8.2 Homework #9 - 27 odd

#9. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n} = \sum_{n=1} \frac{1}{4} \cdot \left(\frac{-3^{n-1}}{4^{n-1}}\right) = \sum_{n=1} \frac{1}{4} \left(\frac{-3}{4}\right)^{n-1}$

convergent since $r < 1$.

$a = \frac{1}{4}$ $r = \frac{-3}{4}$

$\boxed{S = \frac{a}{1-r}} = \frac{\frac{1}{4}}{1 + \left(\frac{3}{4}\right)} = \frac{\frac{1}{4}}{\frac{7}{4}} \quad \frac{\frac{4}{4}}{7} = \boxed{\frac{1}{7}}$

#11. $\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}} = \frac{1}{3} \cdot \frac{\pi^{n-1}}{3^{n-1}} = \frac{1}{3} = a$ $\quad \frac{3.14}{3} = > 1$

$\boxed{\text{divergent.}}$ $\quad \frac{\pi}{3} = r$

#13 $\sum_{n=1}^{\infty} \frac{3^n}{e^{n-1}} = \frac{1}{e} \cdot \frac{3^n}{e^n}$ $\quad a = \frac{1}{e}$ $\quad r = \left(\frac{3}{e}\right)^n = \frac{3}{e}$ $\quad \frac{3}{e} > 1$ which means $\sum_{n=1}^{\infty} \frac{3^n}{e^{n-1}}$ is $\boxed{\text{divergent}}$

#15. $\sum_{n=1}^{\infty} \frac{n-1}{3n-1}$ $\quad \frac{\infty}{\infty} = \boxed{\text{divergent}}$

#17. $\sum_{n=1}^{\infty} \frac{1+2^n}{3^{n+1}} = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n + \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ $\quad \frac{\frac{1}{3} + \frac{2}{3}}{1 - \frac{1}{3} + \frac{1}{1} - \frac{2}{3}} =$

$\frac{\frac{1}{3}}{\frac{2}{3}} + \frac{\frac{2}{3}}{\frac{1}{3}} = \frac{1}{2} + 2 = \boxed{\frac{5}{2}}$ convergent.

8.2 Homework #9-27 odd

#19. $\sum_{n=1}^{\infty} \sqrt[n]{2} = \sum_{n=1}^{\infty} 2^{1/n} = 2^{1/n}$  $a=1$ $r=2$

$r>1$, $\boxed{\text{divergent}}$

#21. $\sum_{n=1}^{\infty} \arctan n = \frac{\pi}{2} \neq 0$ $\boxed{\text{Divergent}}$

#23. $\frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \frac{1}{12} + \frac{1}{15}$

$\sum_{n=1}^{\infty} \frac{1}{3 \cdot n}$ $\frac{1}{3 \cdot \infty} = \frac{1}{\infty} = \infty$ $\boxed{\text{divergent}}$

#25. $\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \frac{(n+1)-(n-1)}{(n-1)(n+1)} = \sum_{n=2}^{\infty} \frac{1}{n-1} - \frac{1}{n+1}$

$\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) =$

$\frac{1}{1} + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$  $1 + \frac{1}{2} - 0 - 0 = \boxed{\frac{3}{2}}$

$\boxed{\text{Convergent}}$

#27. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$  $3 = A(n+3) + B(n)$

$A = 1$

$B = -3$

$= \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+3}\right)$ $\left(\frac{1}{1} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{5}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{7}\right) + \left(\cancel{\frac{1}{5}} - \frac{1}{8}\right) =$

$\left(\cancel{\frac{1}{n-3}} - \frac{1}{n}\right) + \left(\cancel{\frac{1}{n-2}} - \cancel{\frac{1}{n+1}}\right) + \left(\cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n+2}}\right) + \left(\cancel{\frac{1}{n}} - \cancel{\frac{1}{n+3}}\right) = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} \frac{1}{n+1} \ldots$

$\frac{11}{6} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3} = \frac{11}{6} - 0 = \boxed{\frac{11}{6}}$ Convergent