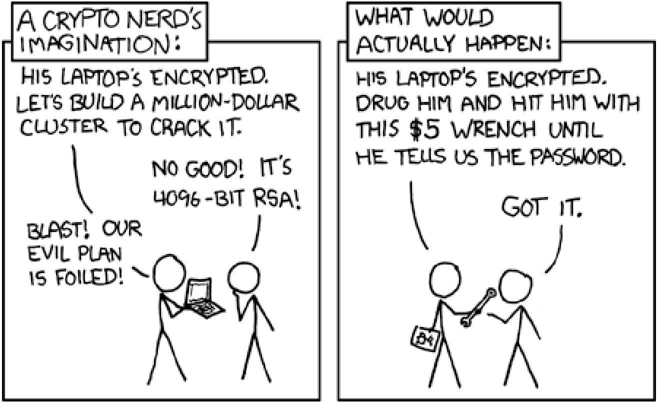
A CRYPTO NERD'S IMAGINATION:
HIS LAPTOP'S ENCRYPTED. LET'S BUILD A MILLION-DOLLAR CLUSTER TO CRACK IT.
NO GOOD! IT'S 4096-BIT RSA!
BLAST! OUR EVIL PLAN IS FOILED!
WHAT WOULD ACTUALLY HAPPEN:
HIS LAPTOP'S ENCRYPTED. DRUG HIM AND HIT HIM WITH THIS $5 WRENCH UNTIL HE TELLS US THE PASSWORD.
GOT IT.

Quantum Key Distribution

Foundational Aspects of Quantum Mechanics

Simon Hirscher & Max Snijders

01-02-2017

Contents

The setting

Alice and Bob

$$\mathrm{ENC} : \underbrace{\{\text{plaintexts}\}}_{\cong\, \mathbb{N}} \xrightarrow{\text{bijective}} \underbrace{\{\text{ciphertexts}\}}_{\cong\, \mathbb{N}}$$

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- Encryption function hard to reverse for a 3rd party.

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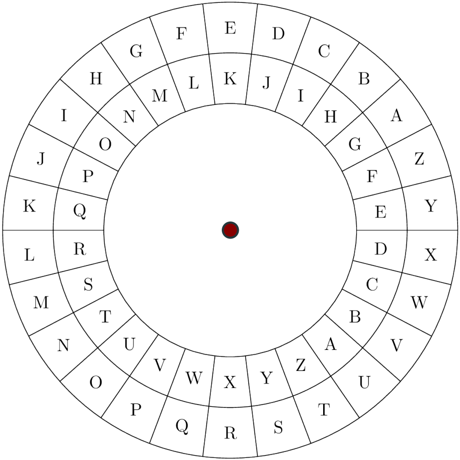
- Symmetric (*shared secret*)
- Asymmetric (*public/private key*)

Shifting Caesar Cipher



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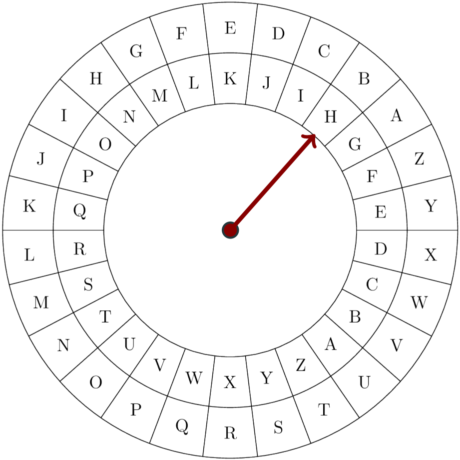

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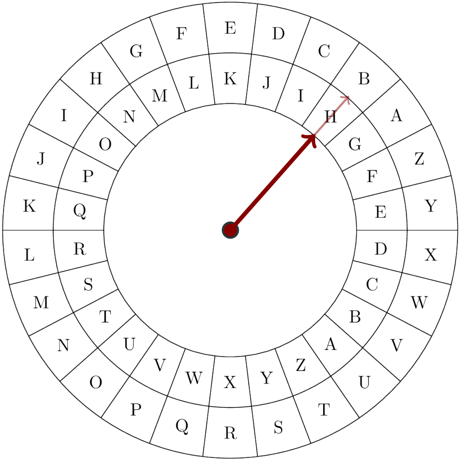

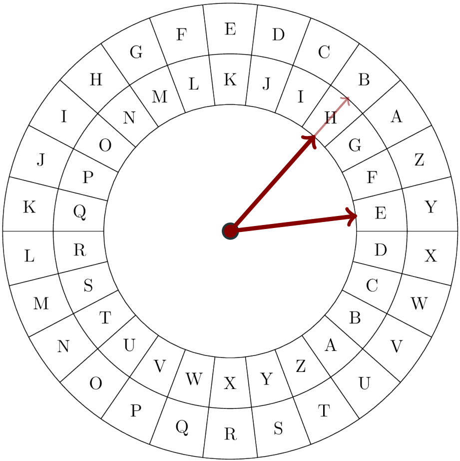
E D C B A Z Y X W V U T S R Q P O N M L K J I H G F
K J I H G F E D C B A Z Y X W V U T S R Q P O N M L

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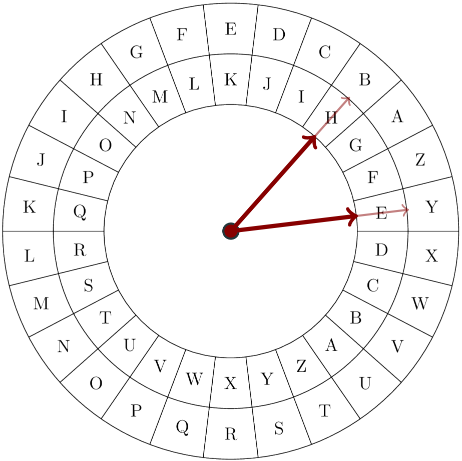

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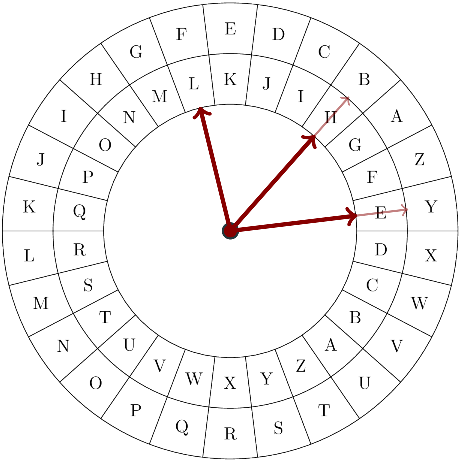

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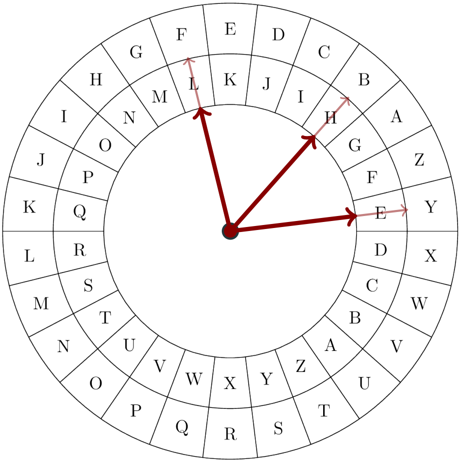

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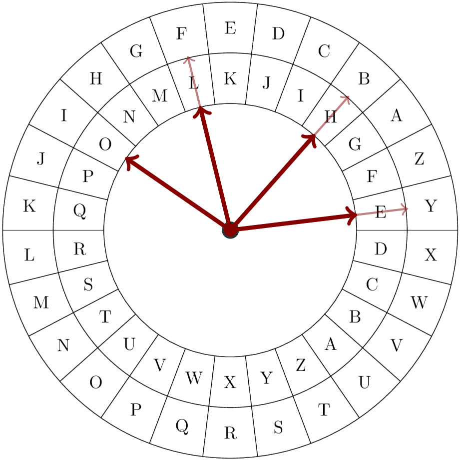

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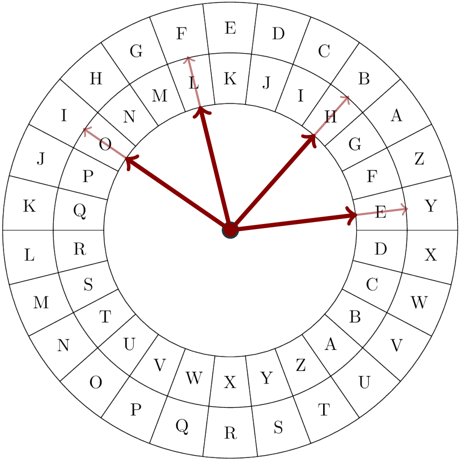

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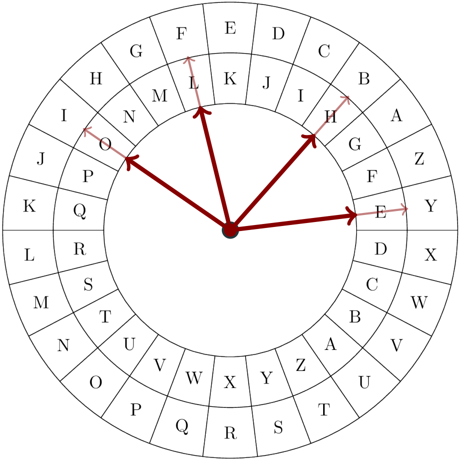

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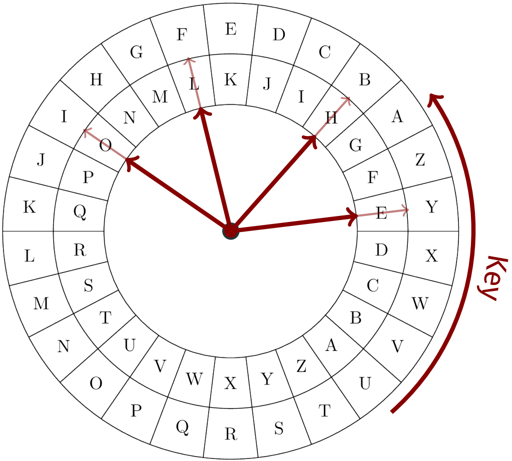
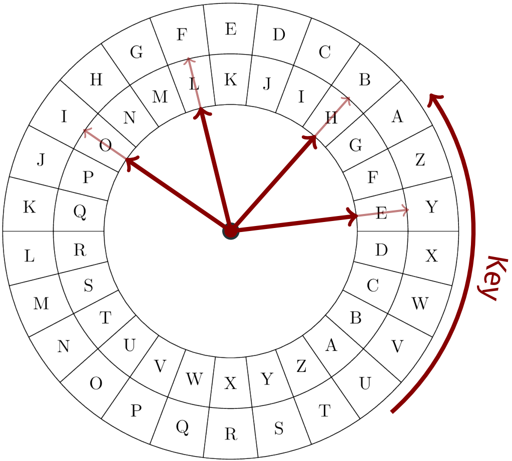
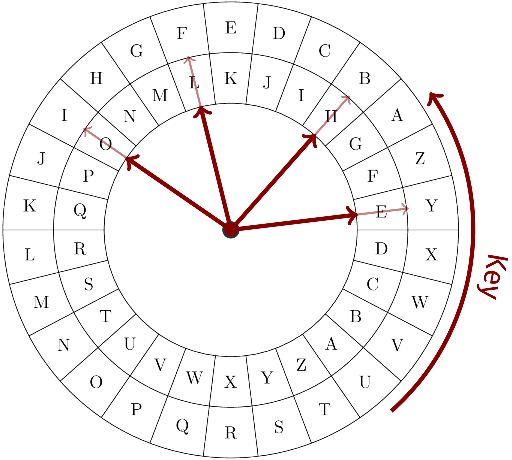
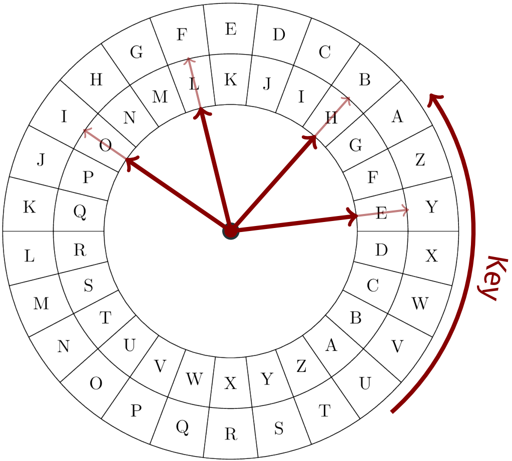




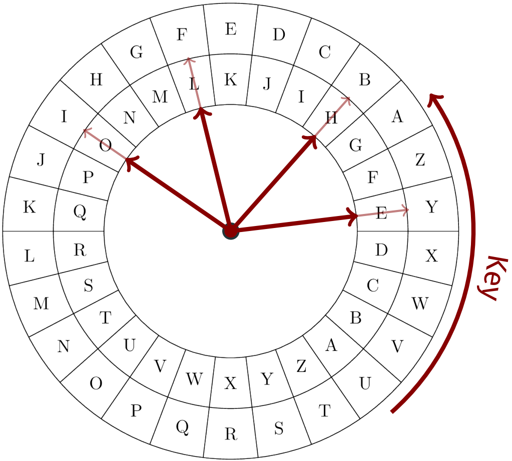
- "HELLO" $\rightarrow$ "BYFFI"



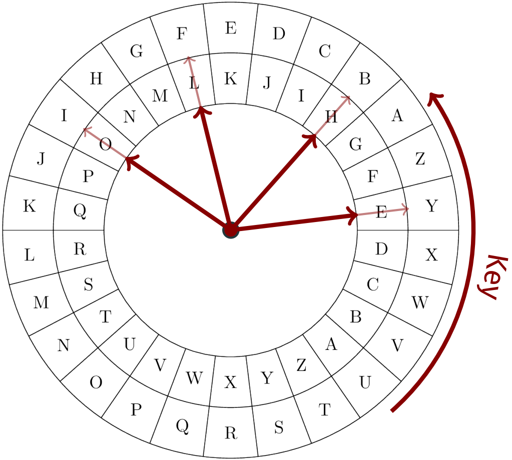
- “HELLO” → “BYFFI”
- 26 options



- “HELLO” → “BYFFI”
- 26 options
- Vulnerabilities

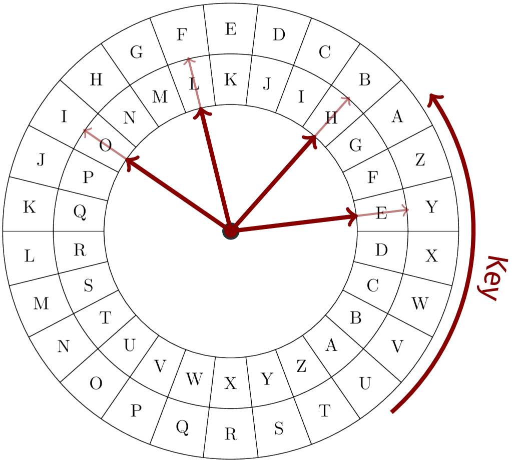


- “HELLO” → “BYFFI”
- 26 options
- Vulnerabilities:
  - Brute-force attacks



- "HELLO" $\rightarrow$ "BYFFI"
- 26 options
- Vulnerabilities:
  - Brute-force attacks
  - Frequency analysis

Shifting Caesar Cipher



- “HELLO” → “BYFFI”
- 26 options
- Vulnerabilities:
    - Brute-force attacks
    - Frequency analysis
    - Known-plaintext attacks

Every character gets mapped to a unique character

Plaintext	Ciphertext

Every character gets mapped to a unique character

Plaintext	Ciphertext
A	G

Every character gets mapped to a unique character

Plaintext	Ciphertext
A	G
B	X

Every character gets mapped to a unique character

Plaintext	Ciphertext
A	G
B	X
C	C

Every character gets mapped to a unique character

Plaintext	Ciphertext
A	G
B	X
C	C
D	J

Every character gets mapped to a unique character

Plaintext	Ciphertext
A	G
B	X
C	C
D	J
⋮	⋮

Every character gets mapped to a unique character

Plaintext	Ciphertext
A	G
B	X
C	C
D	J
⋮	⋮

- "ABBACD" $\rightarrow$ "GXXGCJ"

Every character gets mapped to a unique character

Plaintext	Ciphertext
A	G
B	X
C	C
D	J
⋮	⋮

- "ABBACD" $\rightarrow$ "GXXGCJ"
- $26 \cdot 25 \cdot 24 \cdot \ldots \cdot 1 = 26! \approx 10^{26}$ options

Every character gets mapped to a unique character

Plaintext	Ciphertext
A	G
B	X
C	C
D	J
$\vdots$	$\vdots$

- "ABBACD" $\rightarrow$ "GXXGCJ"
- $26 \cdot 25 \cdot 24 \cdot \ldots \cdot 1 = 26! \approx 10^{26}$ options
- Vulnerabilities

Every character gets mapped to a unique character

Plaintext	Ciphertext
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B	X
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D	J
$\vdots$	$\vdots$

- "ABBACD" $\rightarrow$ "GXXGCJ"
- $26 \cdot 25 \cdot 24 \cdot \ldots \cdot 1 = 26! \approx 10^{26}$ options
- Vulnerabilities:
  - frequency analysis.

Permutation Cipher

Every character gets mapped to a unique character

Plaintext	Ciphertext
A	G
B	X
C	C
D	J
$\vdots$	$\vdots$

- "ABBACD" $\rightarrow$ "GXXGCJ"
- $26 \cdot 25 \cdot 24 \cdot \ldots \cdot 1 = 26! \approx 10^{26}$ options
- Vulnerabilities:
  - frequency analysis.
  - known-plaintext attacks.

XOR

- Key K of n bits
- Successively apply K to blocks A of n bits of plaintext by xor'ing data and key bits: $\text{ENC}_K(A) := A \oplus K$

Bit #	1	2	3	n=4	
Plaintext	1	0	1	1	
Key	1	1	0	1	
Ciphertext	0	1	1	0	

- Decryption: $\text{DEC}_K(\text{ENC}_K(A)) := \text{ENC}_K(A) \oplus K = A \oplus K \oplus K = A$

XOR

- Key K of n bits
- Successively apply K to blocks A of n bits of plaintext by xor'ing data and key bits: $\mathrm{ENC}_K(A) := A \oplus K$

Bit #	1	2	3	n=4	5	6	7	8
Plaintext	1	0	1	1	0	0	0	1
Key	1	1	0	1	1	1	0	1
Ciphertext	0	1	1	0	1	1	0	0

- Decryption: $\mathrm{DEC}_K(\mathrm{ENC}_K(A)) := \mathrm{ENC}_K(A) \oplus K = A \oplus K \oplus K = A$

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Bit #	1	2	3	n=4	5	6	7	8	...
Plaintext	1	0	1	1	0	0	0	1	...
Key	1	1	0	1	1	1	0	1	...
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- Decryption: $\text{DEC}_K(\text{ENC}_K(A)) := \text{ENC}_K(A) \oplus K = A \oplus K \oplus K = A$
- Susceptible to **frequency analysis**, **known-plaintext attacks** and **brute force**.

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Plaintext	1	0	1	1	0	0	0	1	...
Key	1	1	0	1	1	1	0	1	...
Ciphertext	0	1	1	0	1	1	0	0	...

- Decryption: $\text{DEC}_K(\text{ENC}_K(A)) := \text{ENC}_K(A) \oplus K = A \oplus K \oplus K = A$
- Susceptible to **frequency analysis**, **known-plaintext attacks** and **brute force**.
- Moreover: $\text{ENC}_K(A) \oplus \text{ENC}_K(B) = A \oplus K \oplus B \oplus K = A \oplus B$

One-time pad = random key that **is as long as the message**, only used **once**.

Bit #	1	2	3	4
Plaintext	1	0	1	1
Key	1	1	0	1
Ciphertext	0	1	1	0

Unbreakable

One-Time Pad

One-time pad = random key that **is as long as the message**, only used **once**.

Bit #	1	2	3	4	5
Plaintext	1	0	1	1	1
Key	1	1	0	1	0
Ciphertext	0	1	1	0	1

Unbreakable

One-time pad = random key that **is as long as the message**, only used **once**.

Bit #	1	2	3	4	5	6
Plaintext	1	0	1	1	1	0
Key	1	1	0	1	0	1
Ciphertext	0	1	1	0	1	1

Unbreakable

One-time pad = random key that **is as long as the message**, only used **once**.

Bit #	1	2	3	4	5	6	...
Plaintext	1	0	1	1	1	0	...
Key	1	1	0	1	0	1	...
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Unbreakable

One-time pad = random key that **is as long as the message**, only used **once**.

Bit #	1	2	3	4	5	6	...	n
Plaintext	1	0	1	1	1	0	...	1
Key	1	1	0	1	0	1	...	1
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One-time pad = random key that **is as long as the message**, only used **once**.

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Plaintext	1	0	1	1	1	0	...	1
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Unbreakable since:

- No correlation

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Bit #	1	2	3	4	5	6	...	n
Plaintext	1	0	1	1	1	0	...	1
Key	1	1	0	1	0	1	...	1
Ciphertext	0	1	1	0	1	1	...	0

Unbreakable since:

- No correlation
- Any plaintext $\iff$ any ciphertext

One-time pad = random key that **is as long as the message**, only used **once**.

Bit #	1	2	3	4	5	6	...	n
Plaintext	1	0	1	1	1	0	...	1
Key	1	1	0	1	0	1	...	1
Ciphertext	0	1	1	0	1	1	...	0

Unbreakable

The problem with exchanging the key

How do we agree on the key in the first place?

How can we do that securely?

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How can we do that securely?

Two ways:

1. Meet in person every time

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How can we do that securely?

Two ways:

1. Meet in person every time
2. Meet in person once

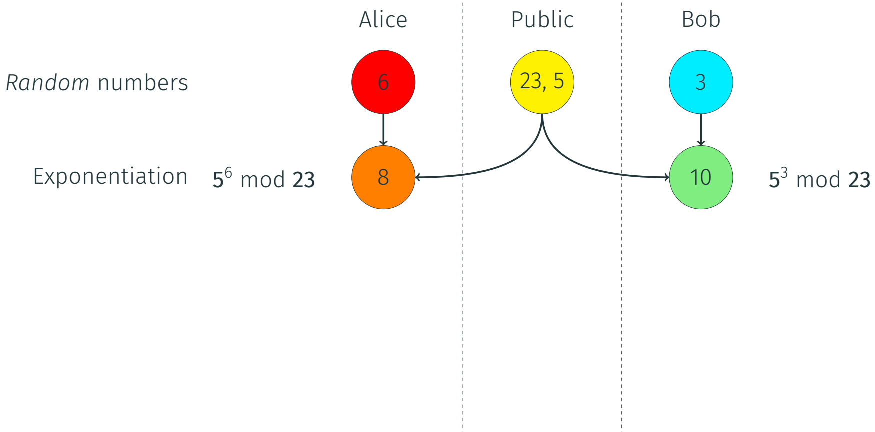
Diffie-Hellman Key Exchange (DHE)

	Alice	Public	Bob
Random numbers		23, 5	

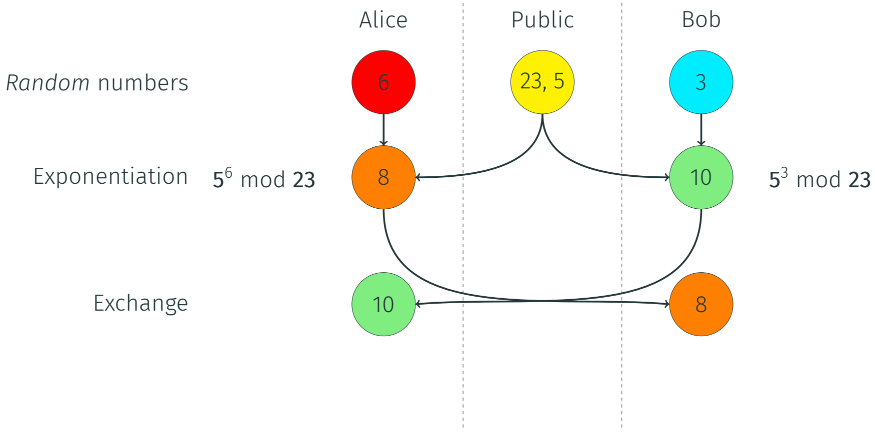
Diffie-Hellman Key Exchange (DHE)

	Alice	Public	Bob
Random numbers	6	23, 5	3

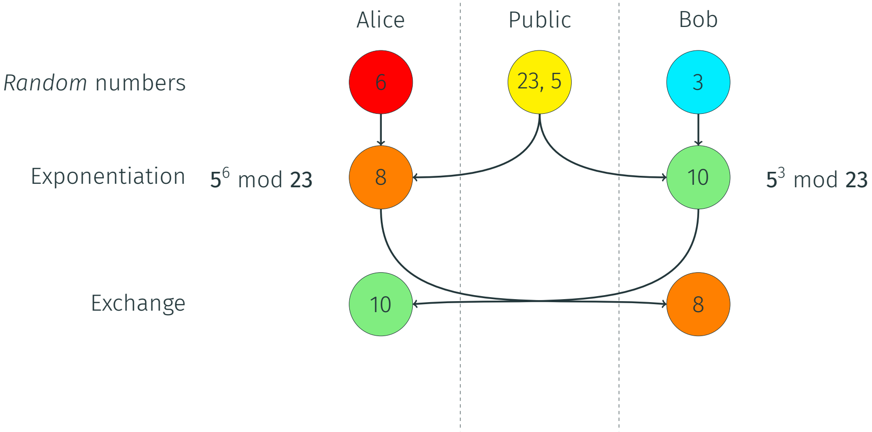
DIFFIE-HELLMAN KEY EXCHANGE (DHE)



Diffie-Hellman Key Exchange (DHE)



Diffie-Hellman Key Exchange (DHE)



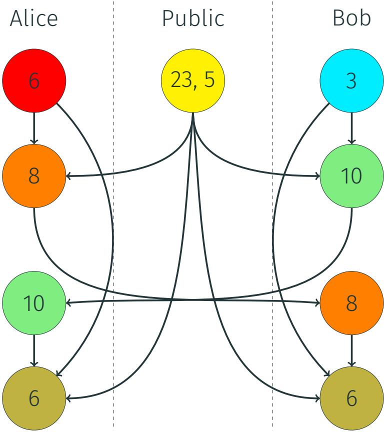
Diffie-Hellman Key Exchange (DHE)

Random numbers

Exponentiation $5^6 \bmod 23$ — $5^3 \bmod 23$

Exchange

Exponentiation $10^6 \bmod 23$ — $8^3 \bmod 23$



- Exponentiation is hard to invert classically ("discrete logarithm problem")

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- Impractical for one-time pad use

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- Impractical for one-time pad use
- Insecure in light of quantum algorithms:
  discrete logarithm $\sim$ integer factorization ($\rightarrow$ last week)

In QKD, quantum states carry the key information.

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QKD makes use of fundamental principles of quantum mechanics:

1. Measurement changes system (unless in eigenstate of observable)

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QKD makes use of fundamental principles of quantum mechanics:

1. Measurement changes system (unless in eigenstate of observable)
   - will prevent Eve from measuring quantum states without being detected
2. No-cloning theorem
   - will prevent Eve from copying the quantum states for later measurement

- Consider two Hilbert spaces $H_A \cong H_B$, $\dim H_A \geq 2$.

No-cloning theorem

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- Want to find unitary operator (time evolution) U such that $\forall\, |\psi\rangle \in H_A, |b\rangle \in H_B : U(|\psi\rangle \otimes |b\rangle) \stackrel{!}{=} |\psi\rangle \otimes |\psi\rangle$ (up to a phase)

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- But then take another $\left|\phi\right\rangle \in H_A$:

$$
\begin{aligned}
\langle\psi|\phi\rangle = \langle\psi|\phi\rangle \langle b|b\rangle &= \big(\langle\psi| \otimes \langle b|\big)\big(\left|\phi\right\rangle \otimes \left|b\right\rangle\big) \\
&= \big(\langle\psi| \otimes \langle b|\big)U^\dagger U\big(\left|\phi\right\rangle \otimes \left|b\right\rangle\big) \\
&= \big(\langle\psi| \otimes \langle\psi|\big)\big(\left|\phi\right\rangle \otimes \left|\phi\right\rangle\big) = \langle\psi|\phi\rangle^2
\end{aligned}
$$

No-cloning theorem

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- Want to find unitary operator (time evolution) U such that $\forall \left|\psi\right\rangle \in H_A, \left|b\right\rangle \in H_B : U(\left|\psi\right\rangle \otimes \left|b\right\rangle) \stackrel{!}{=} \left|\psi\right\rangle \otimes \left|\psi\right\rangle$ (up to a phase)
- But then take another $\left|\phi\right\rangle \in H_A$:

$$\begin{aligned}\langle\psi|\phi\rangle = \langle\psi|\phi\rangle \langle b|b\rangle &= \big(\left\langle\psi\right| \otimes \left\langle b\right|\big)\big(\left|\phi\right\rangle \otimes \left|b\right\rangle\big)\\ &= \big(\left\langle\psi\right| \otimes \left\langle b\right|\big)U^\dagger U\big(\left|\phi\right\rangle \otimes \left|b\right\rangle\big)\\ &= \big(\left\langle\psi\right| \otimes \left\langle\psi\right|\big)\big(\left|\phi\right\rangle \otimes \left|\phi\right\rangle\big) = \langle\psi|\phi\rangle^2\end{aligned}$$

$\implies$ $\langle\psi|\phi\rangle = 1$, i.e. identical, or $\langle\psi|\phi\rangle = 0$
$\implies$ Can never work with different, non-orthogonal states

The BB-84 Protocol

Charles Bennet & Gilles Brassard, 1984

Core concept: measurements affect states

$\rightarrow$ Blackoard

1. Alice chooses bit value $b_i \in \{0, 1\}$

Bit # i	1	2	3	4
Bit b_i	0	1	1	0

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2. Alice picks basis $A_i \in \{+, \times\}$

Bit # i	1	2	3	4
Bit b_i	0	1	1	0
Basis A_i	$\times$	$\times$	$\times$	$+$

1. Alice chooses bit value $b_i \in \{0, 1\}$
2. Alice picks basis $A_i \in \{+, \times\}$
3. Alice encodes b_i as polarized photon using basis

Bit # i	1	2	3	4
Bit b_i	0	1	1	0
Basis A_i	$\times$	$\times$	$\times$	$+$
Alice sends	$\nearrow\!\!\!\!\swarrow$	$\nwarrow\!\!\!\!\searrow$	$\nwarrow\!\!\!\!\searrow$	$\leftrightarrow$

Polarization	$\nearrow\!\!\!\!\swarrow$	$\nwarrow\!\!\!\!\searrow$	$\leftrightarrow$	$\updownarrow$
Bit value	0	1	0	1

1. Alice chooses bit value $b_i \in \{0, 1\}$
2. Alice picks basis $A_i \in \{+, \times\}$
3. Alice encodes b_i as polarized photon using basis
4. Bob picks basis $B_i \in \{+, \times\}$

Bit # i	1	2	3	4
Bit b_i	0	1	1	0
Basis A_i	$\times$	$\times$	$\times$	$+$
Alice sends	$\swarrow\!\!\!\nearrow$	$\nwarrow\!\!\!\searrow$	$\nwarrow\!\!\!\searrow$	$\leftrightarrow$
Basis B_i	$+$	$\times$	$+$	$+$

Polarization	$\swarrow\!\!\!\nearrow$	$\nwarrow\!\!\!\searrow$	$\leftrightarrow$	$\updownarrow$
Bit value	0	1	0	1

1. Alice chooses bit value $b_i \in \{0, 1\}$
2. Alice picks basis $A_i \in \{+, \times\}$
3. Alice encodes b_i as polarized photon using basis
4. Bob picks basis $B_i \in \{+, \times\}$
5. Bob measures photon polarization using the basis

Bit # i	1	2	3	4
Bit b_i	0	1	1	0
Basis A_i	$\times$	$\times$	$\times$	$+$
Alice sends	$\swarrow\!\!\!\nearrow$	$\nwarrow\!\!\!\searrow$	$\nwarrow\!\!\!\searrow$	$\leftrightarrow$
Basis B_i	$+$	$\times$	$+$	$+$
Bob sees	?	1	?	0

Polarization	$\swarrow\!\!\!\nearrow$	$\nwarrow\!\!\!\searrow$	$\leftrightarrow$	$\updownarrow$
Bit value	0	1	0	1

1. Alice chooses bit value $b_i \in \{0,1\}$
2. Alice picks basis $A_i \in \{+,\times\}$
3. Alice encodes b_i as polarized photon using basis
4. Bob picks basis $B_i \in \{+,\times\}$
5. Bob measures photon polarization using the basis
6. Alice & Bob exchange list of bases over classical channel.

Bit # i	1	2	3	4
Bit b_i	0	1	1	0
Basis A_i	$\times$	$\times$	$\times$	$+$
Alice sends	$\swarrow\!\!\!\nearrow$	$\nwarrow\!\!\!\searrow$	$\nwarrow\!\!\!\searrow$	$\leftrightarrow$
Basis B_i	$+$	$\times$	$+$	$+$
Bob sees	?	1	?	0

Polarization	$\swarrow\!\!\!\nearrow$	$\nwarrow\!\!\!\searrow$	$\leftrightarrow$	$\updownarrow$
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1. Alice chooses bit value $b_i \in \{0, 1\}$
2. Alice picks basis $A_i \in \{+, \times\}$
3. Alice encodes b_i as polarized photon using basis
4. Bob picks basis $B_i \in \{+, \times\}$
5. Bob measures photon polarization using the basis
6. Alice & Bob exchange list of bases over classical channel. If bases A_i & B_i match $\implies$ append bit b_i to shared key

Bit # i	1	2	3	4
Bit b_i	0	1	1	0
Basis A_i	$\times$	$\times$	$\times$	$+$
Alice sends	$\nearrow\!\!\!\swarrow$	$\nwarrow\!\!\!\searrow$	$\nwarrow\!\!\!\searrow$	$\leftrightarrow$
Basis B_i	$+$	$\times$	$+$	$+$
Bob sees	?	1	?	0
Shared key	-	1	-	0

Polarization	$\nearrow\!\!\!\swarrow$	$\nwarrow\!\!\!\searrow$	$\leftrightarrow$	$\updownarrow$
Bit value	0	1	0	1

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$\implies$ To detect Eve:

- Keep systematic error rate (**noise level** N) far below 25%
  - Eve will eavesdrop on every n$^{\text{th}}$ bit if error rate is $\frac{25\%}{n}$
- Compute **quantum bit error rate** E
  - $E > N \implies$ discard key
  - $E \sim N \implies$ do error correction and proceed

BB-84 – Privacy amplification

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$\Longrightarrow$ Eve's average knowledge of the key **decreases** at the expense of decreasing the key length. $\rightarrow$ blackboard

- Conceived of by Artur Ekert (Oxford) in 1991
- Difference to BB-84: Source inbetween Alice and Bob produces pairs of entangled photons in state

$$|\psi\rangle := \frac{1}{\sqrt{2}}(\uparrow\uparrow + \rightarrow\rightarrow) = \frac{1}{\sqrt{2}}(\nearrow\nearrow + \nwarrow\nwarrow)$$

- If Alice and Bob choose the same basis ($+$ or $\times$) $\implies$ measurements agree

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- Bell's inequality ($a_i, b_i \in \{\pm 1\}$):

$$
\begin{aligned}
1 \overset{\text{classically}}{\geq} \; & \mathbb{E}(a_1 b_2) + \mathbb{E}(a_2 b_2) - \mathbb{E}(a_2 b_1) \\
&= \cos(2\theta_{a_1 b_2}) + \cos(2\theta_{a_2 b_2}) - \cos(2\theta_{a_2 b_1}) \\
&= \cos(\frac{\pi}{4}) + \cos(\frac{\pi}{4}) - \cos(\frac{\pi}{2}) = \frac{2}{\sqrt{2}} = \sqrt{2} > 1
\end{aligned}
$$

- In practice: Use 4 different bases Z_θ ($\theta =$ rotation w.r.t. $+$):
  - Alice's bases: $a_1 := Z_0 = +, a_2 := Z_{\frac{\pi}{8}}, a_3 := Z_{\frac{\pi}{4}} = \times$
  - Bob's bases: $b_1 := Z_0 = +, b_2 := Z_{\frac{\pi}{8}}, b_3 := Z_{-\frac{\pi}{8}}$
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- CHSH inequality w/ $a := Z_0, a' := Z_{\frac{\pi}{4}}, b := Z_{\frac{\pi}{8}}, b' := Z_{-\frac{\pi}{8}}$

$$
\begin{aligned}
2 \overset{\text{classically}}{\geq} \; & \mathbb{E}(ab) + \mathbb{E}(ab') + \mathbb{E}(a'b) - \mathbb{E}(a'b') \\
&= \cos(2\frac{\pi}{8}) + \cos(2\frac{\pi}{8}) + \cos(2\frac{\pi}{8}) \underbrace{- \cos(2\frac{3\pi}{8})}_{\cos(2\frac{\pi}{8})} \\
&= \frac{4}{\sqrt{2}} = 2\sqrt{2} > 2
\end{aligned}
$$

More vulnerabilities in practice

- Basis choice leak
- Authentication issues
- Pseudo-randomness of basis choice

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- Quantum key distribution allows to generate one-time pads on the fly
- In theory: Principles of quantum mechanics protect Alice and Bob from Eve
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- Channel authentication is yet another issue
- Bandwidth in practice:
  - 1 Mbit/s through 20km of optical fiber (Cambridge, 2008)
  - 10 kbit/s through 100km of optical fiber (Cambridge, 2008)
  - 12.7 kbits/s through 300km of optical fiber (Geneva, 2015)

The end

> "Never underestimate the bandwidth of a station wagon full of tapes hurtling down the highway.
> - *Andrew S. Tanenbaum*"

Public/Private Key Authentication