ECS7012P - Music and Audio Programming
Assignment 1: Synth Filter

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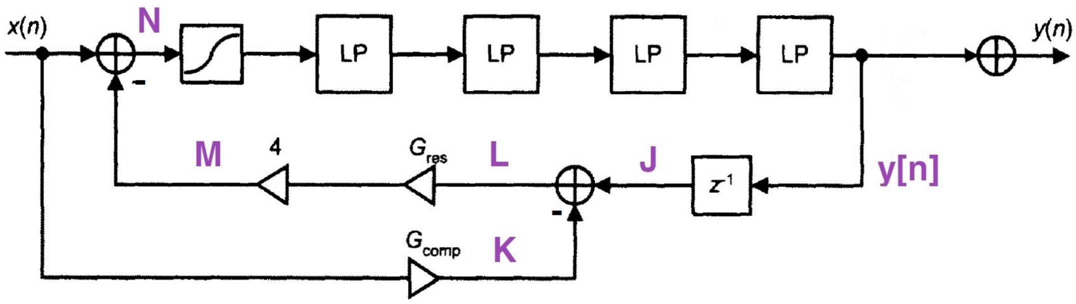


Figure 1: Overall structure of the implemented Moog voltage-controlled filter, adapted from [2]

1 Introduction

This report discusses the implementation of a digital emulation [1] of a Moog voltage-controlled filter. The overall structure of this filter is depicted in Figure 1. It consists out of four first-order low-pass filters, a simple nonlinear function and a feedback loop.

2 First-order filter section

2.1 Derivation

As proposed in [2], one of the four first-order sections of the filter is implemented as depicted in Figure 2. The equations for the individual nodes annotated are given in Equations 1 to 4.

$$A = \frac{1}{1.3} \cdot x[n] \qquad B = x[n-1] \tag{1}$$

$$C = \frac{0.3}{1.3} \cdot B \qquad D = A + C \tag{2}$$

$$E = y[n-1] \qquad F = D - E \tag{3}$$

$$G = g \cdot F \qquad y[n] = E + G \tag{4}$$

The filter equation given in Equation 5 was then derived from these equations.

$$y[n] = g\frac{1}{1.3} \cdot x[n] + g\frac{0.3}{1.3} \cdot x[n-1] + (1-g) \cdot y[n-1] \tag{5}$$

The filter equation was then compared to the standard form of a first-order IIR filter, which is given in Equation 6.

$$y[n] = b_0 \cdot x[n] + b_1 \cdot x[n-1] - a_1 \cdot y[n-1] \tag{6}$$

The resulting filter coefficients b_0, b_1 and a_1 were then calculated and are given in Equation 7.

$$b_0 = g\frac{1}{1.3} \qquad b_1 = g\frac{0.3}{1.3} \qquad a_1 = g - 1 \tag{7}$$

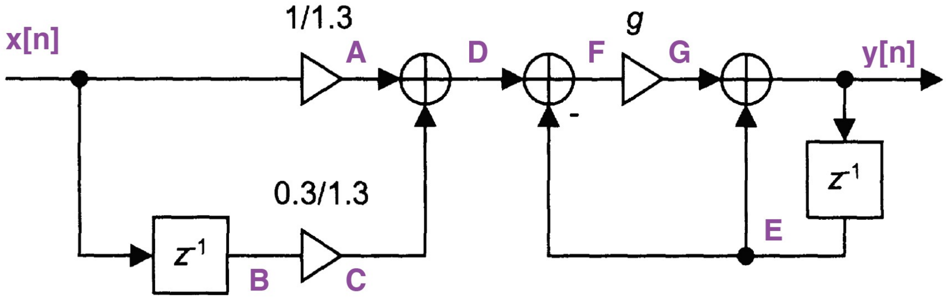


Figure 2: First-order section of the filter depicted in Figure 1, adapted from [2]

2.2 Performance

The parameter g of the filter coefficients firstly calculated simply by $g = 2\pi \cdot f_c/f_s$. The filter frequency response using this formula is given in Figure 3a for the two different cutoff frequencies. It is clearly visible that the intended cutoff frequencies were not met. In Figure 3a, for $f_c = 1\,\text{kHz}$, the response shows its cutoff at approximately 1.1 kHz, resulting in an error of 10 %. For $f_c = 4\,\text{kHz}$, Figure 3a even shows its cutoff frequency at approximately 5.57 kHz - an error of 39.3 %.

To mitigate the nonlinear relation of f_c to the actual cutoff frequency, a polynomial model for the parameter g is proposed in [2]. The model is given in Equation 8, where $\omega_c = 2\pi \cdot f_c/f_s$. The result for the frequency response of the filter is given in Figure 3b. For $f_c = 1\,\text{kHz}$, the error was reduced to 2 % (actual cutoff 1.02 kHz). For $f_c = 4\,\text{kHz}$, the real cutoff frequency is around 4.12 kHz, a significantly improved error of only 3 %.

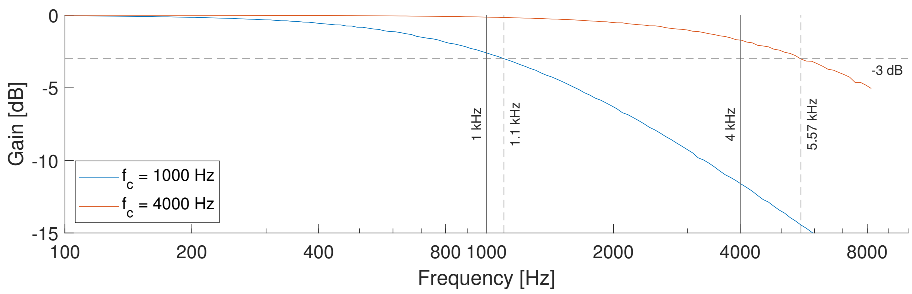
$$g = 0.9892\omega_c - 0.4342\omega_c^2 + 0.1381\omega_c^3 - 0.0202\omega_c^4 \tag{8}$$

3 Fourth-order filter

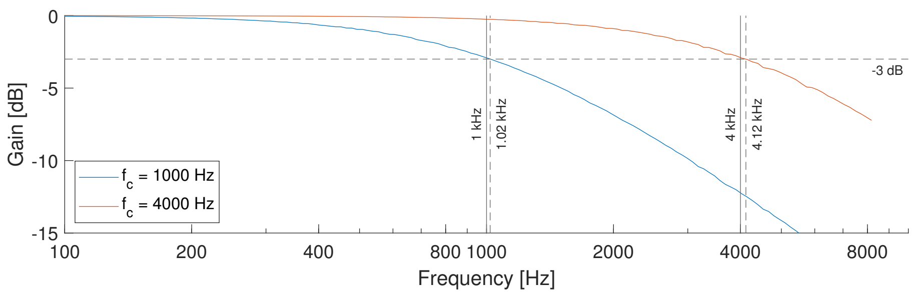
The combination of four first-order filter segments (discussed in Section 2) results in a fourth-order filter. The frequency response of this combination is given in Figure 4. At the $-3\,\text{dB}$-frequency of the first-order section, the fourth-order response shows a gain of $-12\,\text{dB}$. This is expected, as four identical filter elements with $-3\,\text{dB}$ gain are combined.

4 Nonlinearity

The nonlinear function $\tanh(x)$ is applied to the input before filtering, which is depicted in Figure 1. This introduces signal distrotion. Especially for high amplitudes, the peaks of input sine waves are flattened. In the frequency domain, this effect introduces additional harmonics. For low amplitudes however, the function $\tanh(x)$ is very close to linear and does not introduce significant harmonics.



(a) Primitive calculation ($g = 2\pi \cdot f_c/f_s$)



(b) Polynomial fit (see Equation 8)

Figure 3: Filter responses for different calculations of parameter g

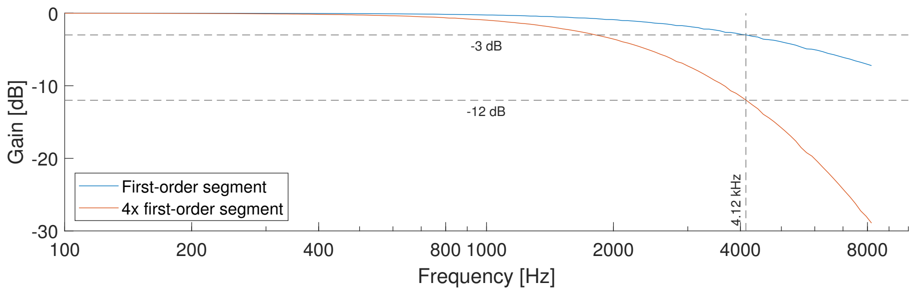


Figure 4: Filter response for four of the first-order sections combined, each using $f_c = 4\,\text{kHz}$

5 Feedback

The feedback path depicted in Figure 1 requires transformation into an equation. Therefore, the equations for the nodes annotated are derived in Equations 9 to 11. For the implementation, the parameter G_{comp} is set to a fixed value of 0.5. An example response is shown in Figure 5, which shows a resonant peak at 870 Hz with a gain of 7.2 dB.

$$J = y[n-1] \qquad K = G_{comp} \cdot x[n] \tag{9}$$

$$L = J - K \qquad M = 4G_{res} \cdot L \tag{10}$$

$$N = x[n] - M = x[n] - 4G_{res} \cdot (y[n-1] - G_{comp} \cdot x[n]) \tag{11}$$

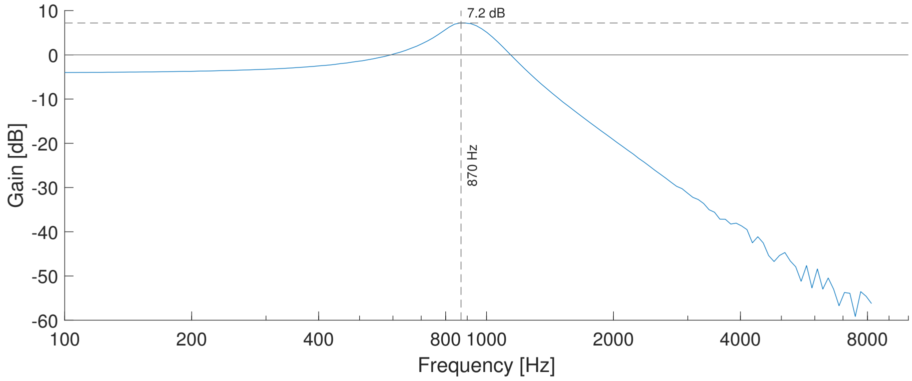


Figure 5: Filter system response using feedback, first-order segments with $f_c = 1\,\text{kHz}$ and $G_{res} = 0.75$

6 Adjustable resonance

Using the parameter $C_{res} \in [0, 1]$, the resonance peak can be adjusted by using Equation 12 for G_{res} [2]. The resonance frequency can simply be adjusted by re-calculating the first-order filter segments coefficients for a different cutoff frequency.

$$G_{res} = C_{res} \cdot (1.0029 + 0.0526\omega_c - 0.0926\omega_c^2 - 0.0218\omega_c^3) \tag{12}$$

7 Experiments

As a simple experiment, a sawtooth oscillator with frequency $f = 100\,\text{Hz}$ and amplitude $A = 0.3$ was used as an input to the implemented system (using $C_{res} = 0.9$ and $f_c = 2\,\text{kHz}$). Figure 6 shows the fast-fourier transformation of this input and the resulting output. The resonant peak is clearly visible at approximately $2\,\text{kHz}$.

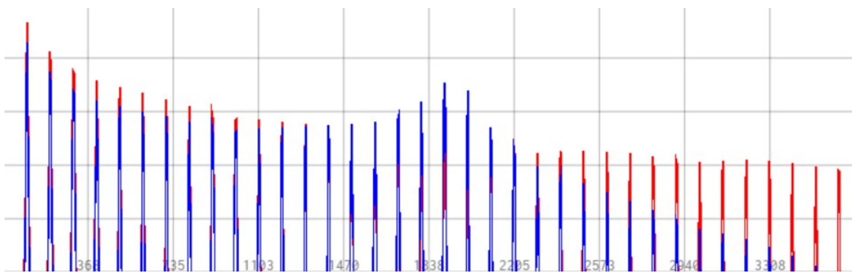


Figure 6: FFT of sawtooth oscillator input with $f = 100\,\text{Hz}$ and amplitude $A = 0.3$ (red) and the resulting output using $C_{res} = 0.9$ and $f_c = 2\,\text{kHz}$ (blue)

Additionally, using the same setting, the resonance parameter was maximised to $C_{res} = 1$. After turning down the input oscillator amplitude to zero, the system output showed its resonance frequency only for approximately $1\,\text{s}$.

References

[1] T. S. Stilson and J. O. Smith, "Analyzing the moog vcf with considerations for digital implementation," 1996.

[2] V. Välimäki and A. Huovilainen, "Oscillator and filter algorithms for virtual analog synthesis," *Computer Music Journal*, vol. 30, no. 2, pp. 19–31, 2006. [Online]. Available: `http://www.jstor.org/stable/3682001`.