5 Homework # 3-19 odd

#3. $\sum_{n=1}^{\infty} (-1)^n n x^n = \dfrac{-1^{n+1} \cdot (n+1) x^{n+1}}{}$

$\left| \dfrac{(n+1)x}{n} \right|$

$\dfrac{-1^{n+1} (n+1) x^{n+1}}{-1^n n x^n} = \cancel{-1^n} \cdot -1 \cdot (n+1) \cdot \cancel{x^n} + x^1 \cdot \dfrac{1}{\cancel{-1^n} n \cancel{x^n}}$

$\dfrac{-1 \cdot x(n+1)}{n} = -1 \cdot 1 + \dfrac{1}{n} \cdot |x|$

By ratio, converges ~~if~~

radius of convergence = 1

interval is -1 to 1

#5. $\sum_{n=1}^{\infty} \dfrac{x^n}{2n-1}$ $\dfrac{x^{n+1}}{2(n+1)-1} \cdot \dfrac{2n-1}{x^n} = \dfrac{\cancel{x^n} + x^1}{2(n+1)-1} \cdot \dfrac{2n-1}{\cancel{x^n}}$

$\dfrac{x}{2}$ $x \cdot \dfrac{2n-1 \div n}{2n+1 \div n} =$ $2n+2-1$

$2n-1+2$

$x \cdot \dfrac{2-\frac{1}{n}}{2+\frac{1}{n}} = x \cdot 1 = |x|$ ~~since $|x| < 1$~~

convergence when $|x| < 1$ so

$R = 1$. $\dfrac{1}{2n-1}$ $\dfrac{1}{2n-1} > \dfrac{1}{n}$ $-1^n \dfrac{1}{2n-1}$ converges

interval is -1 and 1. by Alternating

$[-1, 1)$ Series test.

#7. $\sum_{n=0}^{\infty} \dfrac{x^n}{n!}$ $\dfrac{x^{n+1}}{(n+1)!}$ $\dfrac{\cancel{x^n} \cdot x^1}{(n+1)!} \cdot \dfrac{n!}{\cancel{x^n}} = \dfrac{x^{n+1}}{x^n} \cdot \dfrac{\cancel{n!}}{\cancel{n!}(n+1)!}$

$\dfrac{x}{n+1} = 0 < 1$, converge for every x. $R = \infty$

interval is $(-\infty, \infty)$

8.5 Homework # 3-19 odd

#9. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 x^n}{2^n}$ $\quad \frac{(n+1)^2 x^{n+1}}{2^{n+1}} \cdot \frac{2^n}{n^2 x^n} =$

$\frac{(n+1)^2 \cancel{x^n} x^1}{\cancel{2^n} \cdot 2^1} \cdot \frac{\cancel{2^n} 1}{n^2 \cancel{x^n}} = \frac{1}{2} \cdot \frac{(n+1)^2}{n^2} =$

$\frac{1}{2} \cdot \left(\frac{n+1}{n}\right)^2 = \frac{1}{2} \frac{n+1}{n} = \frac{x}{2} \left(1 + \frac{1}{n}\right)^2 =$

$\lim_{n \to \infty} \frac{x}{2} \cdot (1+0)^2 = \frac{x}{2} \cdot 1 = \frac{|x|}{2}$ $\quad \frac{x}{2} < 1$ convergent

$2 \cdot \frac{x}{2} < 1 \cdot 2$

$\boxed{-2 < x < 2}$ $\boxed{(-2, 2)}$

#11. $\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{4^n \ln n} =$

$\lim_{n \to \infty} \frac{x^{n+1}}{4^{n+1} \ln(n+1)} \cdot \frac{4^n \ln n}{x^n}$

$\lim_{n \to \infty} \frac{\cancel{x^n} x^1}{\cancel{4^n} 4^1 \ln(n+1)} \cdot \frac{\cancel{4^n} \ln n}{\cancel{x^n}}$ $\lim_{n \to \infty} \frac{x \ln(n)}{4 \ln(n+1)} = \frac{x}{4} \cdot \frac{\ln(n)}{\ln(n+1)}$

$\xrightarrow{L} \frac{x}{4} \lim_{n \to \infty} \frac{\frac{1}{n}}{\frac{1}{n+1}} = \frac{x}{4} \lim_{n \to \infty} \frac{n+1}{n} = \frac{x}{4} \lim_{n \to \infty} 1 + \frac{1}{n} =$

$\frac{x}{4} \cdot (1+0) = \frac{|x|}{4}$ $\quad \frac{2}{4} = \frac{1}{2} < 1$ converge by ratio.

$\frac{-4^2}{4^2 \ln(2)} = \sum_{n=2}^{\infty} \frac{1}{\ln n}$ diverge $\quad 4 \cdot \frac{x}{4} < 1$

$-4 < x \leq 4$

converge for 4

$\boxed{(-4, 4]}$

9.5. Homework # 3-19 odd

#13. $\sum_{n=1}^{\infty} \frac{(-3)^n}{n\sqrt{n}} x^n = \lim_{n\to\infty} \frac{(-3)^{n+1}}{n+1\sqrt{n+1}} \cdot \frac{n\sqrt{n}}{-3^n} = \frac{\cancel{-3^n} \cdot -3^1}{n+1\sqrt{n+1}} \cdot \frac{n\sqrt{n}}{\cancel{-3^n}}$

$\lim_{n\to\infty} \frac{-3\, n\sqrt{n}}{n+1\sqrt{n+1}} = -3x \lim_{n\to\infty} \frac{n}{n+1} \cdot \frac{\sqrt{n}}{\sqrt{n+1}} = 1$

$\downarrow \quad \downarrow$

$1 \quad 1$

$|3x| \leq 1 \quad -\frac{1}{3} \cdot \frac{-3^1}{1\sqrt{1}} = \frac{1}{1}$ is convergent

$-\frac{1}{3} < x < \frac{1}{3}$

$\frac{\frac{1}{3} \cdot -3^n}{n\sqrt{n}} = \frac{-1^n}{n\sqrt{n}} = 0$

A.S.T Convergent.

$\boxed{\left[-\frac{1}{3}, \frac{1}{3}\right]}$ interval

#15. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1} = \lim_{n\to\infty} \frac{(x-2)^{n+1}}{(n+1)^2+1} \cdot \frac{n^2+1}{(x-2)^n} =$

$\frac{1\cancel{(x-2)^n} \cdot (x-2)^1}{(n+1)^2+1} \cdot \frac{\cancel{n^2}+1}{\cancel{(x-2)^n}\, 1} = \frac{x-2}{2n+2} \cdot \frac{n^2+1 \div n^2}{(n+1)^2+1 \div n^2}$

$\cancel{n^2}+2n+1$

$|x-2| \cdot \frac{1+\frac{1}{n^2}}{\left(\frac{n+1}{n}\right)^2+\frac{1}{n^2}} =$

$\frac{1+\frac{1}{n^2}}{\left(1+\frac{1}{n}\right)^2+\frac{1}{n^2}} = \frac{1+0}{1+0+0} = x-2$

$x-2<1$
$+1$

$1<x<3$

$\frac{(1-2)^n}{n^2+1} = \frac{-1^n}{n^2+1}$ = convergent.

$\frac{(3-2)^n}{n^2+1} = \frac{1^n}{n^2+1}$ convergent

$\boxed{[1, 3]}$

8.5 Homework # 3-19 odd

#17. $\sum_{n=1}^{\infty} \frac{n}{B^n}(x-a)^n \quad B>0 = \lim_{n\to\infty} \frac{n+1}{B^{n+1}} \cdot (x-a)^{n+1} \cdot \frac{B^n}{n} \cdot \frac{1}{(x-a)^n}$

$\lim_{n\to\infty} \frac{x-a}{B} \cdot \frac{n+1}{n} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{x-a}{B} \frac{(1+\frac{1}{n})}{1} = \left|\frac{x-a}{B}\right| < 1$

$a-B < x < a+B \quad \boxed{R=B} \quad \sum_{n=1}^{\infty} \frac{n}{B^n}(a-B-a)^n = \text{divergent}$

$\boxed{(a-B, a+B)} \quad \sum_{n=1}^{\infty} \frac{n}{B^n}(a+B-a)^n = \text{divergent}$

#19. $\sum_{n=1}^{\infty} n!(2x-1)^n = \lim_{n\to\infty} (n+1)!\,(2x-1)^{n+1} \cdot \frac{1}{n!\,(2x-1)^n} = \frac{2x-1\,(n+1)!}{n!}$

$= 2x-1 \cdot \frac{(n+1)!}{n!} = 2x-1 \cdot \frac{n+1}{1} = 2x-1 \cdot n+1 \to \infty$

$x \neq \frac{1}{2}$ $\boxed{\text{interval is } [\frac{1}{2}, \frac{1}{2}]}$ $\boxed{R=0}$