CS 3530: Assignment 7a

Fall 2022

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Problem 7.6 (10 points)

Problem

Show that P is closed under union and concatenation.

Hint: As we discussed in class, construct the language (e.g. $PCAT = \{\langle P_1, P_2, w \rangle | P_1, P_2 \in P$ and w is a string, $w = x \cdot y$, $x \in P_1$ and $y \in P_2\})$, then provide a deterministic machine that decides the language in polynomial time.

Solution to P closed under union.

Two languages, P_1 and P_2.

Turing machine M that takes $P_1 \bigcup P_2$.

M = on input w length of n:

1. check if w εP_1.

2. if not, check if w εP_2.

3. accept w iff P_1 or P_2 accepts.

4. if both reject then reject.

overall time is polynomial

Solution to P closed under concatenation.

Two languages, P_1 and P_2.

Turing machine M that takes $P_1 P_2$.

M = on input w length of n:

1. w can be split into two strings in n different ways.

2. for each split,

a. check if first substring belongs to P_1.

b. check if second substring belongs to P_2.

if any split succeeds, accept.

overall time is polynomial

Problem 7.5 (10 points)

Is the following formula satisfiable? (Give your reasoning.)

$$(x \lor y) \land (x \lor \overline{y}) \land (\overline{x} \lor y) \land (\overline{x} \lor \overline{y})$$

Solution

The formula is not satisfiable because there are cases where the formula evaluates to 0.

Case 1:

x = 0

y = 1

$(0 \lor 1) \land (0 \lor \overline{1}) \land (\overline{0} \lor 1) \land (\overline{1} \lor \overline{1})$ evaluates to 0

Case 2:

x = 1

y = 0

$(0 \lor 1) \land (0 \lor \overline{1}) \land (\overline{0} \lor 1) \land (\overline{1} \lor \overline{1})$ evaluates to 0

Therefore the formula is not satisfiable.