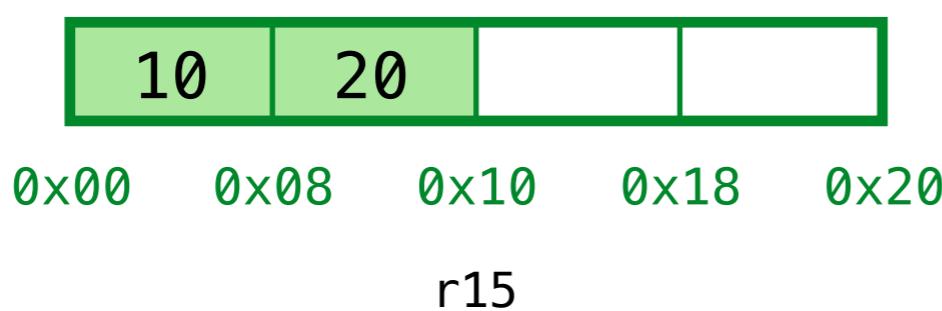
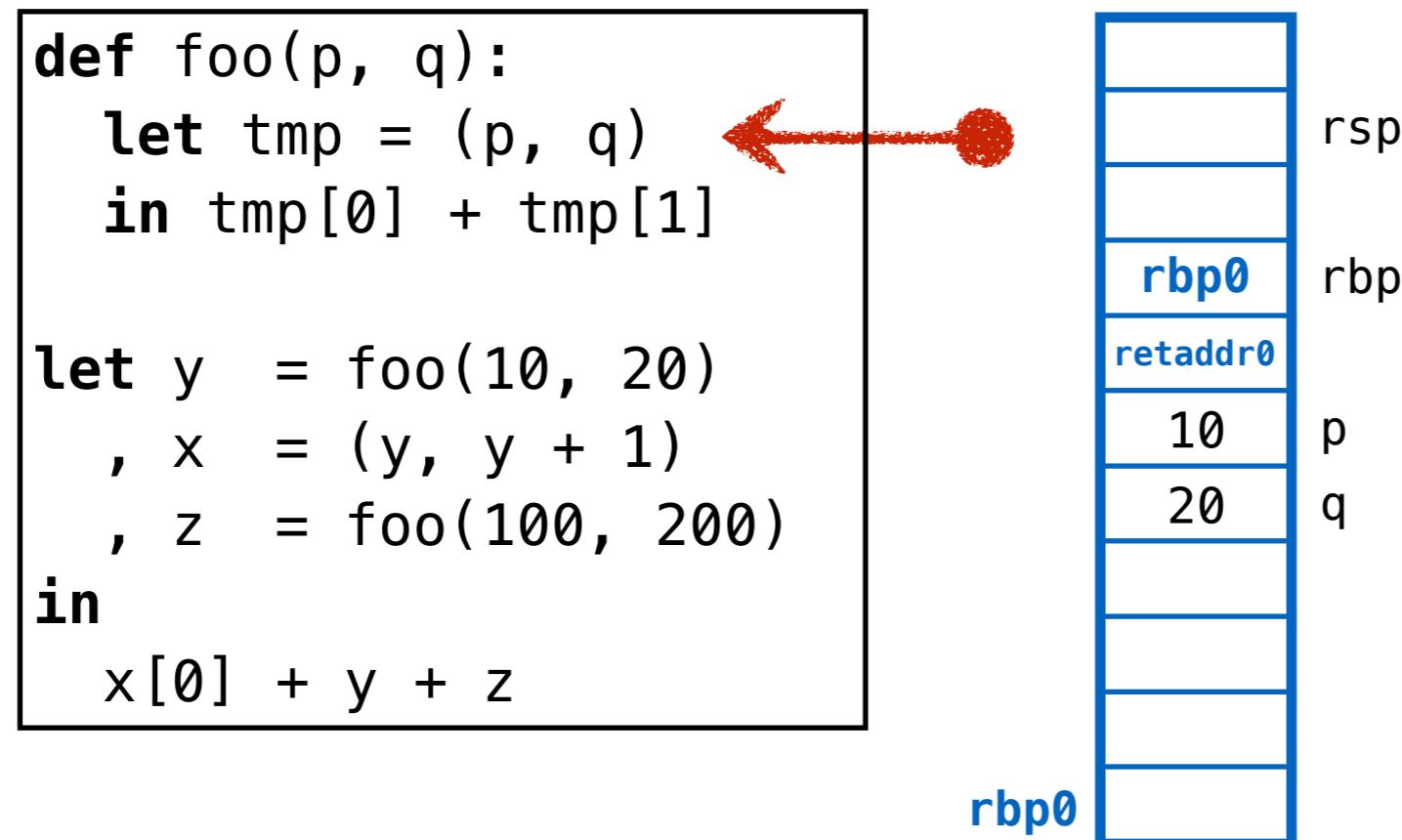
let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + y + z
```



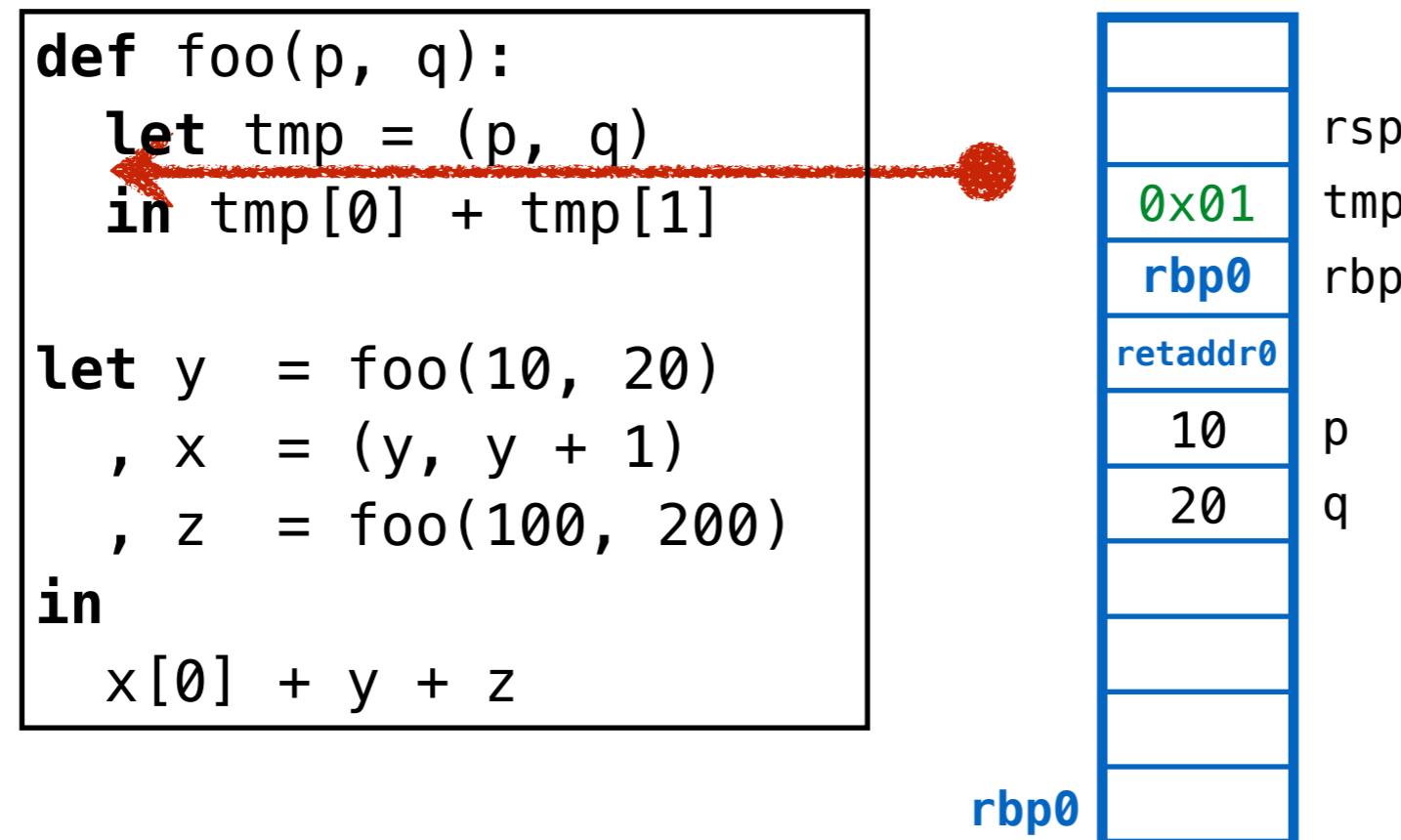
I 1 local var (tmp)



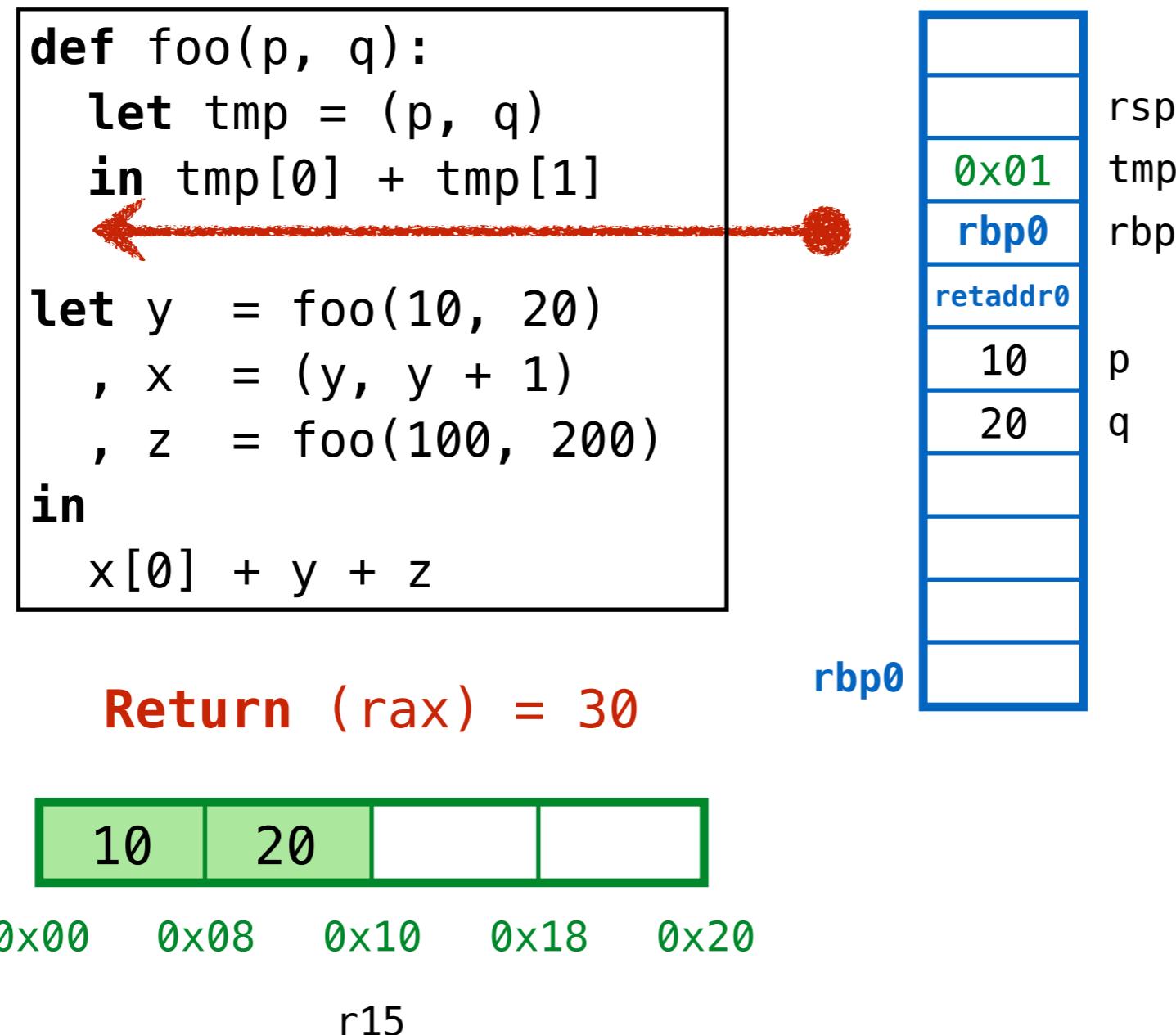
ex3: garbage in the middle (with stack)



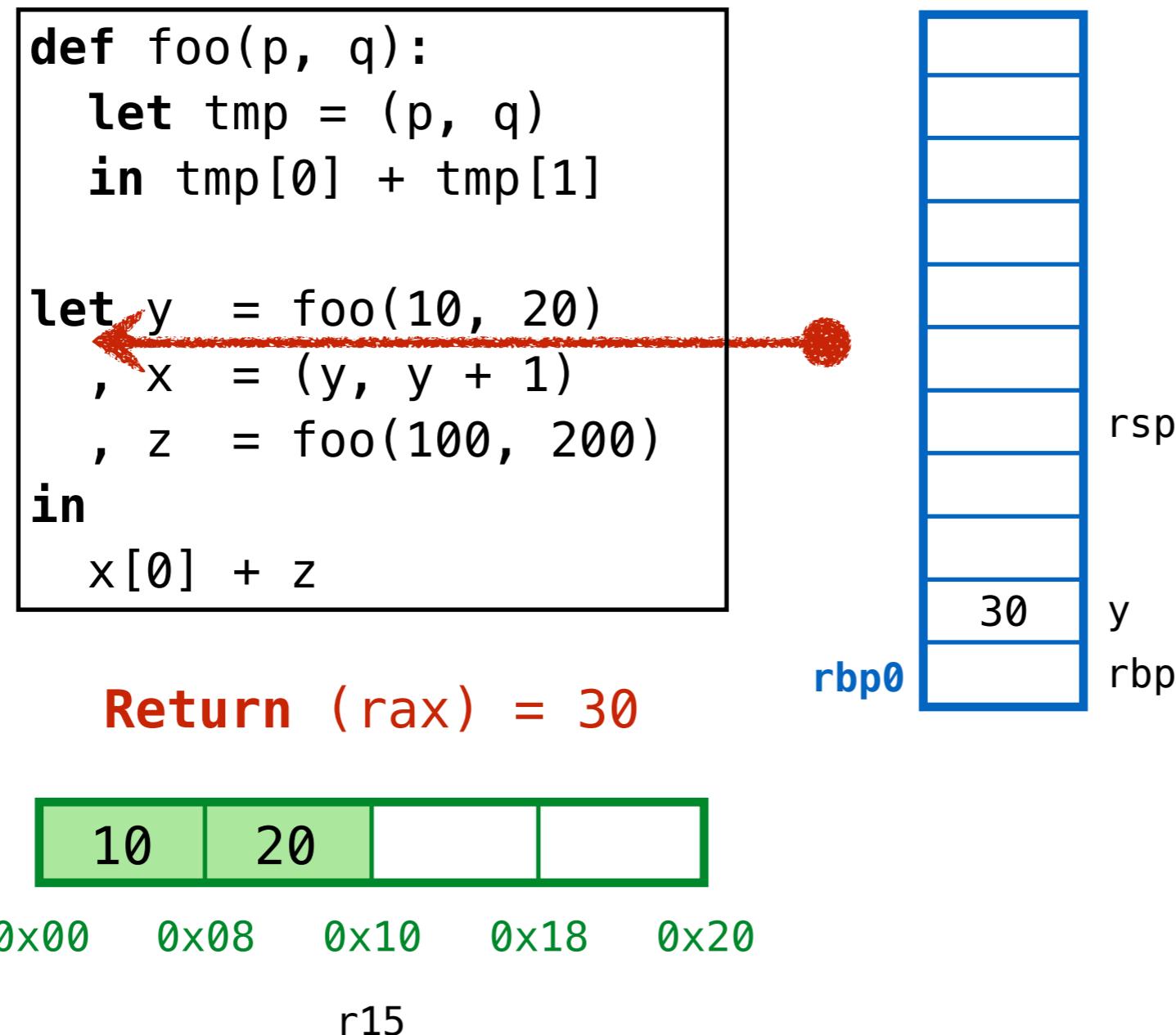
ex3: garbage in the middle (with stack)



ex3: garbage in the middle (with stack)



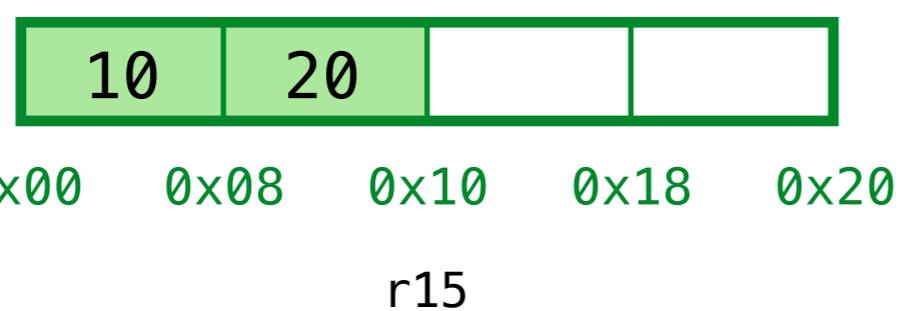
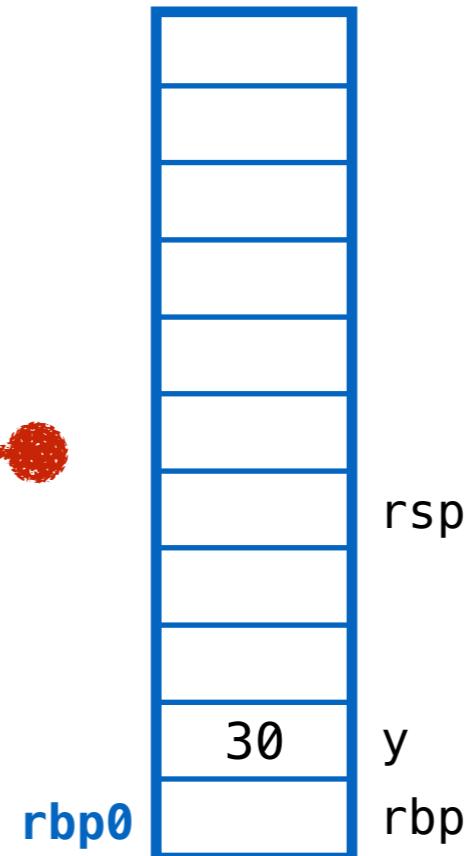
ex3: garbage in the middle (with stack)



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

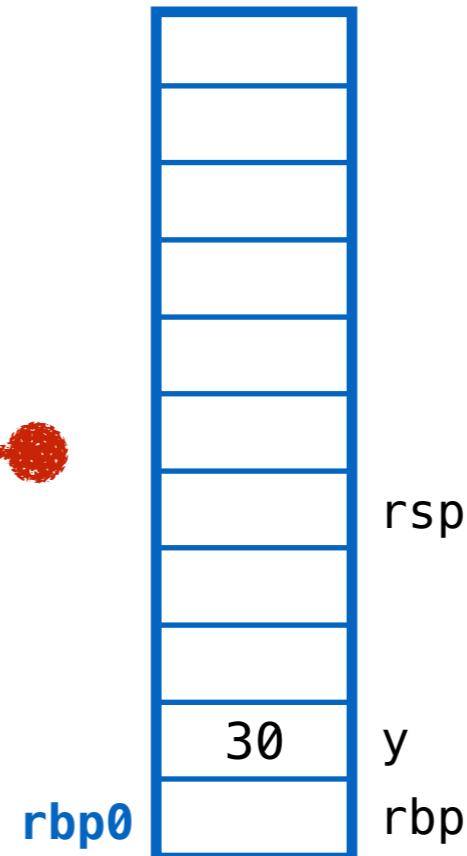
let y  = foo(10, 20)
, x  = (y, y + 1) ←
, z  = foo(100, 200)
in
x[0] + z
```



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1) ←
, z  = foo(100, 200)
in
x[0] + z
```



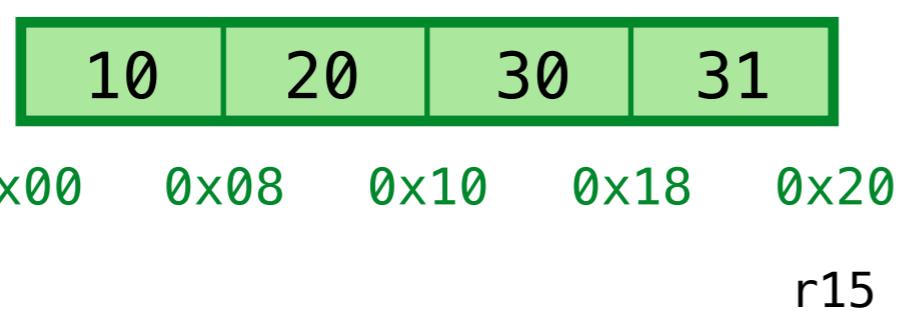
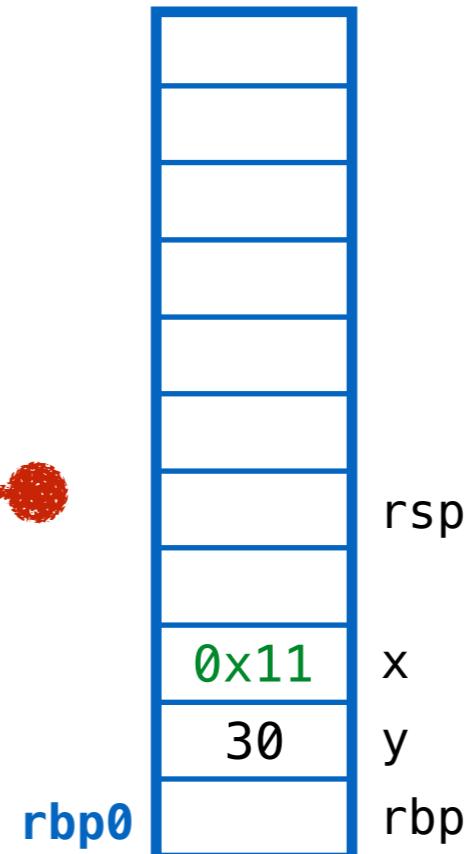
0x00 0x08 0x10 0x18 0x20

r15

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

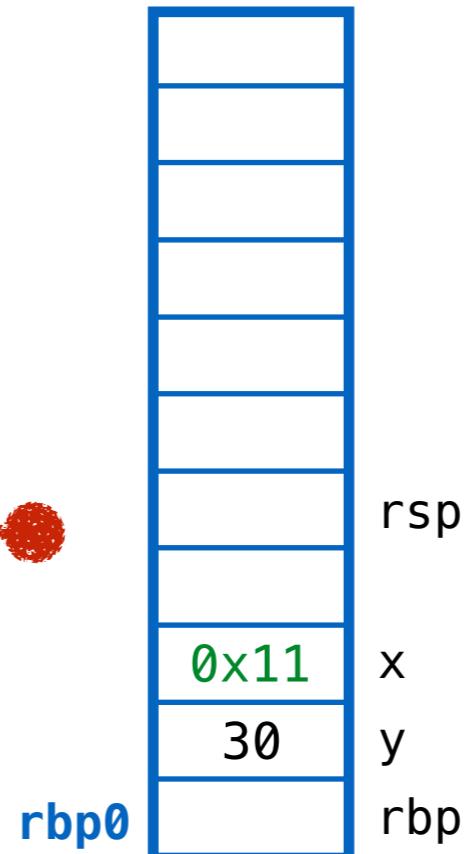
let y  = foo(10, 20)
      ← x = (y, y + 1)
      , z = foo(100, 200)
in
    x[0] + z
```



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200) ←
in
x[0] + z
```



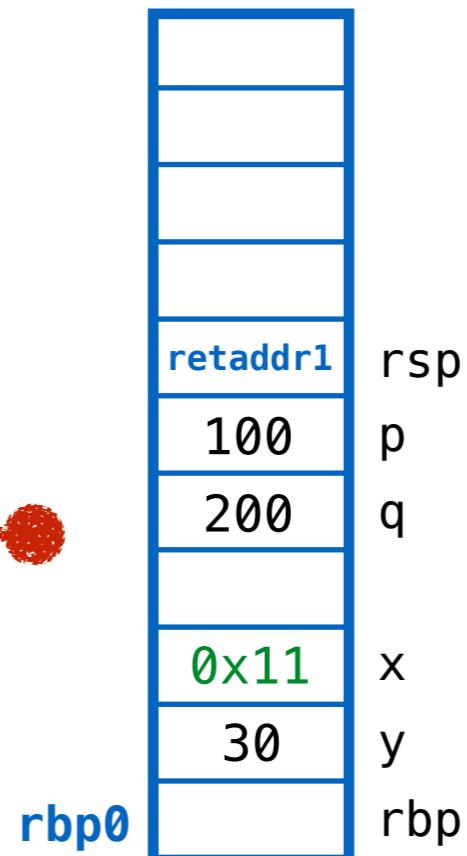
`0x00 0x08 0x10 0x18 0x20`

`r15`

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200) ←
in
x[0] + z
```



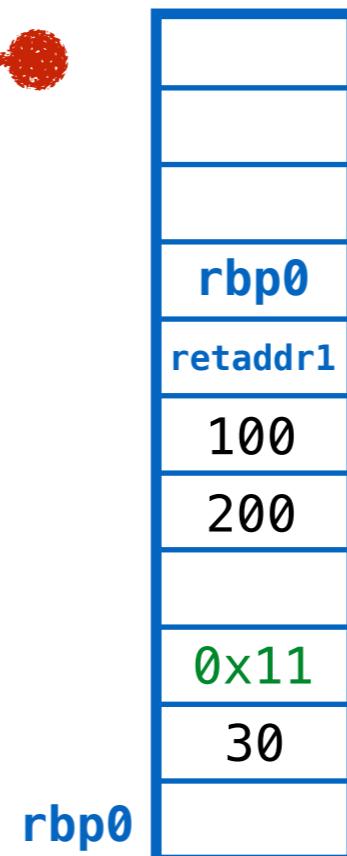
0x00 0x08 0x10 0x18 0x20

r15

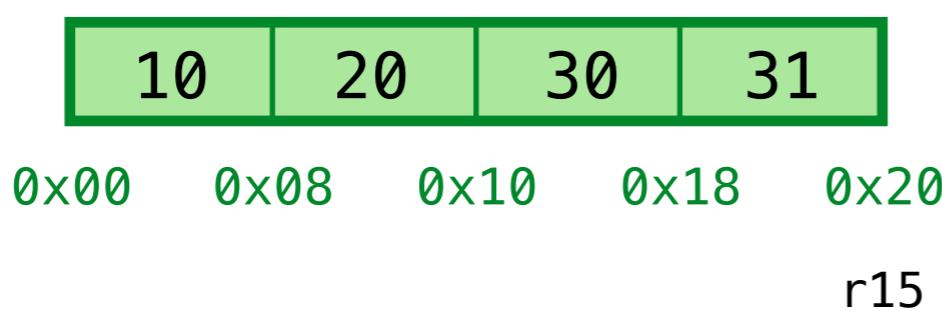
ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

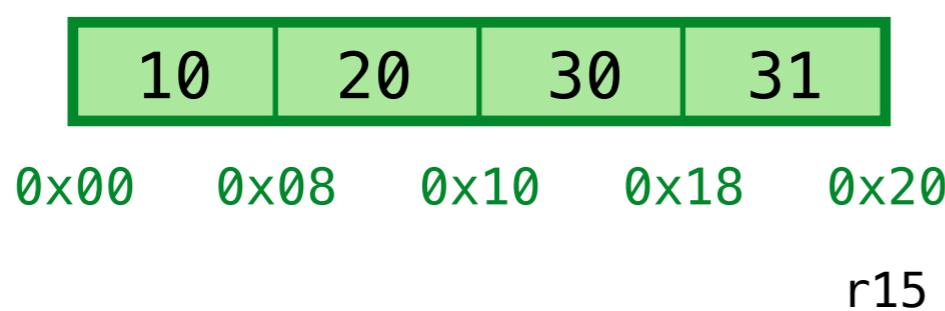
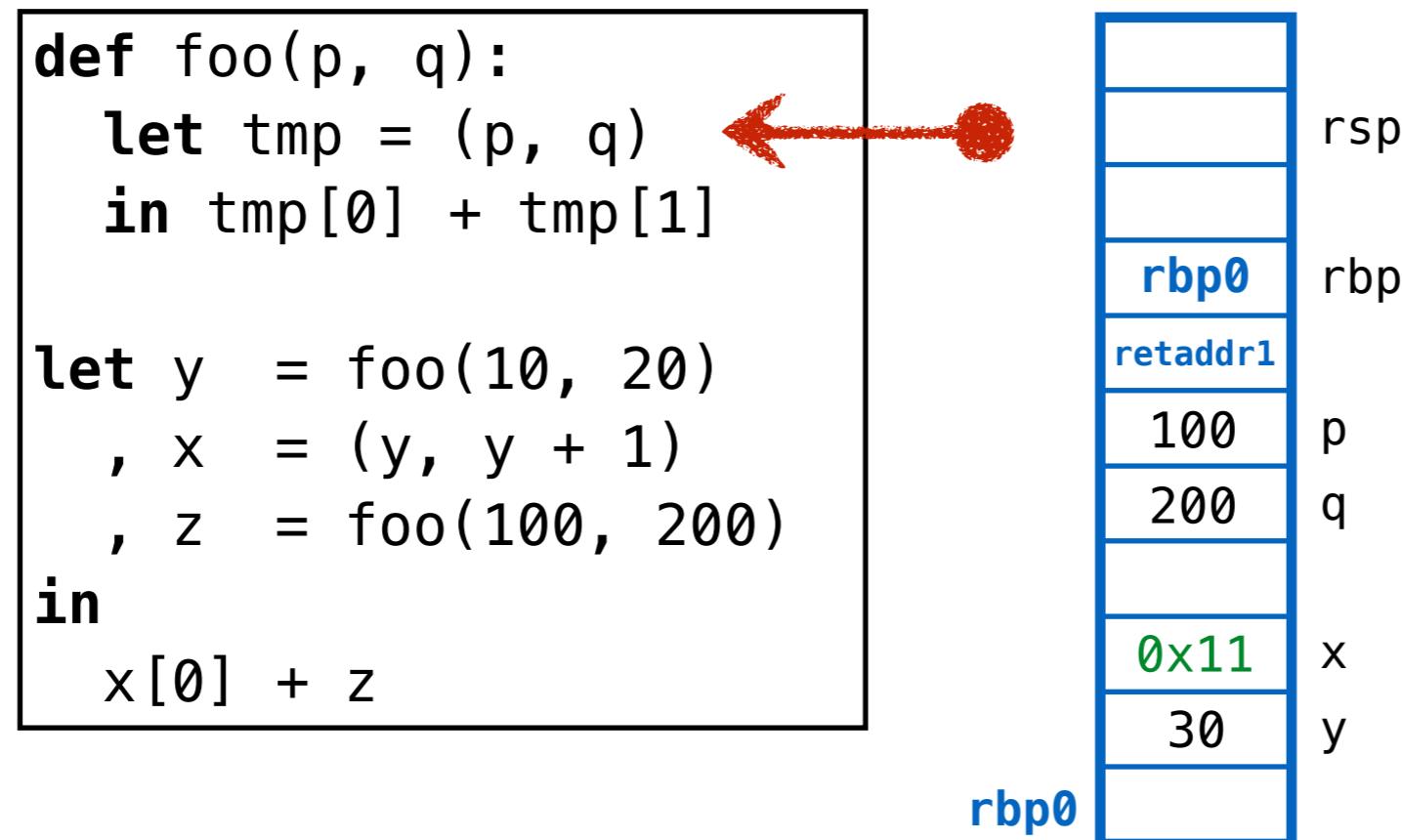
let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```



I 1 local var (tmp)



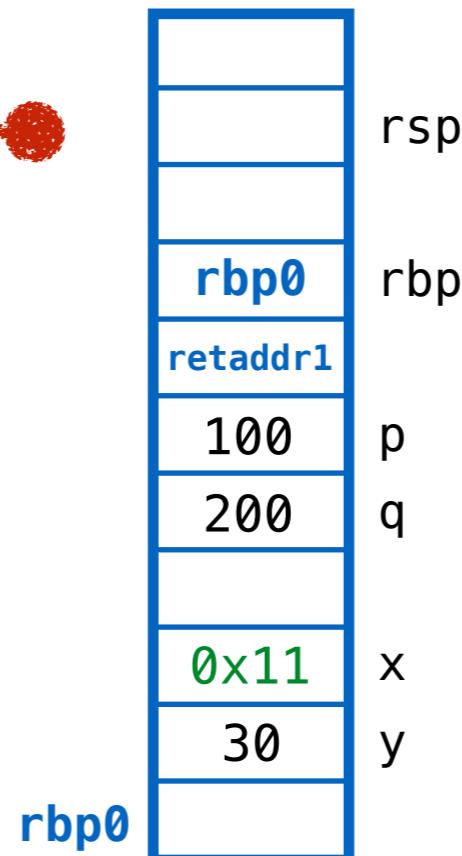
ex3: garbage in the middle (with stack)



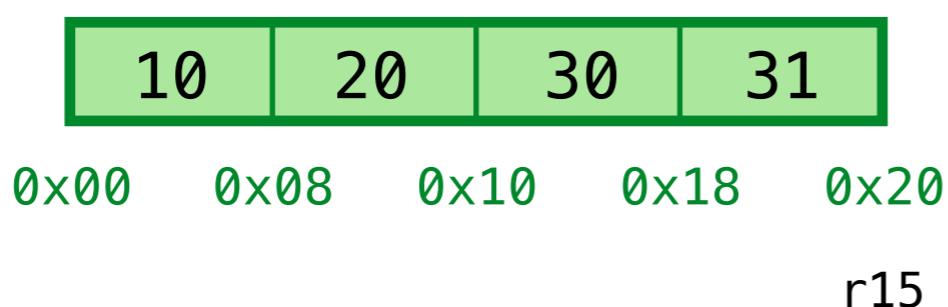
ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

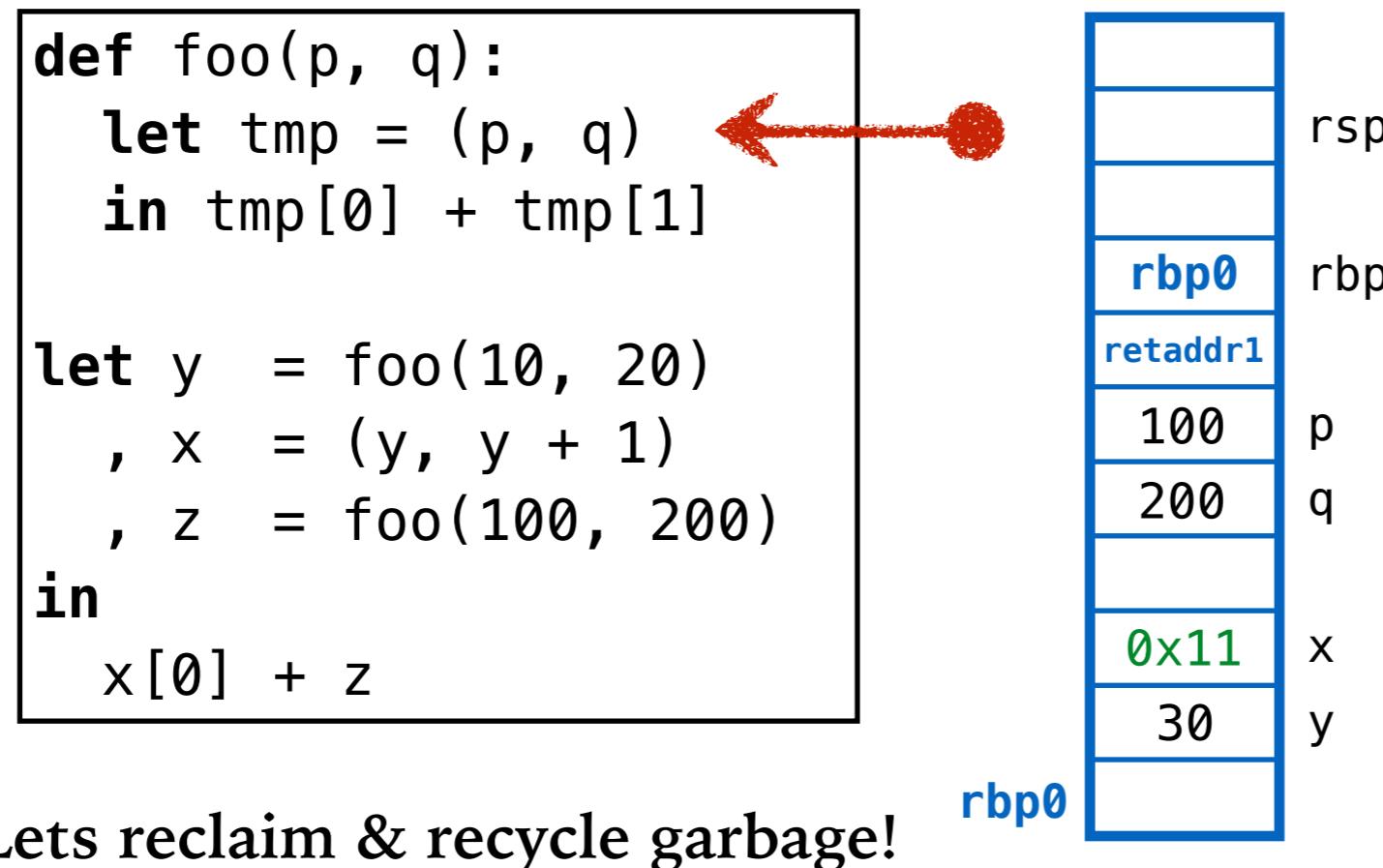
let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```



Lets reclaim & recycle garbage!



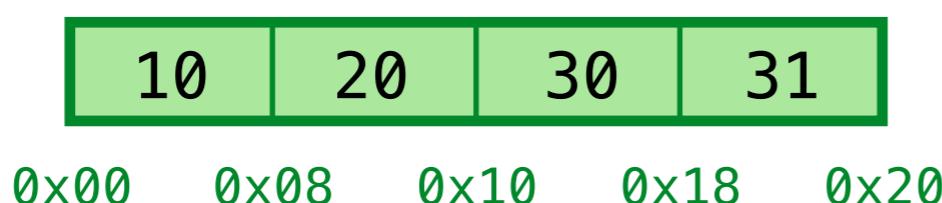
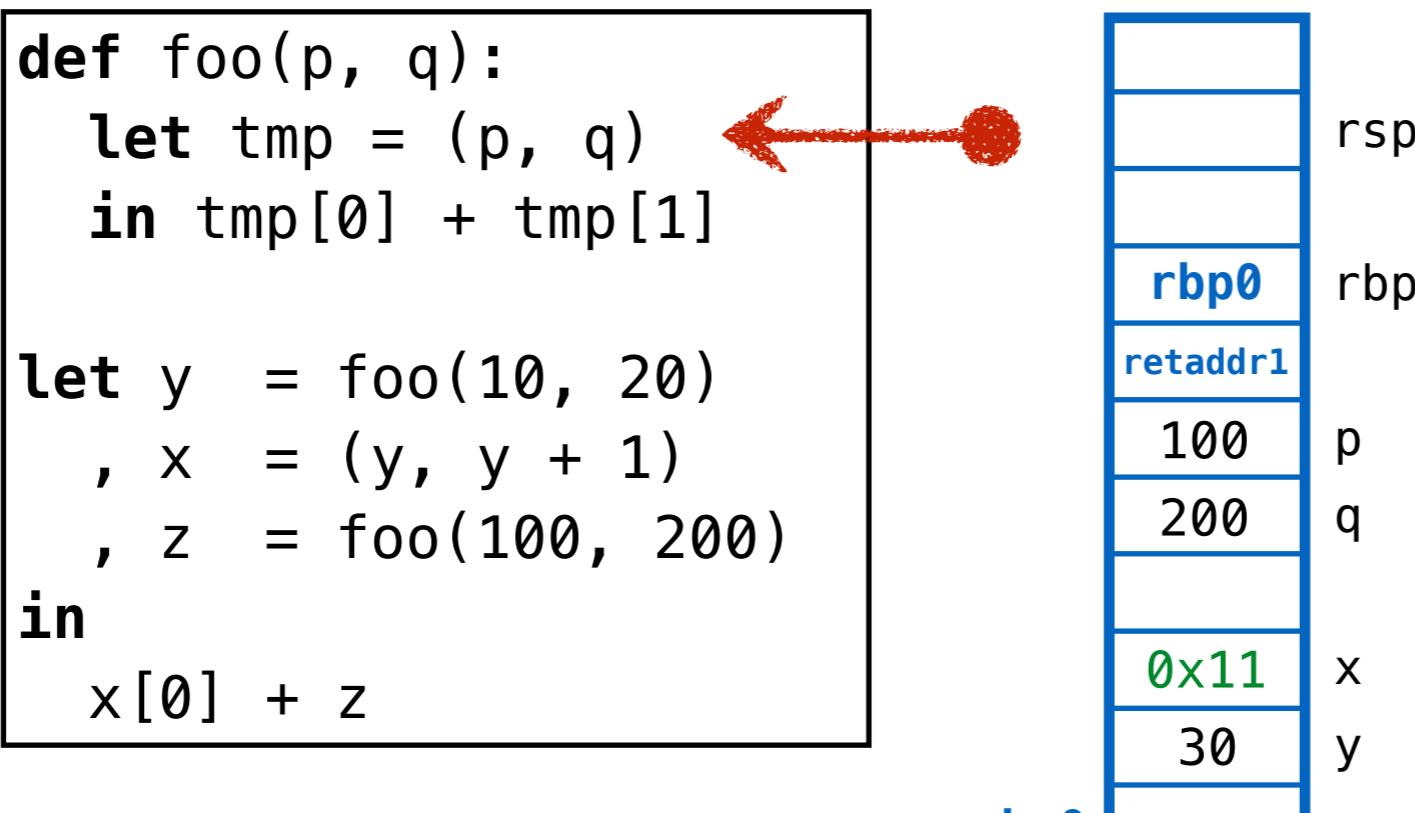
ex3: garbage in the middle (with stack)



QUIZ: Which cells are garbage?

- (A) `0x00, 0x08` (B) `0x08, 0x10` (C) `0x10, 0x18` (D) None (E) All

ex3: garbage in the middle (with stack)



QUIZ: Which cells are garbage?

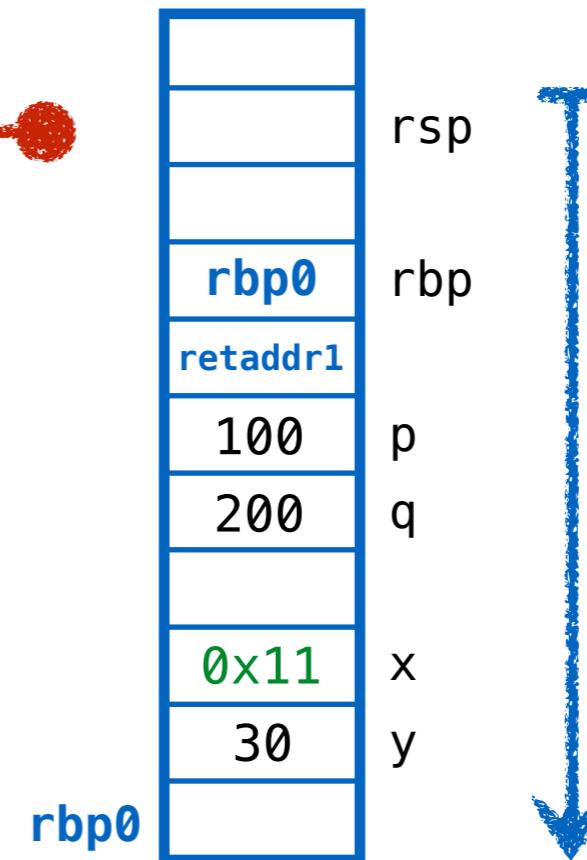
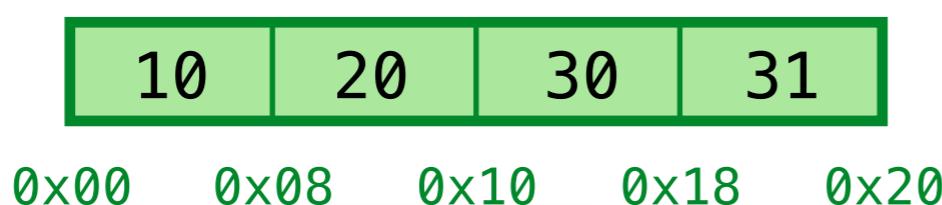
Those that are *not reachable from any stack frame*

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```

Lets reclaim & recycle garbage!



Traverse Stack
from top (rsp)
to bottom (rbp0)
to mark
reachable cells.

QUIZ: Which cells are garbage?

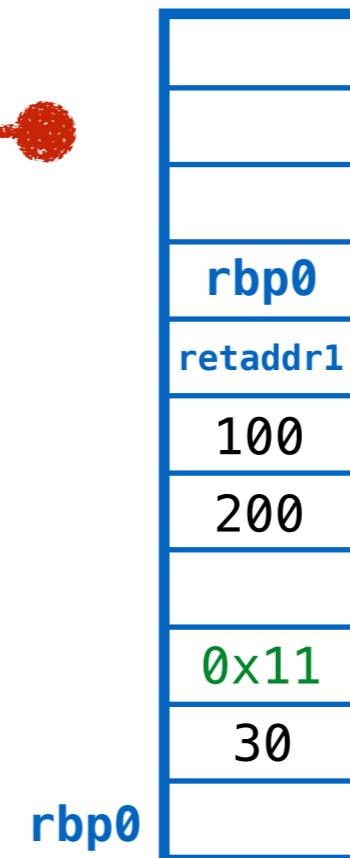
Those that are *not reachable from any stack frame*

ex3: garbage in the middle (with stack)

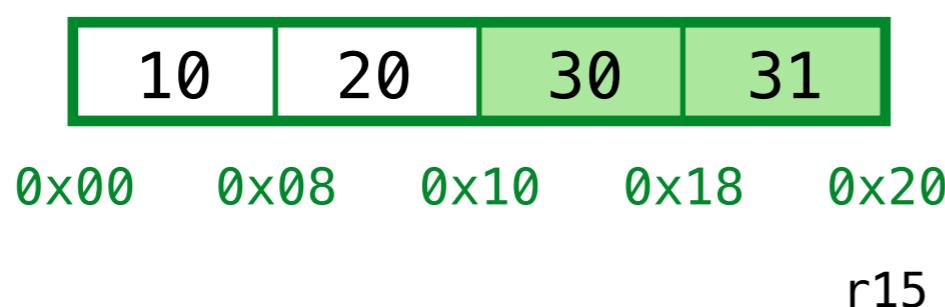
```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```

Lets reclaim & recycle garbage!

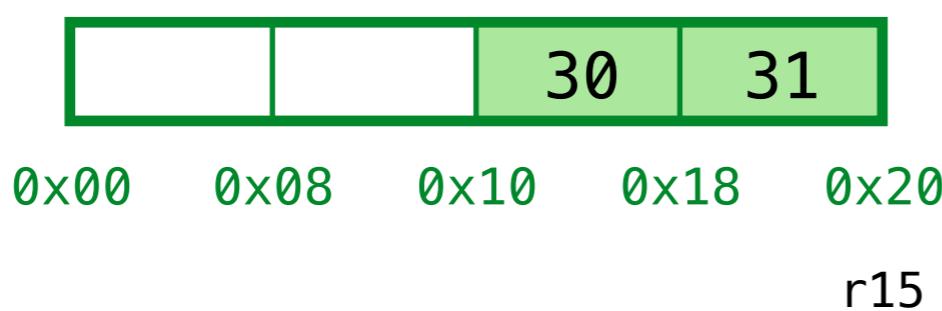
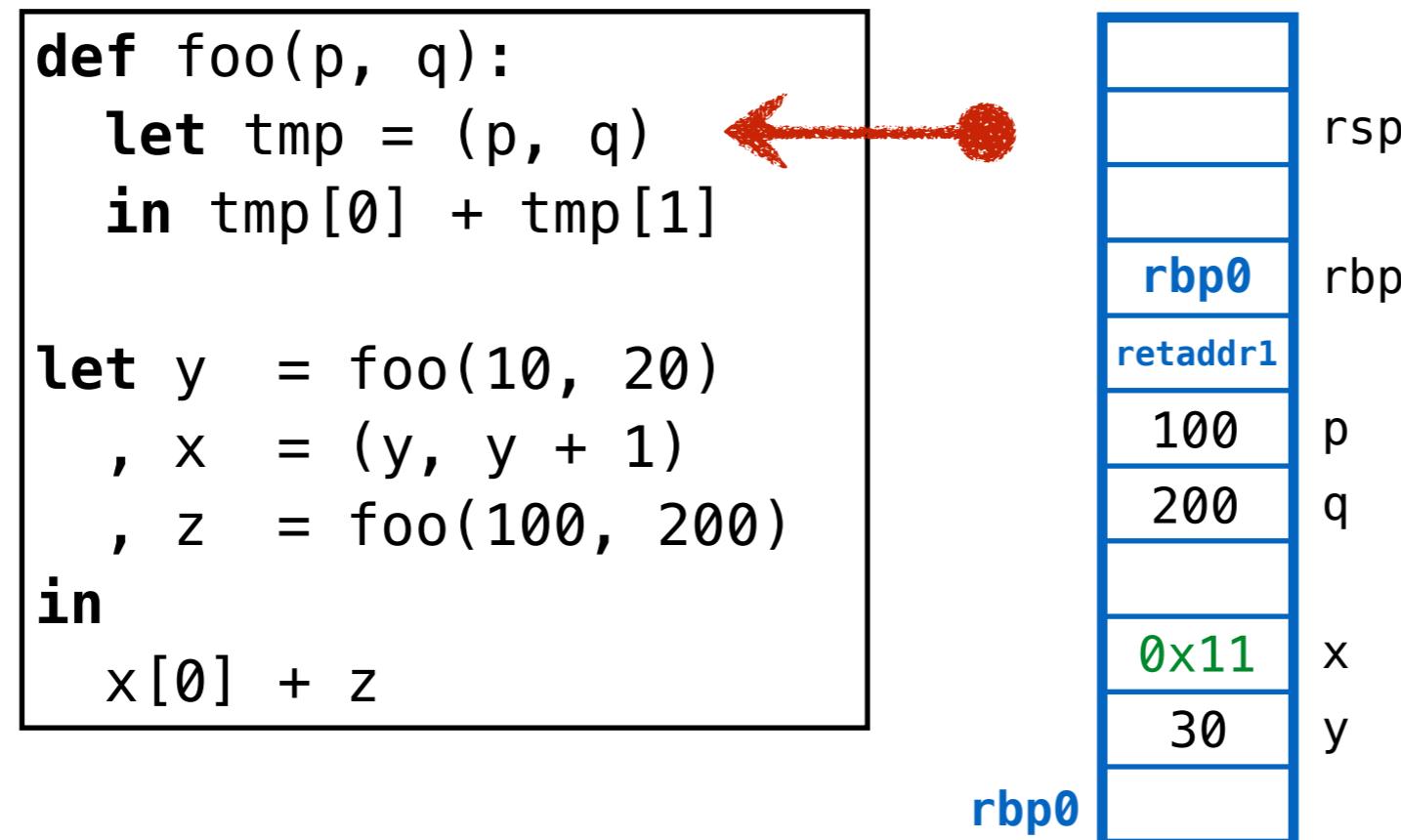


Traverse Stack
from top (rsp)
to bottom (rbp0)
to mark
reachable cells.



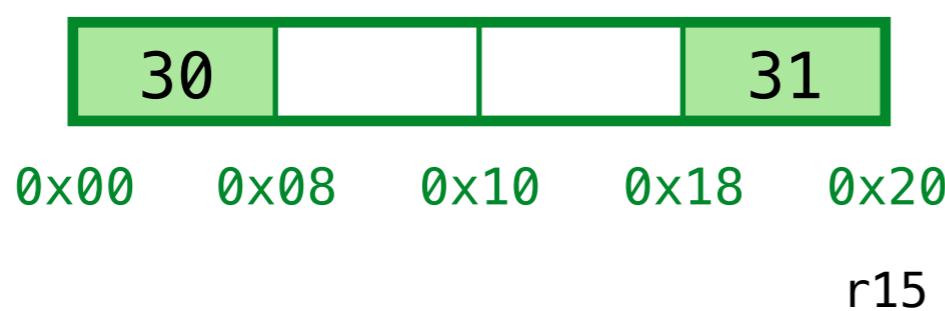
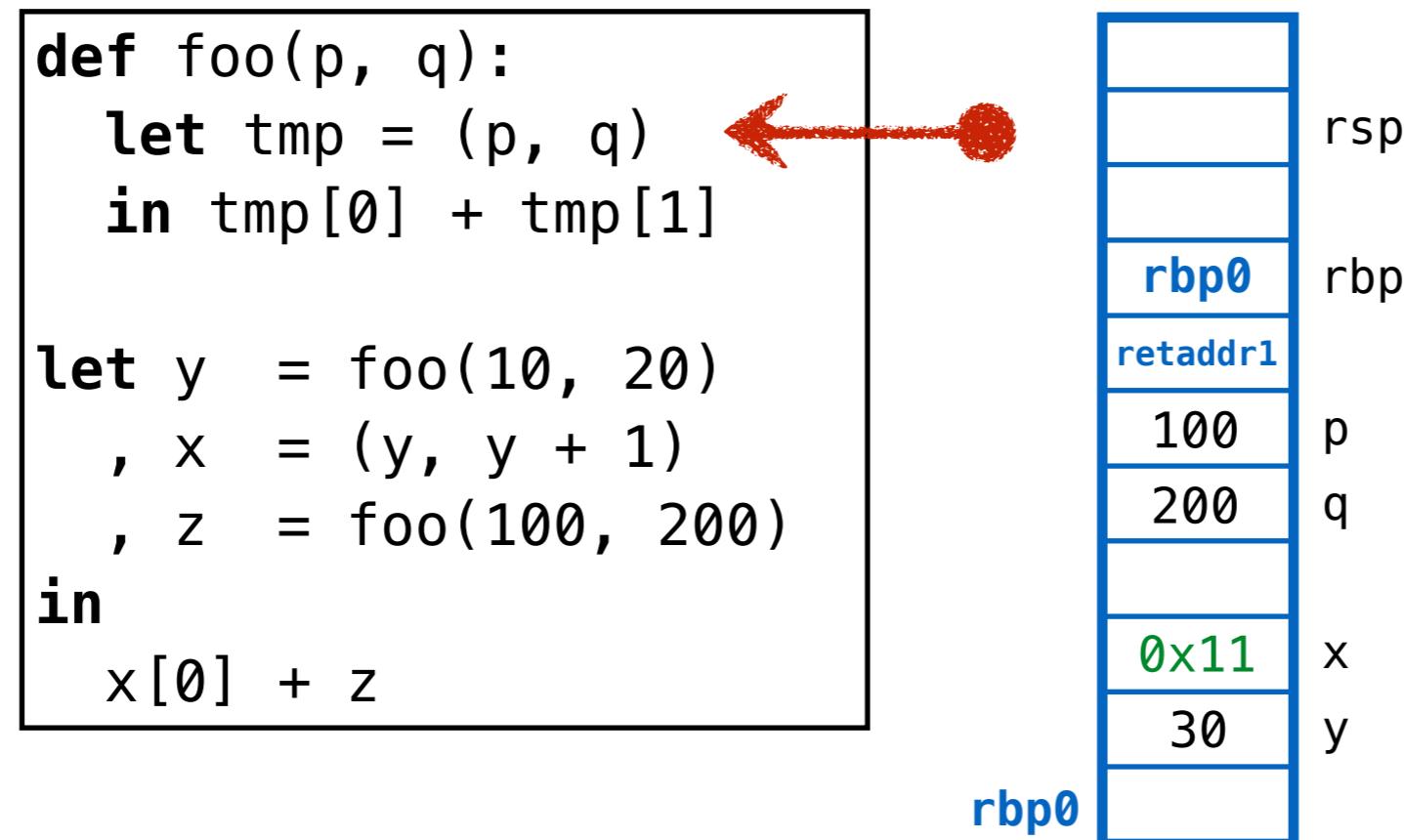
Which cells are garbage?

ex3: garbage in the middle (with stack)



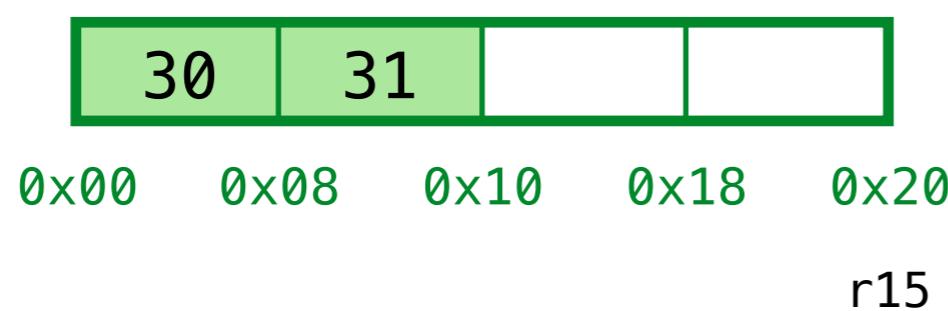
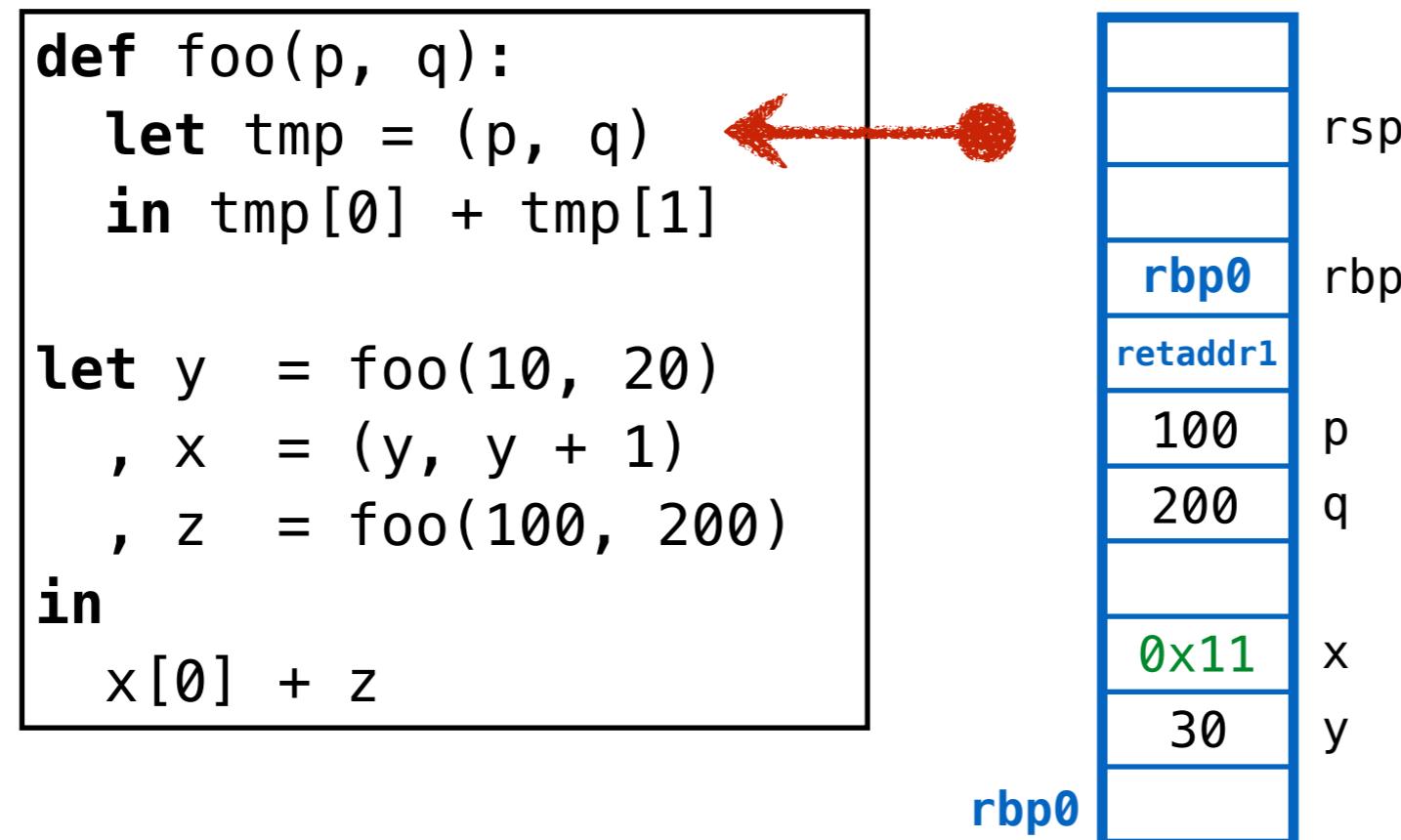
Compact the live cells

ex3: garbage in the middle (with stack)



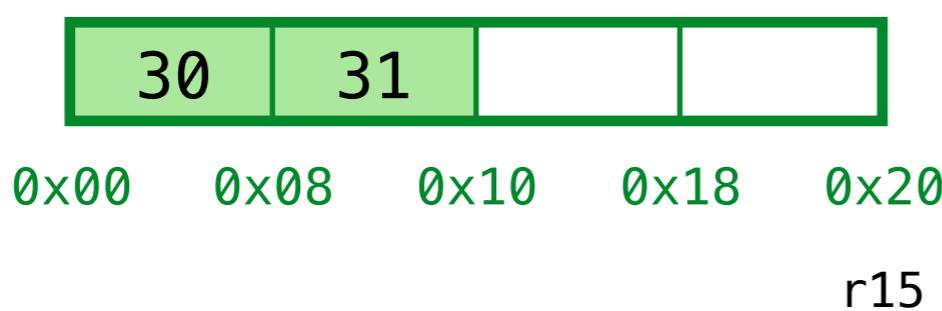
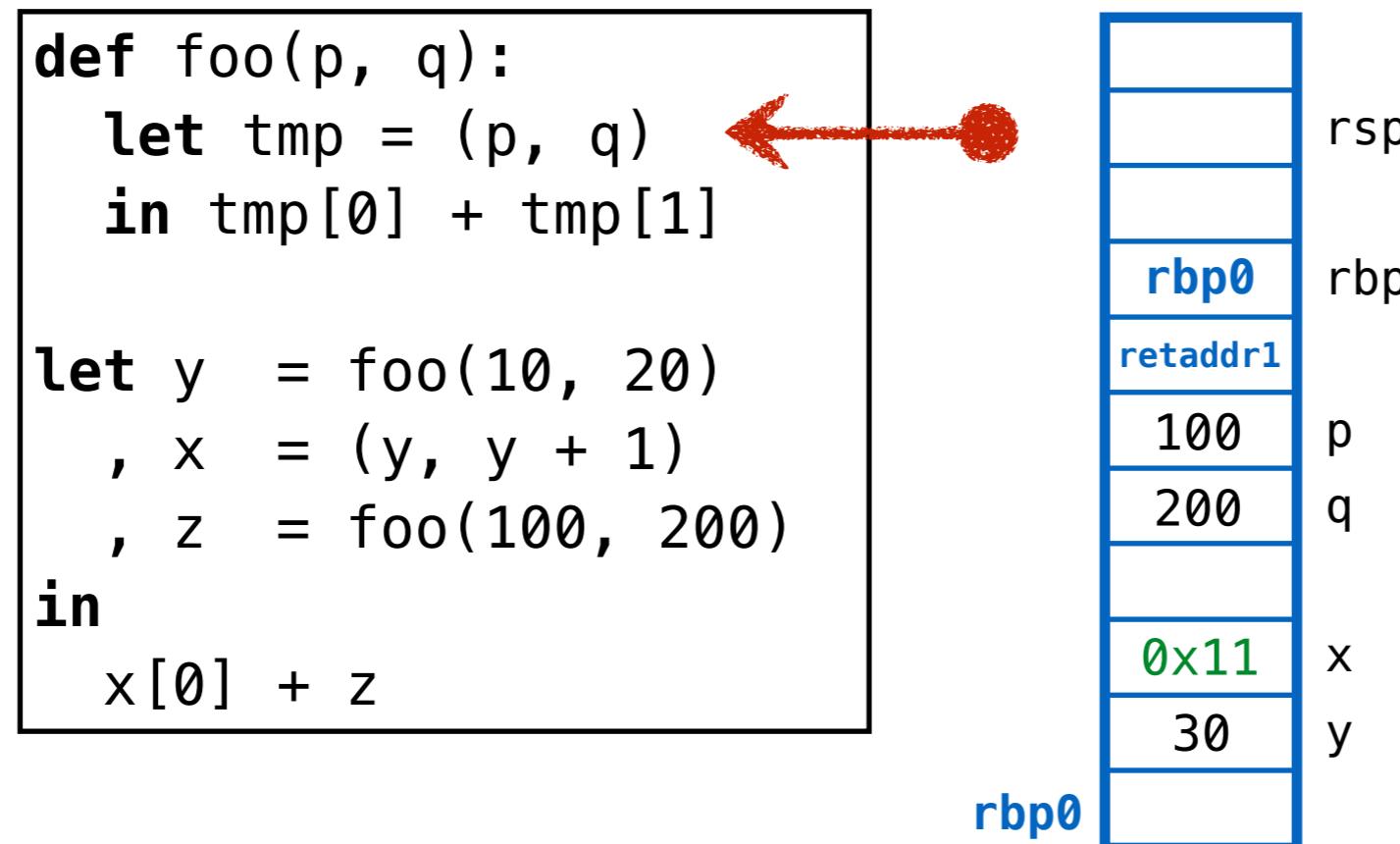
Compact the live cells

ex3: garbage in the middle (with stack)



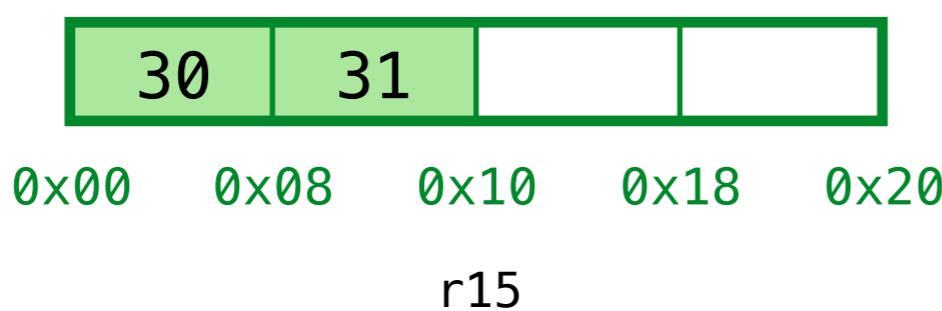
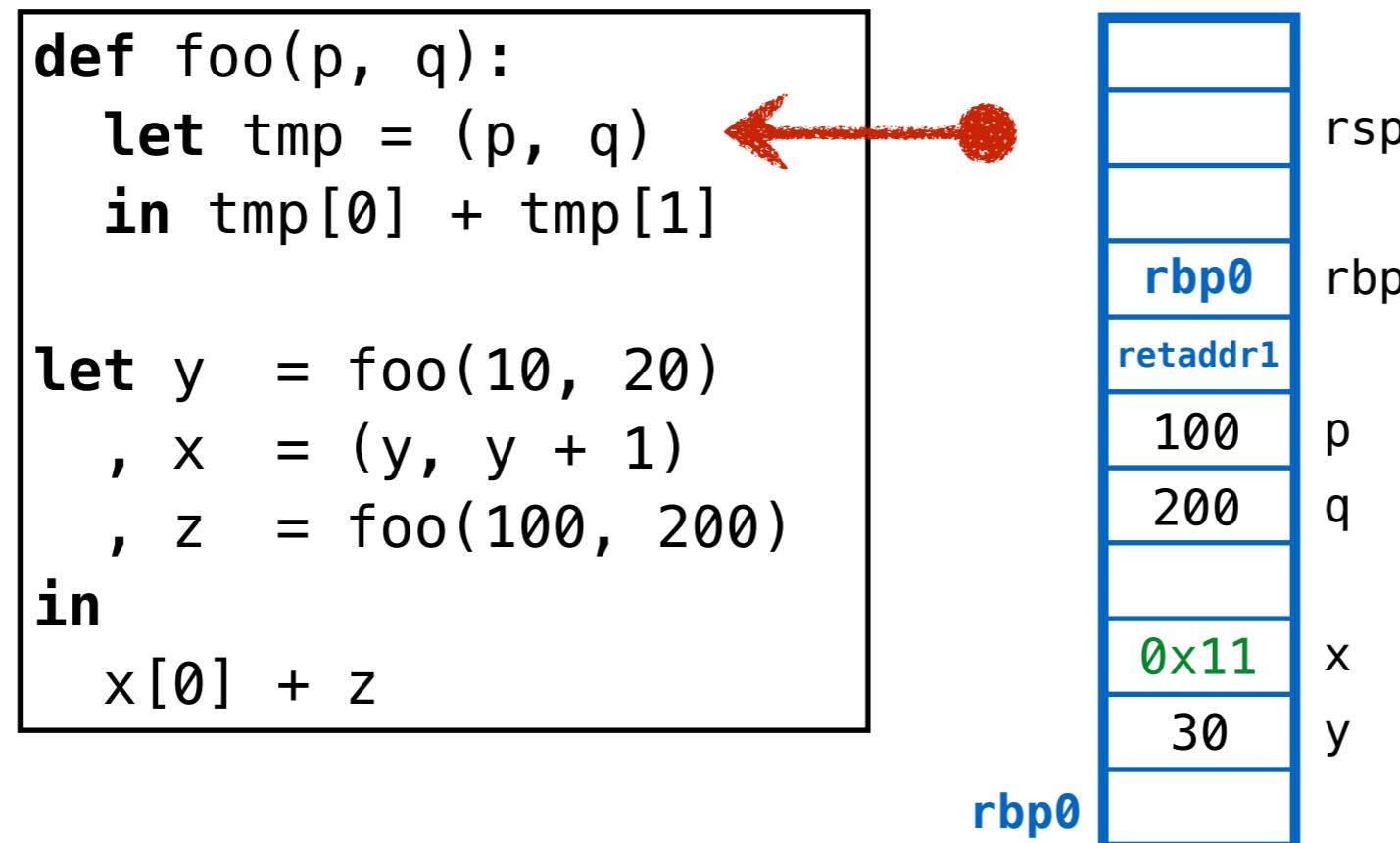
Compact the live cells

ex3: garbage in the middle (with stack)



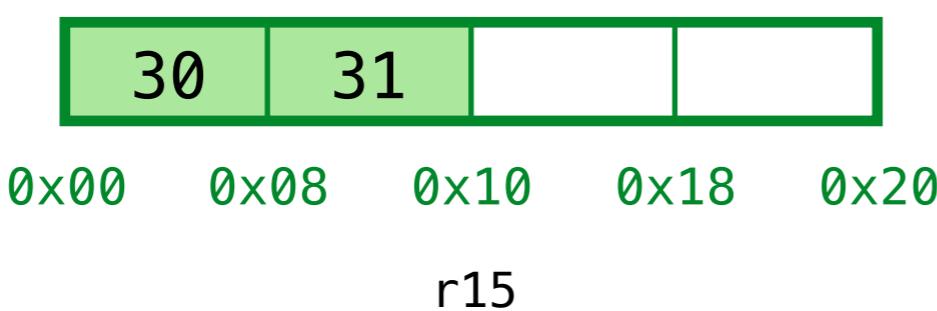
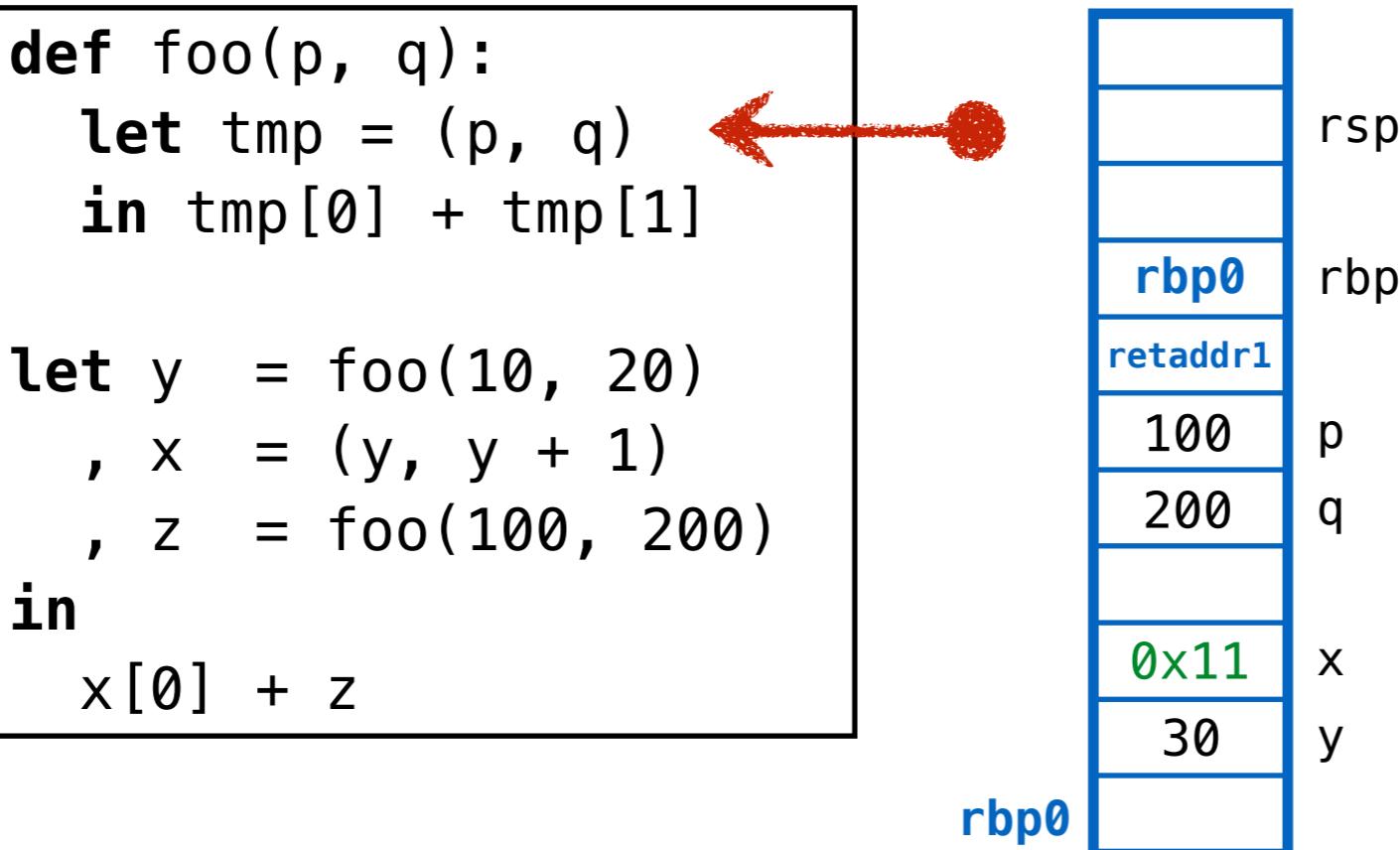
Compact the live cells ... then rewind `r15`

ex3: garbage in the middle (with stack)



Compact the live cells ... then rewind r15

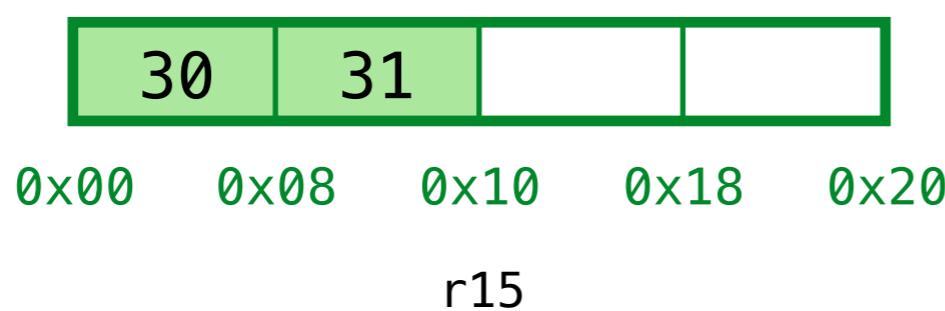
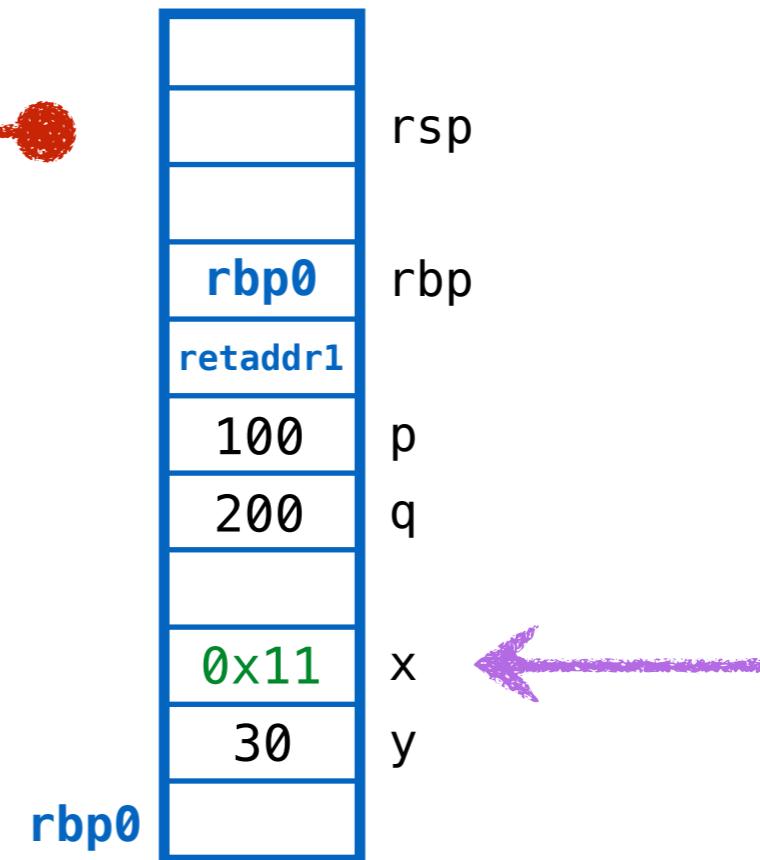
ex3: garbage in the middle (with stack)



Problem???

ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```

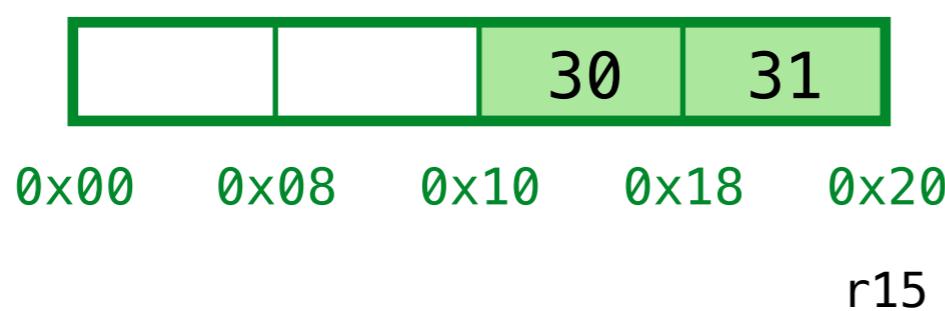
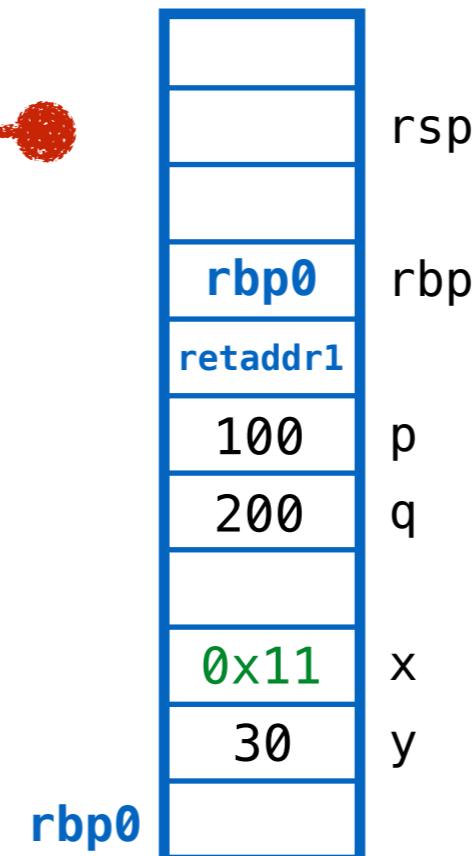


Problem! Have to **REDIRECT** existing pointers

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

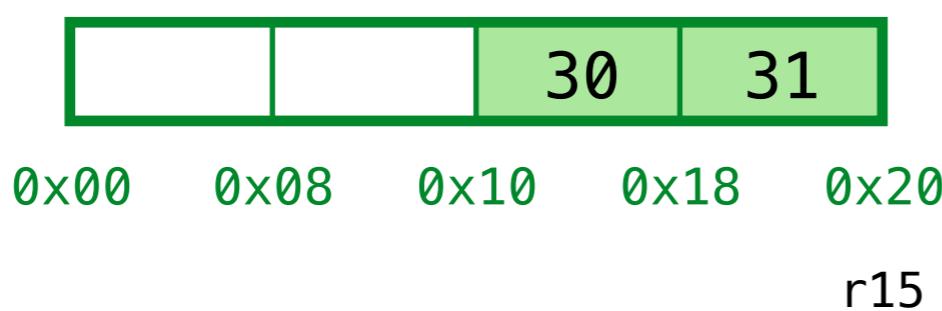
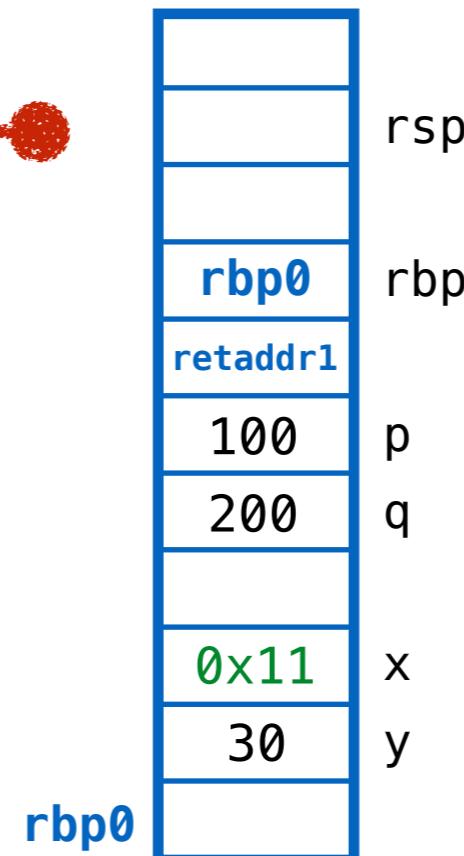
let y = foo(10, 20)
    , x = (y, y + 1)
    , z = foo(100, 200)
in
    x[0] + z
```



ex3: garbage in the middle (with stack)

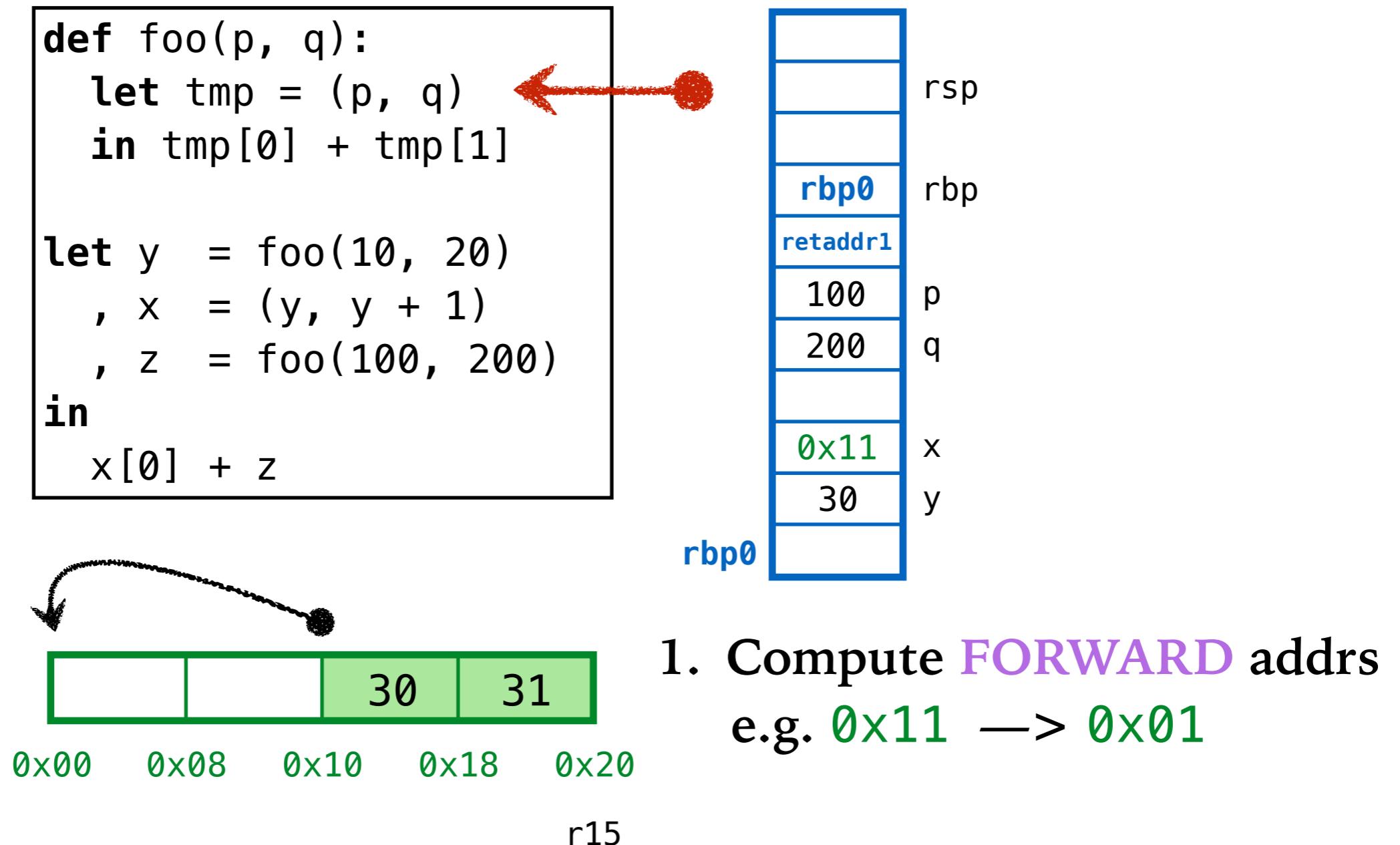
```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```

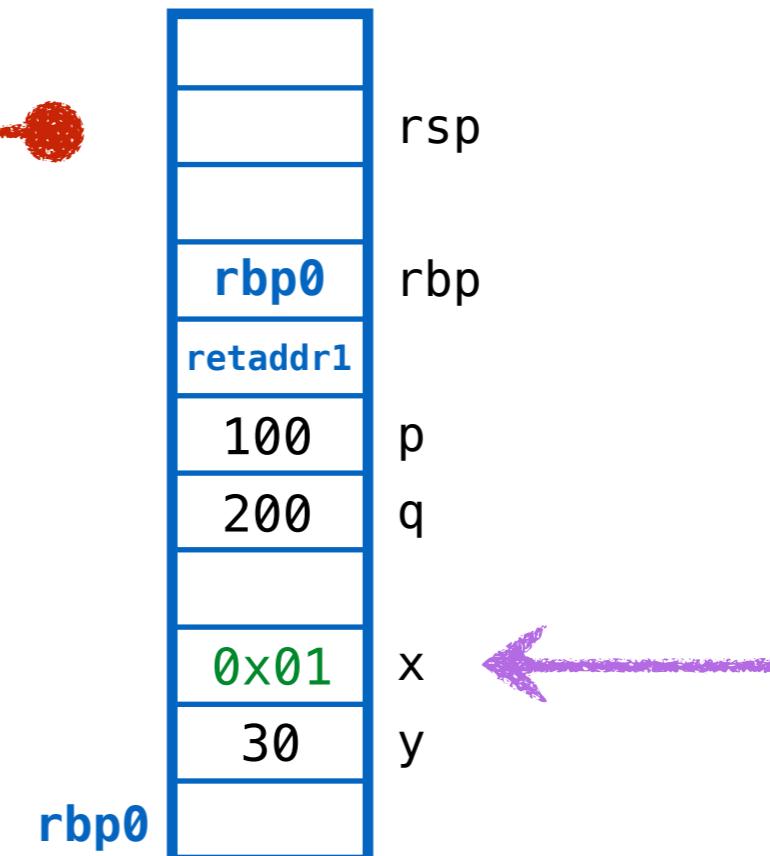
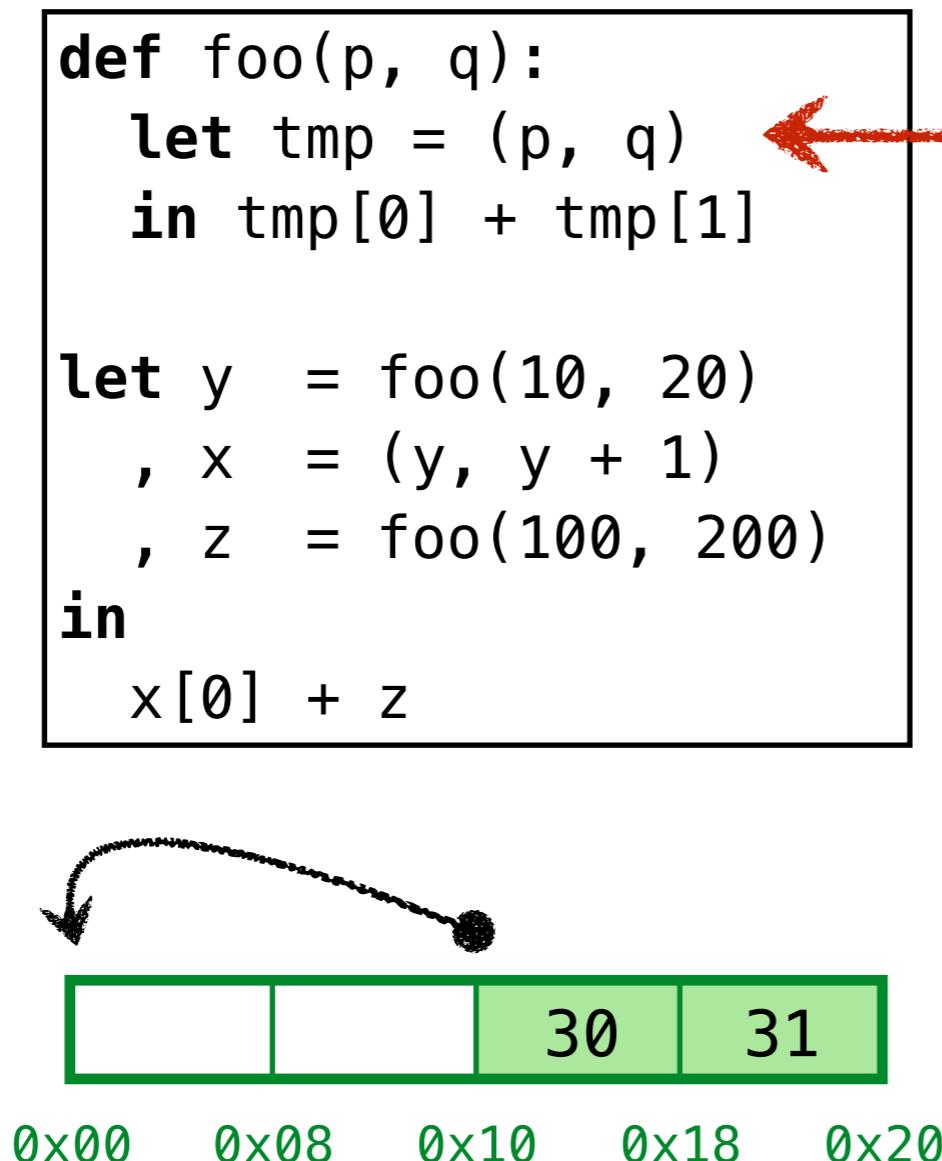


1. Compute **FORWARD** addrs
(i.e. new compacted addrs)

ex3: garbage in the middle (with stack)



ex3: garbage in the middle (with stack)

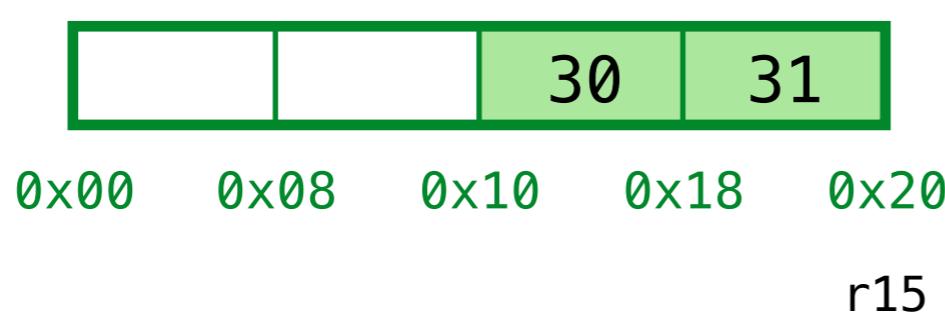
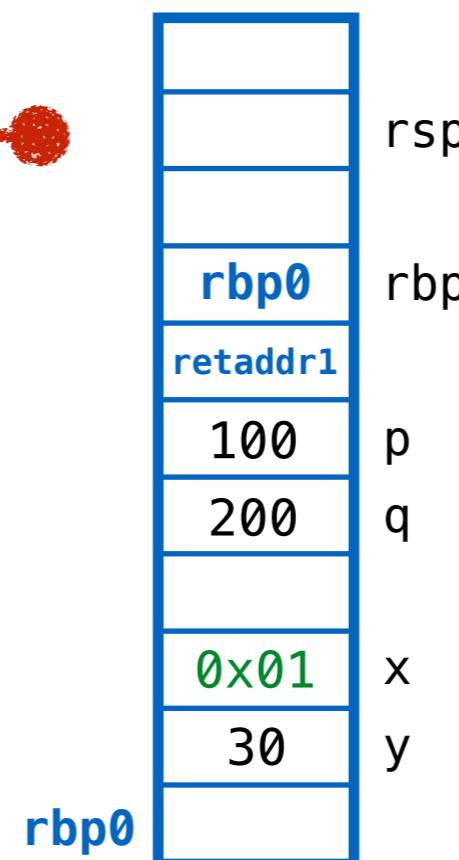


1. Compute **FORWARD** addrs
e.g. `0x11` \rightarrow `0x01`
2. **REDIRECT** addrs on stack

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```



1. Compute **FORWARD** addrs
e.g. 0x11 → 0x01

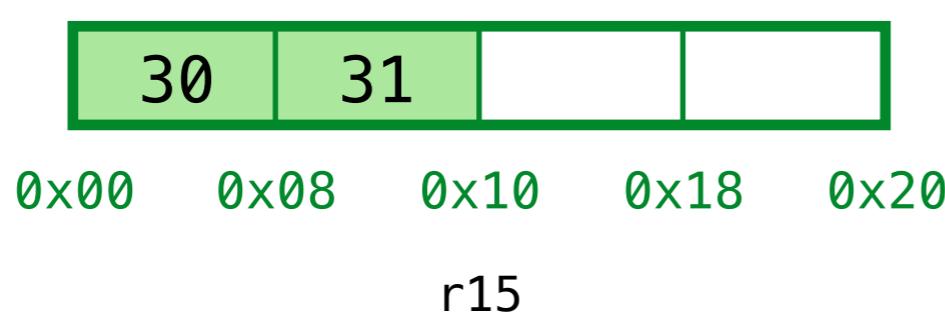
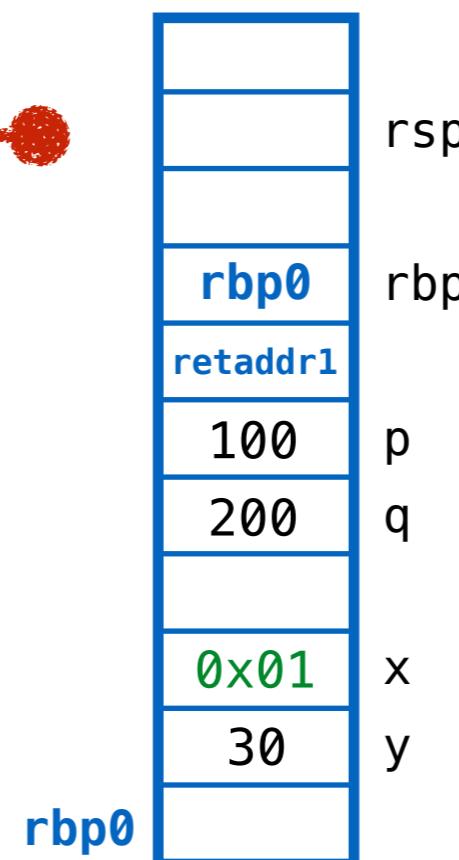
r15 2. **REDIRECT** addrs on stack

3. **COMPACT** cells on heap

ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y = foo(10, 20)
, x = (y, y + 1)
, z = foo(100, 200)
in
x[0] + z
```

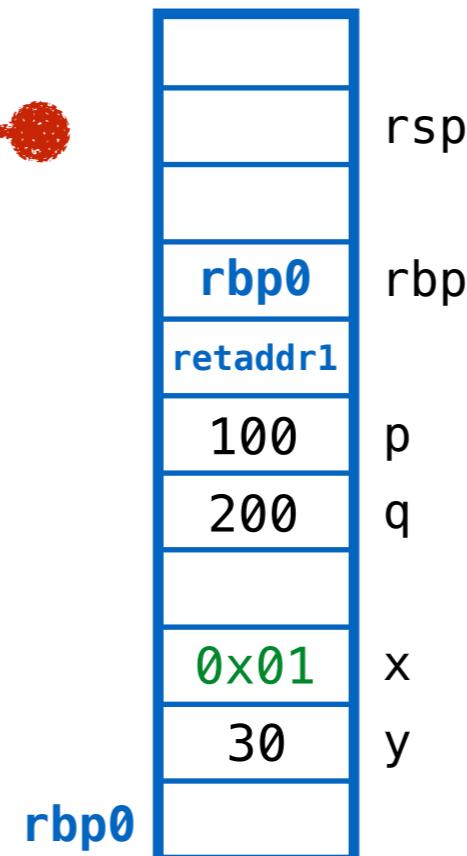


1. Compute **FORWARD** addrs
e.g. **0x11** → **0x01**
2. **REDIRECT** addrs on stack
3. **COMPACT** cells on heap

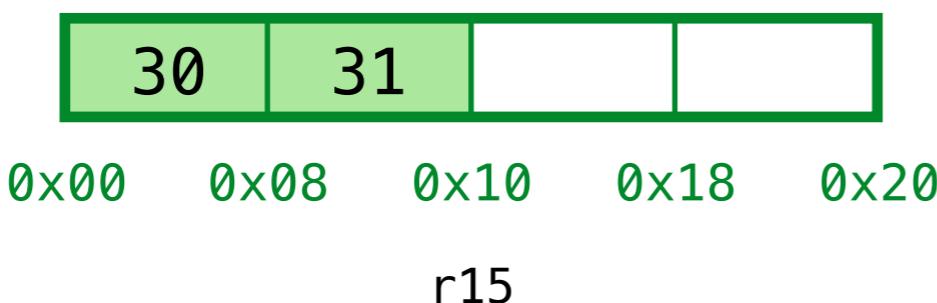
ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```



Yay! Have space for (p, q)

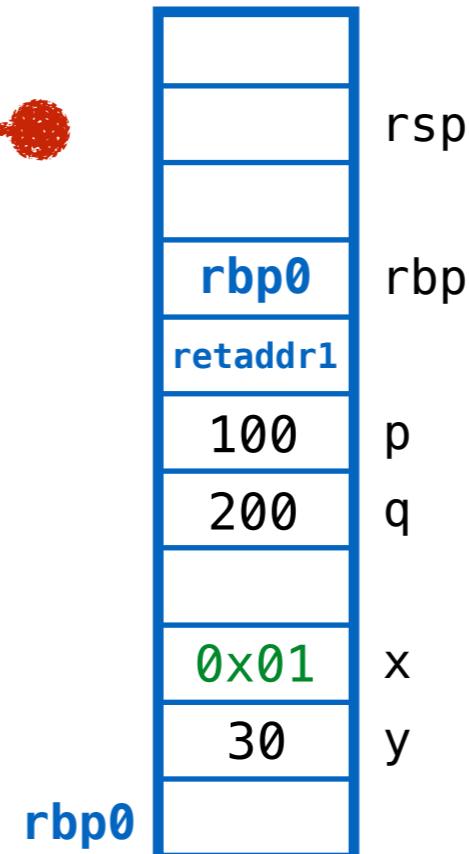


r15

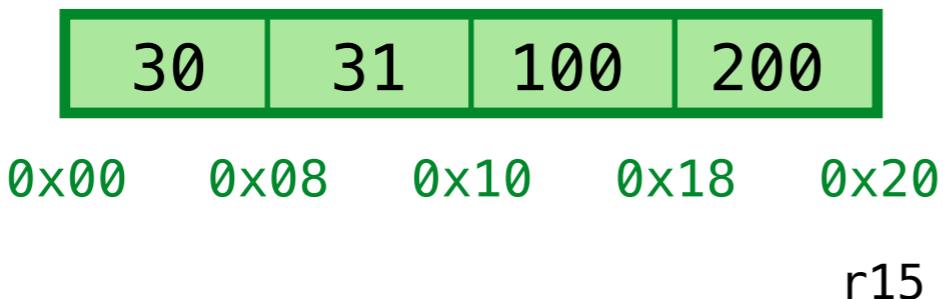
ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q) ←
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```

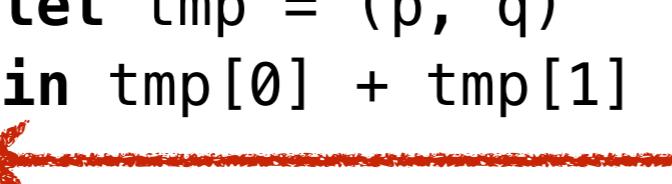


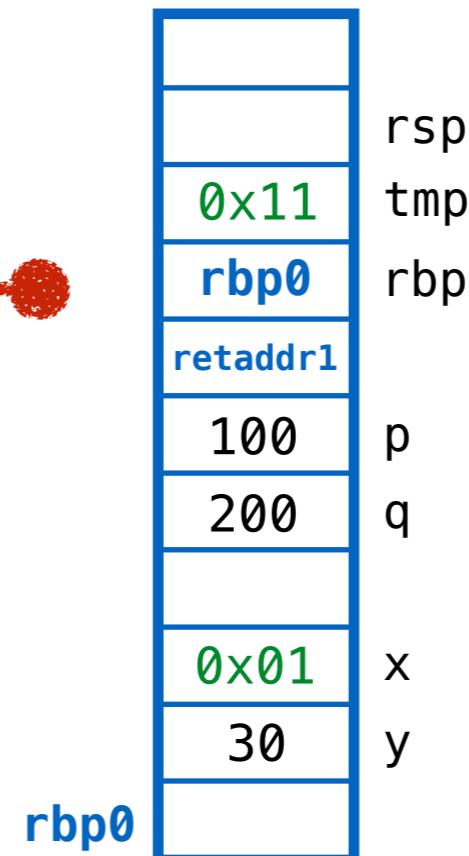
Yay! Have space for (p, q)



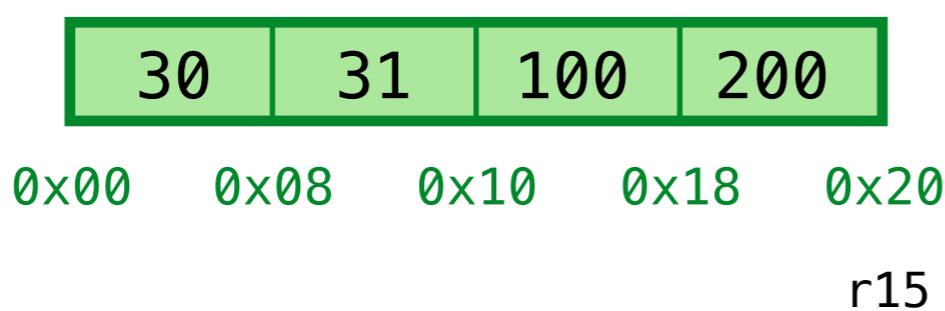
r15

ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
, x = (y, y + 1)  
, z = foo(100, 200)  
in  
x[0] + z
```



Return (rax) = 300

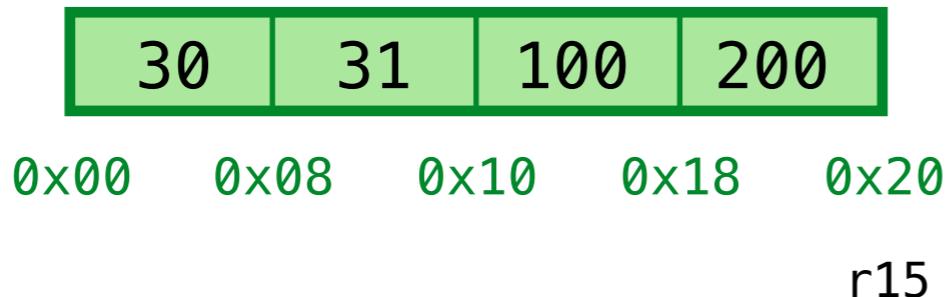
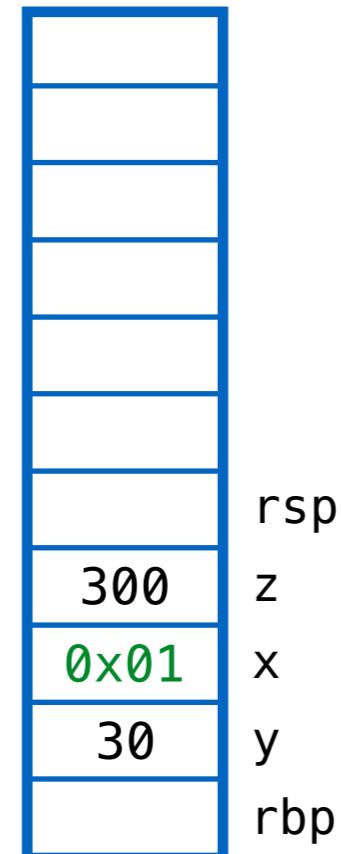


ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
x[0] + z
```

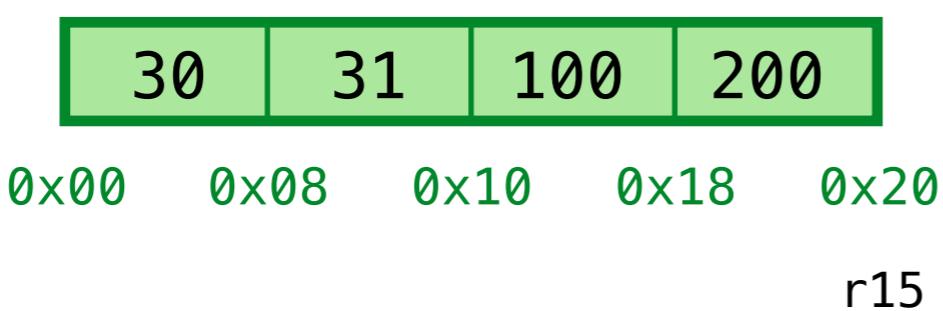
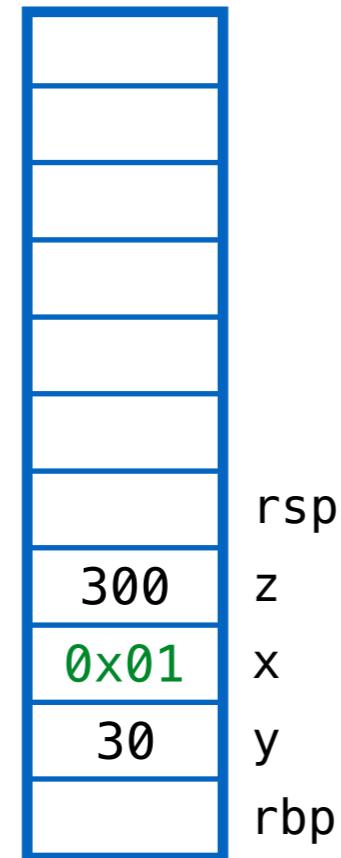
Return (rax) = 300



ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

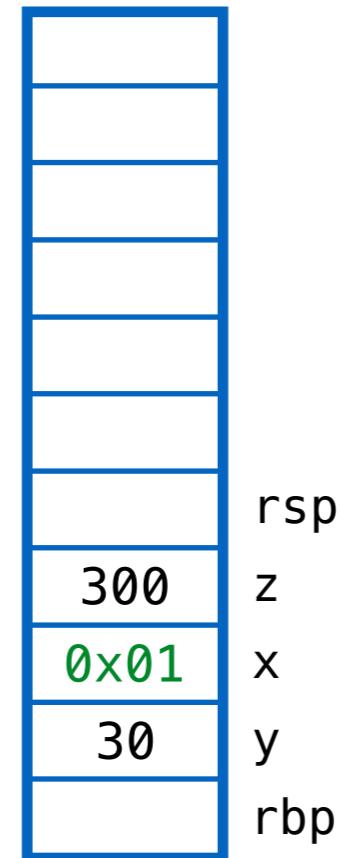
let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```



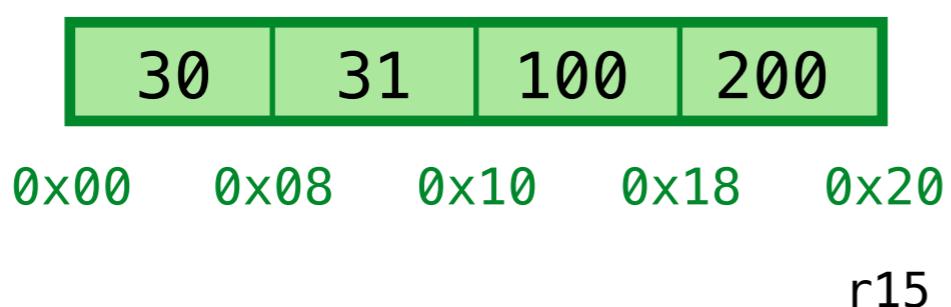
ex3: garbage in the middle (with stack)

```
def foo(p, q):
    let tmp = (p, q)
    in tmp[0] + tmp[1]

let y  = foo(10, 20)
, x  = (y, y + 1)
, z  = foo(100, 200)
in
    x[0] + z
```



Return (rax) = 30+300 = 330



Garter / GC

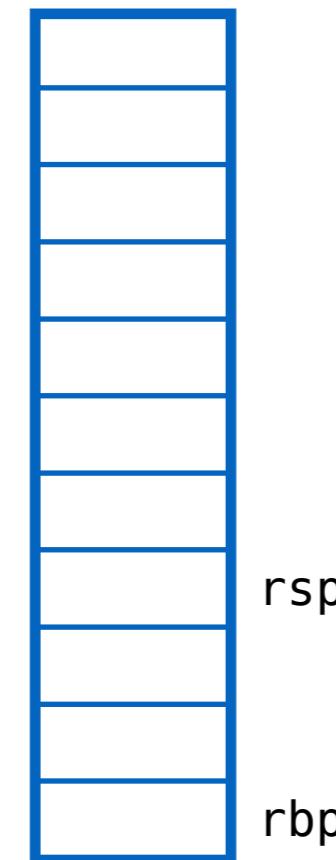
Example 4

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 = ←
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



r15



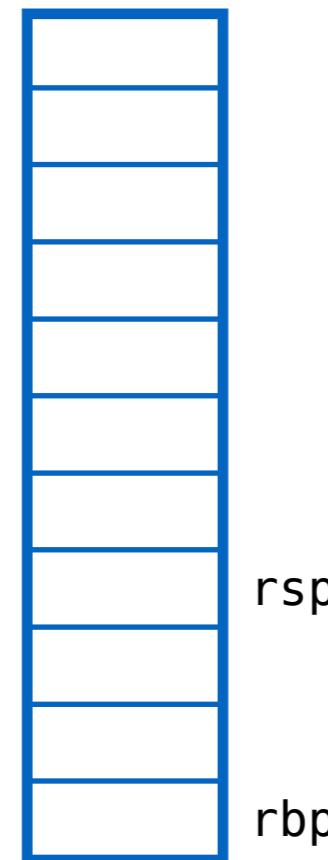
0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3) ←
        in sum(l1)
    , l  = range(t1, t1 + 3)
in
(1000, l)
```



call range(0, 3)

r15



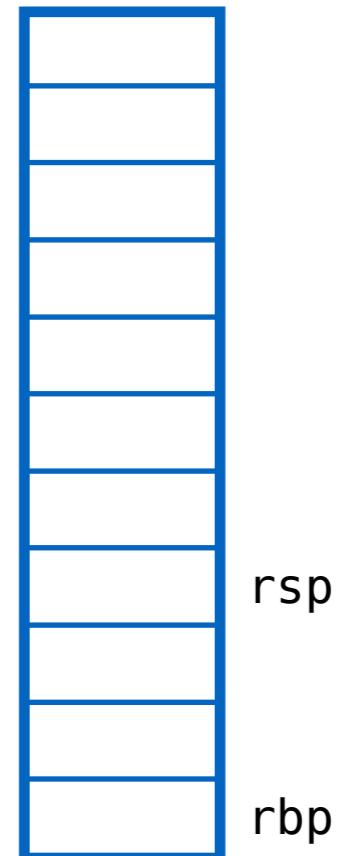
0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3) ←
        in sum(l1)
    , l  = range(t1, t1 + 3)
in
(1000, l)
```



QUIZ: What is heap when range(0,3) returns?

r15

(A)

0	0x11	1	0x21	2	false							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

r15

(B)

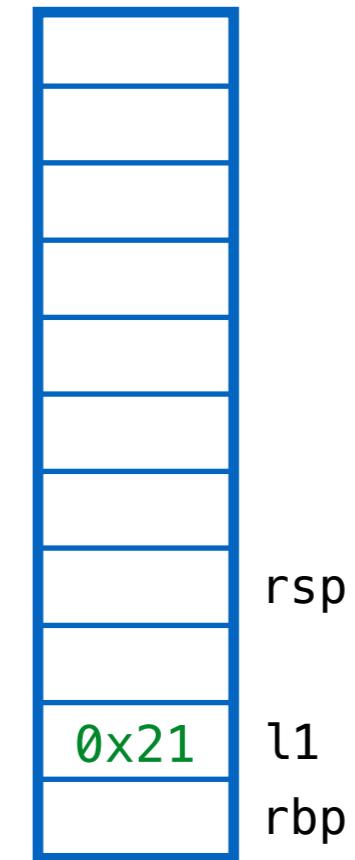
2	false	1	0x01	0	0x11							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



let l1 =
 let l1 = range(0, 3)
 in sum(l1)

, l = range(t1, t1 + 3)

in

(1000, l)

r15

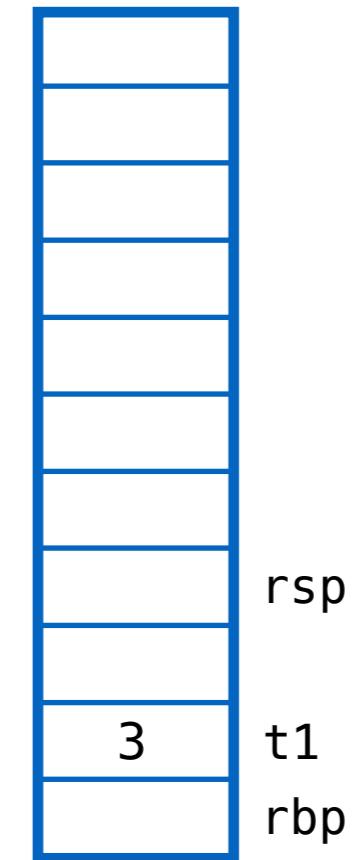
2	false	1	0x01	0	0x11							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1) ←
, l  = range(t1, t1 + 3)
in
(1000, l)
```



Result sum(0x11) = 3

r15

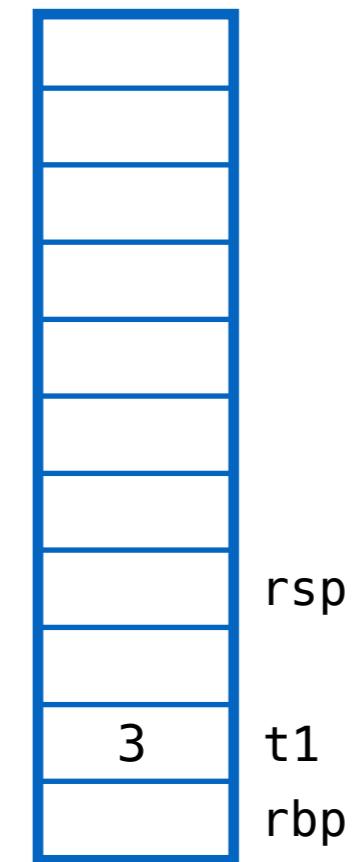
2	false	1	0x01	0	0x11							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, t = range(t1, t1 + 3)
in
(1000, l)
```



r15

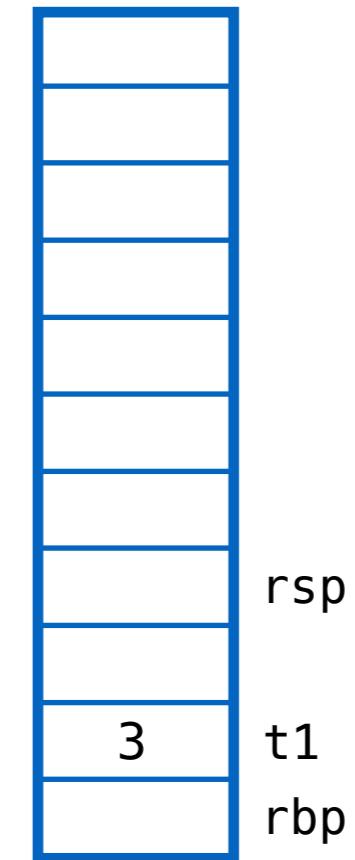
2	false	1	0x01	0	0x11							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3) ←
in
(1000, l)
```



call range(3,6)

r15

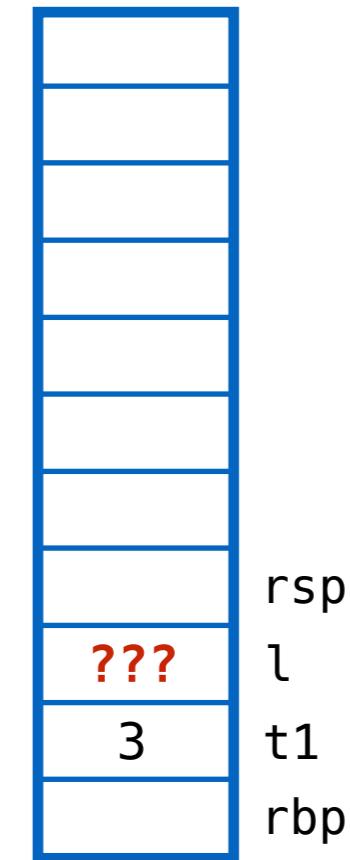
2	false	1	0x01	0	0x11							
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58	0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



call range(3,6)

r15

2	false	1	0x01	0	0x11	5	false	4	0x31	3	0x41
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58

QUIZ: What is the value of l?

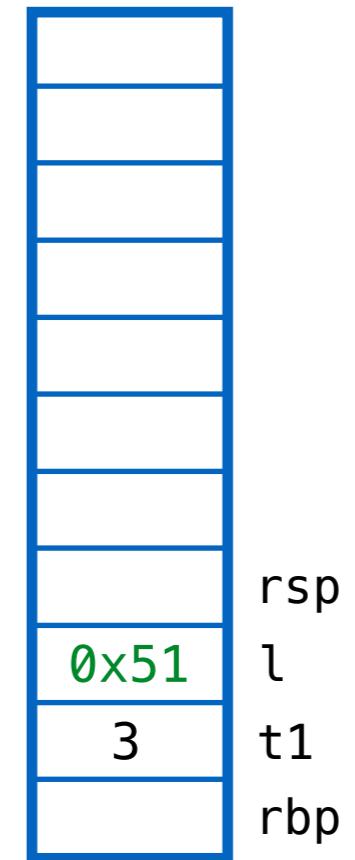
- (A) 0x30 (B) 0x31 (C) 0x50 (D) 0x51 (E) 0x60

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



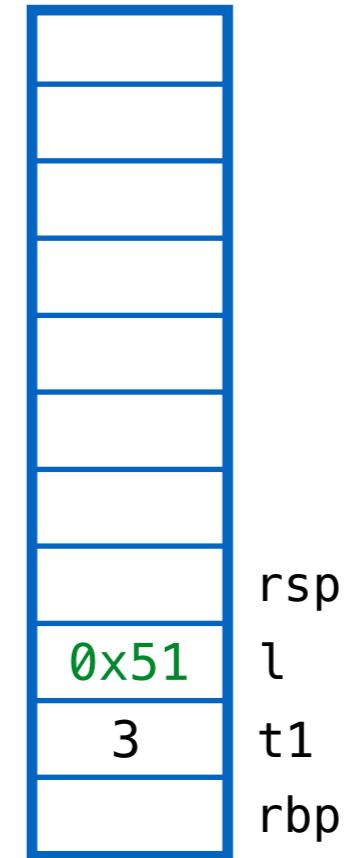
r15

2	false	1	0x01	0	0x11	5	false	4	0x31	3	0x41
0x00	0x08	0x10	0x18	0x20	0x28	0x30	0x38	0x40	0x48	0x50	0x58

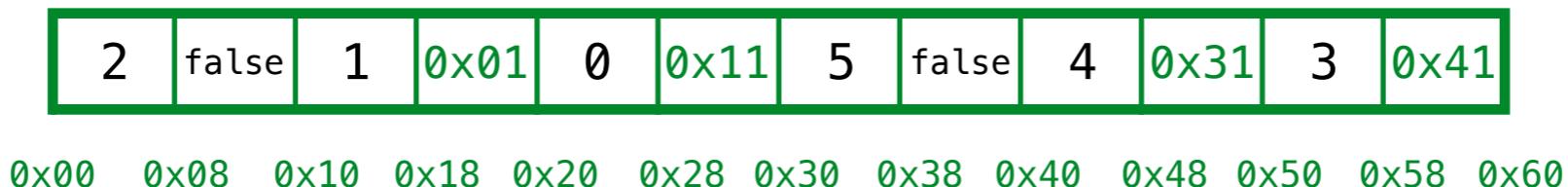
ex4: recursive data

QUIZ: Which cells are “live” on the heap?

- (A) 0x00
- (B) 0x10
- (C) 0x20
- (D) 0x30
- (E) 0x40
- (F) 0x50



r15

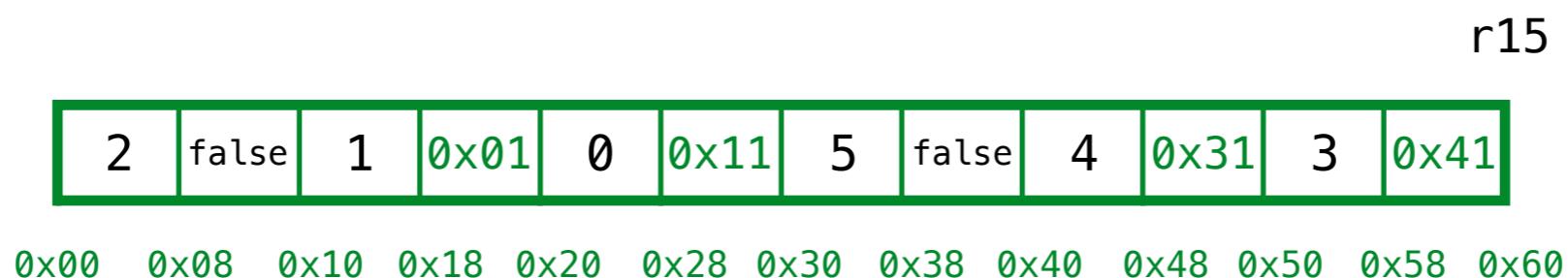
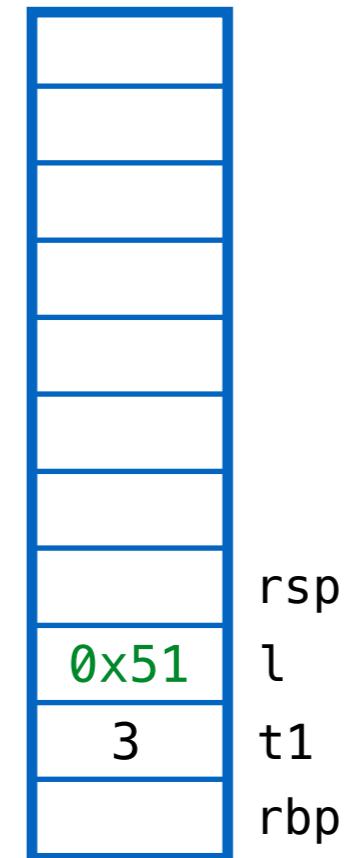


ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



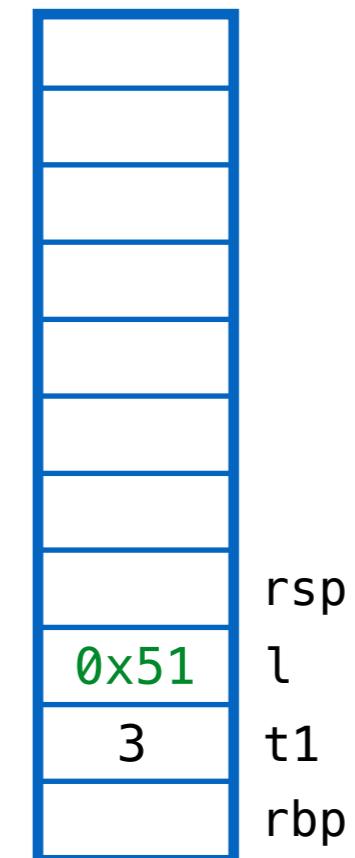
1. **MARK** live addrs
2. Compute **FORWARD** addrs
3. **REDIRECT** addrs on stack
4. **COMPACT** cells on heap

ex4: recursive data

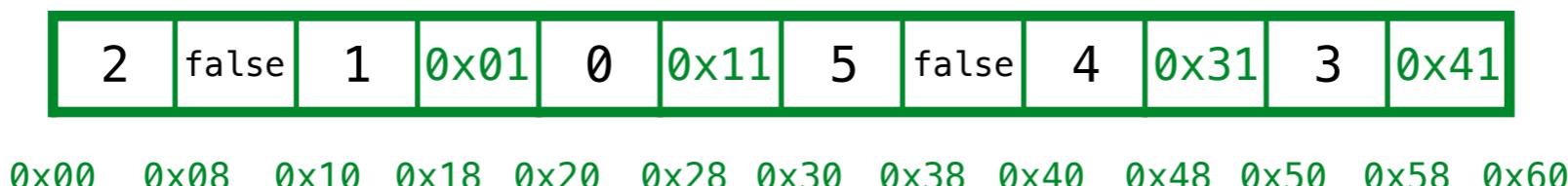
```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



r15



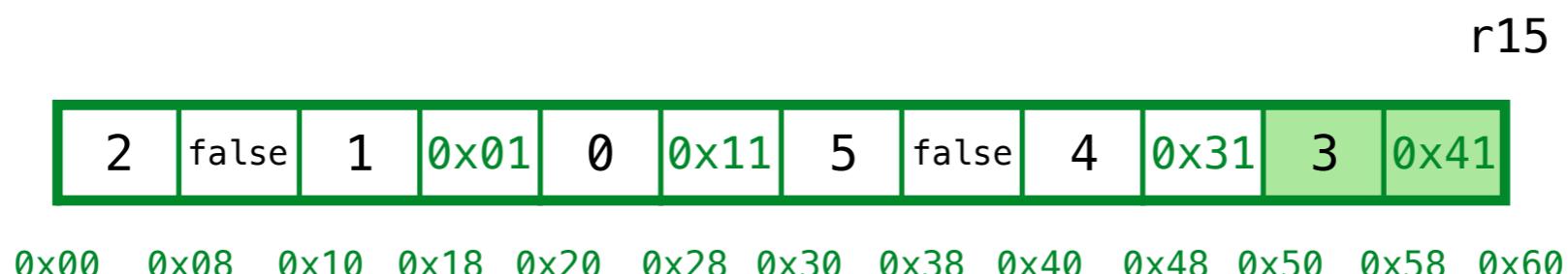
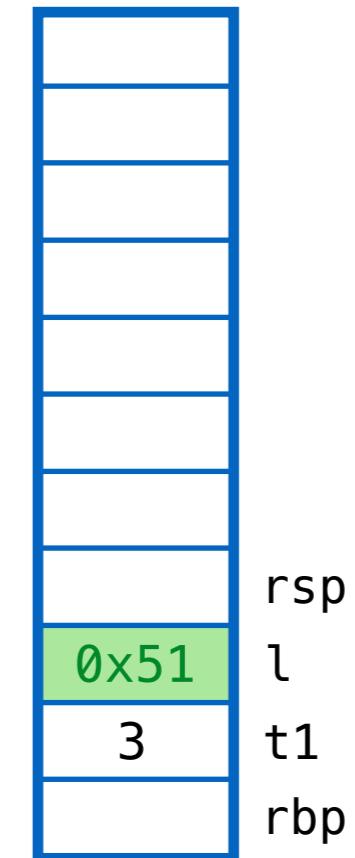
1. **MARK** live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



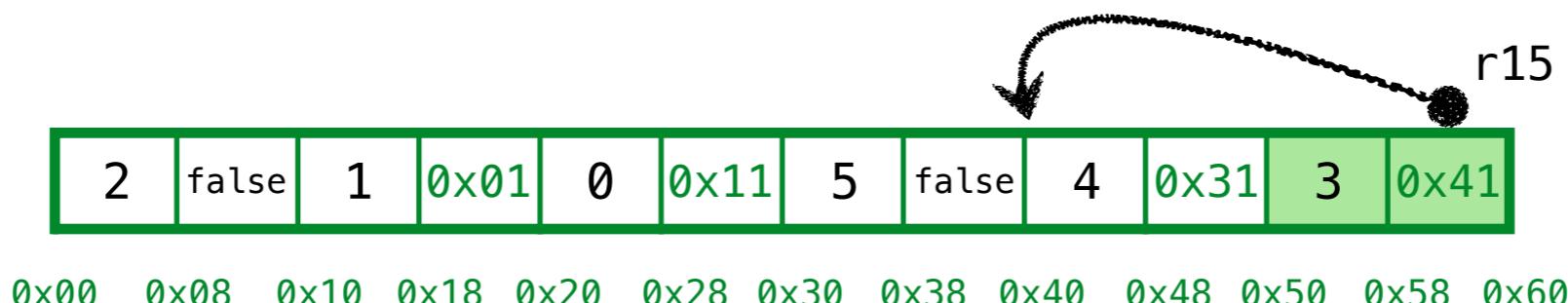
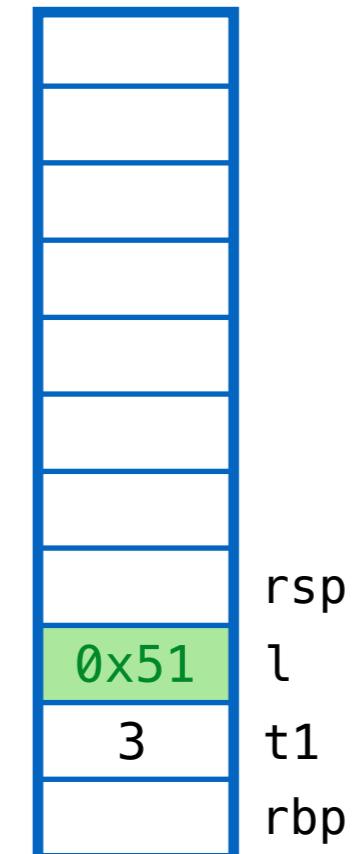
1. **MARK** live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



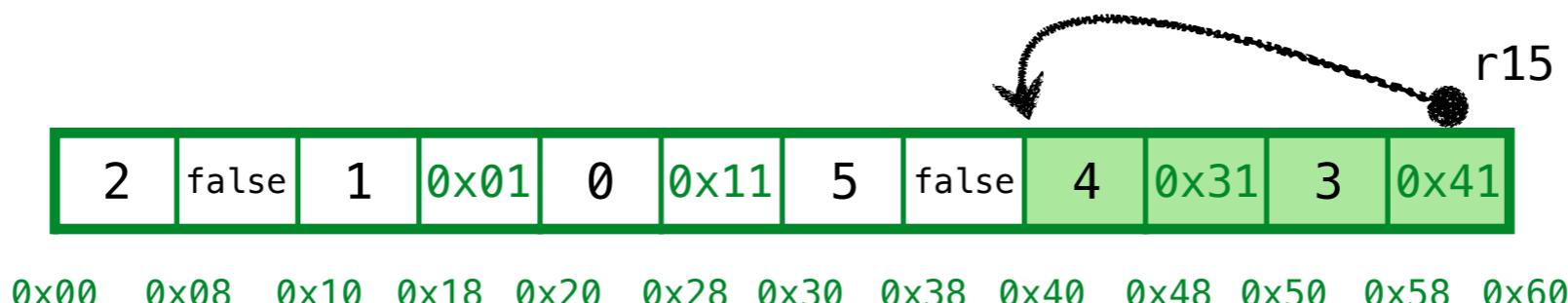
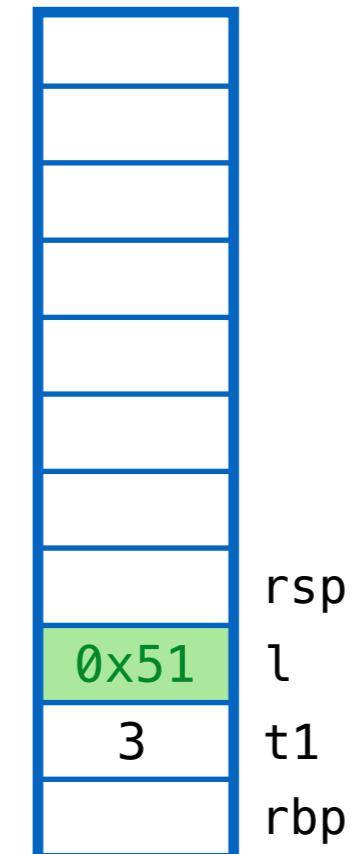
1. MARK live addrs
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ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



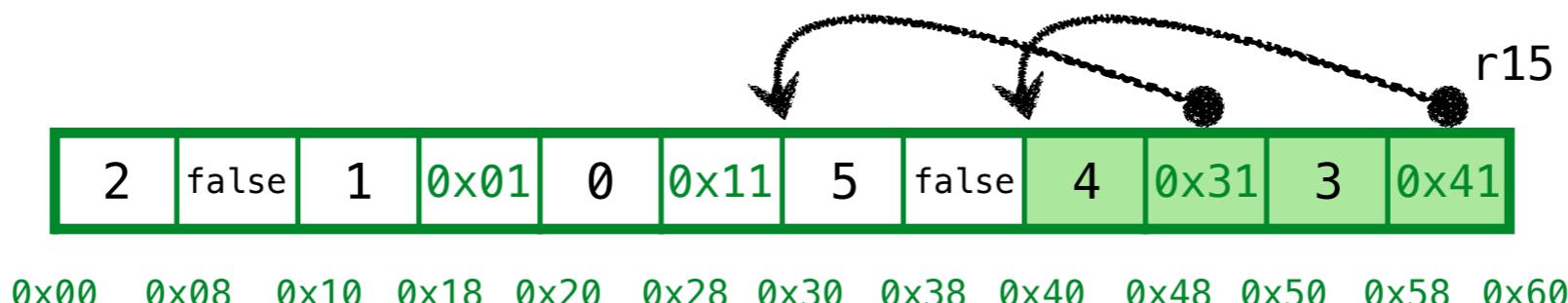
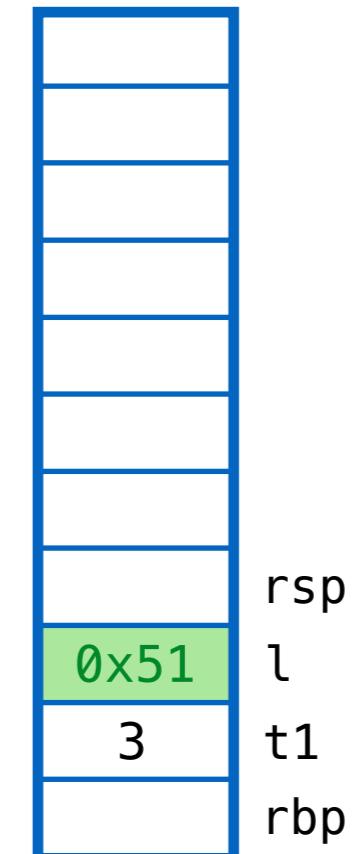
1. **MARK** live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



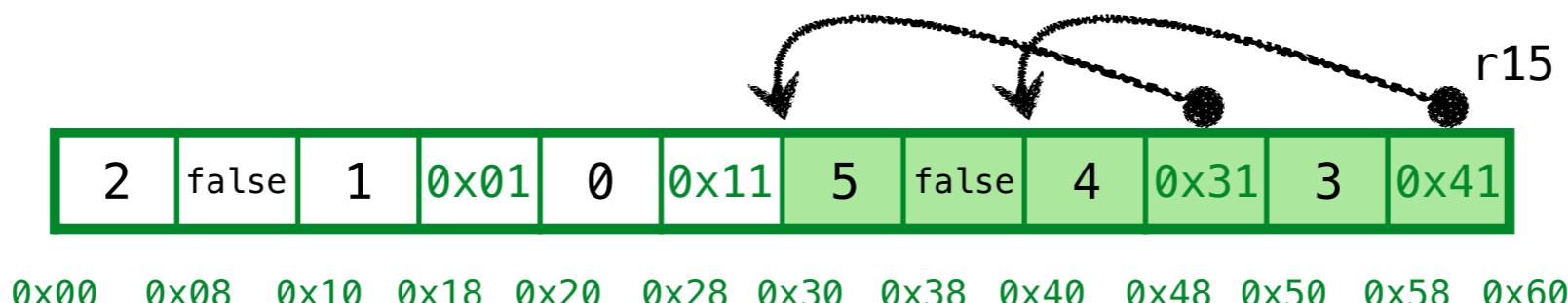
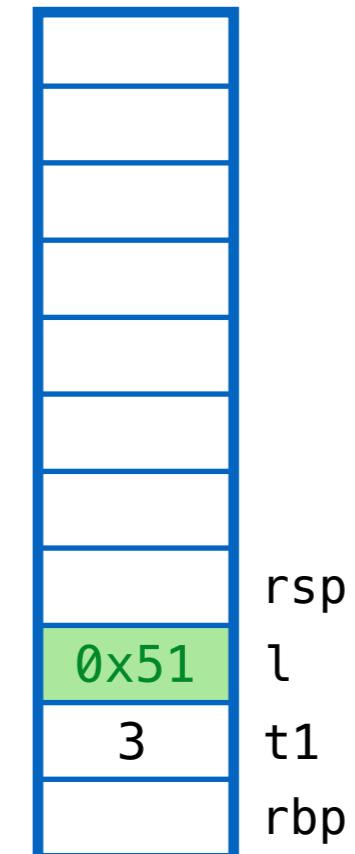
1. MARK live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



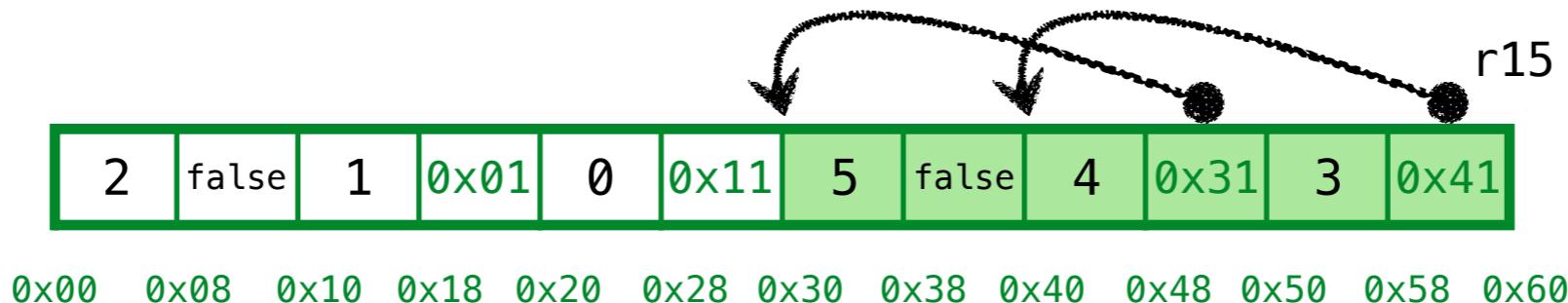
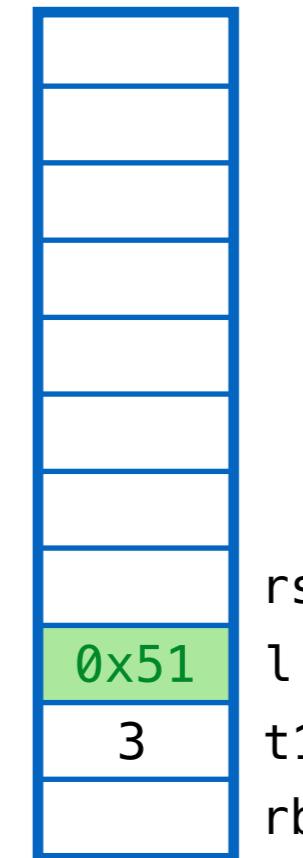
1. MARK live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l  = range(t1, t1 + 3)
in
(1000, l)
```



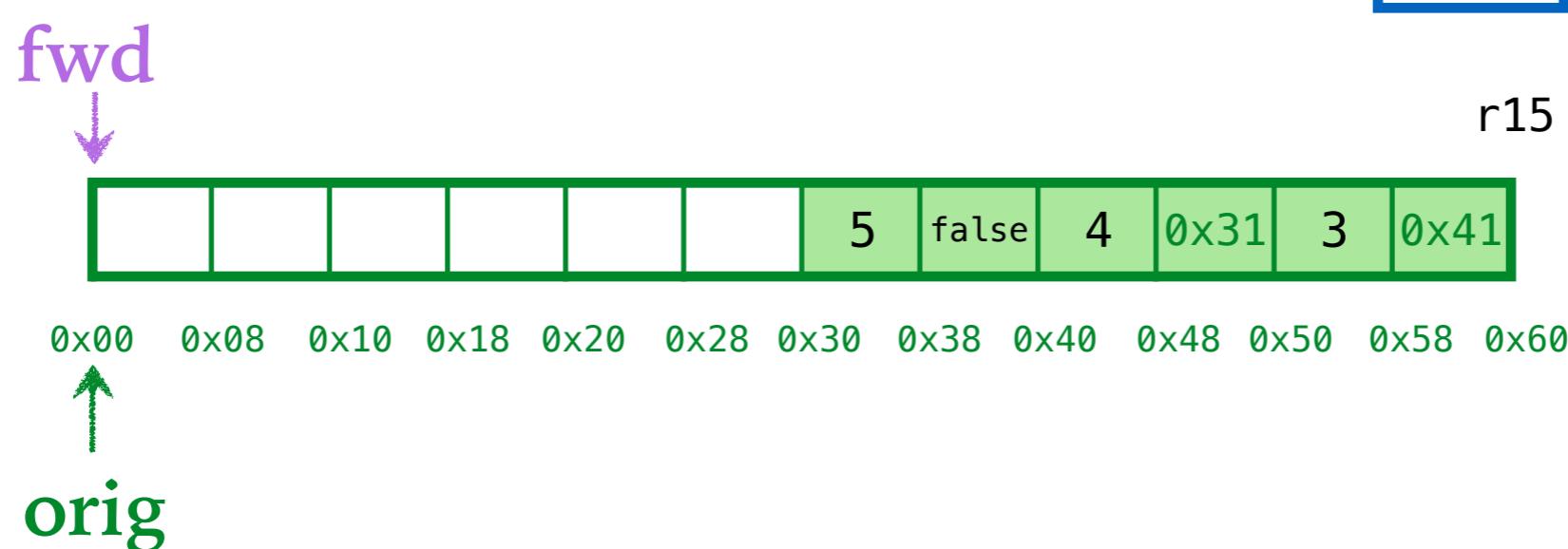
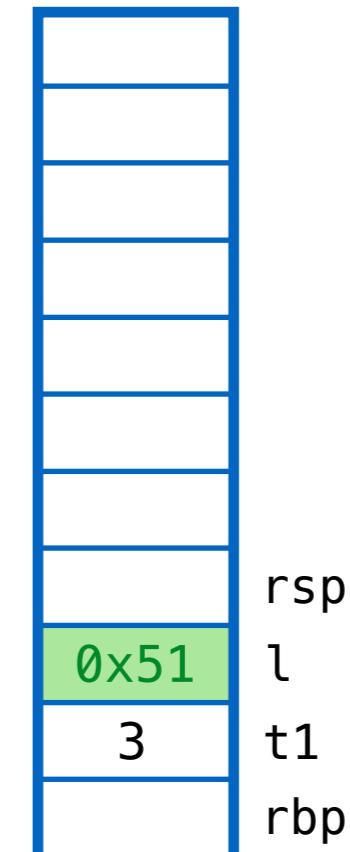
Done!

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



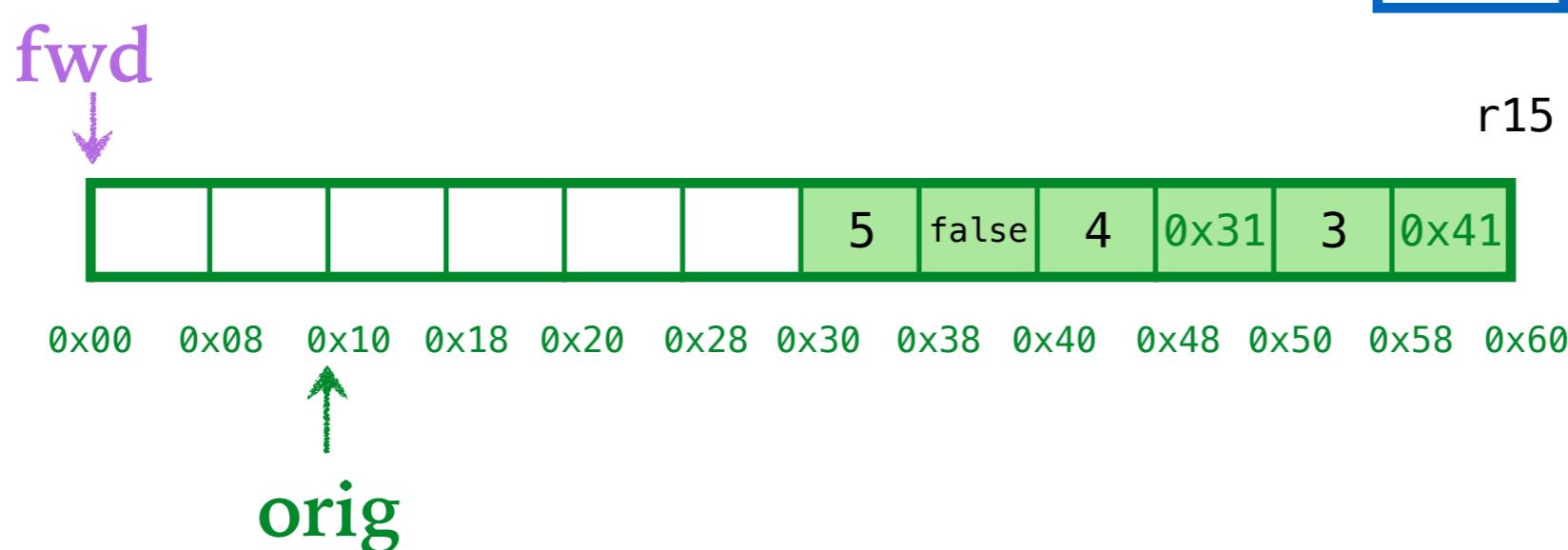
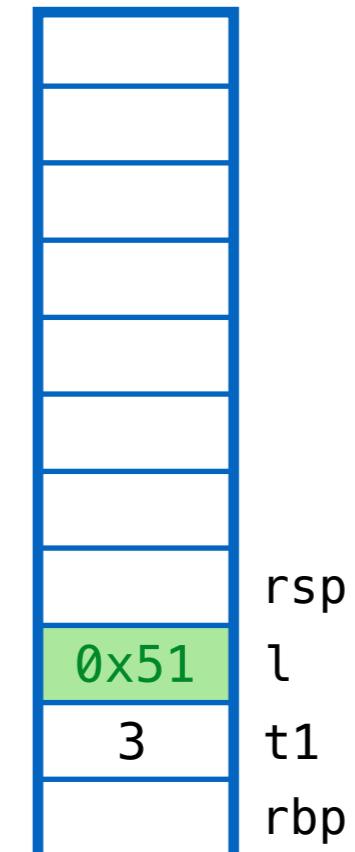
2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



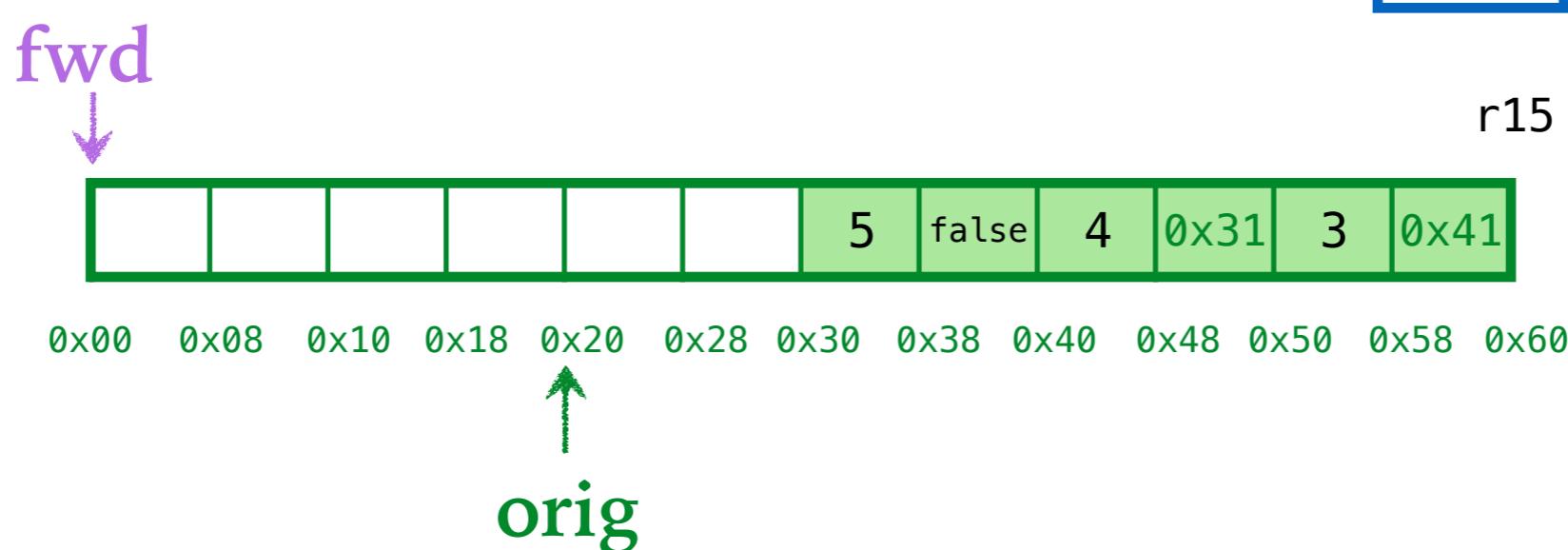
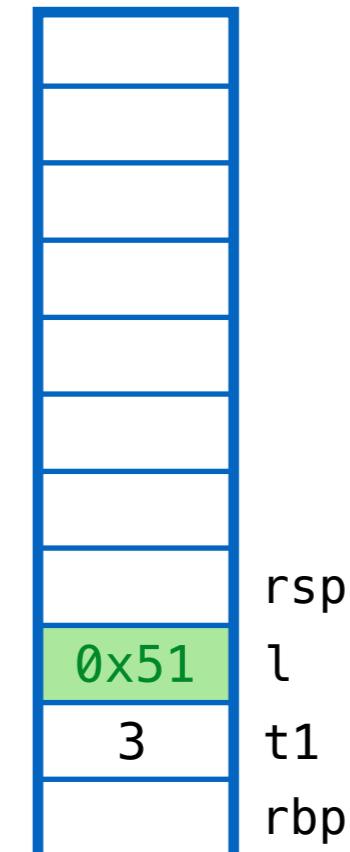
2. Compute FORWARD addrs

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def range(i, j):
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    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



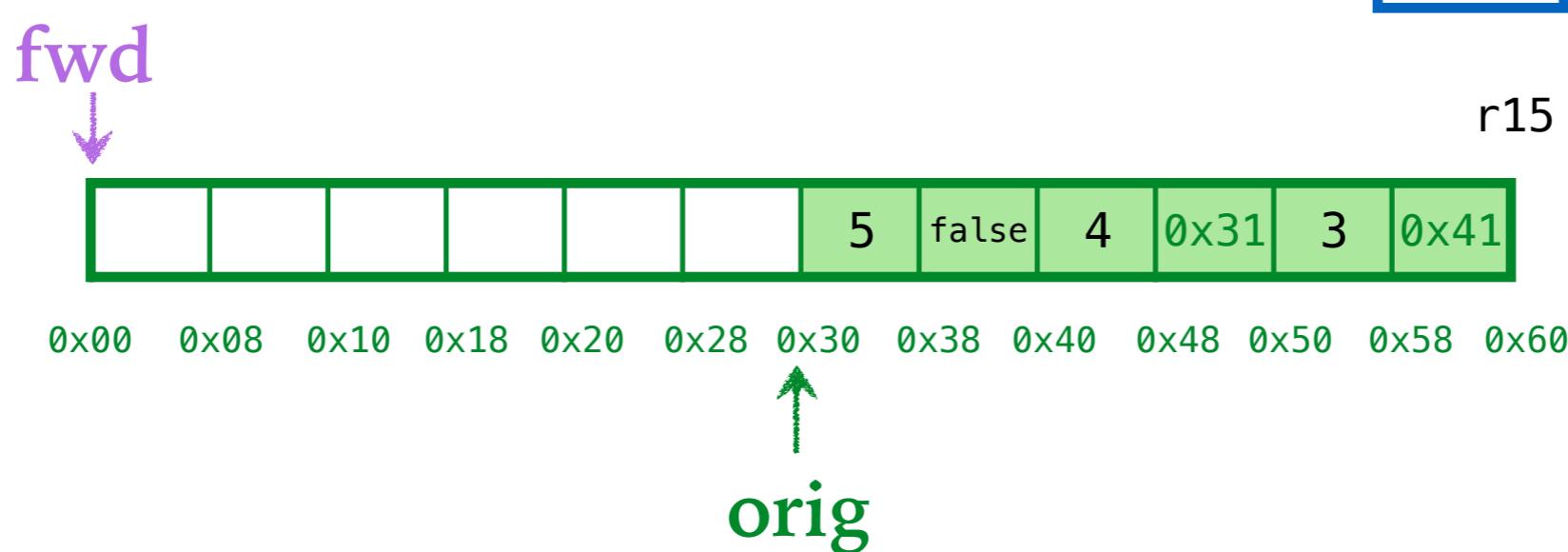
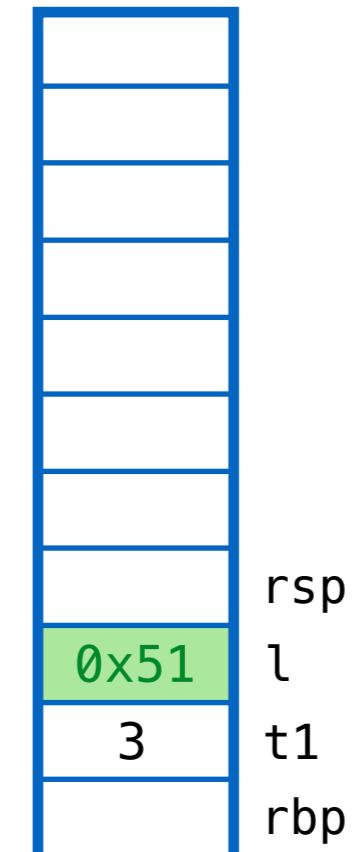
2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

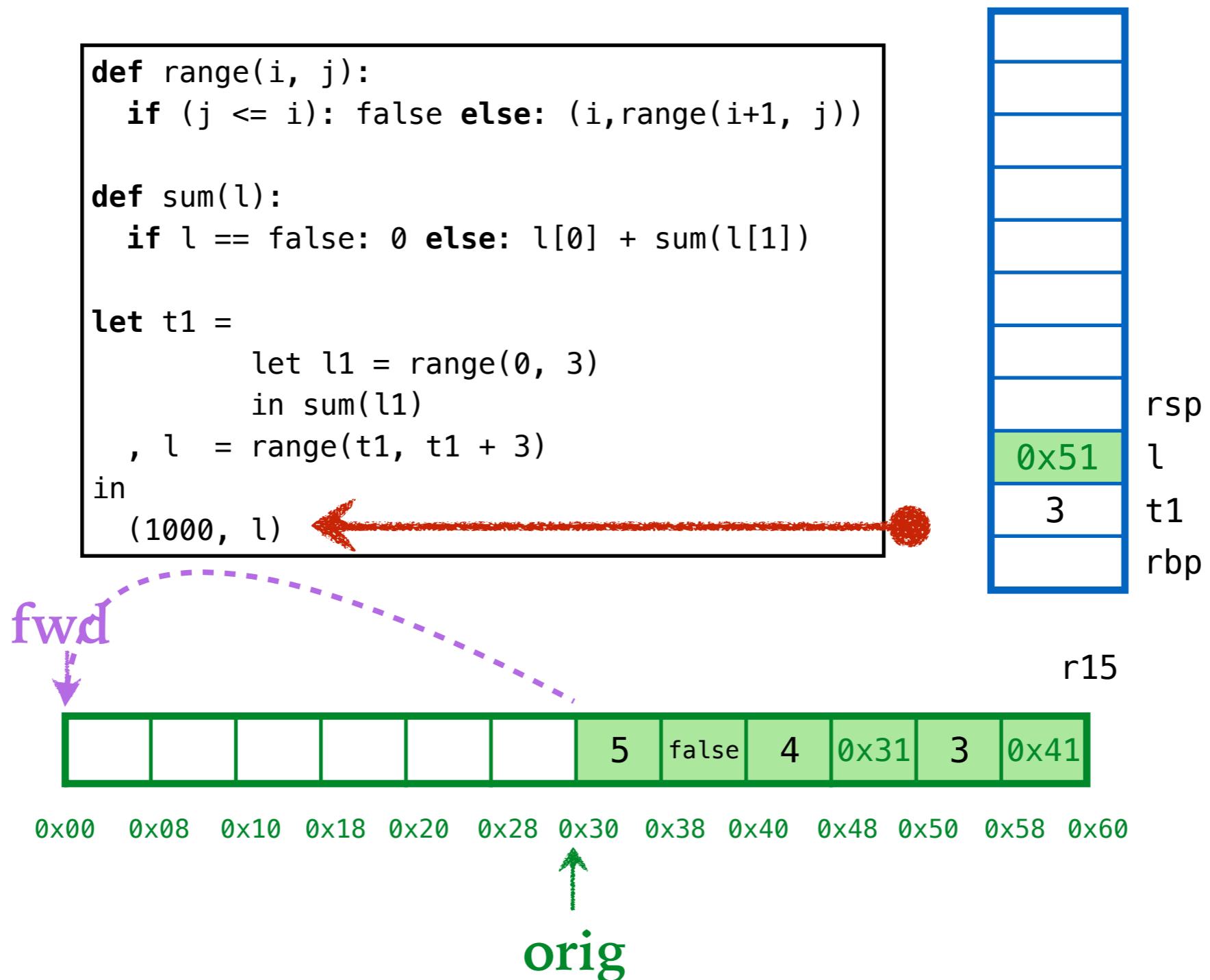
def sum(l):
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let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l) ←
```



2. Compute FORWARD addrs

ex4: recursive data



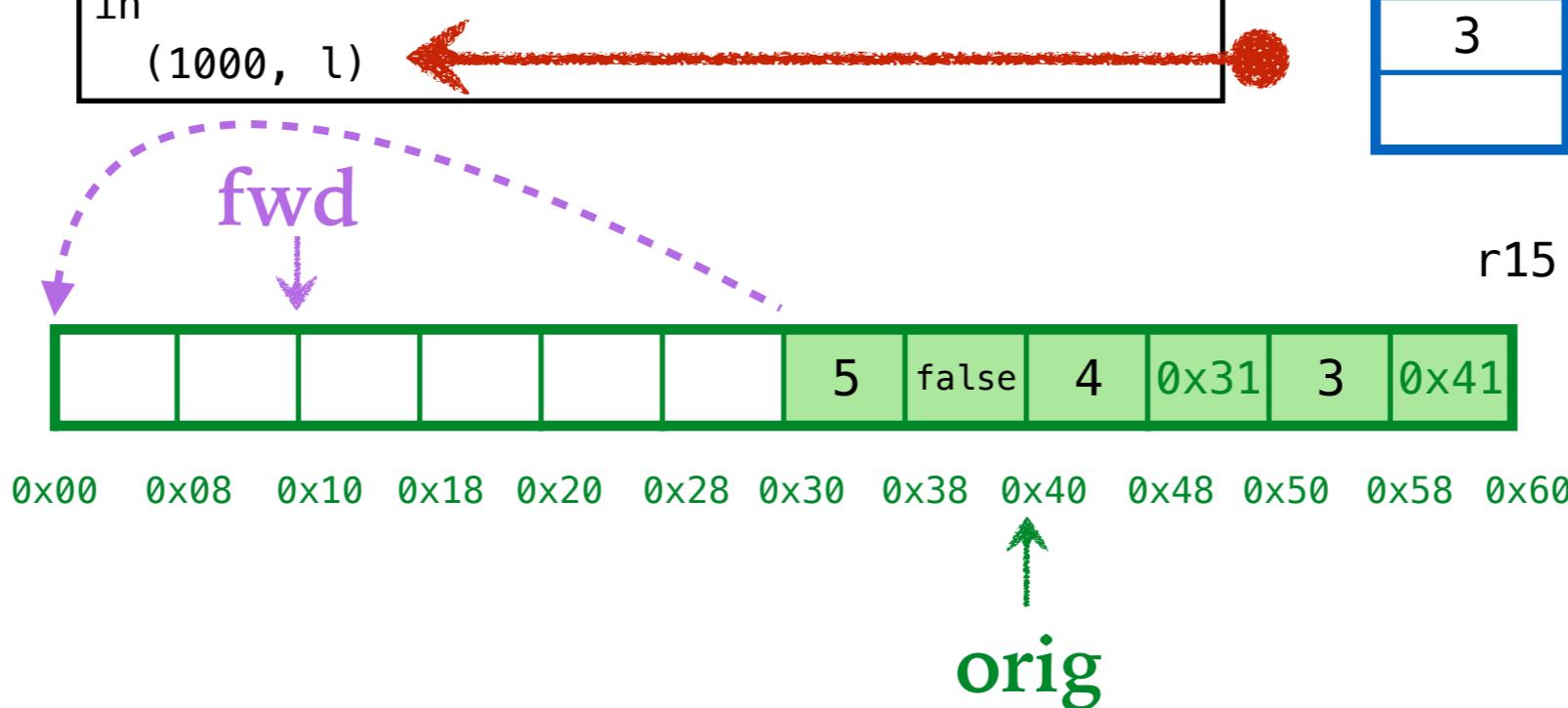
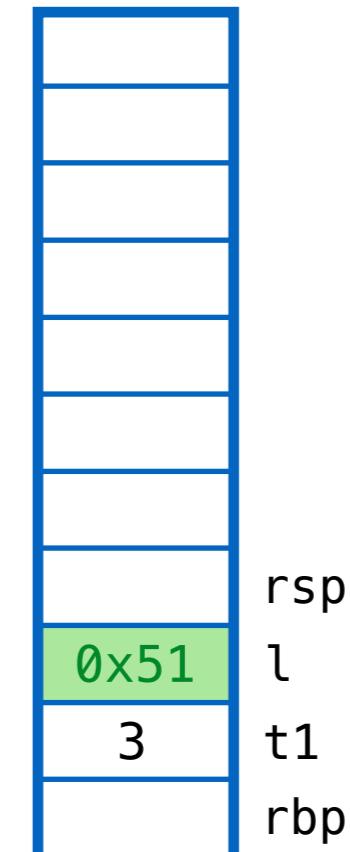
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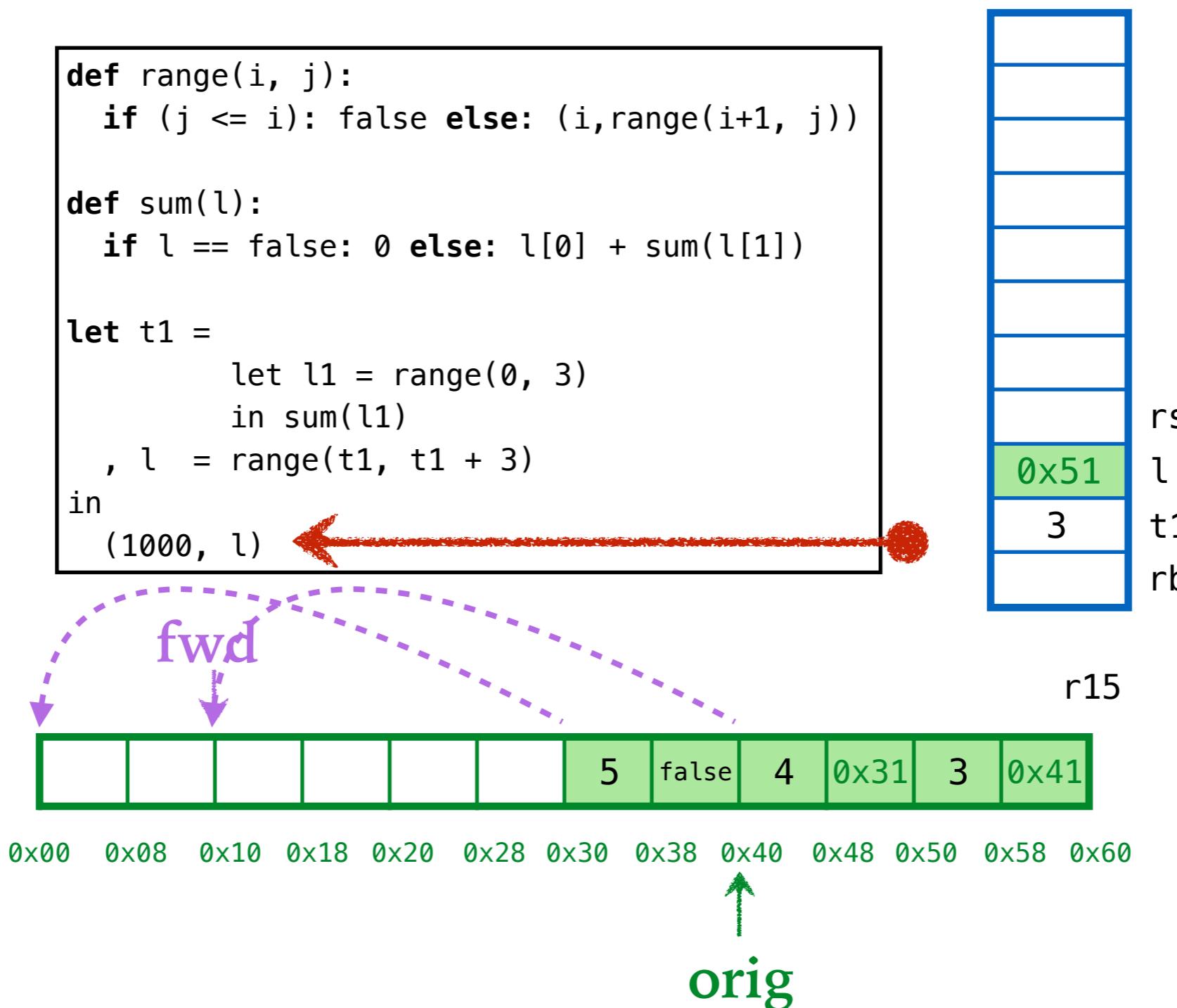
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(1000, l) ←
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2. Compute FORWARD addrs

ex4: recursive data



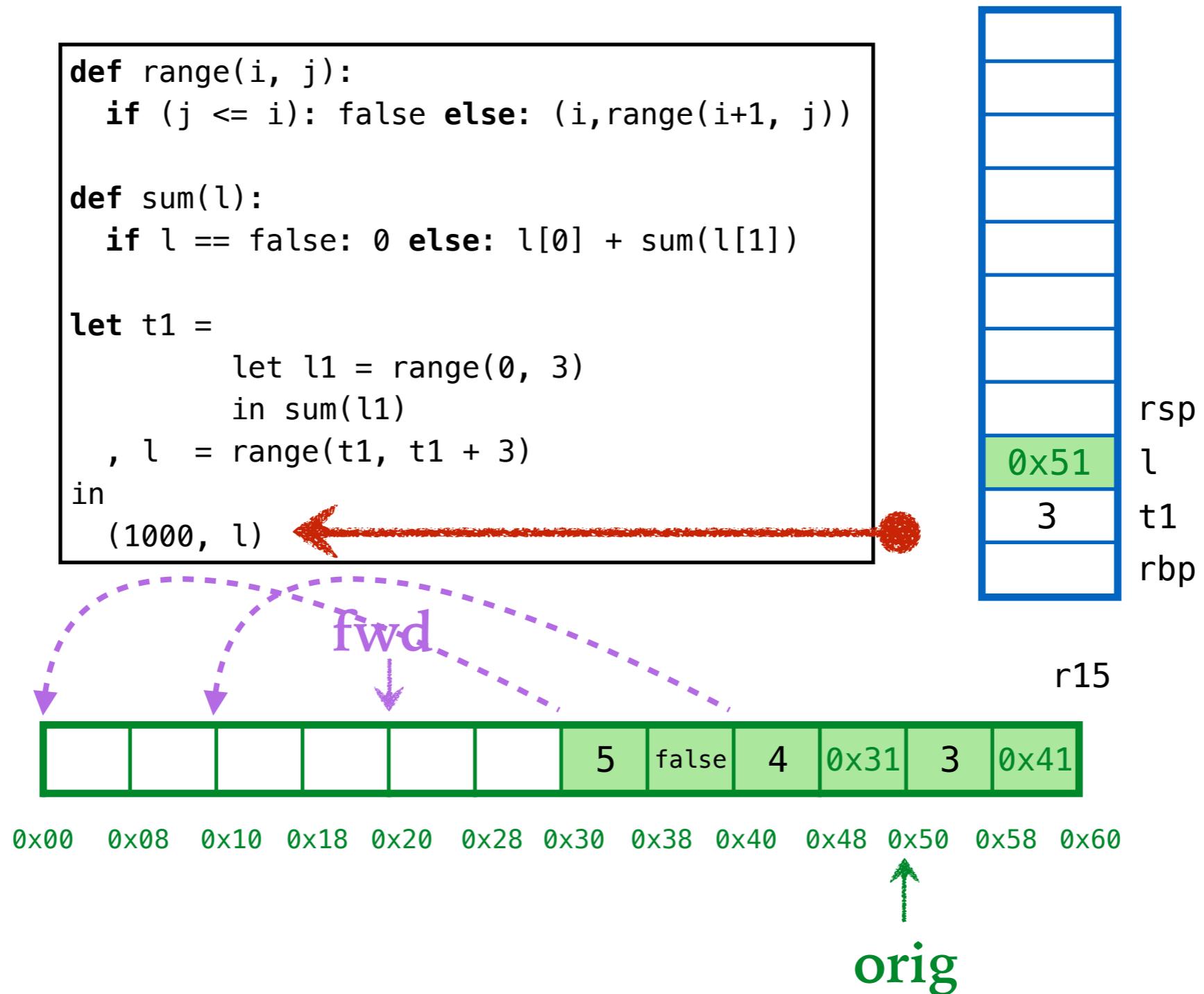
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ex4: recursive data

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def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

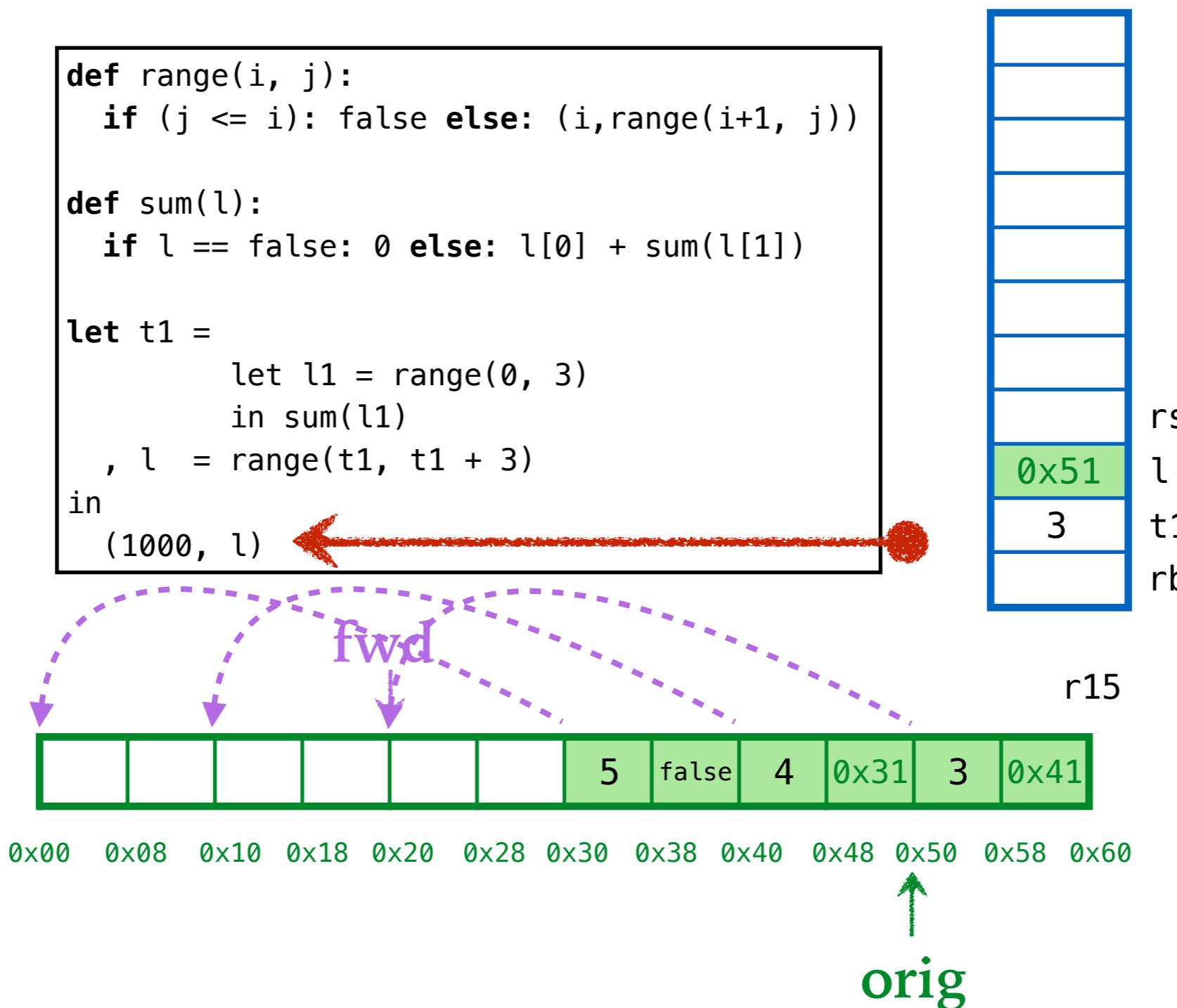
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let t1 =
    let l1 = range(0, 3)
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in
(1000, l) ←
```



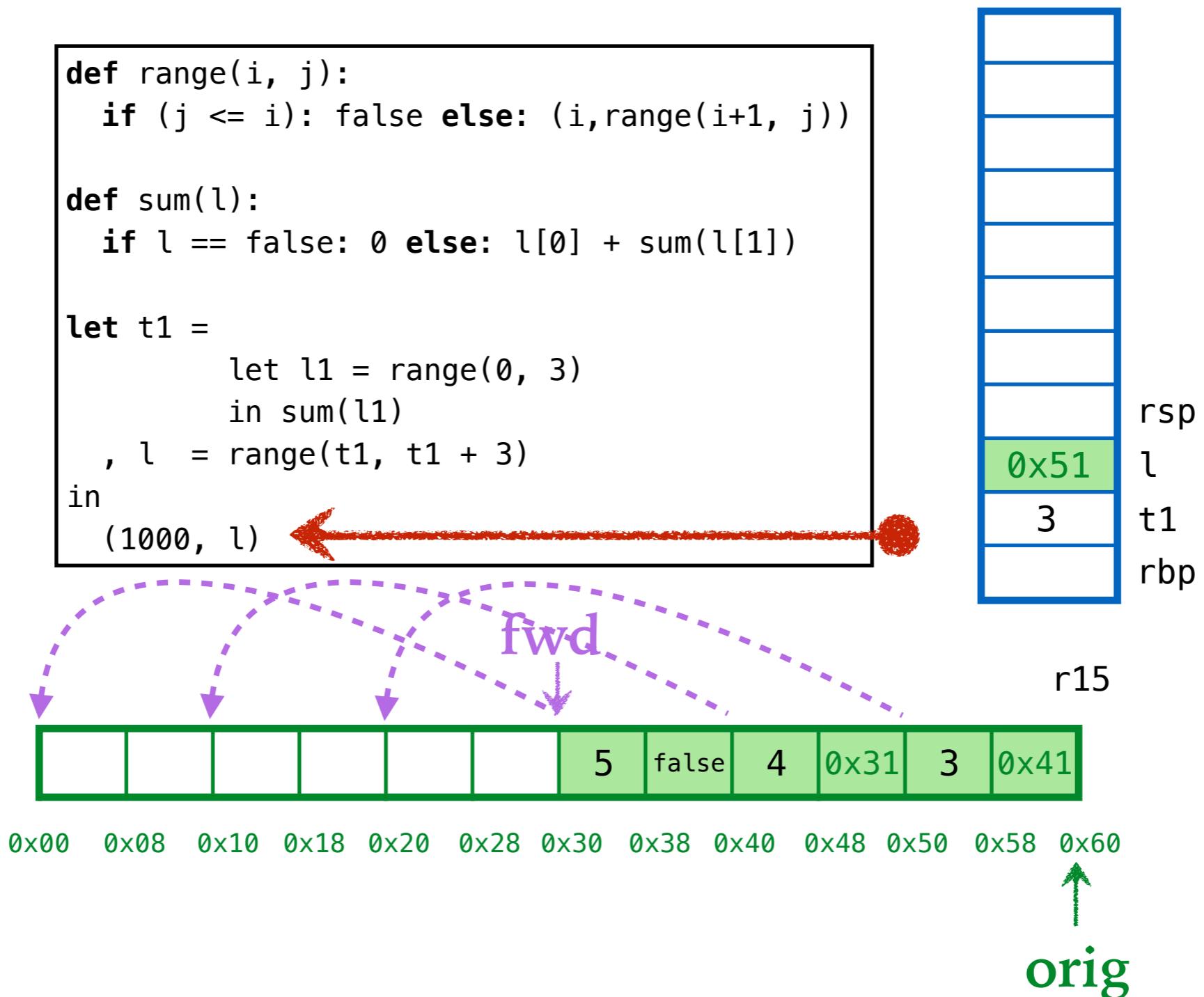
2. Compute FORWARD addrs

ex4: recursive data



2. Compute FORWARD addrs

ex4: recursive data



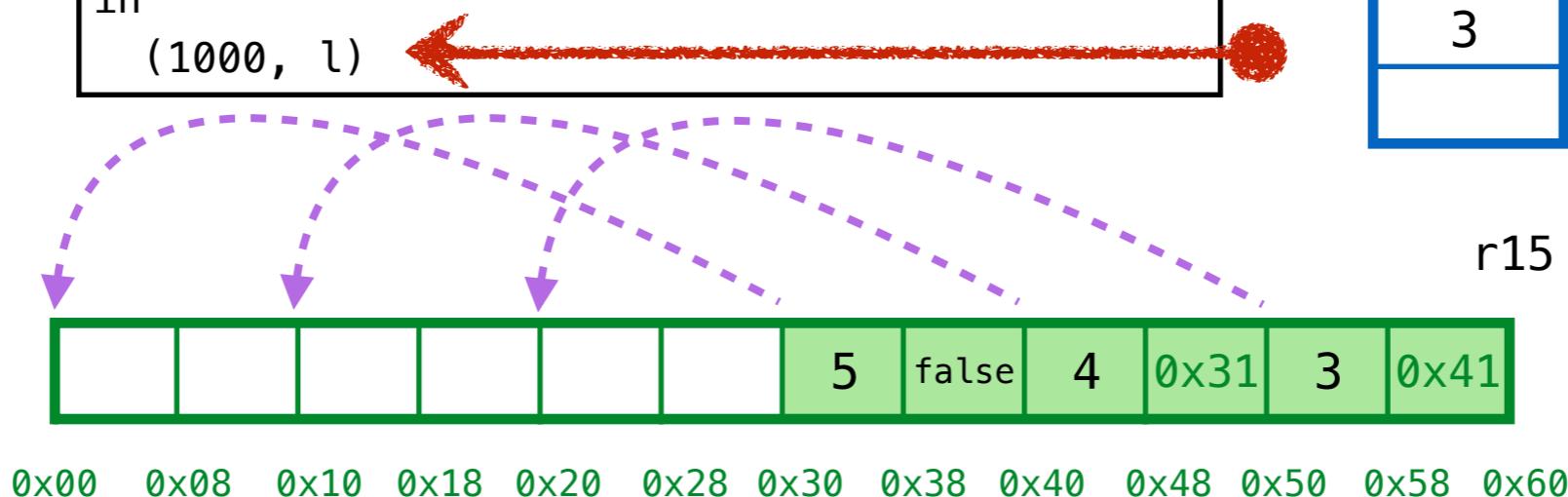
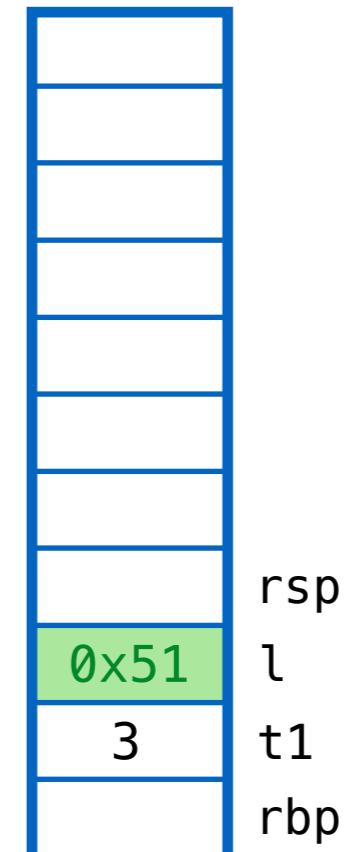
2. Compute FORWARD addrs

ex4: recursive data

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def sum(l):
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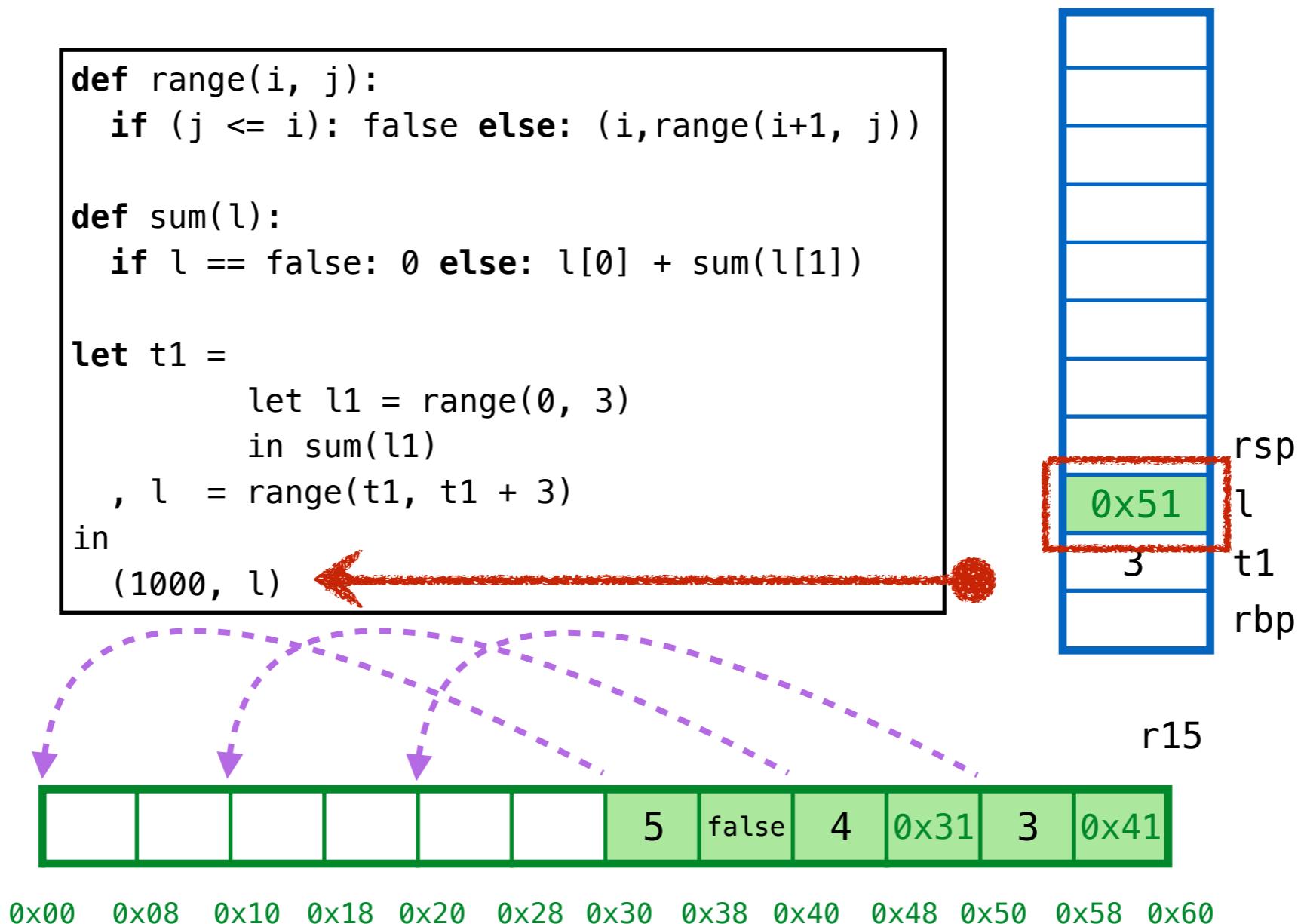
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(1000, l)
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2. Compute FORWARD addrs

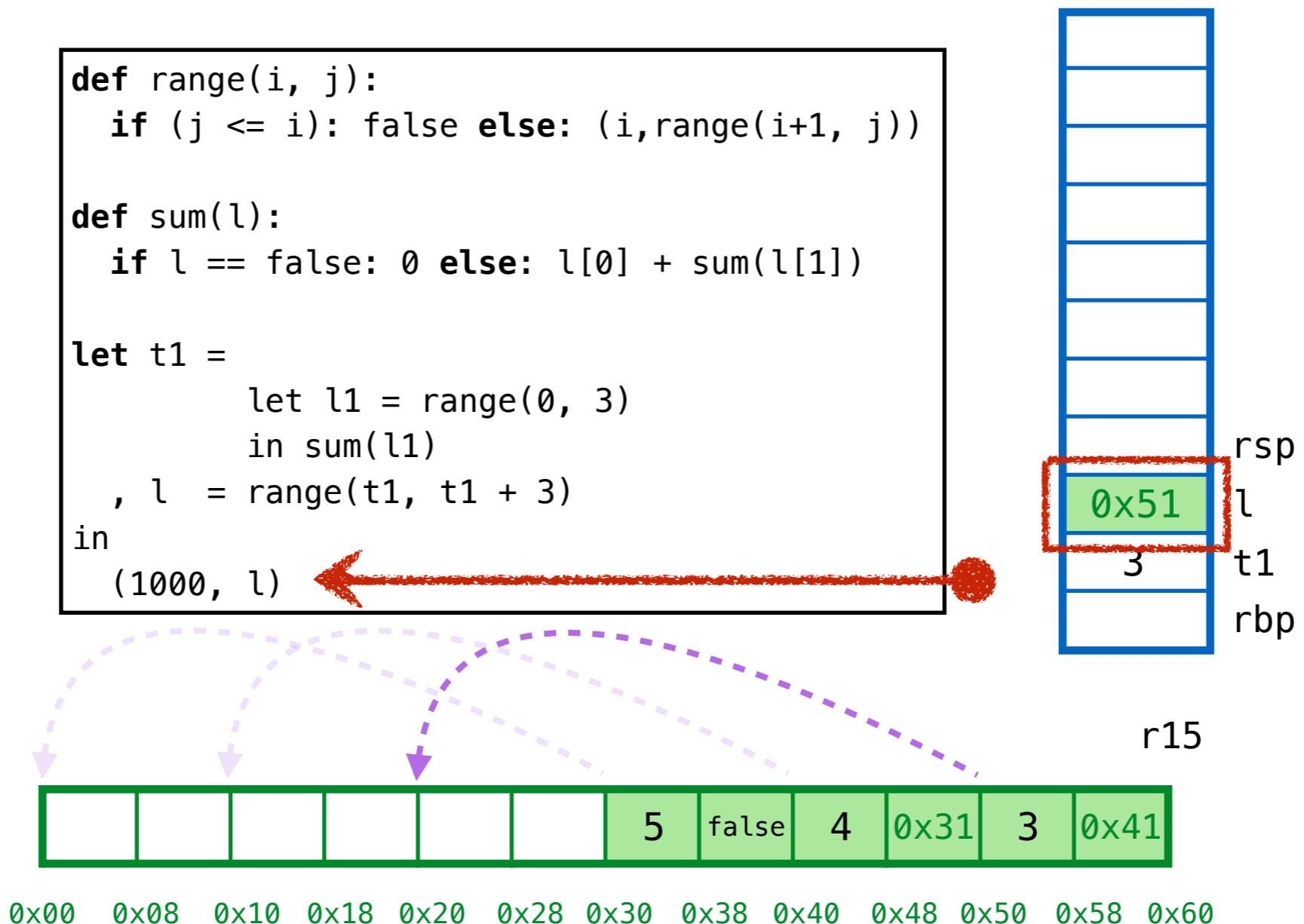
Where should we store the forward addrs?

ex4: recursive data



3. REDIRECT addrs on stack

ex4: recursive data



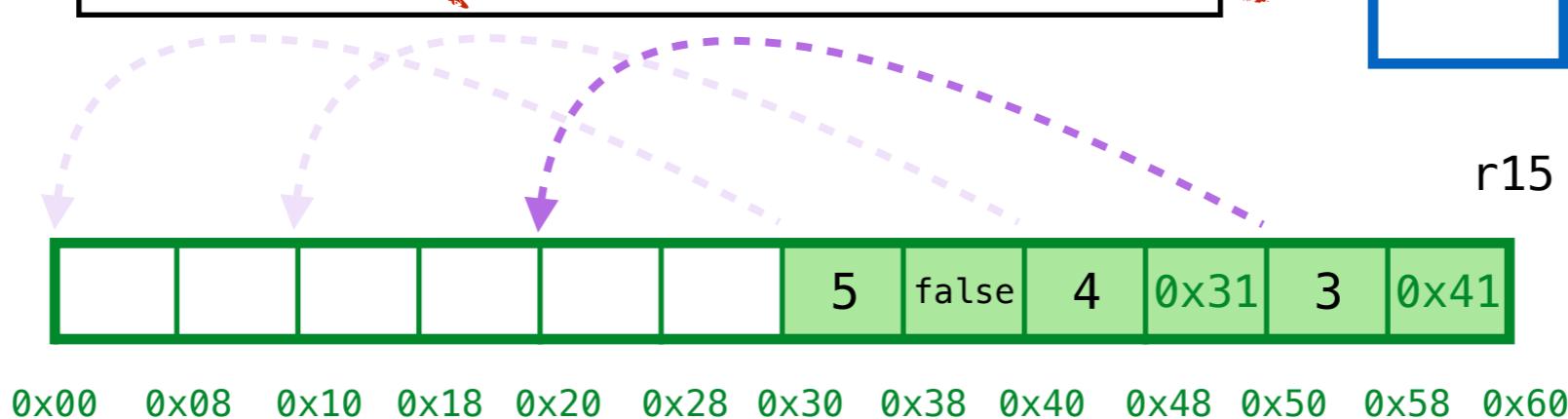
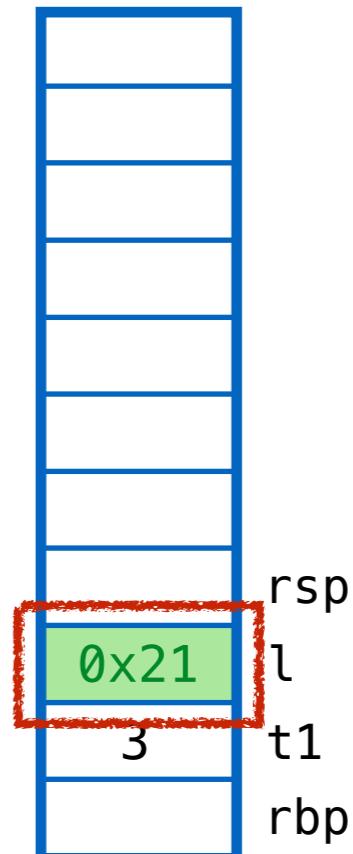
3. REDIRECT addrs on stack

ex4: recursive data

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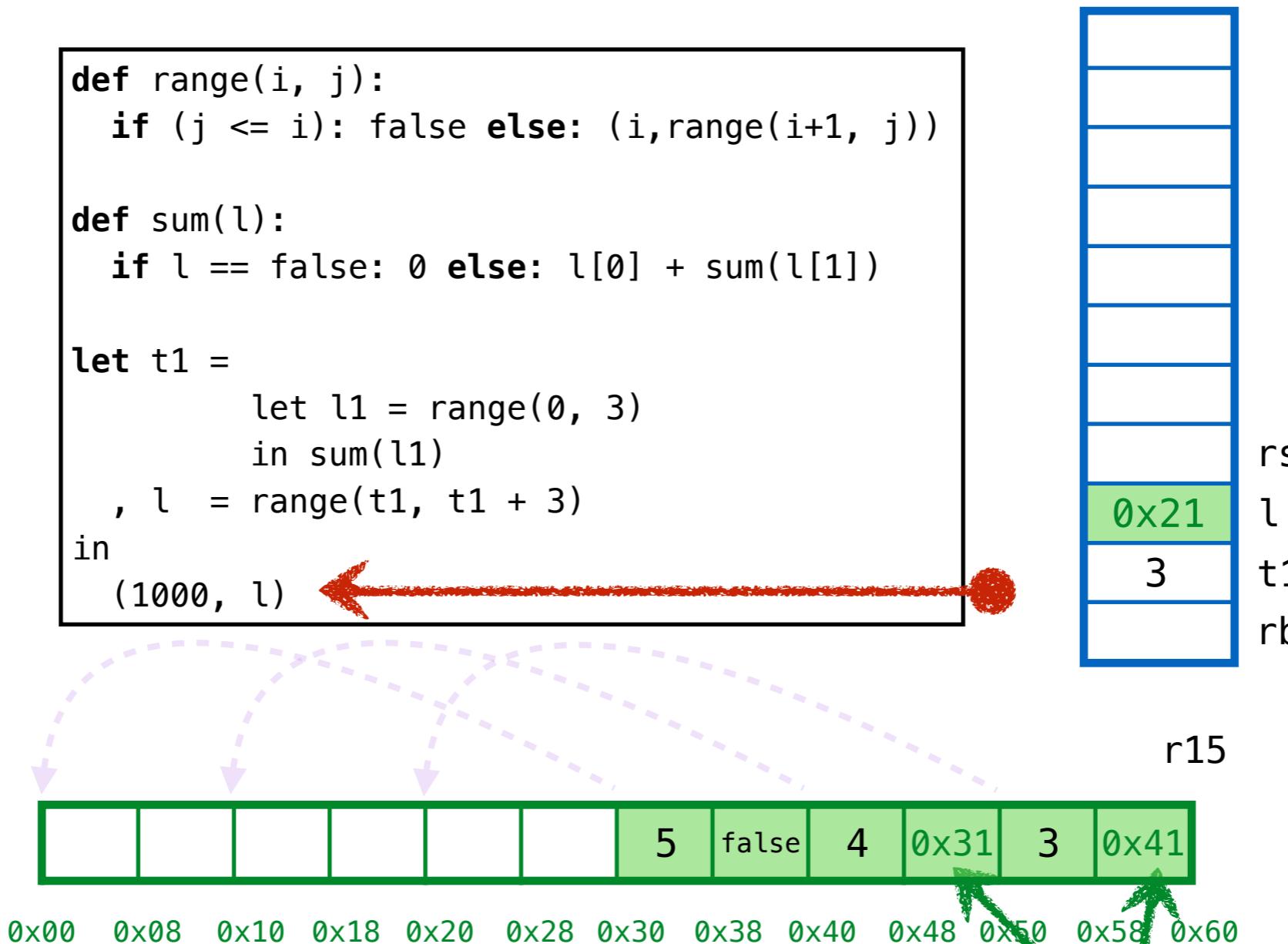
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3. REDIRECT addrs on stack

ex4: recursive data



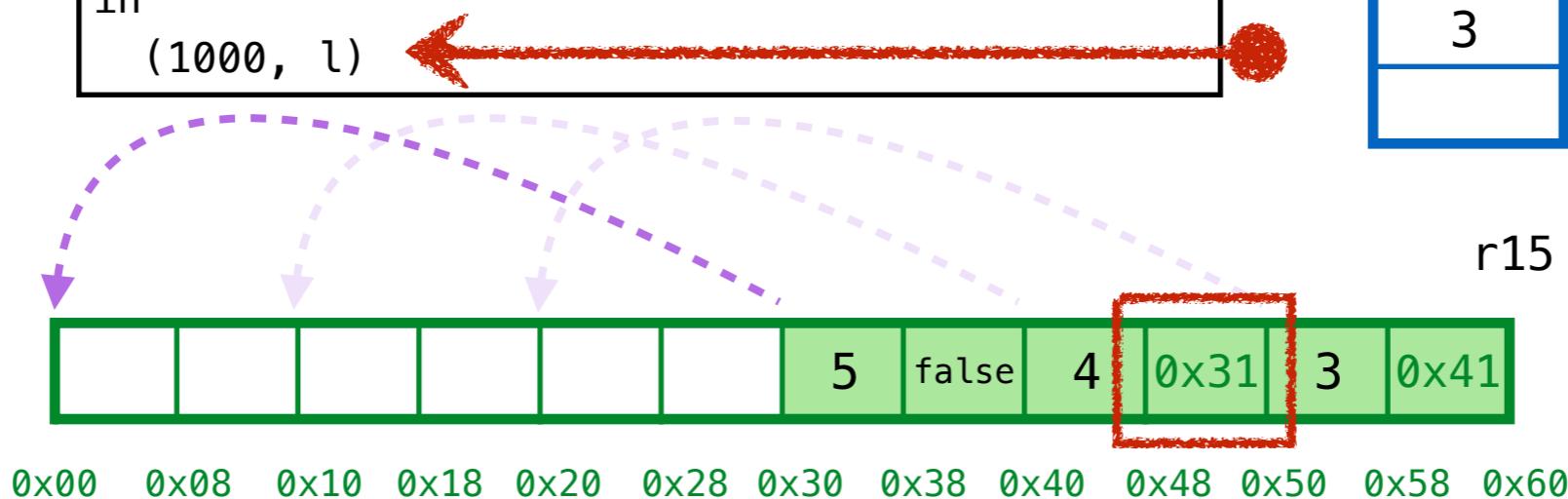
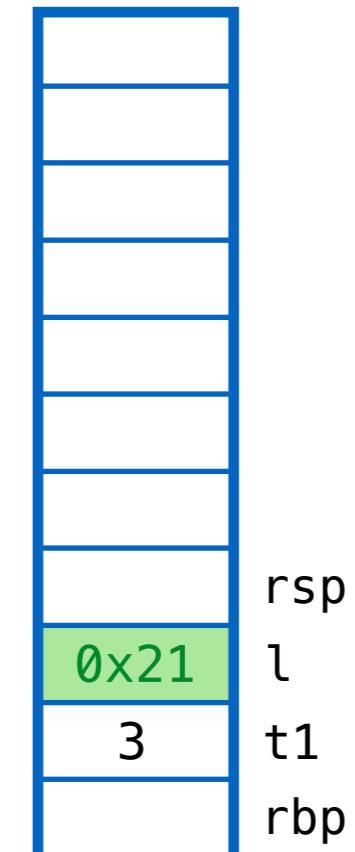
3. **REDIRECT** addrs on stack and heap!

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

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    let l1 = range(0, 3)
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in
(1000, l) ←
```



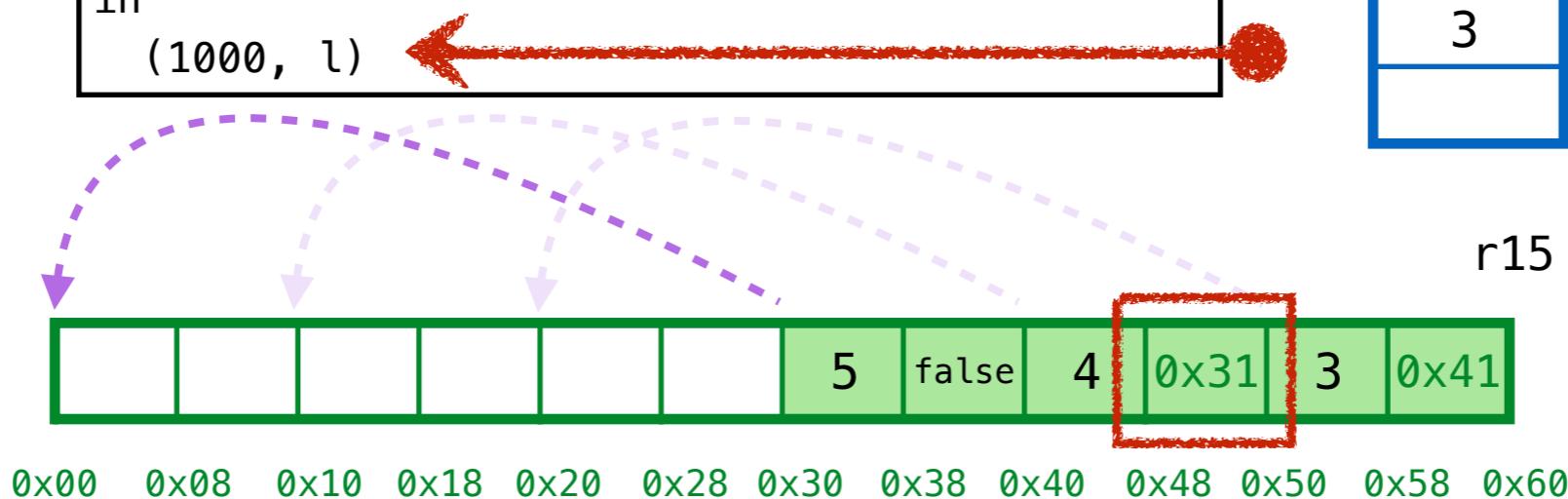
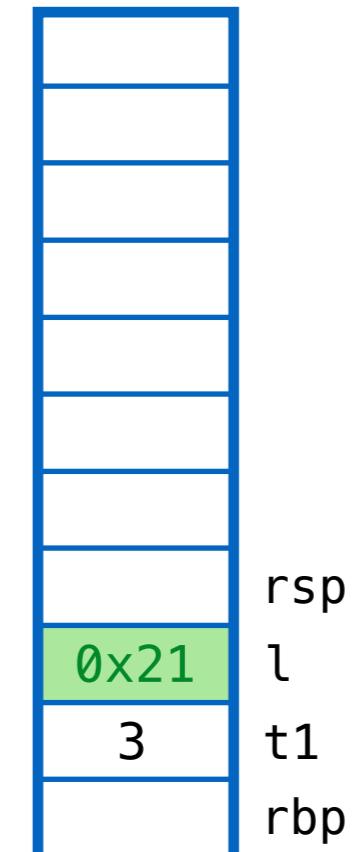
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(1000, l) ←
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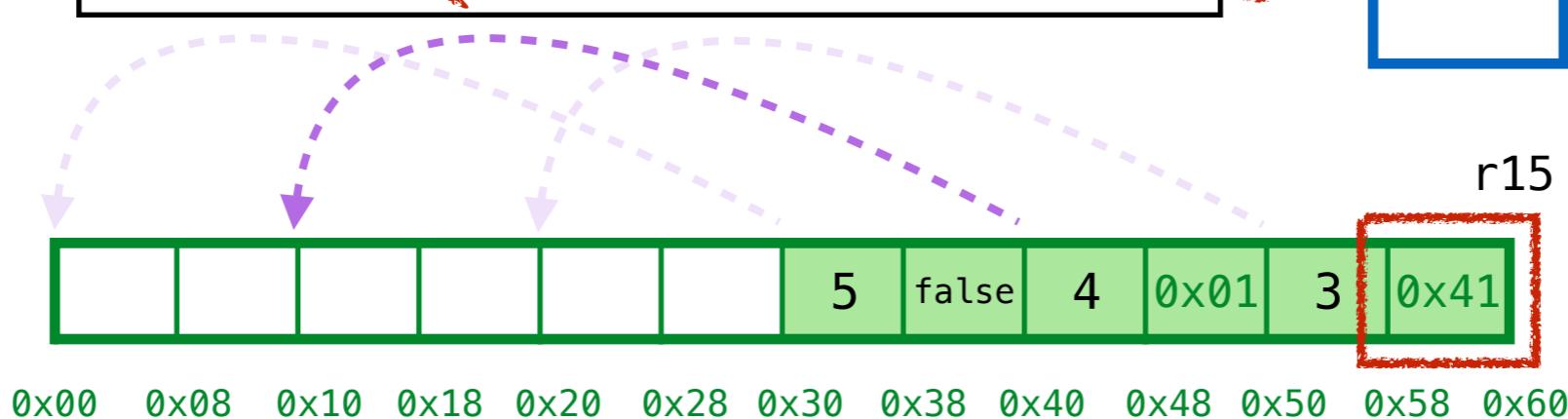
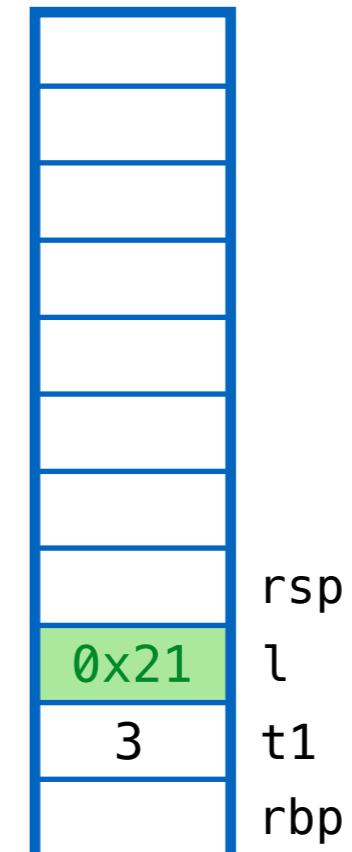
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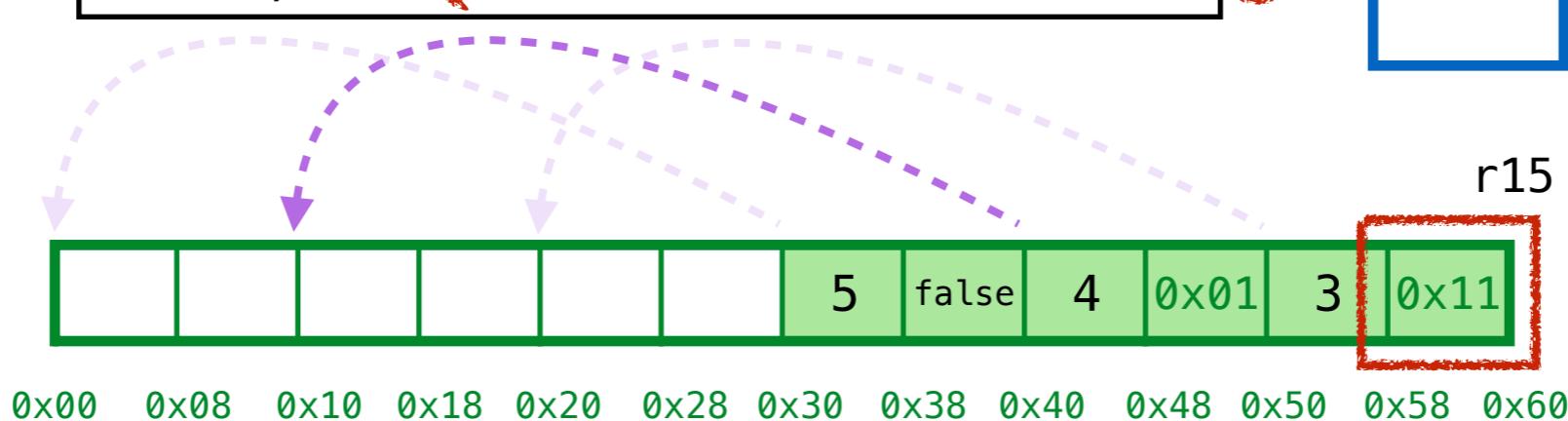
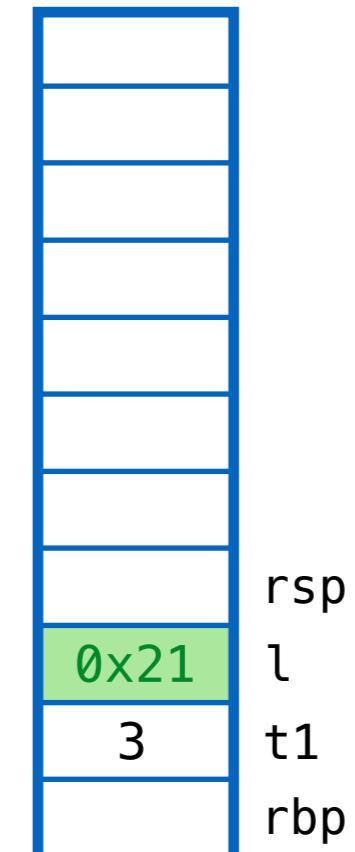
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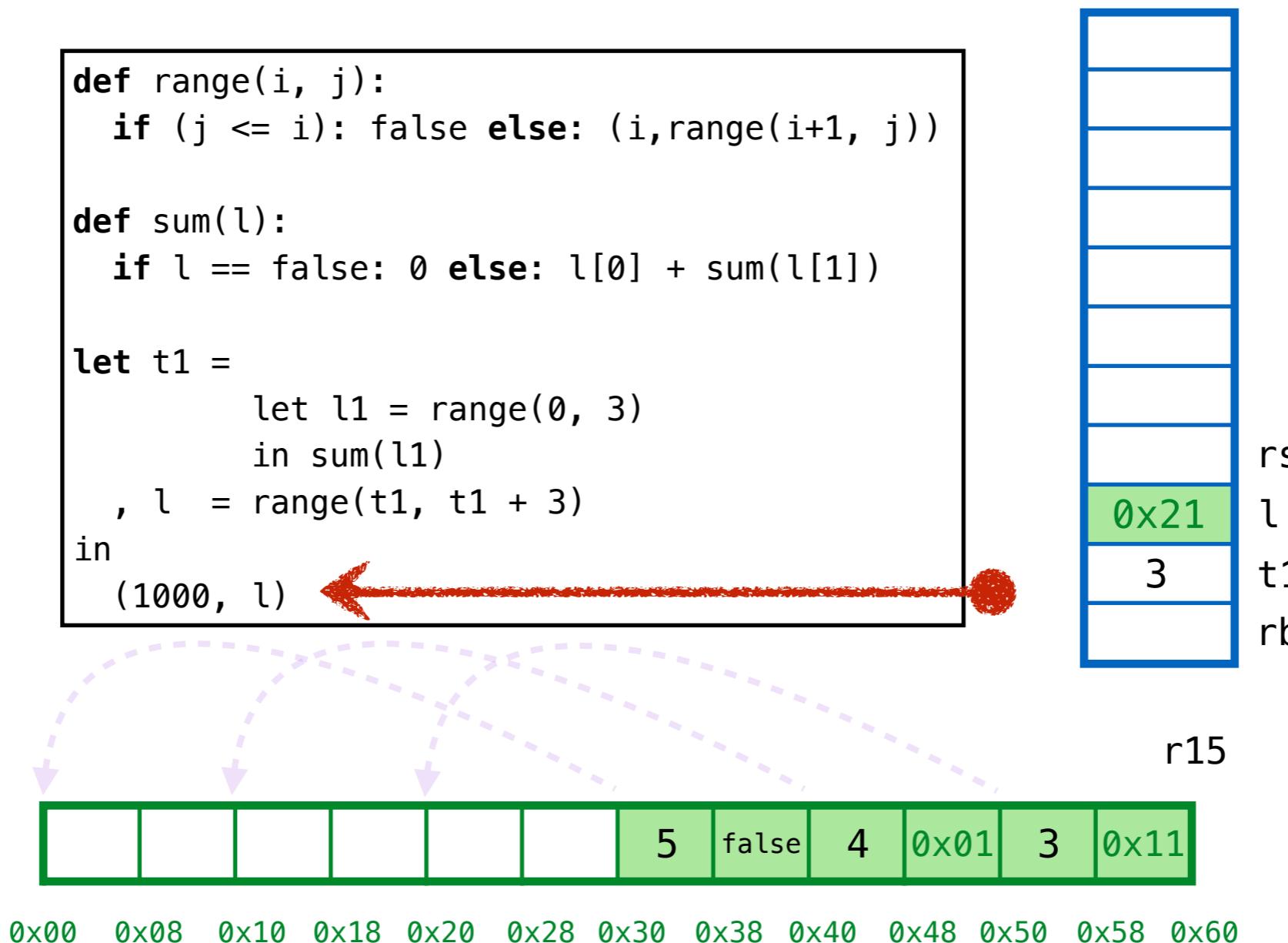
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3. REDIRECT addrs on stack and heap!

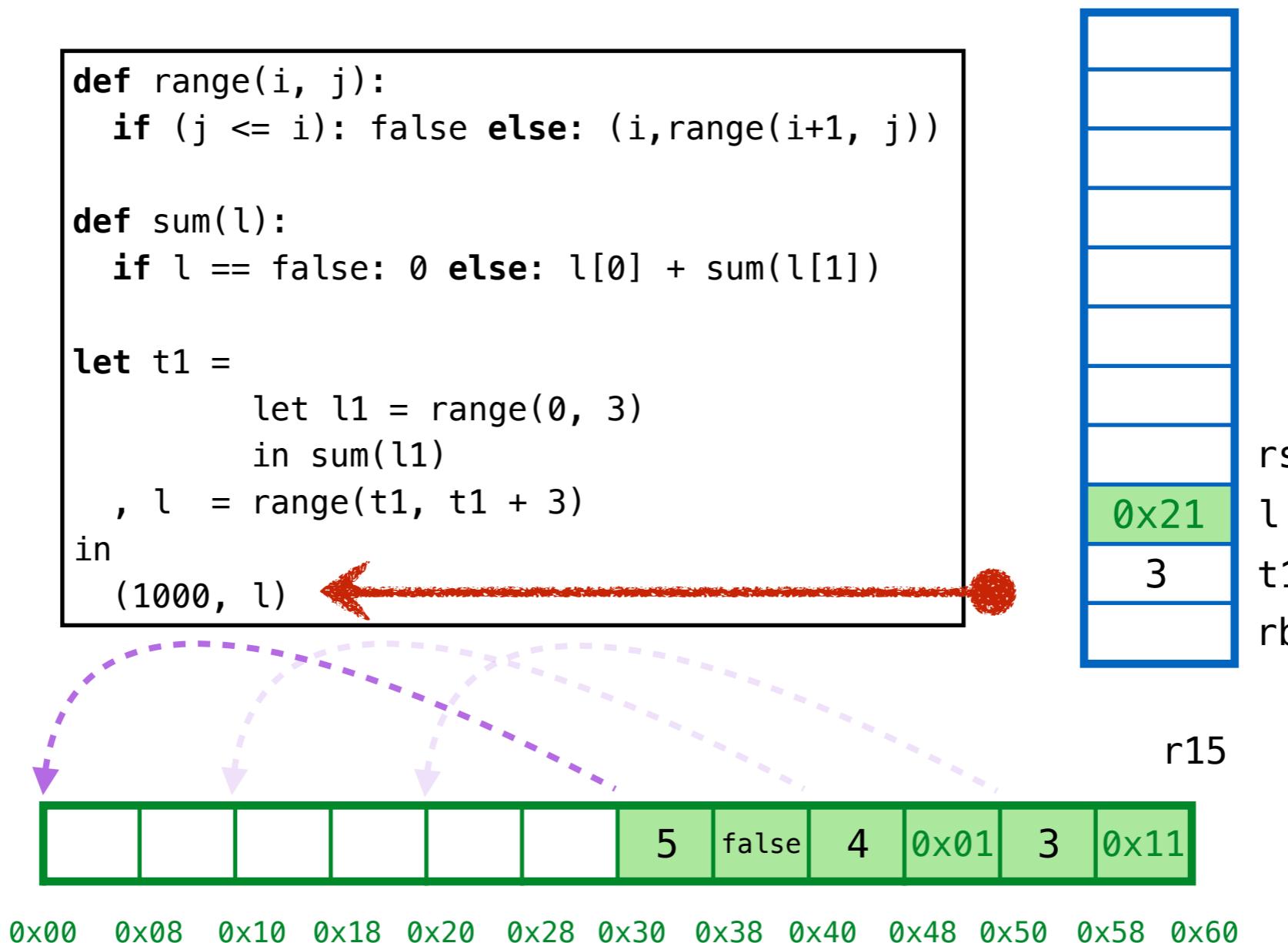
ex4: recursive data



4. COMPACT cells on heap

Copy cell to forward addr!

ex4: recursive data



4. COMPACT cells on heap

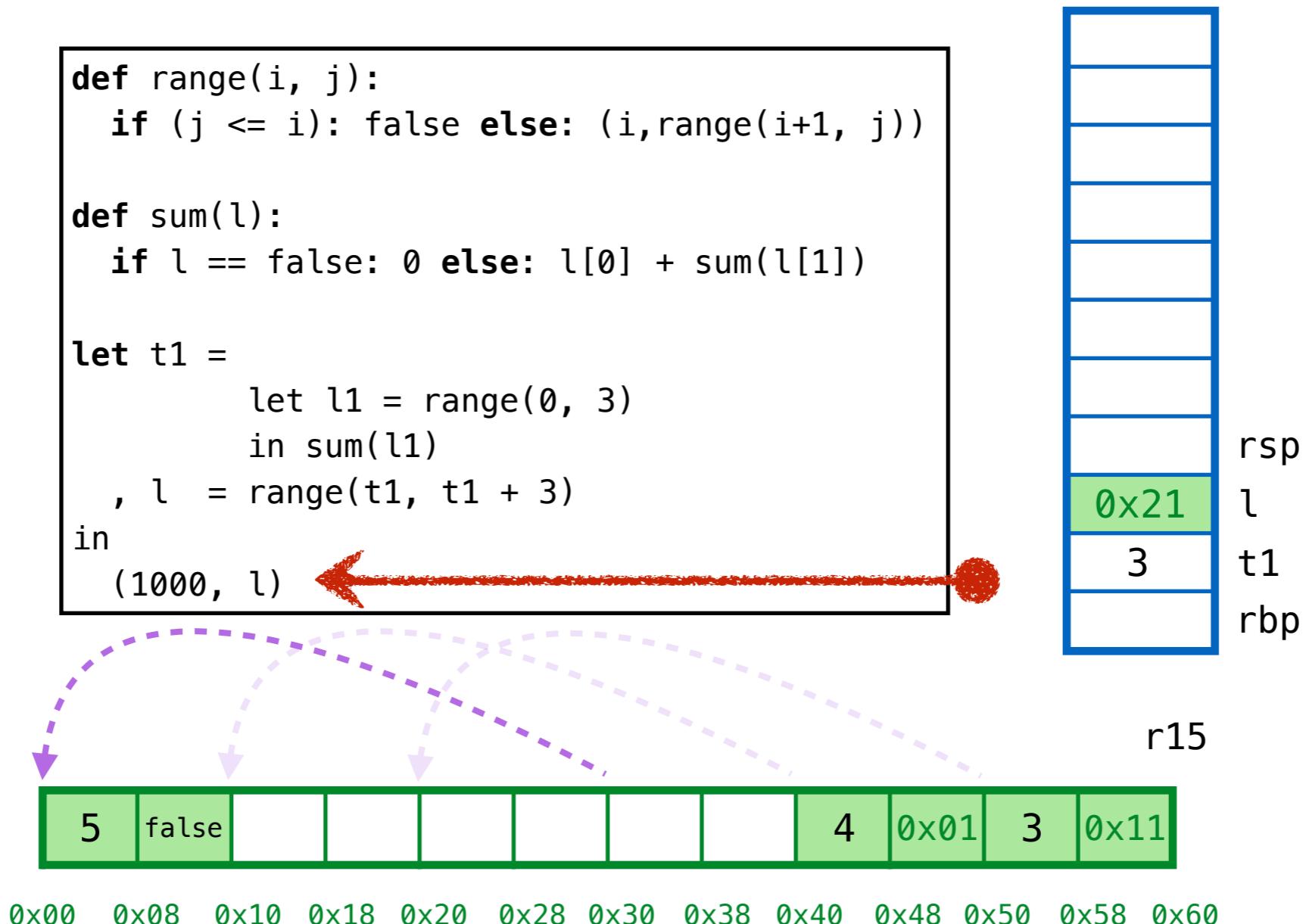
Copy cell to forward addr!

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4. COMPACT cells on heap

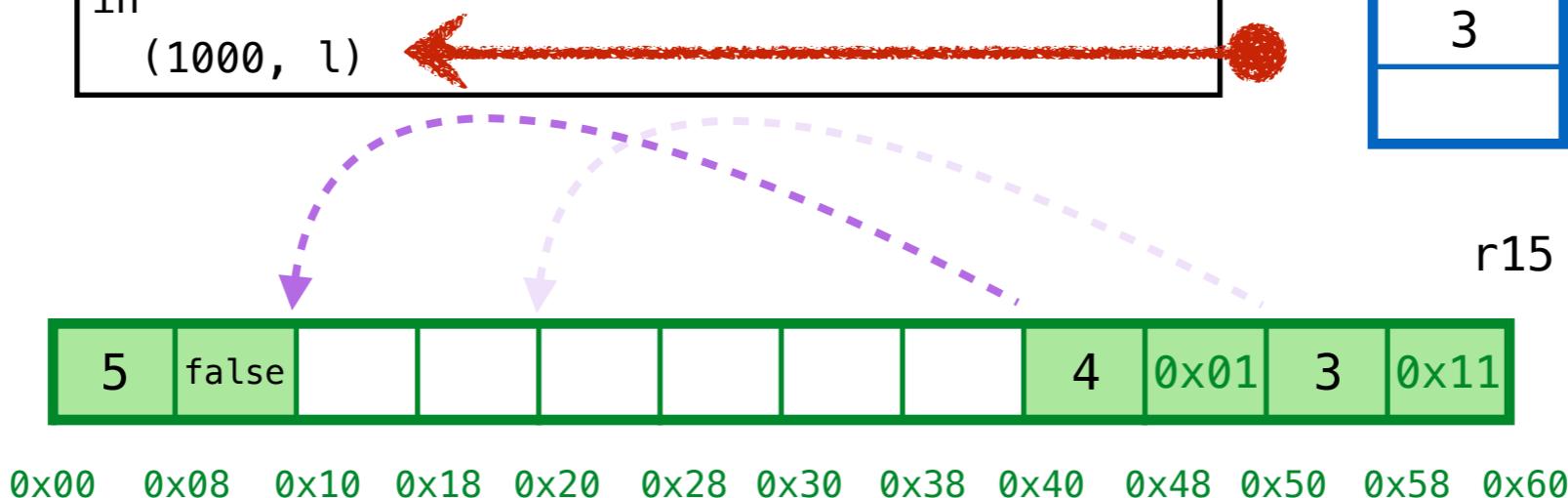
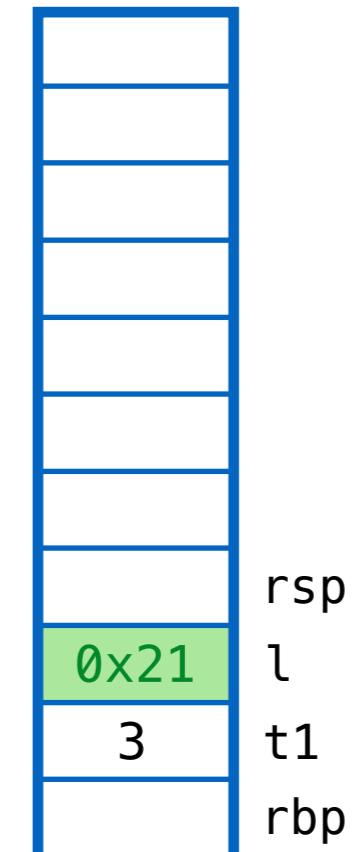
Copy cell to forward addr!

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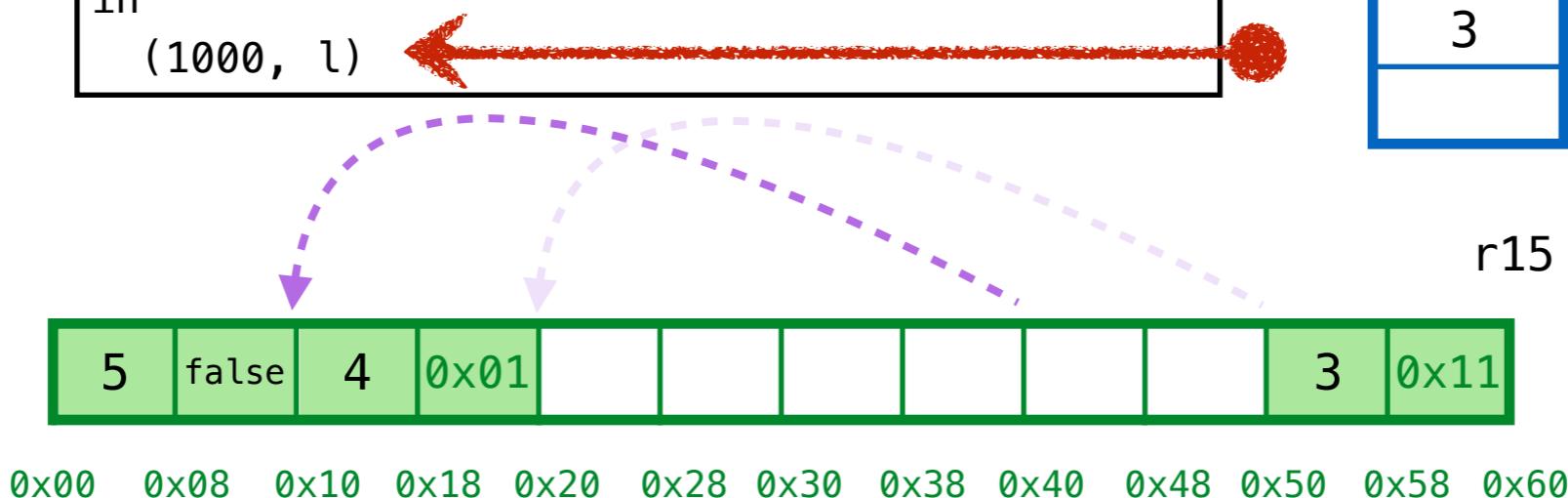
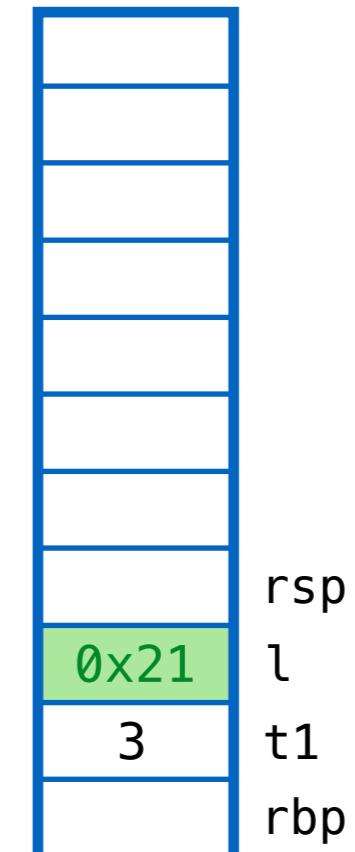
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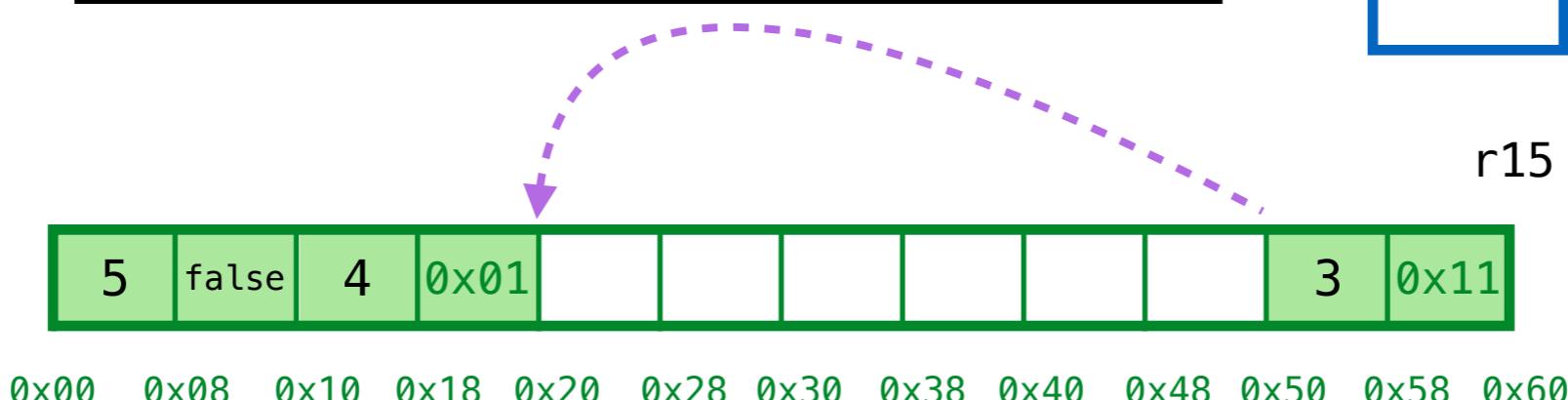
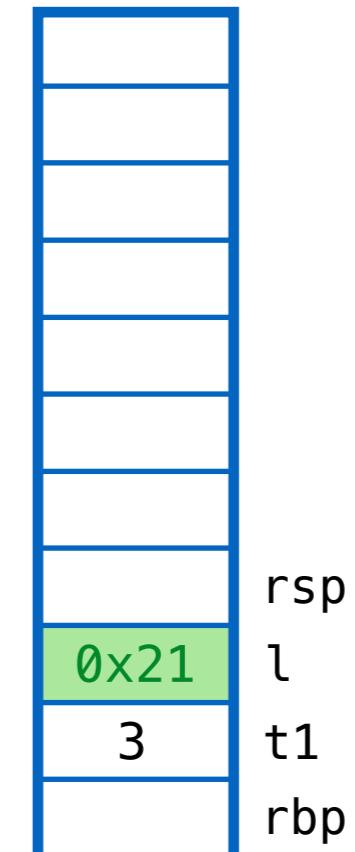
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4. COMPACT cells on heap

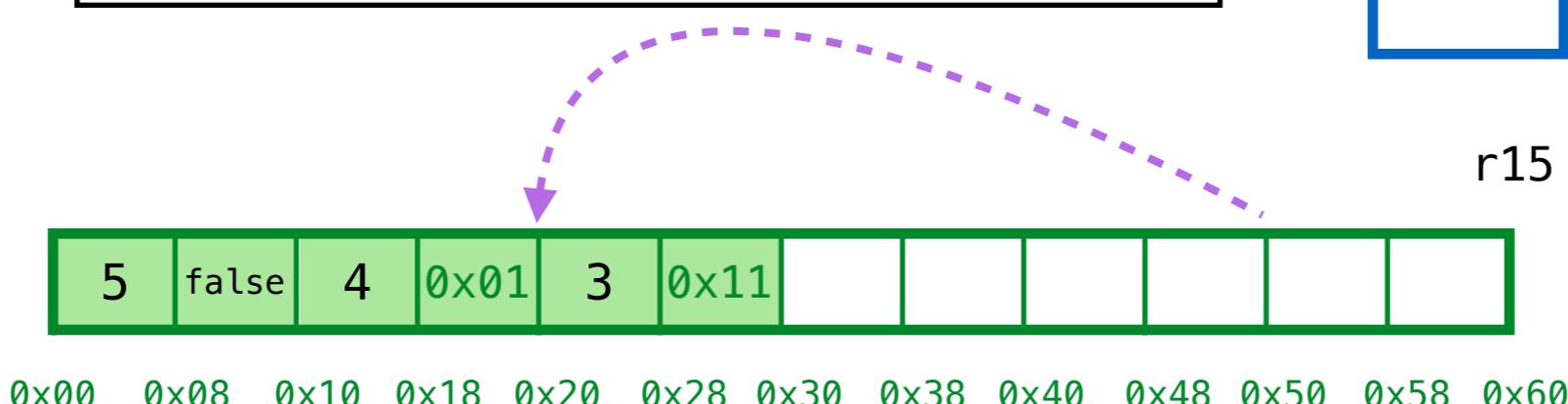
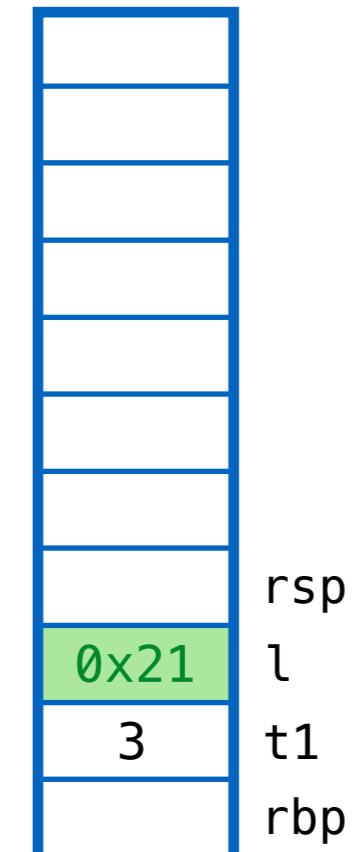
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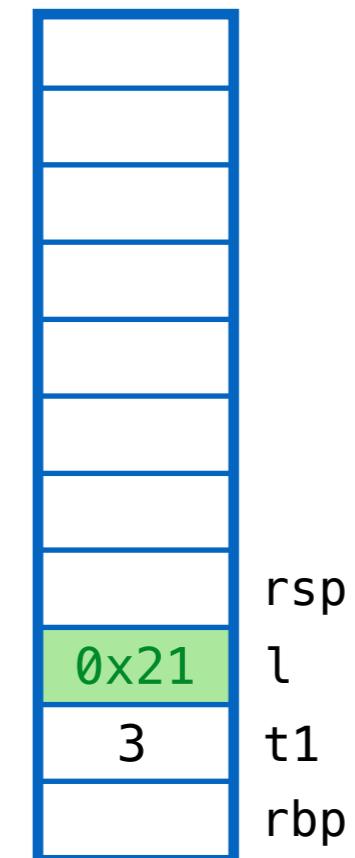
Copy cell to forward addr!

ex4: recursive data

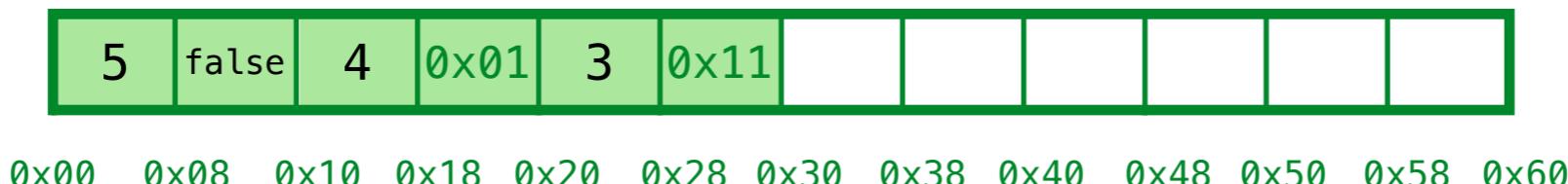
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r15



GC Complete!

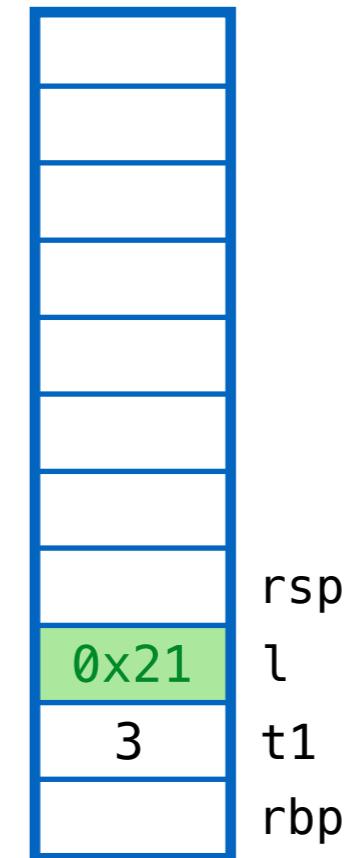
Have space for (1000, l)

ex4: recursive data

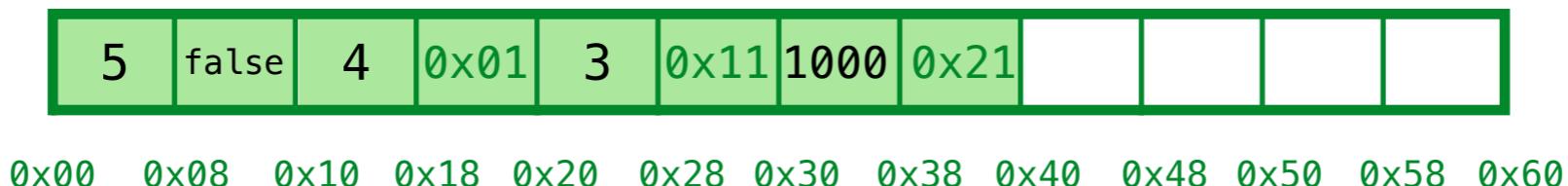
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r15



GC Complete!

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