ECS7012P - Music and Audio Programming
Assignment 1: Synth Filter

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Contents

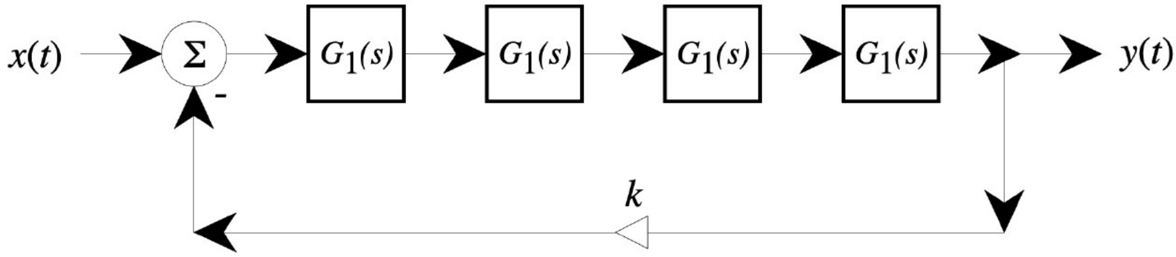


Figure 1: Overall structure of the Moog voltage-controlled filter [1]

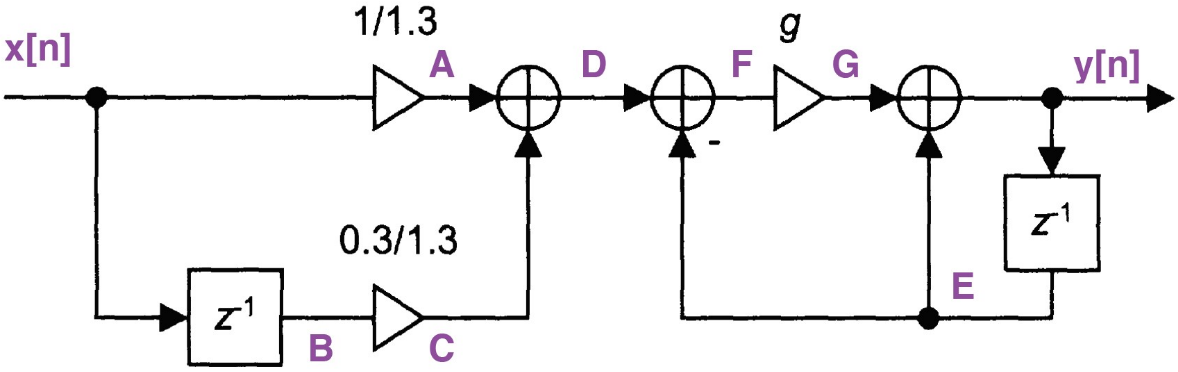


Figure 2: First-order section of the filter depicted in Figure 1 [2]

1 Introduction

This report discusses the implementation of a digital emulation of a Moog voltage-controlled filter. The overall structure of this filter is depicted in Figure 1.

2 First-order filter section

As proposed in [2], one of the four first-order sections of the filter is implemented as depicted in Figure 2. The equations for the individual nodes annotated are given in Equations 1 to 4. The filter equation was derived from these equations to be Equation 5. By comparing the filter equation the standard form of a first-order IIR filter, which is given in Equation 6, the filter coefficients b_0, b_1 and a_1 were calculated. The results are given in Equation 7.

$$A = \frac{1}{1.3} \cdot x[n] \qquad B = x[n-1] \tag{1}$$

$$C = \frac{0.3}{1.3} \cdot B \qquad D = A + C \tag{2}$$

$$E = y[n-1] \qquad F = D - E \tag{3}$$

$$G = g \cdot F \qquad y[n] = E + G \tag{4}$$

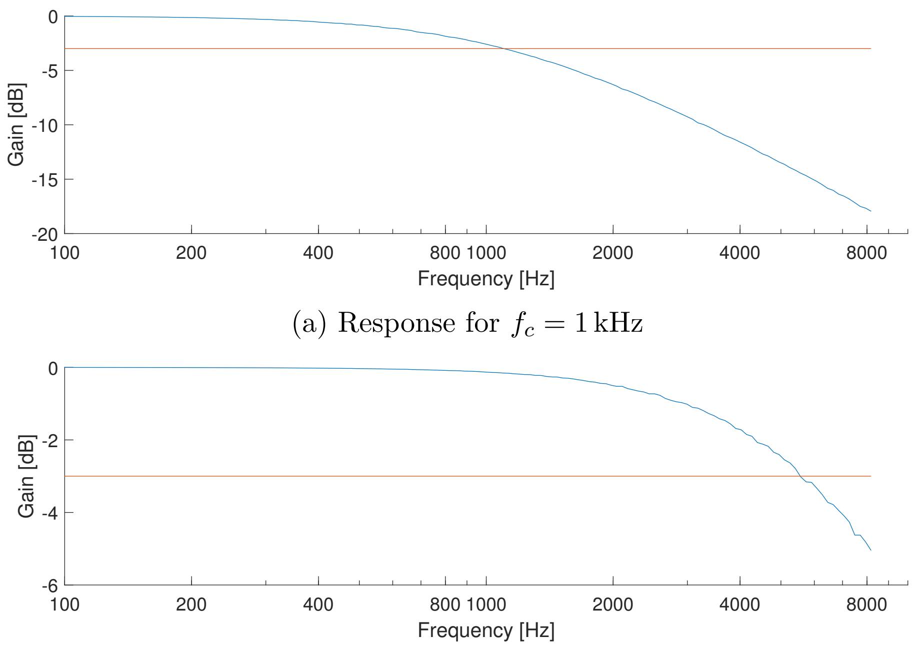
$$y[n] = g\frac{1}{1.3} \cdot x[n] + g\frac{0.3}{1.3} \cdot x[n-1] + (1-g) \cdot y[n-1] \tag{5}$$

$$y[n] = b_0 \cdot x[n] + b_1 \cdot x[n-1] - a_1 \cdot y[n-1] \tag{6}$$

$$b_0 = g\frac{1}{1.3} \qquad b_1 = g\frac{0.3}{1.3} \qquad a_1 = g - 1 \tag{7}$$

2.1 Performance

The parameter g of the filter coefficients firstly calculated simply by $g = 2\pi \cdot f_c/f_s$. The filter frequency response using this formula is given in Figure 3 for the two different cutoff frequencies. It is clearly visible that the intended cutoff frequencies were not met. In Figure 3a, for $f_c = 1\,\text{kHz}$, the response shows its cutoff at approximately 1.1 kHz, resulting in an error of 10 %. For $f_c = 4\,\text{kHz}$, Figure 3b even shows its cutoff frequency at approximately 5.57 kHz - an error of 39.3 %.



(a) Response for $f_c = 1\,\text{kHz}$

(b) Response for $f_c = 4\,\text{kHz}$

Figure 3: Filter responses for primitive calculation of parameter g

To mitigate the nonlinear relation of f_c to the actual cutoff frequency, a polynomial model for the parameter g is proposed in [2]. The model is given in Equation 8, where $\omega_c = 2\pi \cdot f_c/f_s$. The result for the frequency response of the filter is given in Figure 4. The actual cutoff frequency is around 4.12 kHz, a significantly improved error of only 3 %.

$$g = 0.9892\omega_c - 0.4342\omega_c^2 + 0.1381\omega_c^3 - 0.0202\omega_c^4 \tag{8}$$

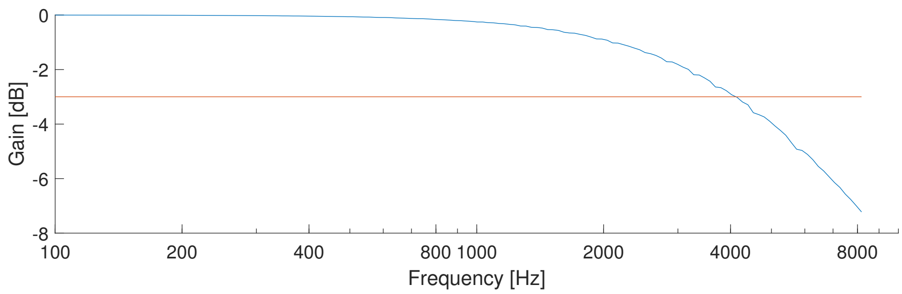


Figure 4: Filter response for polynomial fit of parameter g (see Equation 8)

3 Methods

4 Results

5 Summary and Outlook

References

[1] T. S. Stilson and J. O. Smith, "Analyzing the moog vcf with considerations for digital implementation," 1996.

[2] V. Välimäki and A. Huovilainen, "Oscillator and filter algorithms for virtual analog synthesis," *Computer Music Journal*, vol. 30, no. 2, pp. 19–31, 2006. [Online]. Available: `http://www.jstor.org/stable/3682001`.