CS 3530: Assignment 1c

Fall 2022

Max Stetter

Exercise 1.15 (6 points)

Problem

Give a counterexample to show that the following construction fails to prove Theorem 1.49, the closure of the class of regular languages under the star operation.[1] Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1. Construct $N = (Q_1, \Sigma, \delta, q_1, F)$ as follows. N is supposed to recognize A_1^*.

a The states of N are the states of N_1.

b The start state of N is the same as the start state of N_1.

c $F = \{q_1\} \cup F_1$.
The accept states F are the old accept states plus its start state.

d Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \notin F_1 \text{ or } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon. \end{cases}$$

(Suggestion: Show this construction graphically, as in Figure 1.50.)

Solution

a. States of N_1 are {1, 2, 3} therefore states of N are {1, 2, 3}.

b. Start state of N_1 is {1}. Therefore start of N is {1}.

c. The accept states would be 1 and 3.

d.

$N = (Q_1, \Sigma, \delta, q_1, F)$

$Q =$ set of states $\{q_1,\ q_2,\ ...q_k\}$

$\Sigma = set of alphabet\{a,b\}q_0$ = start state $\{q_0\}$

$F =$ set of final states $\{q_l\}$

[1]In other words, you must present a finite automaton, N_1, for which the constructed automaton N does not recognize the star of N_1's language.

Problem 1.60 (7 points)

Problem

Let $\Sigma = \{a, b\}$. For each $k \geq 1$, let C_k be the language consisting of all strings that contain an a exactly k places from the right-hand end. Thus $C_k = \Sigma^* a \Sigma^{k-1}$. Describe an NFA with $k+1$ states that recognizes C_k, both in terms of a state diagram and a formal description.

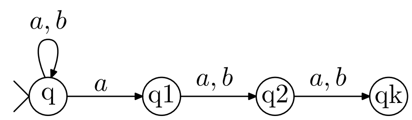
Solution

$N = (Q_1, \Sigma, \delta, q_1, F)$

Q = set of states $\{q_1,\ q_2,\ ...q_k\}$

$\Sigma = setofalphabet\{a, b\}q_0$ = start state $\{q_0\}$

F = set of final states $\{q_l\}$



Problem 1.62 (7 points)

Problem

Let $\Sigma = \{a, b\}$. For each $k \geq 1$, let D_k be the language consisting of all strings that have at least one a among the last k symbols. Thus $D_k = \Sigma^* a(\Sigma \cup \varepsilon)^{k-1}$. Describe a DFA with at most $k+1$ states that recognizes D_k, both in terms of a state diagram and a formal description.

Solution

$N = (Q_1, \Sigma, \delta, q_1, F)$

Q = set of states $\{q_1,\ q_2,\ ...q_k\}$

$\Sigma = setofalphabet\{a, b\}q_0$ = start state $\{q_0\}$

F = set of final states $\{q_l\}$