Proof of the main theorem

Theorem

Let C be a Clifford circuit measuring computing Pauli operators $S_1, \ldots, S_r$. Then, for any subset of qubits L, we have

$$depth(C) \geq \frac{n_{cut}}{64|\partial L|}.$$

Corollary

For families of local-expander quantum LDPC codes of length n, a syndrome extraction circuit C implemented as a local Clifford circuit on a $\sqrt{N} \times \sqrt{N}$ grid of qubits satisfies

$$depth(C) \geq \Omega\left(\frac{n}{\sqrt{N}}\right).$$

Strategy

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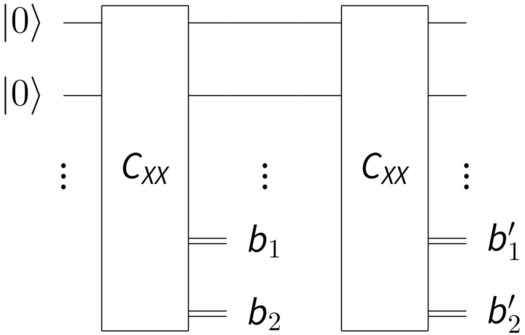
- Lower bound the amount of correlation required between L and R to measure the Pauli operators.
- Upper bound the amount of correlation introduced per operation.
- Combine both arguments to derive a lower bound for the depth of the circuit.

Measuring correlations



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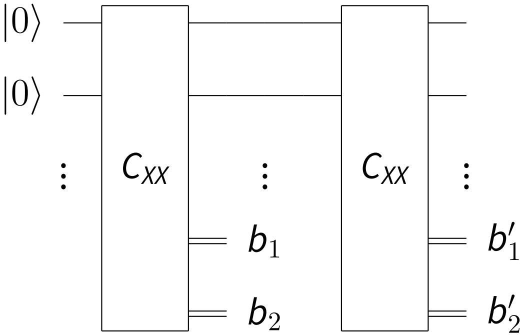

Mutual information

$$I(b_1; b_2) = 0$$

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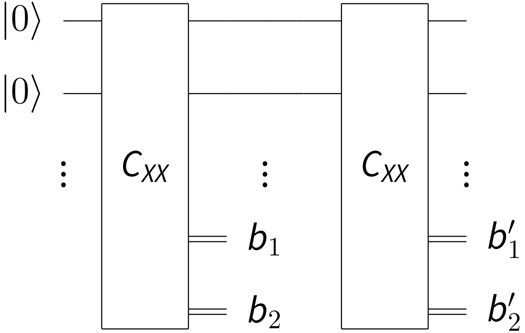

Mutual information

$$I(b_1; b_2) = 0$$

$$I(b_1'; b_2' | b_1, b_2) = 1$$