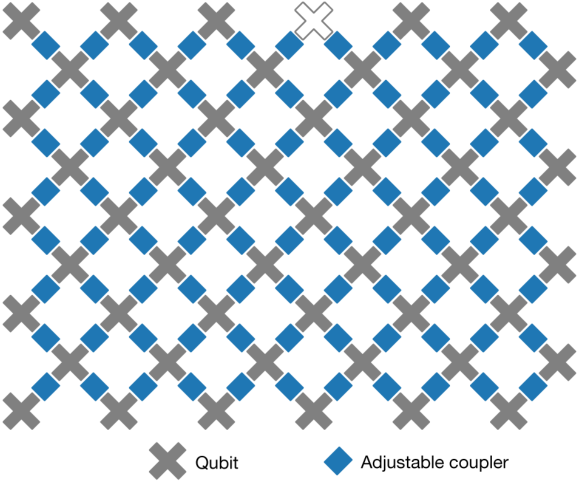
Can we implement good quantum LDPC codes on near-term devices?

Maxime Tremblay[1], Michael Beverland[2], Nicolas Delfosse[2]

[1]Université de Sherbrooke, [2]Microsoft Research

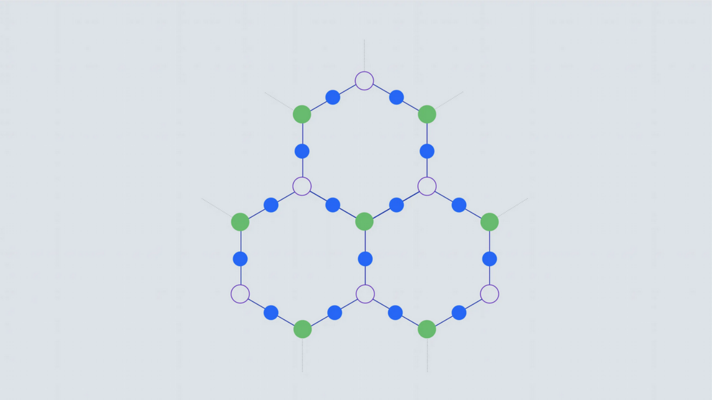
February 4, 2022



Arute et al. Nature 574, 505–510 (2019)

The IBM Quantum heavy hex lattice

As of August 8, 2021, the topology of all active IBM Quantum devices will use the heavy-hex lattice, including the IBM Quantum System One's Falcon processors installed in Germany and Japan.



Near-term quantum computers will be locally connected.

Can we achieve large scale fault-tolerant quantum computing on locally connected devices?

Tradeoffs for reliable quantum information storage in 2D systems

Sergey Bravyi,[1] David Poulin,[2] and Barbara Terhal[1]

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(Dated: September 11, 2018)

Long range interactions from local operations

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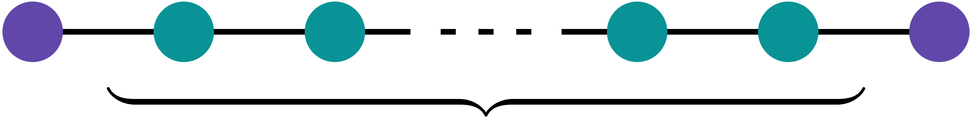

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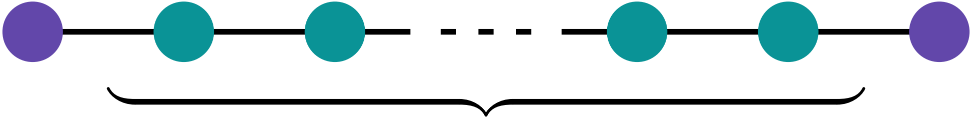
Long range interactions from local operations



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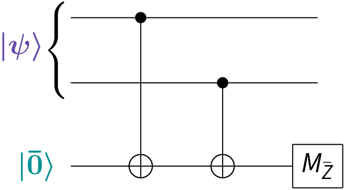

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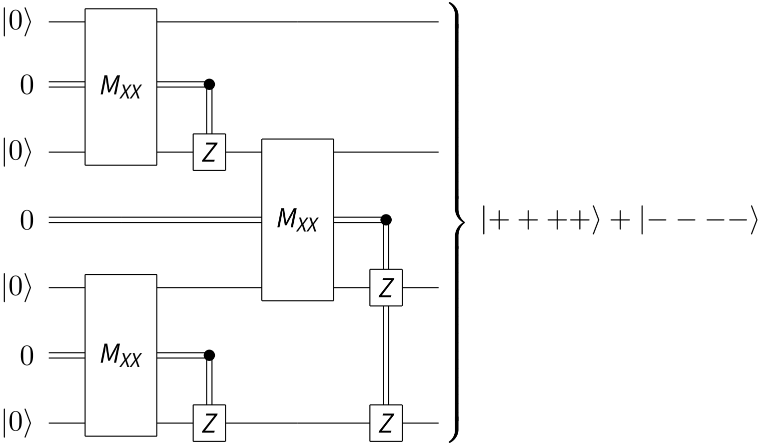

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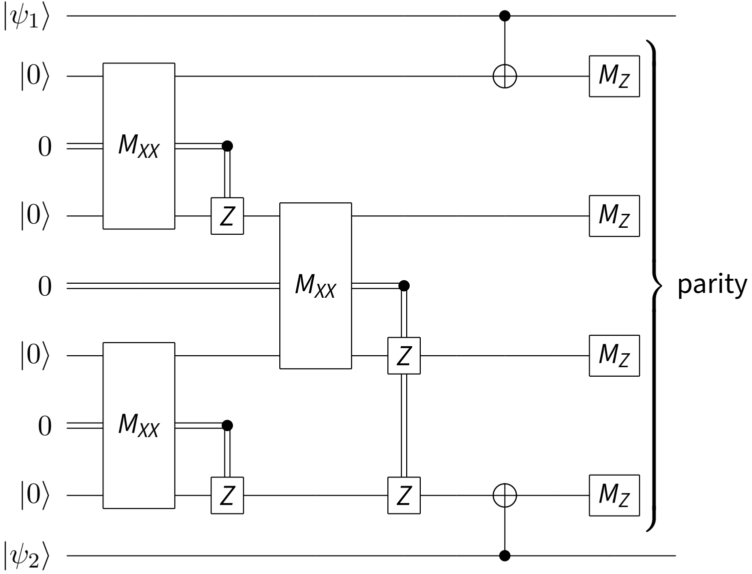

Main results

Theorem

Let C be a Clifford circuit measuring computing Pauli operators $S_1, \ldots, S_r$. Then, for any subset of qubits L, we have

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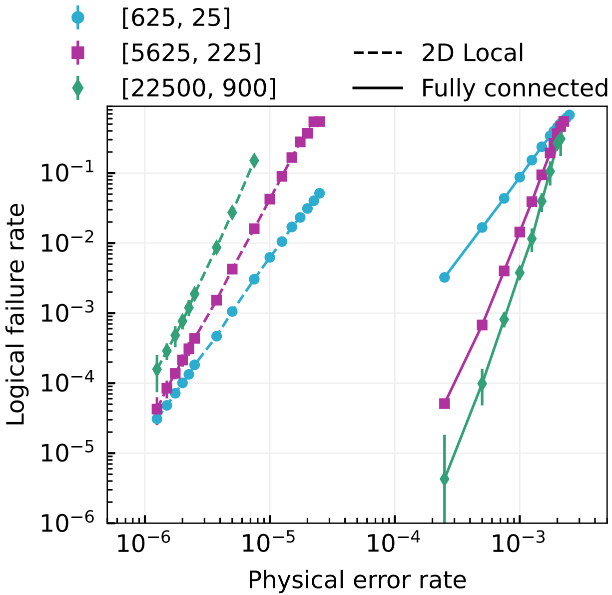
[625, 25]
[5625, 225]
[22500, 900]
2D Local
Fully connected
Logical failure rate
Physical error rate
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-6}
10^{-5}
10^{-4}
10^{-3}

References

- Bounds on stabilizer measurement circuits and obstructions to local implementations of quantum LDPC codes
  arXiv 2109.14599
- Constant-overhead quantum error correction with thin planar connectivity
  arXiv 2109.14609

Outline

1. Background and definitions
2. Proof of the main theorem
3. Circuit implementations
4. Numerical experiments

Background and definitions

Clifford circuit

- Preparations of $|0\rangle$ and $|+\rangle$ and classical bits.

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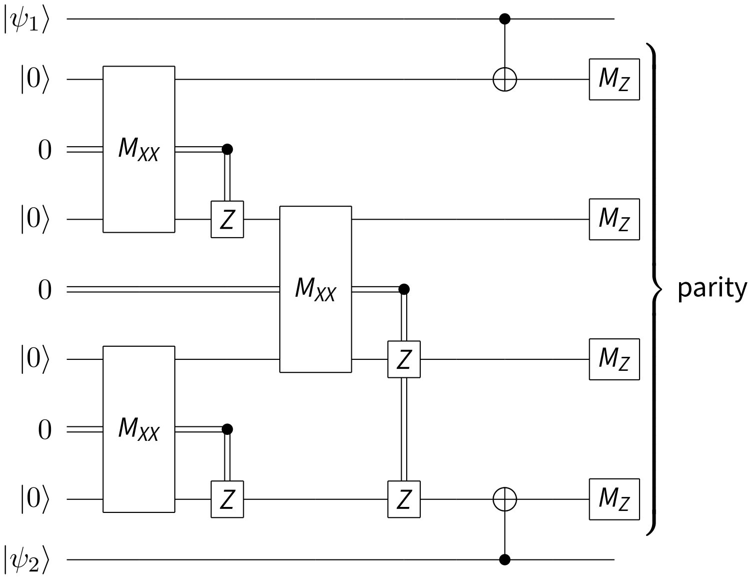
- Classically-controlled Pauli operators.
- Ouput the parity of some subsets of measurement outcomes.

Clifford circuit



Local circuit

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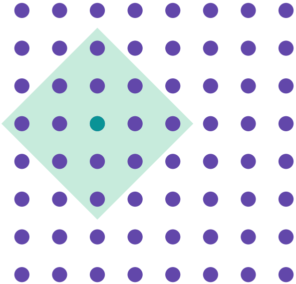
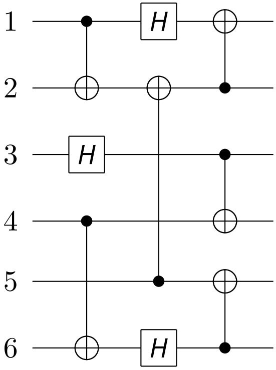


Connectivity graph



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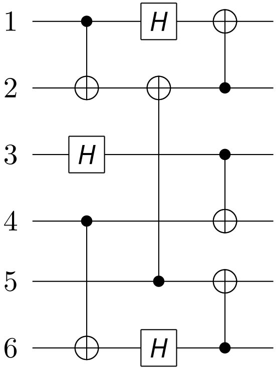

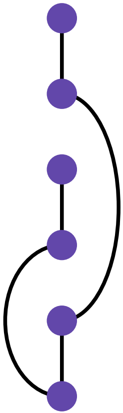

Stabilizer code

Stabilizer group

$$\mathcal{S} = \langle S_1, S_2, \ldots, S_r \rangle \subset \mathcal{P}_n \setminus \{-I\} \text{ such that } S_i S_j = S_j S_i$$

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Example

The five qubits code

$$\mathcal{S} = \langle XXZIZ, ZXXZI, IZXXZ, ZIZXX \rangle$$

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- The circuit use $N = n + a$ qubits where a is the number of ancilla qubits.

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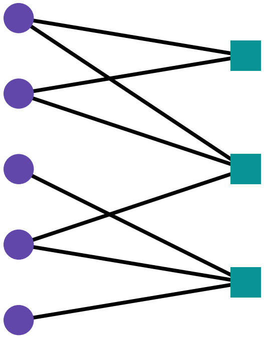

Contracted Tanner graph

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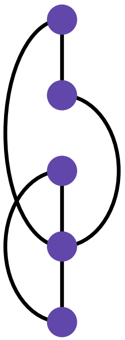

Quantum LDPC codes

A family of quantum LDPC codes $(Q_i)_i$ is a family of stabilizer codes such that the degree of the Tanner graph T_i are bounded by some constant independent of i.

Local-expansion

The Cheeger constant of a graph $G = (V, E)$ is defined as

$$h_\varepsilon(G) = \min_{\substack{L \subseteq V \\ |L| \le \varepsilon |V|/2}} \frac{|\partial L|}{|L|}.$$

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A family of quantum local-expander codes is a family of stabilizer codes with (α, ε)-expander contracted Tanner graphs.

Lemma

Let T be the Tanner graph of a stabilizer code with length n and with r stabilizer generators. Let $\bar{T}$ be its contracted Tanner graph. Then, for all $\varepsilon \in [0, 1]$, we have

$$h_{\varepsilon'}(\bar{T}) \geq \frac{h_{\varepsilon}(T)}{\deg(T)}$$

where

$$\varepsilon' = \left(1 + \frac{r}{n}\right) \frac{\varepsilon}{1 + \deg(T)}.$$

- Show that

$$|\partial_{\bar{T}} L| \geq \frac{|\partial_T (L \cup N_T(L))|}{\deg(T)}.$$

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then

$$\left|\partial_T (L \cup N_T(L))\right| \geq h_\varepsilon(T) \left|L \cup N_T(L)\right| \geq h_\varepsilon(T) \left|L\right| .$$

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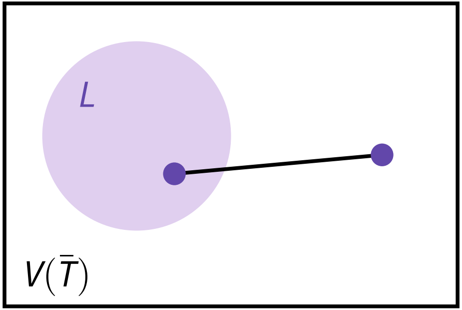
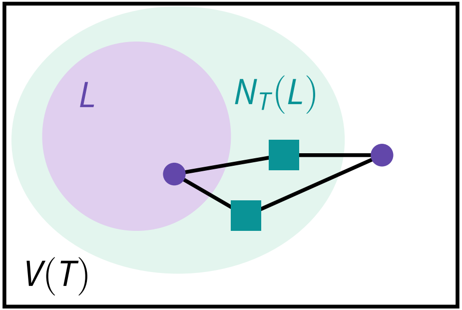
- Combine the two

$$\frac{|\partial_{\bar{T}} L|}{|L|} \geq \frac{h_\varepsilon(T)}{\deg(T)}.$$

$$|\partial_{\bar{T}} L| \,\deg(T) \geq |\partial_T(L \cup N_T(L))|$$





Review

- (Local) Clifford circuits
- Connectivity graphs
- Stabilizer codes and LDPC codes
- Pauli measurement circuits
- (Contracted) Tanner graphs
- Expander codes

Proof of the main theorem

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