CS 3530: Assignment 0d

Fall 2022

Your Name Here

Problem 0.10 (7 points)

Problem

Find the error in the following proof that $2 = 1$.

Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a + b)(a - b) = b(a - b)$, and divide each side by $(a - b)$ to get $a + b = b$. Finally, let a and b equal 1, which shows that $2 = 1$.

Solution

The error in the statement above occurs in the division step. It says to divide each side by (a-b). If a = b, a - b = 0. Division by 0 is not possible.

Exercise 0.11 (13 points)

Problem

Let $S(n) = 1 + 2 + \cdots + n$ be the sum of the first n natural numbers and let $C(n) = 1^3 + 2^3 + \cdots + n^3$ be the sum of the first n cubes. Prove the following equalities by induction on n, to arrive at the curious conclusion that $C(n) = S^2(n)$ for every n.

a. $S(n) = \frac{1}{2}n(n + 1)$.

   Solution

1. $1 = \frac{1}{2}1(1 + 1)$.
2. $1 + 2 = \frac{1}{2}2(2 + 1)$. Therefore:
3. $S(n + 1) = \frac{1}{2}(n + 1)(n + 2)$. = (n+1)(n+2)$\overline{(n+2)}$

   Which is equal to the original equation. When (n + 1) is plugged in it proves that every natural number will work.

b. $C(n) = \frac{1}{4}(n^4 + 2n^3 + n^2) = \frac{1}{4}n^2(n + 1)^2$.

   Solution

1. $\frac{1}{4}n^2(n + 1)^2$

   for n = 1
2. $1^3 = \frac{1}{4}1^2(1 + 1)^2$ or $1 = 1$

   for n = n
3. $n^3 = \frac{1}{4}n^2(n + 1)^2$

   for n = n+1

4. $(n+1)^3 = \frac{1}{4}(n+1)^2(n+2)^2$
   $= \frac{(n+1)^2(n+2)^2}{4}$
   The equation above proves that the sum for of the cube of n numbers is also true for n+1 which means it is correct.