

CS 3530: Assignment 4b

Fall 2022

Max Stetter

Exercise 4.4 (10 points)

Problem

Let $A_{\varepsilon\text{CFG}} = \{\langle G \rangle \mid G \text{ is a CFG that generates } \varepsilon\}$. Show that $A_{\varepsilon\text{CFG}}$ is decidable.

Solution

Create a TM M that decides $A_{\varepsilon\text{CFG}}$.

Run M on input $\langle G \rangle$

If M accepts, Accept.

If M rejects, Reject.

Exercise 4.5abcdef (10 points)

Problem

Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{6, 7, 8, 9, 10\}$. We describe the functions $f : X \rightarrow Y$ and $g : X \rightarrow Y$ in the following tables. Answer each part and give a reason for each answer.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

n	$g(n)$
1	10
2	9
3	8
4	7
5	6

a. Is f one-to-one?

b. Is f onto?

c. Is f a correspondence?

d. Is g one-to-one?

e. Is g onto?

f. Is g a correspondence?

Solution

- a. No because multiple inputs lead to some of the same answers.
- b. No because the numbers 8, 9 and 10 do not have a corresponding X.
- c. No because it is not one-to-one and it is not onto.
- d. Yes because each X maps to a unique Y.
- e. Yes because each Y value is included.
- f. Yes because it is one-to-one and it is onto.