ECS7012P - Music and Audio Programming
Assignment 1: Synth Filter

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Contents

1 Introduction

This report discusses the implementation of a digital emulation [1] of a Moog voltage-controlled filter [2]. The overall structure of this filter is depicted in Figure 1. It consists out of four first-order low-pass filters, a simple nonlinear function and a feedback loop.

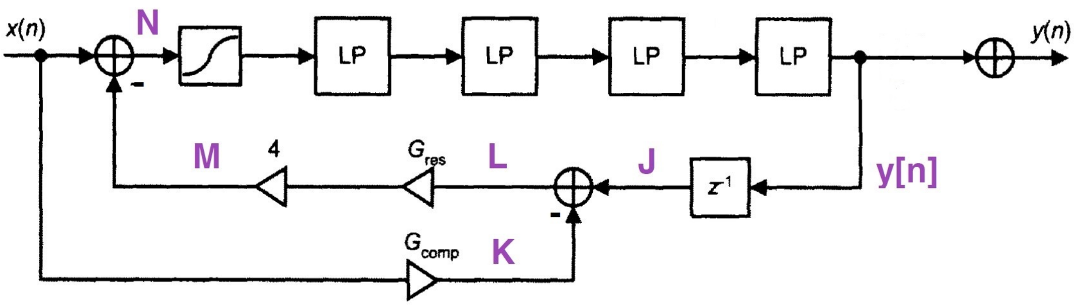


Figure 1: Overall structure of the implemented Moog voltage-controlled filter, adapted from [3]

2 First-order filter section

2.1 Derivation

As proposed in [3], one of the four first-order sections of the filter is implemented as depicted in Figure 2. The equations for the individual nodes annotated are given in Equations 1 to 4.

$$A = \frac{1}{1.3} \cdot x[n] \qquad B = x[n-1] \tag{1}$$
$$C = \frac{0.3}{1.3} \cdot B \qquad D = A + C \tag{2}$$
$$E = y[n-1] \qquad F = D - E \tag{3}$$
$$G = g \cdot F \qquad y[n] = E + G \tag{4}$$

The filter equation given in Equation 5 was then derived from these equations.

$$y[n] = g\frac{1}{1.3} \cdot x[n] + g\frac{0.3}{1.3} \cdot x[n-1] + (1-g) \cdot y[n-1] \tag{5}$$

The filter equation was then compared to the standard form of a first-order IIR filter, which is given in Equation 6.

$$y[n] = b_0 \cdot x[n] + b_1 \cdot x[n-1] - a_1 \cdot y[n-1] \tag{6}$$

The resulting filter coefficients b_0, b_1 and a_1 were then calculated and are given in Equation 7.

$$b_0 = g\frac{1}{1.3} \qquad b_1 = g\frac{0.3}{1.3} \qquad a_1 = g - 1 \tag{7}$$

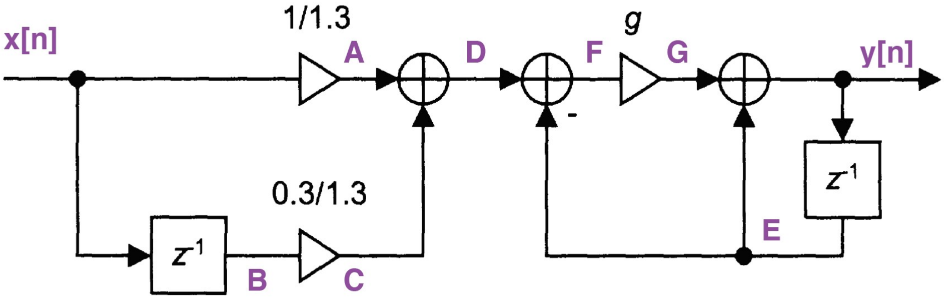


Figure 2: First-order section of the filter depicted in Figure 1, adapted from [3]

2.2 Performance

The parameter g of the filter coefficients is firstly calculated simply by $g = 2\pi \cdot f_c/f_s$. The filter frequency response using this formula is given in Figure 3a for the two different cutoff frequencies. It is clearly visible that the intended cutoff frequencies were not met accurately. In Figure 3a, for $f_c = 1\,\mathrm{kHz}$, the response shows its cutoff at approximately 1.1 kHz, resulting in an error of 10 %. For $f_c = 4\,\mathrm{kHz}$, Figure 3a even shows its cutoff frequency at approximately 5.57 kHz - an error of 39.3 %.

To mitigate the nonlinear relation of f_c to the actual cutoff frequency, a polynomial model for the parameter g is proposed in [3]. The model is given in Equation 8, where $\omega_c = 2\pi \cdot f_c/f_s$. The result for the frequency response of the filter is given in Figure 3b. For $f_c = 1\,\mathrm{kHz}$, the error was reduced to 2 % (actual cutoff 1.02 kHz). For $f_c = 4\,\mathrm{kHz}$, the real cutoff frequency is around 4.12 kHz, a significantly improved error of only 3 %.

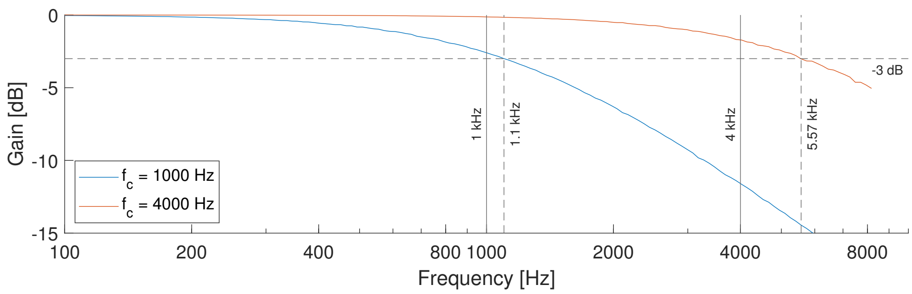
$$g = 0.9892\omega_c - 0.4342\omega_c^2 + 0.1381\omega_c^3 - 0.0202\omega_c^4 \tag{8}$$

3 Fourth-order filter

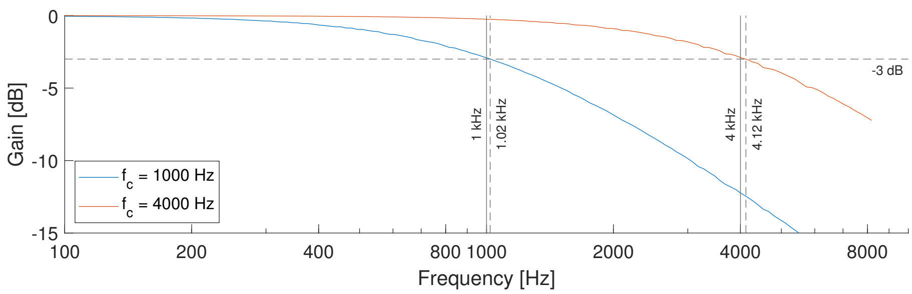
The combination of four first-order filter segments (discussed in Section 2) results in a fourth-order filter. The frequency response of this combination is given in Figure 4. At the -3 dB-frequency of the first-order section, the fourth-order response shows a gain of -12 dB. This is expected, as four identical filter elements with -3 dB gain are combined.

4 Nonlinearity

The nonlinear function $\tanh(x)$ is applied [4] to the input before filtering, which is depicted in Figure 1. This introduces signal distrotion. Especially for high amplitudes, the peaks of input sine waves are flattened. In the frequency domain, this effect introduces additional harmonics which increase in significance for high input amplitudes. For low amplitudes however, no significant harmonics are introduced. This is due to the shape of the function $\tanh(x)$, which is very close to being linear for low and increasingly nonlinear for high input amplitudes.



(a) Primitive calculation ($g = 2\pi \cdot f_c/f_s$)



(b) Polynomial fit (see Equation 8)

Figure 3: Filter responses for different calculations of parameter g

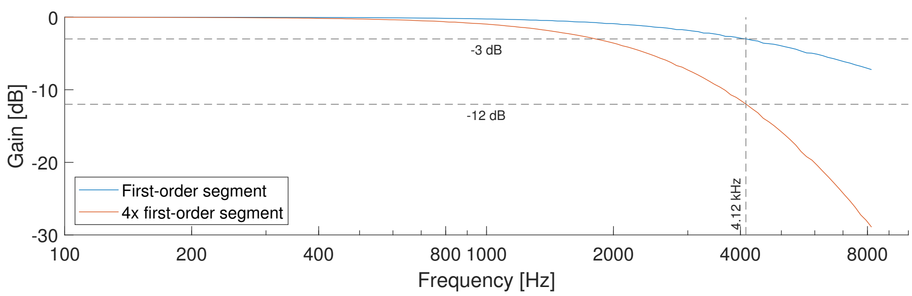


Figure 4: Filter response for four of the first-order sections combined, each using $f_c = 4\,\text{kHz}$

5 Feedback

The feedback path depicted in Figure 1 requires transformation into an equation. Therefore, the equations for the nodes annotated are derived in Equations 9 to 11. For the implementation, the parameter G_{comp} is set to a fixed value of 0.5. An example response for $G_{res} = 0.75$ is shown in Figure 5. It shows a resonant peak at 870 Hz with a maximum gain of 7.2 dB.

$$J = y[n-1] \qquad K = G_{comp} \cdot x[n] \tag{9}$$

$$L = J - K \qquad M = 4G_{res} \cdot L \tag{10}$$

$$N = x[n] - M = x[n] - 4G_{res} \cdot (y[n-1] - G_{comp} \cdot x[n]) \tag{11}$$

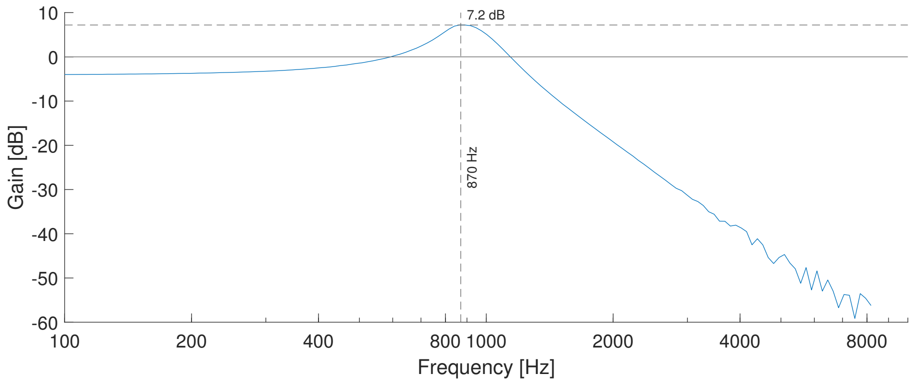


Figure 5: Filter system frequency response using feedback, first-order segments with $f_c = 1\,\text{kHz}$ and $G_{res} = 0.75$

6 Adjustable resonance

Using the parameter $C_{res} \in [0, 1]$, the resonance peak can be adjusted by using Equation 12 for G_{res} [3]. The resonance frequency can simply be adjusted by re-calculating the first-order filter segments coefficients for a different cutoff frequency. The influence of this parameter is observed in Section 7.3.

$$G_{res} = C_{res} \cdot (1.0029 + 0.0526\omega_c - 0.0926\omega_c^2 - 0.0218\omega_c^3) \tag{12}$$

7 Experiments

7.1 Effect on sawtooth spectrum

As a simple experiment, a sawtooth oscillator with frequency $f = 100\,\text{Hz}$ and amplitude $A = 0.3$ was used as an input to the implemented system (using $C_{res} = 0.9$ and $f_c = 2\,\text{kHz}$). Figure 6 shows the Fast Fourier Transformation (FFT) of this input and the resulting output. The resonant peak is clearly visible at approximately 2 kHz. Changing the resonance frequency is furthermore clearly audible, as this specific frequency of the sawtooths spectrum is especially amplified.

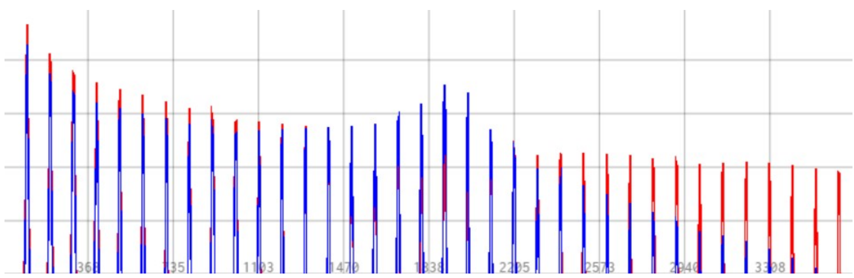


Figure 6: FFT of sawtooth oscillator input with $f = 100\,\text{Hz}$ and amplitude $A = 0.3$ (red) and the resulting output using $C_{res} = 0.9$ and $f_c = 2\,\text{kHz}$ (blue)

7.2 Sudden input removal

This experiment uses the same setting as before, only the resonance parameter was maximised to $C_{res} = 1$. After turning down the input oscillator amplitude to zero, the system output showed only its resonance frequency for approximately 1 s. The high gain at this frequency, combined with feeding back the output to the filters input, results in this behaviour.

7.3 Resonance adjustment

In the final experiment, a bode plot of the complete system is made for five different values of C_{res}. The filter cutoff frequency is set to $f_c = 1\,\text{kHz}$. The results are depicted in Figure 7. It is interesting that the parameter C_{res} does not only influence the height of the resonance peak, but also the resonance frequency and the gain for low frequencies.

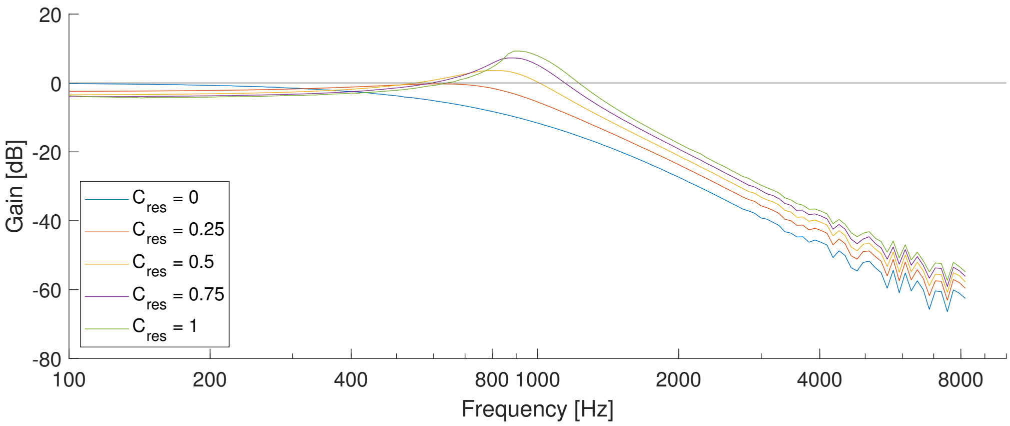


Figure 7: Frequency response of filter system with $f_c = 1\,\text{kHz}$ and different values for C_{res}

References

[1] T. S. Stilson and J. O. Smith, "Analyzing the moog vcf with considerations for digital implementation," 1996.

[2] R. A. Moog, "A voltage-controlled low-pass high-pass filter for audio signal processing," in *Audio Engineering Society Convention 17*, Oct. 1965. [Online]. Available: `http://www.aes.org/e-lib/browse.cfm?elib=1027`.

[3] V. Välimäki and A. Huovilainen, "Oscillator and filter algorithms for virtual analog synthesis," *Computer Music Journal*, vol. 30, no. 2, pp. 19–31, 2006. [Online]. Available: `http://www.jstor.org/stable/3682001`.

[4] A. Huovilainen, "Non-linear digital implementation of the moog ladder filter," in *Proceedings of the International Conference on Digital Audio Effects (DAFx-04)*, 2004, pp. 61–64.