Introductory Real Analyis Exercises

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Chaper _ Exercises

Problem 1

Let X be an uncountable set, and let $\mathscr{R}$ be the ring consisting of all finite subsets of X and their complements. Is $\mathscr{R}$ a $\sigma-$ring?

Problem 2

Are open intervals Borel sets?

Problem 3

Let $y = f(x)$ be a function defined on a set M and taking values in a set N. Let $\mathcal{M}$ be a system of subsets of M, and let $f(\mathcal{M})$ denote the system of all images $f(A)$ of sets $A \in \mathcal{M}$. Moreover, let $\mathcal{N}$ be a system of subsets of N, and let $f^{-1}(\mathcal{N})$ denote the system of all preimages $f^{-1}(B)$ of sets $B \in \mathcal{N}$. Prove that

a) If $\mathcal{N}$ is a ring, so is $f^{-1}(\mathcal{N})$.

b) If $\mathcal{N}$ is an algebra, so is $f^{-1}(\mathcal{N})$.

c) If $\mathcal{N}$ is a B-algebra, so is $f^{-1}(\mathcal{N})$.

d) $\mathscr{R}(f^{-1}(\mathcal{N})) = f^{-1}(\mathscr{R}(\mathcal{N}))$.

e) $\mathscr{B}(f^{-1}(\mathcal{N})) = f^{-1}(\mathscr{B}(\mathcal{N}))$.

Which of these assertions remain true if $\mathcal{N}$ is replaced by $\mathcal{M}$ and f^{-1} by f?