ECS7012P - Music and Audio Programming
Assignment 1: Synth Filter

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Contents

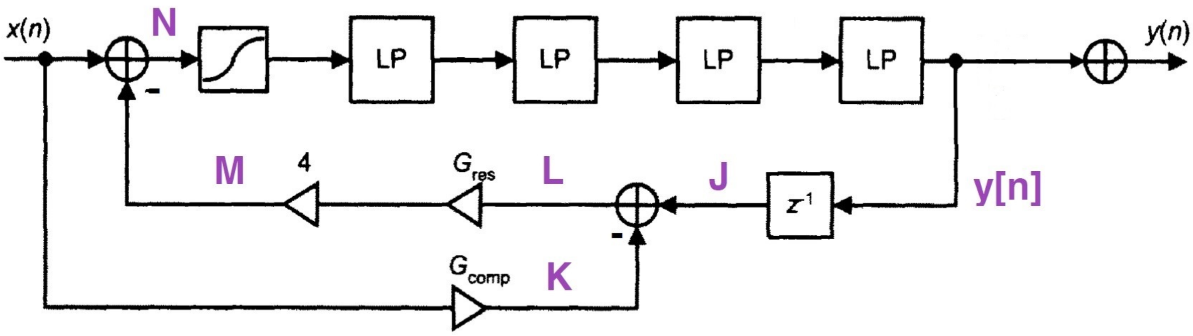

Figure 1: Overall structure of the implemented Moog voltage-controlled filter, adapted from [2]

1 Introduction

This report discusses the implementation of a digital emulation [1] of a Moog voltage-controlled filter. The overall structure of this filter is depicted in Figure 1.

2 First-order filter section

2.1 Derivation

As proposed in [2], one of the four first-order sections of the filter is implemented as depicted in Figure 2. The equations for the individual nodes annotated are given in Equations 1 to 4.

$$A = \frac{1}{1.3} \cdot x[n] \qquad B = x[n-1] \tag{1}$$

$$C = \frac{0.3}{1.3} \cdot B \qquad D = A + C \tag{2}$$

$$E = y[n-1] \qquad F = D - E \tag{3}$$

$$G = g \cdot F \qquad y[n] = E + G \tag{4}$$

The filter equation given in Equation 5 was then derived from these equations.

$$y[n] = g\frac{1}{1.3} \cdot x[n] + g\frac{0.3}{1.3} \cdot x[n-1] + (1-g) \cdot y[n-1] \tag{5}$$

The filter equation was then compared to the standard form of a first-order IIR filter, which is given in Equation 6.

$$y[n] = b_0 \cdot x[n] + b_1 \cdot x[n-1] - a_1 \cdot y[n-1] \tag{6}$$

The resulting filter coefficients b_0, b_1 and a_1 were then calculated and are given in Equation 7.

$$b_0 = g\frac{1}{1.3} \qquad b_1 = g\frac{0.3}{1.3} \qquad a_1 = g - 1 \tag{7}$$

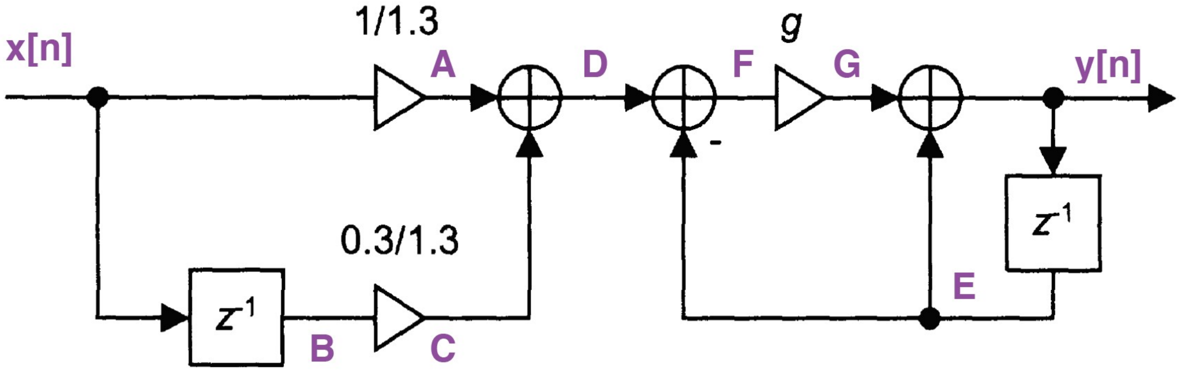

Figure 2: First-order section of the filter depicted in Figure 1 [2]

2.2 Performance

The parameter g of the filter coefficients firstly calculated simply by $g = 2\pi \cdot f_c/f_s$. The filter frequency response using this formula is given in Figure 3 for the two different cutoff frequencies. It is clearly visible that the intended cutoff frequencies were not met. In Figure 3, for $f_c = 1$ kHz, the response shows its cutoff at approximately 1.1 kHz, resulting in an error of 10 %. For $f_c = 4$ kHz, Figure 3 even shows its cutoff frequency at approximately 5.57 kHz - an error of 39.3 %.

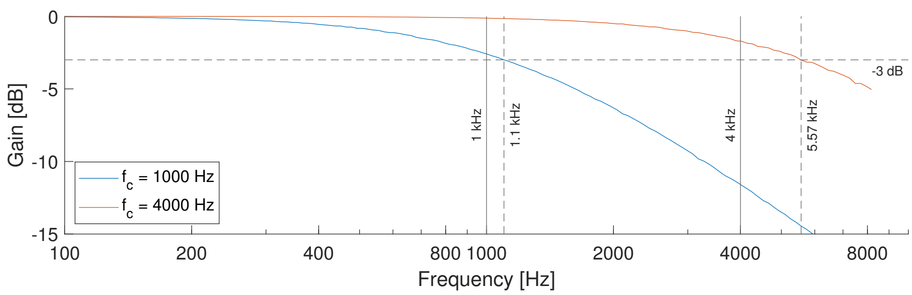

Figure 3: Filter responses for primitive calculation of parameter g

To mitigate the nonlinear relation of f_c to the actual cutoff frequency, a polynomial model for the parameter g is proposed in [2]. The model is given in Equation 8, where $\omega_c = 2\pi \cdot f_c/f_s$. The result for the frequency response of the filter is given in Figure 4. For $f_c = 1$ kHz, the error was reduced to 2 % (actual cutoff 1.02 kHz). For $f_c = 4$ kHz, the real cutoff frequency is around 4.12 kHz, a significantly improved error of only 3 %.

$$g = 0.9892\omega_c - 0.4342\omega_c^2 + 0.1381\omega_c^3 - 0.0202\omega_c^4 \tag{8}$$

3 Fourth-order filter

The combination of four first-order filter segments (discussed in Section 2) results in a fourth-order filter. The frequency response of this combination is given in Figure 5. At the -3 dB-frequency of the first-order section, the fourth-order response shows a gain of -12 dB. This is expected, as four identical filter elements with -3 dB gain are combined.

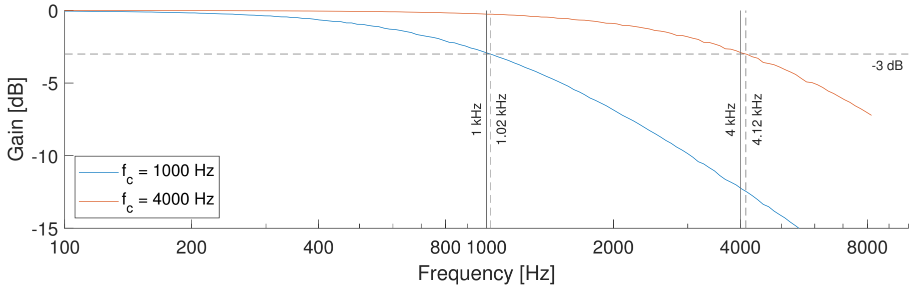

Figure 4: Filter responses for polynomial fit of parameter g (see Equation 8)

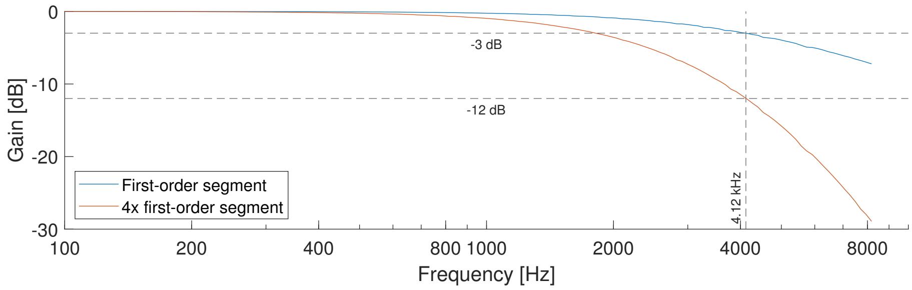

Figure 5: Filter response for four of the first-order sections combined, each using $f_c = 1\,\text{kHz}$

4 Nonlinearity

The nonlinear function $\tanh(x)$ is applied to the input before filtering, which is depicted in Figure 1. This introduces signal distrotion. Especially for high amplitudes, the peaks of input sine waves are flattened. In the frequency domain, this effect introduces additional harmonics. For low amplitudes however, the function $\tanh(x)$ is very close to linear and does not introduce significant harmonics.

5 Feedback

The feedback path depicted in Figure 1 requires transformation into an equation. Therefore, the equations for the nodes annotated are derived in Equations 9 to 11. For the implementation, the parameter G_{comp} is set to a fixed value of 0.5. An example response is shown in Figure 6, which shows a resonant peak at 870 Hz with a gain of 7.2 dB.

$$J = y[n-1] \qquad K = G_{comp} \cdot x[n] \tag{9}$$
$$L = J - K \qquad M = 4G_{res} \cdot L \tag{10}$$

$$N = x[n] - M = x[n] - 4G_{res} \cdot (y[n-1] - G_{comp} \cdot x[n]) \tag{11}$$

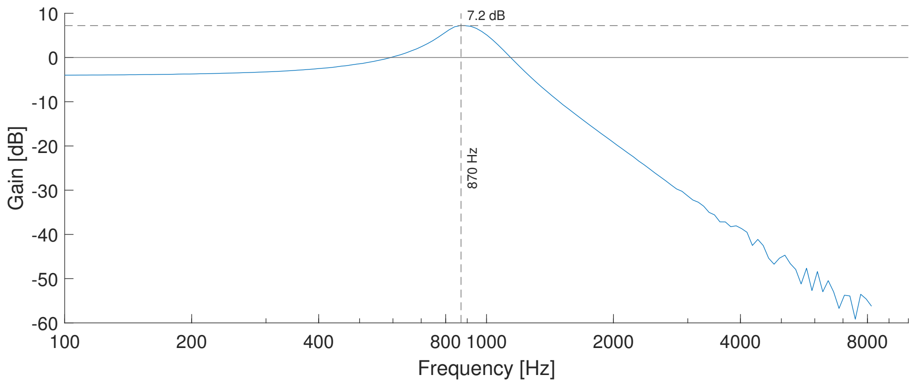

Figure 6: Filter system response using feedback, first-order segments with $f_c = 1\,\text{kHz}$ and $G_{res} = 0.75$ (peak gain $7.2\,\text{dB}$)

References

[1] T. S. Stilson and J. O. Smith, “Analyzing the moog vcf with considerations for digital implementation,” 1996.

[2] V. Välimäki and A. Huovilainen, “Oscillator and filter algorithms for virtual analog synthesis,” *Computer Music Journal*, vol. 30, no. 2, pp. 19–31, 2006. [Online]. Available: `http://www.jstor.org/stable/3682001`.