Introductory Real Analyis Exercises

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Chaper 1.2 Exercises

Problem 1

Prove that a set with an uncountable subset is itself uncountable.

Given a set M with an uncountable subset A we can see that the set M is uncountable since any counting of M must eventually count all the elements of A, which is impossible.

Problem 2

Let M be any infinite set and A any countable set. Prove that M $M \cup A$.

$\square$

Problem 3

Prove that each of the following sets is countable:

$\square$

Problem 4

A number α is called *algebraic* if it is the roof of a polynomial equation with rational coefficients. Prove that the set of all algebraic numbers is countable.

$\square$

Problem 5

Prove the existance of uncountably many *transcendental* numbers.

$\square$

Problem 6

Prove that the set of all real functions defined on a set M is of power greater than the power of M. In particular, prove that the power of the set of all real functions defined in the unit interval is greater tha c. (Use the fact that the set of charachteristic functions on M is equivalent to the set of all subsets of M.

$\square$

Problem 7

Give an indirect proof of the equivalence of the closed interval $[a, b]$, the open interval (a, b), and the half open interval $(a, b]$ or $[b, a)$.

$\square$

Problem 8

Prove that the union of a finite or countable number of set each of power c is itself of power c.

□

Problem 9

Proce that each of the following sets has the power of the continuum:

a) The set of all infinite sequences of positive integers.

b) The set of all ordered n-tuples of real numbers.

c) The set of all infinite sequences of real numbers.

□

Problem 10

Develop a contradiction inherent in the notion of the ”set of all sets which are not members of themselves.”'

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