Introductory Real Analyis Exercises

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Chaper 1.3 Exercises

Problem 1

Exhibit both a partial ordering and a simple ordering of the set of all complex numbers.

Problem 2

What is the minimal element of the set of all subset of a given set X, partially ordered by set inclusion? What is the maximal element.

Problem 3

A partially ordered set M is said to be a *directed set* if, given any two elements $a, b \in M$, there is an element $c \in M$ such that $a \leq c, b \leq c$. Are the partially ordered sets in Examples 1-4, Sec 3.1 all directed sets?

Problem 4

Prove that the set of all subsets of a given set X, ordered by set inclusion, is a lattice. What is the set theoretic meaning of the greatest lower bound and least upper bound of two elements of this set?

Problem 5

Prove that an order preserving mapping of one ordered set onto another is automatically an isomorphism.

Problem 6

Prove that ordered sums and products of ordered sets are associative.

Problem 7

Construct well ordered sets with ordinals

$$\omega + n, \omega + \omega, \omega + \omega + n, \omega + \omega + \omega, \ldots$$

Problem 8

Construct well ordered sets with ordinals

$$\omega \cdot n, \omega^2 \cdot n, \omega^3, \dots$$

Show that the sets are all countable.

Problem 9

Show that

$$\omega + \omega = \omega \cdot 2, \omega + \omega + \omega = \omega \cdot 3, \dots$$

Problem 10

Prove that the set $W(\alpha)$ of all ordinals less that a given ordinal α is well-ordered.

Problem 11

Prove that any nonempty set of ordinals is well-ordered.

Problem 12

Prove that the set M of all ordinals corresponding to a countable set is itself uncountable.

Problem 13

Let $\aleph_1$ be the power of the set M in the preceding problem. Prove that there is no power m such that $\aleph_0 < m < \aleph N_1$.