CO 250 Spring 2023: Tutorial 4 problems

Due: Friday June 2

In this tutorial, we practise nonlinear programming formulations, outcomes of LPs and LPs in SEF.

T4-1. We are required to draw rectangles R_1,..., R_n (without rotations) on a rectangular sheet, such that no two rectangles intersect but can potentially touch. For $i = 1, \ldots, n$ the rectangle R_i has size $a_i \times b_i$.

Formulate an NLP to find a rectangular sheet of smallest possible area such that all rectangles can be drawn on it without intersections.

T4-2. Given $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, consider the following LP.

$$\begin{aligned} &\min \quad c^T x \\ &\text{subject to} \\ &A\,x \geq b \\ &x \geq \mathbf{0}\,, \end{aligned}$$

Let $b \leq \mathbf{0}$ and let there exist a vector $z \in \mathbb{R}^m$, $z \geq \mathbf{0}$ such that $c \geq A^T z$. Determine the outcome of the above LP and prove correctness of your answer.

T4-3. Given $c \in \mathbb{R}^n$, $r \in \mathbb{R}^m$, $b \in \mathbb{R}^q$, $d \in \mathbb{R}^g$, $A \in \mathbb{R}^{q\times n}$, $B \in \mathbb{R}^{q\times m}$, $C \in \mathbb{R}^{g\times n}$, $D \in \mathbb{R}^{g\times m}$, convert the following LP into Standard Equality Form.

$$\begin{aligned} &\min \quad c^T x + r^T y \\ &\text{subject to} \\ &\begin{array}{ccccc} Ax & + & By & \geq & b \\ Cx & + & Dy & \leq & d \end{array} \\ &x \leq \mathbf{0},\ y \text{ free}\,, \end{aligned} \tag{P}$$

where $x = (x_1, x_2, \ldots, x_n)^T$ and $y = (y_1, y_2, \ldots, y_m)^T$.

Call the new LP from part (that is in SEF): (P'). Given a feasible solution of (P') show how to obtain the corresponding feasible solution of (P). Further, given a feasible solution of (P) show how to obtain a corresponding feasible solution of (P').