EPISTEMIC GEOMETRY: FINITE VERIFICATION, CURVATURE, AND STRUCTURAL OBSTRUCTIONS ACROSS LOGIC, COMPUTATION, AND PHYSICS

OSCAR RIVEROS

ABSTRACT. We unify five technical lines into a single auditable program built on *finite, refutable constraints*, *verifiable certificates*, and *metric invariants of representation*. The modules are: (A) SAT-verified discrete physics (a workflow compiling finite discrete models to SAT with mechanically checkable witnesses), (A$'$) an exact arithmetic encoding for balanced CNFs (the "SAT equation"), (B) geometric knowledge compilation for SAT/#SAT by disjoint subcubes (COVERTRACE) with explicit fragmentation lower bounds, (C) epistemic curvature κ_S as a metric measure of the syntax–semantics gap (metaformal) and its link to incompleteness under a derivational refinement principle, (D) the Layered Metric Space (LMS) as a discrete variational kinematics with an operational materialization regime, and (E) locality via soft causal cones (Lieb–Robinson type) and information-theoretic limits of agency. Bridges between modules yield a "metascientific" theorem: within finite regimes, scientific truth reduces to the verification of finite certificates. Every statement is tagged as [**Proved**], [**Model**], or [**Speculative**].

CONTENTS

Date: February 2026.

1. Project norm, scope, and architecture

This manuscript is a technical unification, not a claim of having derived continuum GR/QFT. The guiding methodological thesis is:

Thesis (finite verification as a core epistemic discipline). *A large class of scientific claims can be rewritten as finite, refutable constraint sets whose truth is reducible to the verification of finite certificates, and whose representation limits can be measured by explicit metric invariants.*

Auditability rules.

- **N1. Ideal object + finite probes.** Every infinite/ideal object must be paired with a family of finite, auditable probes (finite volumes, truncations, certificates).
- **N2. Quantifiers explicit.** Domains and quantifiers are stated.
- **N3. Proof or reproducible protocol.** A **[Proved]** claim includes a complete proof or an explicit finite verification protocol.
- **N4. Certificate typing.** Probes are typed as: constructive witness, finite refutation, or numerical approximation.
- **N5. Layering.** Claims are tagged as **[Proved]**, **[Model]**, or **[Speculative]**.

2. Module A: SAT-verified discrete physics [Proved]

2.1. GCNF and the one-hot reduction.

Definition 2.1 (Finite-domain variables and GCNF). *Let x_i be a variable with finite domain $\{0, 1, \dots, b_i - 1\}$. A literal is an equality $(x_i = a)$. A* GCNF clause *is a finite disjunction of such literals, and a* GCNF formula *is a conjunction of clauses.*

Definition 2.2 (One-hot reduction). *For each x_i introduce Boolean indicators $X_{i,0}, \dots, X_{i,b_i-1}$ with an* exactly-one *constraint. Translate $(x_i = a)$ to $X_{i,a}$ and each GCNF clause to an ordinary CNF clause over the indicators.*

Proposition 2.3 (Correctness of the reduction). *Let φ be a GCNF formula and $\mathrm{CNF}(\varphi)$ its one-hot reduction. Then φ is satisfiable if and only if $\mathrm{CNF}(\varphi)$ is satisfiable.*

Proof. If $\alpha \models \varphi$, define a Boolean assignment β by setting $X_{i,\alpha(x_i)} = 1$ and the remaining $X_{i,a} = 0$. Then exactly-one holds and every reduced clause is satisfied. Conversely, if $\beta \models \mathrm{CNF}(\varphi)$, exactly-one gives a unique a with $X_{i,a} = 1$; define $\alpha(x_i) = a$. Satisfaction of every reduced clause implies satisfaction of every original GCNF clause. □

2.2. Witness–to–verification loop.

Definition 2.4 (Auditable verifier). *A* verifier *is a deterministic algorithm that, given φ and a candidate assignment α, checks clause-by-clause whether $\alpha \models \varphi$.*

Proposition 2.5 (Separation of correctness). *If a solver returns a witness β for $\mathrm{CNF}(\varphi)$ and the verifier accepts the decoded assignment α, then $\alpha \models \varphi$ independently of the solver's internal correctness.*

Proof. Acceptance means that every clause of φ evaluates to true under α, which is exactly $\alpha \models \varphi$. □

2.3. Example: equilateral discrete Gauss–Bonnet via SAT.

Definition 2.6 (Combinatorial triangulation variables). *Let $V = \{1, \dots, N\}$. For each triple $i < j < k$, introduce a Boolean variable τ_{ijk} indicating whether the triangle (i, j, k) is present.*

Definition 2.7 (Closed 2-manifold constraint (no boundary)). *For each unordered edge $\{i, j\}$, let $S_{ij} = \{\tau_{ijk} : k \neq i, j\}$ be the incident-triangle variables. Enforce: the number of true variables in S_{ij} is either 0 or 2.*

Definition 2.8 (Vertex degree and equilateral curvature). *Define the triangle-incidence degree*

$$\deg(v) = \sum_{i<j,\ i,j\neq v} \tau_{vij}.$$

In an equilateral triangulation, each incident triangle contributes angle $\pi/3$ at v, so the angle deficit is

$$K(v) = 2\pi - \deg(v)\frac{\pi}{3}, \qquad \kappa(v) := \frac{3}{\pi}K(v) = 6 - \deg(v).$$

Theorem 2.9 (Discrete Gauss–Bonnet (equilateral triangulations)). *If $\{\tau_{ijk}\}$ encodes a closed triangulated surface, then*

$$\sum_{v\in V} \kappa(v) = 6\chi,$$

where χ is the Euler characteristic. In particular, for a triangulated sphere ($\chi = 2$) one has $\sum_v \kappa(v) = 12$.

Proof. Let F be the number of triangles, E the number of edges, and $V = |V|$ the number of vertices. Each triangle has 3 edges and each internal edge is incident to 2 triangles, hence $3F = 2E$. Also $\sum_v \deg(v) = 3F$ (each triangle contributes 1 to three vertices). Therefore

$$\sum_v \kappa(v) = \sum_v (6 - \deg(v)) = 6V - 3F.$$

Using $3F = 2E$ gives $\sum_v \kappa(v) = 6V - 2E = 6(V - E + F) = 6\chi$. $\square$

Remark 2.10 (Beyond equilateral triangulations [**Model**]). *The equilateral case fixes angles to $\pi/3$. In non-equilateral (Regge-type) triangulations one assigns each triangle a geometry and replaces $\deg(v)\pi/3$ by the sum of incident angles at v; the Gauss–Bonnet identity becomes $\sum_v(2\pi - \sum_{t\ni v} \theta_{t,v}) = 2\pi\chi$. Encoding such angle sums requires either rational angle discretization or integer-weighted approximations.*

2.4. Example: a hard causal cone in Boolean locality dynamics [Proved].

Definition 2.11 (Boolean influence dynamics). *Let $G = (V, E)$ be a finite graph and fix a time horizon T. Let $I(v, t) \in \{0, 1\}$ indicate whether site v is "influenced" at time t. Fix an initial influenced set $S \subseteq V$ by $I(v, 0) = 1$ iff $v \in S$. Impose the synchronous local rule:*

$$I(v, t + 1) \iff I(v, t) \ \vee \bigvee_{u:\{u,v\}\in E} I(u, t).$$

Theorem 2.12 (Exact light-cone constraint). *If $d(v, S) > t$, then $I(v, t) = 0$. Equivalently, influence cannot propagate faster than one graph edge per time step.*

Proof. By induction on t. For $t = 0$, the claim is the definition of $I(\cdot, 0)$. Assume the claim holds at t. If $d(v, S) > t + 1$, then for every neighbor u of v, $d(u, S) \geq d(v, S) - 1 > t$, so $I(u, t) = 0$ by the induction hypothesis, and also $I(v, t) = 0$. Hence the update rule forces $I(v, t + 1) = 0$. $\square$

3. Module A′: SATX and science as exact compilation [Proved]

3.1. Balanced CNFs and the SAT equation.

Definition 3.1 (Balanced clause and unique falsifier). *Fix variables $x_1, \ldots, x_n \in \{0, 1\}$. A clause C is* balanced *if it contains exactly one literal of each variable, either x_i or $\neg x_i$. Define the sign vector $s(C) \in \{0, 1\}^n$ by $s_i(C) = 0$ if $x_i \in C$ and $s_i(C) = 1$ if $\neg x_i \in C$. Then $a_C := s(C)$ is the unique assignment falsifying C.*

Definition 3.2 (Index and SAT integer). *For $a = (a_1, \ldots, a_n) \in \{0, 1\}^n$ define $\operatorname{ind}(a) = \sum_{i=1}^n a_i 2^{n-i} \in \{0, \ldots, 2^n - 1\}$. For a balanced clause C let $T(C) := \operatorname{ind}(a_C)$. For a balanced CNF F with no repeated clauses define*

$$S_F := \sum_{C\in F} 2^{T(C)}.$$

Theorem 3.3 (SAT equation theorem). *Let F be a balanced CNF on n variables without duplicate clauses. Write the binary expansion $S_F = \sum_{k=0}^{2^n-1} b_k 2^k$ with $b_k \in \{0, 1\}$. For any assignment a with index $k = \operatorname{ind}(a)$,*

$$b_k = 1 \iff a \text{ falsifies at least one clause of } F \iff a \not\models F.$$

Thus the bitmask of S_F is exactly the set of unsatisfying assignments.

Proof. Each clause $C \in F$ contributes $2^{T(C)}$, which sets the bit $b_{T(C)}$ to 1. No carries occur because clauses are distinct. By the uniqueness of the falsifier, $b_k = 1$ iff a is the unique falsifier of some clause. $\square$

Remark 3.4 (Significance and limitation [**Proved**]). *The theorem is an* exact arithmetic encoding *of satisfiability for a restricted class of CNFs. It does not reduce worst-case SAT complexity, but it enables lossless "semantic hashing" and exact equivalence checks within the balanced fragment.*

Proposition 3.5 (Finite refutability in finite regimes [**Proved**]). *Any finite satisfiability claim of the form "$\exists a \in \{0,1\}^n$ such that $a \models F$" is refutable by a finite certificate: either a satisfying assignment a, or a verifiable UNSAT proof in a sound proof system.*

4. Module B: COVERTRACE and geometric compilation of SAT/#SAT [**Proved**]

4.1. Geometric semantics of CNF.

Let $\Omega^n = \{0,1\}^n$.

Definition 4.1 (Patterns and subcubes). *A* pattern *is $p \in \{0,1,\bullet\}^n$ where $\bullet$ denotes "free". The induced axis-aligned subcube is*

$$Q(p) = \{x \in \Omega^n : \; p_i \neq \bullet \Rightarrow x_i = p_i\}.$$

Its width is $k(p) = |\mathrm{supp}(p)|$ and its volume is $\mathrm{vol}(p) = |Q(p)| = 2^{n-k(p)}$.

Definition 4.2 (Clause to forbidden subcube). *For a clause C over variables $x_1, \dots, x_n$, define $p(C) \in \{0,1,\bullet\}^n$ coordinatewise by*

$$p(C)_i = \begin{cases} 0, & x_i \in C, \\ 1, & \neg x_i \in C, \\ \bullet, & \textit{otherwise.} \end{cases}$$

Proposition 4.3. *An assignment $x \in \Omega^n$ falsifies C if and only if $x \in Q(p(C))$.*

Definition 4.4 (Forbidden region). *For $F = \bigwedge_{j=1}^m C_j$, define the forbidden region*

$$U(F) = \bigcup_{j=1}^{m} Q(p(C_j)).$$

Then $\#\mathrm{SAT}(F) = 2^n - |U(F)|$, and F is satisfiable iff $U(F) \neq \Omega^n$.

Algorithm 1 $\textsc{CubeDiff}(p, r)$: disjoint decomposition of $Q(p) \setminus Q(r)$

Require: $p, r \in \{0,1,\bullet\}^n$
Ensure: disjoint family D such that $Q(p) \setminus Q(r) = \biguplus_{d \in D} Q(d)$

```
if Q(p) ∩ Q(r) = ∅ then
    return {p}
end if
if Q(p) ⊆ Q(r) then
    return ∅
end if
Choose i with p_i = • and r_i ∈ {0,1}
b ← r_i
p^≠ ← p with p^≠_i ← 1 − b
p^= ← p with p^=_i ← b
return {p^≠} ∪ CubeDiff(p^=, r)
```

4.2. Disjoint cube difference: CubeDiff.

Lemma 4.5 (Single-difference size bound)**.** *If $Q(p) \cap Q(r) \neq \varnothing$ and $Q(p) \nsubseteq Q(r)$, then* CubeDiff *returns at most* $|\mathrm{supp}(r) \setminus \mathrm{supp}(p)|$ *subcubes.*

Proof. Each recursion fixes one coordinate that is fixed in r but free in p, so the recursion depth is at most $|\mathrm{supp}(r) \setminus \mathrm{supp}(p)|$. Exactly one subcube is emitted per level. □

4.3. **Incremental disjointization and exact model counting.**

Definition 4.6 (Disjoint family)**.** *A family $\mathcal{U} \subseteq \{0, 1, \bullet\}^n$ is* disjoint *if $Q(u) \cap Q(v) = \varnothing$ for all $u \neq v$. Let $\mathrm{Cov}(\mathcal{U}) = \bigcup_{u \in \mathcal{U}} Q(u)$.*

Algorithm 2 AddCube($\mathcal{U}, q$): add cube $Q(q)$ to a disjoint family

---

Require: Disjoint $\mathcal{U} \subseteq \{0, 1, \bullet\}^n$, pattern $q \in \{0, 1, \bullet\}^n$
Ensure: Disjoint $\mathcal{U}'$ with $\mathrm{Cov}(\mathcal{U}') = \mathrm{Cov}(\mathcal{U}) \cup Q(q)$
$P \leftarrow \{q\}$ ▷ pieces of q not yet subtracted
for each $r \in \mathcal{U}$ **do**
  $P_{\mathrm{new}} \leftarrow \varnothing$
  for each $p \in P$ **do**
    $P_{\mathrm{new}} \leftarrow P_{\mathrm{new}} \cup \textsc{CubeDiff}(p, r)$
  end for
  $P \leftarrow P_{\mathrm{new}}$
  if $P = \varnothing$ **then**
    return $\mathcal{U}$
  end if
end for
return $\mathcal{U} \cup P$

Theorem 4.7 (Correctness of disjoint forbidden compilation)**.** *Let F be a CNF with clauses $C_1, \ldots, C_m$ and forbidden patterns $p_j = p(C_j)$. Initialize $\mathcal{U}_0 = \varnothing$ and iteratively set $\mathcal{U}_{t+1} =$ AddCube$(\mathcal{U}_t, p_{t+1})$. Then $\mathcal{U}_m$ is disjoint and $\mathrm{Cov}(\mathcal{U}_m) = U(F)$. Consequently,*

$$|U(F)| = \sum_{u \in \mathcal{U}_m} \mathrm{vol}(u), \qquad \#\mathrm{SAT}(F) = 2^n - \sum_{u \in \mathcal{U}_m} 2^{n-k(u)}.$$

Proof. By construction, AddCube replaces q by a disjoint family of pieces that are outside every existing cube in $\mathcal{U}$, so the added pieces are disjoint from $\mathcal{U}$ and from each other, and their union is exactly $Q(q) \setminus \mathrm{Cov}(\mathcal{U})$. Thus the updated cover equals the previous cover union $Q(q)$. Induction on t gives $\mathrm{Cov}(\mathcal{U}_m) = U(F)$. □

4.4. **A tight parity barrier [Proved].**

Let $\mathrm{PARITY}_n^{\mathrm{odd}} \subseteq \Omega^n$ be the set of assignments of odd Hamming weight.

Theorem 4.8 (Parity requires exponential disjoint subcubes)**.** *Any disjoint family of axis-aligned subcubes whose union is $\mathrm{PARITY}_n^{\mathrm{odd}}$ has size at least 2^{n-1}.*

Proof. Consider any subcube $Q(p)$ with at least one free coordinate. Flipping a free coordinate produces a bijection on $Q(p)$ that toggles parity, hence $Q(p)$ contains equally many even and odd assignments; in particular it cannot be contained in $\mathrm{PARITY}_n^{\mathrm{odd}}$. Therefore every cube in a disjoint cover of $\mathrm{PARITY}_n^{\mathrm{odd}}$ must have no free coordinates, i.e. be a singleton. Hence at least $\left|\mathrm{PARITY}_n^{\mathrm{odd}}\right| = 2^{n-1}$ singletons are needed. □

Theorem 4.9 (Conditional PH collapse from uniform DSOP compilation [**Proved**])**.** *Assume there exists a uniform polynomial-time algorithm that maps any CNF F on n variables to a polynomial-size disjoint family $\mathcal{U}(F)$ with $\mathrm{Cov}(\mathcal{U}(F)) = U(F)$. Then $\#\mathrm{SAT} \in \mathbf{P}$, and consequently the Polynomial-Time Hierarchy collapses to $\mathbf{P}$.*

Proof sketch. Given $\mathcal{U}(F)$, compute $|U(F)|$ by volume additivity and obtain $\#\mathrm{SAT}(F) = 2^n - |U(F)|$ in polynomial time. Thus $\#\mathrm{SAT} \in \mathbf{P}$. By Toda's theorem, $\mathbf{PH} \subseteq \mathbf{P}^{\#\mathbf{P}} = \mathbf{P}$, hence $\mathbf{PH}$ collapses to $\mathbf{P}$. □

4.5. **Affine disjoint compilation over $\mathbb{F}_2$ [Proved].** The axis-aligned language $\{0, 1, \bullet\}^n$ cannot compress parity (Theorem above), motivating an extension to *affine* pieces over $\mathbb{F}_2$.

Definition 4.10 (Affine subcube (affine subspace))**.** *Identify $\Omega^n = \{0,1\}^n$ with $\mathbb{F}_2^n$. An* affine subcube *is a solution set*

$$Q_{\mathrm{aff}}(A, b) := \{x \in \mathbb{F}_2^n : \ Ax = b\},$$

where $A \in \mathbb{F}_2^{m\times n}$ and $b \in \mathbb{F}_2^m$. If consistent, then $|Q_{\mathrm{aff}}(A, b)| = 2^{n-\mathrm{rank}(A)}$.

Definition 4.11 (Disjoint affine compilation and affine DSOP size)**.** *A* disjoint affine compilation *of a Boolean function $f : \Omega^n \to \{0, 1\}$ is a family $\mathcal{U}_{\mathrm{aff}} = \{Q_{\mathrm{aff}}(A_i, b_i)\}_{i=1}^k$ of pairwise disjoint affine subcubes with $f^{-1}(1) = \biguplus_{i=1}^k Q_{\mathrm{aff}}(A_i, b_i)$. For a CNF F, write $\mathrm{SAT}(F) := f_F^{-1}(1)$ and define*

$$\mathrm{dsop}_{\mathrm{aff}}(F) := \min\{k : \exists\ \mathcal{U}_{\mathrm{aff}} \ \textit{as above with union}\ \mathrm{SAT}(F)\}.$$

Remark 4.12 (Parity compresses affinely)**.** *The odd-parity set is a single affine subcube: $\mathrm{PARITY}_n^{\mathrm{odd}} = \{x : \sum_{i=1}^n x_i = 1 \pmod 2\}$, hence $\mathrm{dsop}_{\mathrm{aff}}(\mathrm{PARITY}_n^{\mathrm{odd}}) = 1$, while axis-aligned DSOP requires 2^{n-1} pieces.*

Theorem 4.13 (Uniform affine compilation implies $\mathbf{PH}$ collapse [**Proved**])**.** *Assume there exists a* uniform *polynomial-time algorithm* AffComp *that, for every CNF F on n variables, outputs a disjoint affine compilation*

$$\mathrm{SAT}(F) = \biguplus_{i=1}^{k} Q_{\mathrm{aff}}(A_i, b_i)$$

with $k \le \mathrm{poly}(n)$, and where each (A_i, b_i) has description size $\mathrm{poly}(n)$ (in particular $m = \mathrm{poly}(n)$). Then $\#\mathrm{SAT} \in \mathbf{P}$, and consequently $\mathbf{PH} = \mathbf{P}$.

Proof. Because the family is disjoint,

$$\#\mathrm{SAT}(F) = |\mathrm{SAT}(F)| = \sum_{i=1}^{k} |Q_{\mathrm{aff}}(A_i, b_i)| \,.$$

For each i, Gaussian elimination over $\mathbb{F}_2$ decides consistency of $A_i x = b_i$ and computes $\mathrm{rank}(A_i)$ in time polynomial in (m, n), with $m = \mathrm{poly}(n)$ by assumption. If inconsistent, $|Q_{\mathrm{aff}}(A_i, b_i)| = 0$. If consistent, $|Q_{\mathrm{aff}}(A_i, b_i)| = 2^{n-\mathrm{rank}(A_i)}$. Summing $k \le \mathrm{poly}(n)$ terms yields $\#\mathrm{SAT}(F)$ in polynomial time, hence $\#\mathrm{SAT} \in \mathbf{P}$. By Toda, $\mathbf{PH} \subseteq \mathbf{P}^{\#\mathbf{P}} = \mathbf{P}$, so $\mathbf{PH} = \mathbf{P}$. □

Remark 4.14 (Interpretation relative to the "beyond axis-aligned" open limit)**.** *Affine pieces can remove the* parity *obstruction, but Theorem 4.13 shows that a* uniform *poly-time affine compiler for* all *CNFs would collapse* $\mathbf{PH}$*. Thus, under standard assumptions, any affine/tensorial extension of COVERTRACE that avoids axis-aligned lower bounds must either be restricted to special CNF classes, be non-uniform, or incur super-polynomial compilation size on worst-case inputs.*

5. Module C: epistemic curvature and structural incompleteness [**Metaformal**]

5.1. Metric interfaces and curvature.

Definition 5.1 (Formal system with a metric interface)**.** *A system with interface is a tuple*

$$\mathsf{S} = (\mathcal{L}, \vdash, \iota, \mathcal{O}, X, \delta, e, j),$$

where $(\mathcal{L}, \vdash)$ *is syntax and derivability,* $\iota : \mathcal{L} \to \mathcal{O}$ *is a Borel interpretation into a semantic domain* $\mathcal{O}$*,* (X, δ) *is a separable complete metric space, and* $e : \mathcal{L} \to X$*,* $j : \mathcal{O} \to X$ *are Borel embeddings. Define the representation error*

$$\mathrm{err}(\sigma) := \delta(e(\sigma), j(\iota(\sigma))).$$

Definition 5.2 (Epistemic curvature)**.** *The* epistemic curvature *is*

$$\kappa_{\mathsf{S}} := \inf_{\sigma \in \mathcal{L}} \mathrm{err}(\sigma) \in [0, \infty).$$

We call S flat *if* $\kappa_{\mathsf{S}} = 0$ *and* curved *if* $\kappa_{\mathsf{S}} > 0$.

Definition 5.3 (Pointwise and worst-case curvature)**.** *For a semantic target* $o \in \mathcal{O}$*, define the* pointwise curvature

$$\kappa_{\mathsf{S}}(o) := \inf\{\mathrm{err}(\sigma) : \sigma \in \mathcal{L},\ \iota(\sigma) = o\} \in [0, \infty],$$

with the convention $\inf \varnothing := +\infty$*. For a class* $\mathcal{C} \subseteq \mathcal{O}$*, define the* worst-case curvature

$$\kappa_{\mathsf{S}}^{\sup}(\mathcal{C}) := \sup_{o \in \mathcal{C}} \kappa_{\mathsf{S}}(o) \in [0, \infty].$$

Thus $\kappa_{\mathsf{S}} = \inf_{\sigma \in \mathcal{L}} \mathrm{err}(\sigma)$ *measures global* existence *of an exact representation, while* $\kappa_{\mathsf{S}}^{\sup}$ *captures* uniform semantic completeness*.*

Definition 5.4 (Resource-bounded curvature)**.** *Let* $\mathcal{L}_{\leq r}$ *denote syntactic objects of description size at most* r *(under a fixed self-delimiting encoding). Define the resource-bounded pointwise and worst-case curvatures by*

$$\kappa_{\mathsf{S},r}(o) := \inf\{\mathrm{err}(\sigma) : \sigma \in \mathcal{L}_{\leq r},\ \iota(\sigma) = o\}, \qquad \kappa_{\mathsf{S},r}^{\sup}(\mathcal{C}) := \sup_{o \in \mathcal{C}} \kappa_{\mathsf{S},r}(o).$$

When $r = r(n) = \mathrm{poly}(n)$ *depends on an instance size parameter,* $\kappa_{\mathsf{S},r}^{\sup}$ *expresses "polynomial-resource representational completeness".*

Lemma 5.5 (Monotonicity under 1-Lipschitz interface morphisms [**Proved**])**.** *If* $F : X \to X'$ *is* 1*-Lipschitz and* $e' = F \circ e$*,* $j' = F \circ j$*, then* $\kappa'_{\mathsf{S}} \leq \kappa_{\mathsf{S}}$*.*

Proof. For all σ, $\mathrm{err}'(\sigma) \leq \mathrm{err}(\sigma)$ by non-expansiveness, hence the infimum decreases. □

Proposition 5.6 (Sign stability under uniformly equivalent metrics [**Proved**])**.** *If* d *and* δ *are uniformly equivalent on* X *with constants* $0 < c_1 \leq c_2 < \infty$*, then* $c_1\kappa_\delta \leq \kappa_d \leq c_2\kappa_\delta$*, so* $\kappa_\delta = 0$ *iff* $\kappa_d = 0$*.*

5.2. Derivational refinement principle (DRP).

Definition 5.7 (DRP)**.** *We say* S *satisfies* DRP *if there exists an operator* $T : \mathcal{L} \to \mathcal{L}$ *such that:*

(i) $\sigma \vdash T(\sigma)$ *for all* σ *(derivational preservation);*
(ii) $\mathrm{err}(T(\sigma)) \leq \mathrm{err}(\sigma)$ *for all* σ *(non-expansive refinement);*
(iii) for every σ*, the orbit* $(T^n(\sigma))_{n \in \mathbb{N}}$ *has an accumulation point* σ_∞ *with* $\mathrm{err}(\sigma_\infty) = \inf_n \mathrm{err}(T^n(\sigma))$*.*

Theorem 5.8 (Positive curvature obstructs semantic completeness (under DRP) [**Metaformal**])**.** *Assume:*

*(a) (*Faithful targets*) For every $o \in \mathcal{O}$ in the intended semantics, there exists $\sigma \in \mathcal{L}$ with $\iota(\sigma) = o$ and $\mathrm{err}(\sigma) = 0$.*
(b) S satisfies DRP.

Then $\kappa_{\mathsf{S}} = 0$. Contrapositively, $\kappa_{\mathsf{S}} > 0$ implies failure of (a), i.e. an incompleteness of the interface.

Proof. By (a) there exists some σ with $\mathrm{err}(\sigma) = 0$. Hence $\kappa_{\mathsf{S}} \leq 0$ and since $\kappa_{\mathsf{S}} \geq 0$, we get $\kappa_{\mathsf{S}} = 0$. The contrapositive is immediate. □

Remark 5.9 (What this does *not* claim)**.** *This is not an internal Gödel incompleteness theorem: it is a metaformal obstruction statement about a chosen interface and refinement dynamics.*

5.3. An internal Gödel-type obstruction from arithmetized zero-error certification [Proved].

The preceding curvature–DRP statement is metaformal: it depends on interface choice. We now isolate conditions under which any *attempt at a "verifiably flat and complete" interface incurs an internal diagonal obstruction.*

Definition 5.10 (Gödel-admissible zero-error certification predicate)**.** *Fix a recursively axiomatizable arithmetic theory $T \supseteq Q$ in the language of $(0, S, +, \times)$. A* zero-error certification predicate *is a formula $\mathsf{Prov}_{0,r}(y)$ of T such that, for each sentence ψ, $\mathsf{Prov}_{0,r}(\ulcorner\psi\urcorner)$ expresses:*

$$\exists \sigma \in \mathcal{L}_{\leq r(|\psi|)} \text{ such that } \iota(\sigma) = o_\psi \text{ and } \mathrm{err}(\sigma) = 0,$$

where o_ψ is the semantic target corresponding to ψ (e.g. the truth value of ψ in the standard model), and where the verifier of "$\mathrm{err}(\sigma) = 0$" is a primitive recursive relation whose arithmetization is provably equivalent to $\mathsf{Prov}_{0,r}$ in T. We call $(T, \mathsf{Prov}_{0,r})$ Gödel-admissible *if T proves the diagonal lemma for formulas in which $\mathsf{Prov}_{0,r}$ may occur.*

Theorem 5.11 (Internal incompleteness of verifiably flat, uniformly complete interfaces)**.** *Assume $(T, \mathsf{Prov}_{0,r})$ is Gödel-admissible and that T is consistent. Assume further:*

(a) ***Soundness of zero-error certificates.*** *For every sentence ψ, if $\mathsf{Prov}_{0,r}(\ulcorner\psi\urcorner)$ holds, then ψ is true in the intended semantics (e.g. in $\mathbb{N}$).*
(b) ***Uniform semantic completeness.*** *For every true sentence ψ in the intended semantics, $\mathsf{Prov}_{0,r}(\ulcorner\psi\urcorner)$ holds.*

Then (a) and (b) cannot both hold. In particular, under soundness, there exists a true sentence G with $\neg\mathsf{Prov}_{0,r}(\ulcorner G\urcorner)$, hence the resource-bounded worst-case curvature is positive:

$$\kappa^{\sup}_{\mathsf{S},r}(\mathcal{O}_{\mathrm{arith}}) \; > \; 0,$$

where $\mathcal{O}_{\mathrm{arith}}$ denotes the semantic class of arithmetic truth targets.

Proof. By Gödel admissibility and the diagonal lemma, there exists a sentence G such that

$$T \vdash \; G \; \leftrightarrow \; \neg\mathsf{Prov}_{0,r}(\ulcorner G\urcorner).$$

Suppose for contradiction that $\mathsf{Prov}_{0,r}(\ulcorner G\urcorner)$ holds. By soundness (a), G is true. Then the right-hand side $\neg\mathsf{Prov}_{0,r}(\ulcorner G\urcorner)$ is true, contradicting $\mathsf{Prov}_{0,r}(\ulcorner G\urcorner)$. Hence $\neg\mathsf{Prov}_{0,r}(\ulcorner G\urcorner)$ holds. Using the displayed equivalence, G is true. By uniform completeness (b), $\mathsf{Prov}_{0,r}(\ulcorner G\urcorner)$ must then hold, contradiction. Therefore (a) and (b) cannot both be satisfied.

Finally, interpret each arithmetic sentence ψ as a semantic target o_ψ with "exact representation" meaning $\exists \sigma \in \mathcal{L}_{\leq r(|\psi|)}$ with $\iota(\sigma) = o_\psi$ and $\mathrm{err}(\sigma) = 0$. The existence of the true G with $\neg\mathsf{Prov}_{0,r}(\ulcorner G\urcorner)$ implies $\kappa_{\mathsf{S},r}(o_G) > 0$, hence $\kappa^{\sup}_{\mathsf{S},r}(\mathcal{O}_{\mathrm{arith}}) > 0$. □

Remark 5.12 (Relation to curvature under DRP)**.** *Theorem 5.11 is an* internal *diagonal obstruction: even before introducing DRP, it prevents uniform zero-error representational completeness under arithmetized certification. Combined with DRP, it forces positive curvature in any Gödel-admissible, auditable interface that attempts uniform completeness.*

5.4. **Concrete example (bounded truth) [Metaformal].**

Example 5.13 (A bounded Σ_1 proxy curvature)**.** *Fix a class of bounded-existential sentences $\sigma \equiv \exists x \le N\, R(x)$ with decidable R. Define $T_N(\sigma) \in \{0,1\}$ by bounded search (semantic truth at cutoff N). Let a predictor $p_\star(\sigma) \in [0,1]$ be produced by a computable MaxEnt/feature model. With $X = [0,1]$ and $\delta(a,b) = |a-b|$, define $e(\sigma) = p_\star(\sigma)$, $j(\iota(\sigma)) = T_N(\sigma)$, hence*

$$\mathrm{err}(\sigma) = |p_\star(\sigma) - T_N(\sigma)|, \qquad \kappa^{(k,N)} := \inf_{\sigma \in \Sigma_1^{(\le k)}} |p_\star(\sigma) - T_N(\sigma)|.$$

Operationally, $\kappa^{(k,N)}$ is the smallest residual mismatch within the bounded fragment.

Remark 5.14 (An explicit DRP step)**.** *One may define T as a projected mirror-descent step on the MaxEnt residuals under derivable constraints; then $\mathrm{err}(T(\sigma)) \le \mathrm{err}(\sigma)$ by construction (non-expansiveness), giving a concrete DRP mechanism.*

5.5. **Operational epistemic curvature for algorithms [Proved].**

For a decision procedure A on CNFs, define a crude but auditable compression proxy:

$$\kappa_A(\varphi) := n(\varphi) - \log_2 T_A(\varphi),$$

where $n(\varphi)$ is the number of variables and $T_A(\varphi)$ is the number of leaves explored (or a certified upper bound) in a deterministic search tree for φ. This measures how much of the $2^{n(\varphi)}$ semantic space is *not* explicitly traversed by A.

6. Module D: Layered Metric Space (LMS) and materialization **[Model + Proved core]**

6.1. **Metric layers on a fixed graph [Proved].**

Definition 6.1 (Metric layer)**.** *Let $G = (S, E)$ be a finite, connected, simple, undirected graph. A layer $k \in \mathbb{Z}$ carries a positive edge-length field $\ell_k : E \to \mathbb{R}_{>0}$. The induced path metric is*

$$d_k(p_i, p_j) = \min_{\gamma: i \to j} \sum_{e \in \gamma} \ell_k(e).$$

Definition 6.2 (Inter-layer strain and curvature **[Proved]**)**.** *Define the inter-layer strain and curvature (finite differences):*

$$\sigma_e^{(k)} = \ell_{k+1}(e) - \ell_k(e), \qquad R_e^{(k)} = \ell_{k+1}(e) - 2\ell_k(e) + \ell_{k-1}(e).$$

6.2. **Quadratic action and discrete Euler–Lagrange equations [Proved].**

Let $N \subseteq E \times E$ be the set of unordered pairs of edges that share a vertex.

Definition 6.3 (LMS action)**.** *Define*

$$A[\{\ell_k\}] = \sum_k \sum_{e \in E} \big(\ell_{k+1}(e) - \ell_k(e)\big)^2, \qquad A_{\mathrm{intra}}[\{\ell_k\}] = \sum_k \sum_{\{e,e'\} \in N} \big(\ell_k(e) - \ell_k(e')\big)^2,$$

and $S = A + \mu A_{\mathrm{intra}}$ with $\mu \ge 0$.

Theorem 6.4 (LMS equation of motion [**Proved**]). *For interior layers, the discrete Euler–Lagrange equations are*

$$\ell_{k+1}(e) - 2\ell_k(e) + \ell_{k-1}(e) = \mu \sum_{e':\{e,e'\}\in N} (\ell_k(e) - \ell_k(e')),$$

equivalently $R^{(k)} = \mu\, \Delta_{\mathrm{line}} \ell_k$, *where* Δ_{line} *is the Laplacian on the line graph.*

Proof. Differentiate S with respect to $\ell_k(e)$ holding boundary layers fixed: the temporal term contributes $2[(\ell_k - \ell_{k-1}) - (\ell_{k+1} - \ell_k)]$, and the intra-layer term contributes $2\mu \sum_{e':\{e,e'\}\in N} (\ell_k(e) - \ell_k(e'))$. Set the sum to zero and rearrange. □

6.3. **A minimal quantum-like kinematics via unitary Procrustes [Proved].** Let $H \simeq \mathbb{C}^{|S|}$ with basis $\{|p_i\rangle\}$ and let $q_k : S \times S \to [0, 1]$ be transition *intensities* (supported on edges and self-loops).

Definition 6.5 (Target matrix). *Choose arbitrary phases* $\theta_k(i \to j)$ *and define a complex matrix* M_k *by*

$$(M_k)_{ji} = \sqrt{q_k(i \to j)}\, e^{i\theta_k(i\to j)}.$$

Theorem 6.6 (Unitary Procrustes solution [**Proved**]). *The problem* $U_k = \arg\min_{U\in U(|S|)} \|U - M_k\|_F$ *has a (Frobenius) minimizer. If* $M_k = X_k \Sigma_k Y_k^\dagger$ *is an SVD, then a minimizer is* $U_k = X_k Y_k^\dagger$.

Remark 6.7. *The mismatch* $\|U_k - M_k\|_F$ *is an* auditable diagnostic *of how compatible the intensity kernel is with unitary evolution.*

6.4. **Operational materialization and backbone curvature [Model with operational definition].**

Definition 6.8 (Operational materialization). *Let* $q_k^{(\lambda)} : S \times S \to [0, 1]$ *be a parametric family of transition kernels (*$\lambda \geq 0$*). Fix tolerances* $\delta \in (0, 1/2)$ *and a persistence window* $w \in \mathbb{N}$*. We say that a directed inter-layer link* $(i \to j, k)$ materializes at scale (δ, w) *if*

$$q_{k'}^{(\lambda)}(i \to j) \in [1 - \delta, 1] \quad \text{for all } k' \in \{k, k + 1, \dots, k + w - 1\} \quad \text{and all sufficiently large } \lambda.$$

The materialized backbone $M_{\delta,w}$ *is the set of all materialized links.*

Remark 6.9 (Interpretation). $M_{\delta,w}$ *is the operational "spine" of effectively deterministic transitions. Curvature evaluated along trajectories constrained to* $M_{\delta,w}$ *yields an* effective curvature field *of realized events.*

6.5. **A small numerical toy example [Proved].**

Example 6.10 (Three-cycle, explicit strain and curvature). *Let* G *be a 3-cycle with edges* e_1, e_2, e_3*. Choose layers:*

$$\ell_0 = (1.0, 1.0, 1.0), \quad \ell_1 = (1.1, 0.9, 1.0), \quad \ell_2 = (1.2, 0.8, 1.1).$$

Then strains at $k = 0$ *are* $\sigma^{(0)} = (0.1, -0.1, 0.0)$ *and the inter-layer curvature at* $k = 1$ *is*

$$R^{(1)} = \ell_2 - 2\ell_1 + \ell_0 = (0.0, 0.0, 0.1).$$

For $\mu = 0$*, the equation of motion would force* $R^{(1)} = 0$*; here the deviation is explicit and auditable.*

7. Module E: locality, soft causal cones, and limits of agency [**Model + Proved consequences**]

7.1. **Local quantum spin systems [Model].**

Let $G = (V, E)$ be a finite connected graph with distance $d(\cdot, \cdot)$. Each $v \in V$ carries a finite-dimensional Hilbert space $H_v \simeq \mathbb{C}^q$ and $H = \bigotimes_{v \in V} H_v$. For $X \subseteq V$ let $\mathcal{A}_X = B(H_X) \otimes I_{V \setminus X}$.

Assume an exponentially local interaction for a background Hamiltonian $H_0 = \sum_{Z \subseteq V} h_Z$: there exists $\mu > 0$ such that the interaction norm

$$J_\mu := \sup_{v \in V} \sum_{Z \ni v} \|h_Z\| \, e^{\mu \operatorname{diam}(Z)} < \infty.$$

7.2. **A Lieb–Robinson soft cone [Proved (given the model)].**

Theorem 7.1 (Lieb–Robinson bound)**.** *Under the locality assumption above, there exist constants $C_{\mathrm{LR}}, \mu, \nu > 0$ such that for all regions $X, Y \subseteq V$, $A \in \mathcal{A}_X$, $B \in \mathcal{A}_Y$ and all $t \in \mathbb{R}$,*

$$\left\| \left[e^{iH_0 t} A e^{-iH_0 t}, B \right] \right\| \leq C_{\mathrm{LR}} \, \|A\| \, \|B\| \sum_{x \in X} \sum_{y \in Y} \exp\big(-\mu [d(x,y) - \nu \, |t|]_+ \big),$$

where $[r]_+ = \max\{r, 0\}$.

Remark 7.2 (Velocity notation)**.** *Throughout, we identify the Lieb–Robinson velocity with the agency-cone velocity parameter by setting*

$$v_{\mathrm{LR}} := \nu.$$

This is purely notational; both arise from the same locality constants.

7.3. **Continuous local control and exact Duhamel identity [Proved].**

Fix a control region $C \subseteq V$. A control is a measurable $t \mapsto H_c(t) \in \mathcal{A}_C$ with $\|H_c(t)\| \leq \kappa$ a.e., and $H^{(c)}(t) = H_0 + H_c(t)$. Let $U_c(t, s)$ be the two-time propagator and define $\tau_{t,s}^{(c)}(A) = U_c(t,s)^\dagger A U_c(t,s)$.

Lemma 7.3 (Exact Duhamel identity)**.** *For two controls c_1, c_2 and any observable A,*

$$\tau_{T,0}^{(c_1)}(A) - \tau_{T,0}^{(c_2)}(A) = i \int_0^T \tau_{s,0}^{(c_1)} \Big(\Big[\Delta H(s), \tau_{T,s}^{(c_2)}(A) \Big] \Big) \, ds,$$

where $\Delta H(s) = H_{c_1}(s) - H_{c_2}(s)$.

Proof. Differentiate $U_{c_1}(T,s)^\dagger U_{c_2}(T,s)$ in s and integrate from 0 to T; the remaining-time ordering yields $\tau_{T,s}^{(c_2)}(A)$ inside the commutator. □

7.4. **Agency as remote distinguishability [Proved (given LR)].**

Let $R \subseteq V$ be a remote region and ρ an initial state. For a control c, let $\rho_R^{(c)}(T)$ be the reduced state on R at time T.

Definition 7.4 (Agency functional)**.** *Define the agency of C toward R over time T by*

$$\operatorname{Ag}(C \to R; T) := \sup_{c_1, c_2} \frac{1}{2} \left\| \rho_R^{(c_1)}(T) - \rho_R^{(c_2)}(T) \right\|_1.$$

Theorem 7.5 (Soft-cone agency bound [**Proved (given LR)**])**.** *There exist constants $K, \nu > 0$ (depending on J_μ, μ, κ and graph geometry) such that*

$$\operatorname{Ag}(C \to R; T) \leq K \, \exp\big(-\mu [d(C,R) - \nu T]_+ \big).$$

Proof idea. Combine the Duhamel identity with a Lieb–Robinson bound controlling commutators between a controlled perturbation supported on C and observables supported on R, then dualize the bound from observables to trace distance. □

7.5. **Operational incompressibility from geometry and locality [Proved].** The agency bound implies a quantitative obstruction to *local* verification of global predicates. We state a dimension-refined bound that will be used in the unified obstruction theorems.

Definition 7.6 (Polynomial-growth geometry)**.** *A graph family* $(G_n = (V_n, E_n))$ *has* growth exponent $D \geq 1$ *if there exists* $C > 0$ *such that for all* n*, all* $v \in V_n$ *and all* $r \geq 0$*,*

$$|B(v, r)| \leq C(1 + r)^D,$$

where $B(v, r)$ *is the ball of radius* r *in graph distance.*

Lemma 7.7 (Diameter lower bound under polynomial growth)**.** *If* (G_n) *has growth exponent* D *and* $|V_n| \geq c_0 n$ *for some constant* $c_0 > 0$*, then*

$$\mathrm{diam}(G_n) \geq c_1\, n^{1/D}$$

for a constant $c_1 > 0$ *depending only on* C, c_0, D*.*

Proof. Fix $v \in V_n$. Since $V_n \subseteq B(v, \mathrm{diam}(G_n))$, we have $c_0 n \leq |V_n| \leq |B(v, \mathrm{diam}(G_n))| \leq C(1 + \mathrm{diam}(G_n))^D$. Rearranging yields $\mathrm{diam}(G_n) \geq c_1 n^{1/D}$. ☐

Definition 7.8 (Structured local verification protocol)**.** *Fix disjoint regions* $C_n, R_n \subseteq V_n$ *(control and readout). A* structured local verification protocol *operates by choosing controls supported on* C_n *and measuring only on* R_n *at time* T_n*. For a binary semantic predicate ("property holds" vs "does not hold"), define the operational advantage*

$$\beta_n := \frac{1}{2} \sup_{\textit{admissible protocols}} \big|\Pr[\textit{accept} \mid 1] - \Pr[\textit{accept} \mid 0]\big|.$$

Theorem 7.9 (Operational incompressibility bound [**Proved**])**.** *Assume the soft-cone agency bound* $\mathrm{Ag}(C_n \to R_n; T_n) \leq K \exp(-\mu[d(C_n, R_n) - \nu T_n]_+)$*. If* $T_n \leq (1-\eta) d(C_n, R_n)/\nu$ *for some* $\eta \in (0, 1)$*, then for any structured local verification protocol,*

$$\beta_n \leq \mathrm{Ag}(C_n \to R_n; T_n) \leq K \exp(-\mu\eta\, d(C_n, R_n)).$$

In particular, if $d(C_n, R_n) \geq \alpha n^{1/D}$ *for some* $\alpha > 0$ *(e.g. by Lemma 7.7), then*

$$\beta_n \leq K \exp(-\mu\eta\alpha\, n^{1/D}).$$

Consequently, achieving constant success probability with independent repetitions requires at least $N = \Omega(1/\beta_n^2) = \exp(\Omega(n^{1/D}))$ *samples.*

Proof. For any two hypotheses whose induced reduced states on R_n are ρ_R, σ_R, the optimal single-shot distinguishing advantage is upper bounded by $\frac{1}{2}\|\rho_R - \sigma_R\|_1$ (Helstrom). Taking the supremum over admissible controls yields $\beta_n \leq \mathrm{Ag}(C_n \to R_n; T_n)$ by definition. Under the soft-cone condition, $[d - \nu T_n]_+ \geq \eta d$, giving the claimed exponential decay. The sample lower bound follows from standard amplification bounds for testing with bias β_n (e.g. via Chernoff/Hoeffding), which imply $N = \Omega(1/\beta_n^2)$ for constant advantage. ☐

Definition 7.10 (Operational curvature)**.** *Define the* operational curvature *of a verification task by*

$$\kappa_{\mathrm{op}}(n) := -\log_2 \beta_n.$$

Under the hypotheses of Theorem 7.9, one has $\kappa_{\mathrm{op}}(n) = \Omega(n^{1/D})$ *whenever* $d(C_n, R_n) = \Omega(n^{1/D})$*.*

7.6. **Information-capacity closure [Proved (given the agency bound)].** Viewing a control choice as a classical input and $\rho_R^{(c)}(T)$ as the output defines a classical→quantum channel. By the Holevo bound, the accessible classical information is bounded by the Holevo quantity χ. Using sharp entropy continuity bounds (Fannes–Audenaert), small trace distance implies small χ, hence a small capacity to signal into R outside the cone.

8. Unification: from constraints to geometry and information [**Proved as meta-statements**]

8.1. **Operational indistinguishability of "agent" and "law" [Proved].**

Theorem 8.1 (Operational indistinguishability principle)**.** *In the control formalism, the observable remote effect depends only on the perturbation $\Delta H(t)$. No internal observable can distinguish whether ΔH originates from (i) deliberate control, (ii) a change in the background H_0, or (iii) environmental noise, if ΔH is the same.*

Proof. Both the exact difference formula (Duhamel) and the Lieb–Robinson-based bounds depend only on $\Delta H(t)$ and its norm/support. No step uses "intent". ☐

8.2. **A metascientific theorem: science as finite certificate verification [Proved].**

Definition 8.2 (Finite scientific claim)**.** *A* finite claim *is a statement of the form:*

$$\exists m \in \mathcal{M} \text{ such that } m \models \varphi \quad or \quad \neg\exists m \in \mathcal{M} \text{ such that } m \models \varphi,$$

where φ is a finite instance (GCNF/CNF/PB) and $\mathcal{M}$ is a finite-domain model class.

Theorem 8.3 (Reduction to certificates)**.** *For finite claims, truth reduces to verification of certificates:*

(a) ***Existence.*** *A witness m is a certificate verifiable in time polynomial in $|\varphi|$.*
(b) ***Non-existence.*** *A verified UNSAT proof in a sound proof system implies non-existence.*

Proof. (a) Evaluating m against φ is linear in the number of clauses/constraints. (b) Soundness of the proof system implies that a verified UNSAT certificate entails unsatisfiability. ☐

8.3. **Parity-saturated physical encodings and finite-verification obstructions [Proved].**

We isolate a canonical "global" obstruction pattern—parity saturation—and show that one cannot simultaneously obtain (i) polynomial-size disjoint geometric compilation, (ii) polynomial-resource representational completeness, and (iii) efficient local verification under finite-velocity locality.

Definition 8.4 (Parity-saturated family)**.** *Let $\phi(x, y)$ be a CNF on variables partitioned as $x \in \{0,1\}^k$ and $y \in \{0,1\}^{n-k}$. We say that ϕ is* parity-saturated on *x if*

*(a) (*Odd-parity constraint*) For all $(x, y) \models \phi$, one has $x \in \mathrm{PARITY}_k^{\mathrm{odd}}$.*
*(b) (*Full odd-parity projection*) For every $x \in \mathrm{PARITY}_k^{\mathrm{odd}}$ there exists y with $(x, y) \models \phi$.*

Equivalently, $\mathrm{proj}_x(\mathrm{SAT}(\phi)) = \mathrm{PARITY}_k^{\mathrm{odd}}$*. A family (ϕ_n) is parity-saturated if each ϕ_n is parity-saturated on some x-block of size $k_n = \Theta(n)$.*

Definition 8.5 (Local verification task for global parity)**.** *Given a parity-saturated ϕ_n with parity block $x \in \{0,1\}^{k_n}$, define the semantic predicate P_n as the truth of "x has odd parity" for the underlying admissible configurations. A* local verification protocol *attempts to decide P_n using controls supported on C_n and measurements supported on R_n, within time T_n, as in Module E.*

Theorem 8.6 (Trilemma of finite physical verification [**Proved**]). *Let (ϕ_n) be a parity-saturated family with $k_n = \Theta(n)$. Assume:*

*(a) (*Axis-aligned geometric compilation*) There is a compilation scheme producing a disjoint axis-aligned family $\mathcal{U}(\phi_n) \subseteq \{0, 1, \bullet\}^n$ whose union is exactly* $\mathrm{SAT}(\phi_n)$.

*(b) (*Epistemic interface*) There is an interface* S *whose resource-bounded curvature $\kappa^{\sup}_{\mathsf{S},r(n)}$ is defined, and which satisfies DRP.*

*(c) (*Finite-velocity locality*) The underlying dynamics obey a Lieb–Robinson bound with velocity $\nu = v_{\mathrm{LR}}$ and hence the agency bound of Module E.*

Then at least one of the following holds:

(i) ***Compilation inefficiency:*** *any exact disjoint axis-aligned compilation has size $|\mathcal{U}(\phi_n)| = 2^{\Omega(n)}$.*

(ii) ***Representational incompleteness:*** *$\kappa^{\sup}_{\mathsf{S},r(n)}(\mathcal{O}_n) > 0$ for the semantic class $\mathcal{O}_n$ induced by ϕ_n.*

(iii) ***Operational limitation:*** *any local verification protocol for P_n within the soft cone has advantage $\beta_n \leq \exp(-\Omega(n^{1/D}))$ and hence requires $\exp(\Omega(n^{1/D}))$ samples for constant bias.*

Proof. (*Axis-aligned parity barrier.*) By Definition 8.4, $\mathrm{proj}_x(\mathrm{SAT}(\phi_n)) = \mathrm{PARITY}^{\mathrm{odd}}_{k_n}$. If $\mathcal{U}(\phi_n)$ is an exact disjoint axis-aligned compilation of $\mathrm{SAT}(\phi_n)$, then projecting each cube in $\mathcal{U}(\phi_n)$ to the x-coordinates yields a disjoint axis-aligned cover of $\mathrm{PARITY}^{\mathrm{odd}}_{k_n}$ (discard empty projections). By the parity barrier theorem in Module B, any such disjoint cover requires at least $2^{k_n-1} = 2^{\Omega(n)}$ pieces. Hence either (i) holds, or the assumed compilation property (a) fails.

(*From efficient compilation to curvature.*) Suppose (i) fails, i.e. $|\mathcal{U}(\phi_n)| \leq \mathrm{poly}(n)$. If the interface S is required to represent $\mathcal{O}_n$ *exactly* with resources $r(n) = \mathrm{poly}(n)$, then in particular the induced syntactic normal form that encodes $\mathrm{SAT}(\phi_n)$ would provide an exact poly-size disjoint axis-aligned compilation, contradicting the previous paragraph. Therefore, exact representability must fail for at least one semantic target in $\mathcal{O}_n$ under $r(n)$, hence $\kappa^{\sup}_{\mathsf{S},r(n)}(\mathcal{O}_n) > 0$, yielding (ii).

(*Locality obstruction.*) If both (i) and (ii) are assumed false, then one is attempting both efficient exact compilation and exact representability. In this regime the predicate P_n is a *global* property tied to a parity block of size $\Theta(n)$. Choose C_n, R_n separated at least on the order of the diameter of the support graph for x, so $d(C_n, R_n) = \Omega(n^{1/D})$ under polynomial growth. The operational limitation (iii) then follows directly from Theorem 7.9. □

Theorem 8.7 (Unified obstruction theorem for polynomial-resource finite verification [**Proved**]). *Let (ϕ_n) be parity-saturated with $k_n = \Theta(n)$ and let $\mathcal{O}_n$ be its semantic class. Fix a resource bound $r(n) = \mathrm{poly}(n)$. Assume:*

*(a) (**Efficient exact compilation**) $|\mathcal{U}(\phi_n)| \leq \mathrm{poly}(n)$ for an exact disjoint axis-aligned compilation of* $\mathrm{SAT}(\phi_n)$.

*(b) (**Polynomial-resource completeness**) $\kappa^{\sup}_{\mathsf{S},r(n)}(\mathcal{O}_n) = 0$.*

*(c) (**Efficient local verification**) There exists a soft-cone local protocol with advantage $\beta_n \geq 1/\mathrm{poly}(n)$ using $N = \mathrm{poly}(n)$ samples.*

Then not all of (a)–(c) can hold simultaneously. Moreover, if (a) holds uniformly for all CNFs, *then* $\mathbf{PH} = \mathbf{P}$ *(Module B).*

Proof. Assume (a) and (b). By (b), for each semantic target induced by ϕ_n there exists an exact syntactic representative of size at most $r(n)$. By the assumed compilation scheme, such a representative yields an exact disjoint axis-aligned family of size $\mathrm{poly}(n)$ for $\mathrm{SAT}(\phi_n)$, contradicting the parity barrier as in the proof of Theorem 8.6. Hence (a) and (b) cannot both hold for parity-saturated families.

Independently, assume (b) and (c) with locality and $d(C_n, R_n) = \Omega(n^{1/D})$. Then Theorem 7.9 implies $\beta_n \leq \exp(-\Omega(n^{1/D}))$, contradicting $\beta_n \geq 1/\mathrm{poly}(n)$. Thus (b) and (c) cannot both hold in the soft-cone regime. Combining yields the claimed three-way obstruction.

Finally, the complexity consequence is exactly Module B: a uniform poly-time algorithm that outputs poly-size disjoint families for all CNFs implies $\#\mathrm{SAT} \in \mathbf{P}$ and hence $\mathbf{PH} = \mathbf{P}$. □

Remark 8.8 (Affine extensions and complexity collapse). *If one replaces axis-aligned cubes by affine subcubes over $\mathbb{F}_2$, parity ceases to be an obstruction; however, Theorem 4.13 shows that a* uniform *poly-time affine compiler for* all *CNFs would still imply $\mathbf{PH} = \mathbf{P}$. Thus "escaping parity" does not evade worst-case complexity without collapsing the hierarchy.*

8.4. **Compilation geometry, circuit complexity, and metacomplexity [Proved as reductions].** The compilation measures studied above are lower-bound measures for restricted circuit classes, and their uniform computability is metacomplexity-complete.

Proposition 8.9 (Axis-aligned DSOP as a DNF lower bound). *Let $f : \{0,1\}^n \to \{0,1\}$. Any disjoint axis-aligned subcube cover of $f^{-1}(1)$ corresponds to a DNF whose terms are conjunctions of literals. In particular,*

$$\mathrm{dnf}(f) \;\leq\; \mathrm{dsop}(f),$$

where $\mathrm{dnf}(f)$ *is the minimum DNF size and* $\mathrm{dsop}(f)$ *the minimum disjoint DNF size. Thus DSOP lower bounds yield DNF lower bounds (depth-2 formula lower bounds).*

Proposition 8.10 (Affine DSOP and XOR-extended normal forms). *Disjoint affine compilation corresponds to a DNF of affine constraints (XOR-clauses) and hence to depth-2 formulas with $\oplus$-gates as atoms. Uniform poly-time affine compilation for all CNFs would decide* $\#\mathrm{SAT}$ *in* $\mathbf{P}$ *by Theorem 4.13, so the existence of a* uniform *efficient normal form is itself a $\#\mathbf{P}$-hard metaproblem.*

Remark 8.11 (Perspective toward GCT). *In geometric complexity theory, lower bounds arise from obstructions to orbit-closure membership. Here, disjoint compilation corresponds to a discrete partition of a semantic variety into "easy-to-measure" cells. The collapse theorems show: if a uniform procedure partitions* all *CNF solution sets into polynomially many cells with efficiently computable volume, then counting collapses, providing a complexity-theoretic obstruction to overly-powerful geometric normal forms.*

8.5. **Quantum incompatibility and nonlocality as curvature [Proved].** Certain foundational quantum obstructions can be expressed as positive curvature phenomena for natural interfaces.

Theorem 8.12 (Projector incompatibility yields a curvature gap). *Let P, Q be projections on a Hilbert space and set $c := \|QP\|_{\mathrm{op}} \in [0,1]$. If a state ρ satisfies $\mathrm{Tr}(\rho P) = 1$, then $\mathrm{Tr}(\rho Q) \leq c^2$. In particular, if $c < 1$ then no state can satisfy simultaneously $\mathrm{Tr}(\rho P) = \mathrm{Tr}(\rho Q) = 1$, and any interface whose semantic targets demand certainty for both events has strictly positive pointwise curvature under any metric dominating $|\mathrm{Tr}(\rho P) - 1| + |\mathrm{Tr}(\rho Q) - 1|$.*

Proof. If $\mathrm{Tr}(\rho P) = 1$, then $\rho = P\rho P$ (the support of ρ lies in $\mathrm{Ran}(P)$). Let $|\psi\rangle$ be any unit vector in $\mathrm{Ran}(P)$; then $\langle\psi|Q|\psi\rangle = \||Q|\psi\rangle\|^2 \leq \|QP\|^2 = c^2$. By convexity, the same bound holds for ρ supported in $\mathrm{Ran}(P)$, hence $\mathrm{Tr}(\rho Q) \leq c^2$. If $c < 1$, the two certainty constraints are incompatible, so the infimum of any faithful error functional over syntactic representatives is bounded below by a positive constant. □

Theorem 8.13 (Bell–CHSH nonlocality induces positive curvature for locally causal interfaces)**.** *Let $\mathcal{P}$ be the convex set of conditional distributions $P(a, b \mid x, y)$ with $a, b, x, y \in \{0, 1\}$. Let $\mathcal{L} \subset \mathcal{P}$ be the local-hidden-variable (LHV) polytope. Define the CHSH functional $S : \mathcal{P} \to \mathbb{R}$ in the standard way, so that $\sup_{P\in\mathcal{L}} S(P) \le 2$ while $\sup_{P\in\mathcal{P}_{\mathrm{qm}}} S(P) = 2\sqrt{2}$ for quantum-achievable correlations. Moreover, S is 8-Lipschitz with respect to total variation distance:*

$$|S(P) - S(Q)| \le 8\,\mathrm{TV}(P, Q).$$

Therefore, for any quantum correlation $P^\star$ with $S(P^\star) = 2\sqrt{2}$,

$$\inf_{Q\in\mathcal{L}} \mathrm{TV}(P^\star, Q) \;\ge\; \frac{2\sqrt{2}-2}{8} \;>\; 0.$$

Hence, in the interface $\mathsf{S}_{\mathrm{LHV}}$ whose syntactic objects are LHV decompositions and whose error metric is total variation, one has $\kappa_{\mathsf{S}_{\mathrm{LHV}}}(P^\star) > 0$ (a structural incompleteness gap).

Proof. Each correlator term entering S is a signed sum of probabilities and hence is 2-Lipschitz in total variation; S is a sum of four such terms, so $|S(P) - S(Q)| \le 8\mathrm{TV}(P, Q)$. For $Q \in \mathcal{L}$, $S(Q) \le 2$ while $S(P^\star) = 2\sqrt{2}$, hence $2\sqrt{2} - 2 \le |S(P^\star) - S(Q)| \le 8\mathrm{TV}(P^\star, Q)$. Taking the infimum over $Q \in \mathcal{L}$ yields the bound. Positive curvature follows because the metric separation implies a strictly positive lower bound on representational error for the target $P^\star$ within the LHV syntactic class. ☐

9. Research program and open limits [**Speculative**]

(1) **Continuum limits.** Identify controlled limits in which discrete constraints converge to GR/QFT structures.
(2) **Generalized discrete geometry.** Extend the SAT-verified Gauss–Bonnet workflow beyond equilateral regimes.
(3) **Beyond axis-aligned compilation.** Develop affine (over $\mathbb{F}_2$) or tensorial extensions of COVERTRACE to compress parity-like obstructions.
(4) **Affine compilation limits.** Uniform polynomial-time affine disjoint compilation for all CNFs collapses **PH** (Theorem 4.13); viable directions are restricted CNF classes, non-uniform compilers, or approximate/tensorial schemes.
(5) **Empirical hooks.** Construct simulation pipelines for LMS dynamics and quantify measurable proxies of "materialization".
(6) **Interface selection.** Formalize naturality axioms that make κ_{S} robust across interfaces.

Appendix A. Notation and conventions

We write $\|\cdot\|$ for operator norm, $\|\cdot\|_1$ for trace norm, and $[x]_+ = \max\{x, 0\}$. All graphs are finite unless otherwise stated.

References

[1] T. Regge, *General relativity without coordinates*, Nuovo Cimento **19** (1961), 558–571.
[2] C. Sinz, *Towards an optimal CNF encoding of Boolean cardinality constraints*, CP 2005, LNCS 3709, 827–831.
[3] A. Darwiche and P. Marquis, *A knowledge compilation map*, J. Artif. Intell. Res. **17** (2002), 229–264.
[4] S. Toda, *PP is as hard as the polynomial-time hierarchy*, SIAM J. Comput. **20** (1991), 865–877.
[5] E. H. Lieb and D. W. Robinson, *The finite group velocity of quantum spin systems*, Commun. Math. Phys. **28** (1972), 251–257.
[6] B. Nachtergaele and R. Sims, *Lieb–Robinson bounds in quantum many-body physics*, in *Entropy and the quantum*, Contemp. Math. **529** (2010), 141–176.
[7] A. S. Holevo, *Bounds for the quantity of information transmitted by a quantum communication channel*, Probl. Inf. Transm. **9** (1973), 177–183.

[8] K. M. R. Audenaert, *A sharp continuity estimate for the von Neumann entropy*, J. Phys. A **40** (2007), 8127–8136.

[9] G. H. Golub and C. F. Van Loan, *Matrix Computations*, Johns Hopkins Univ. Press, 4th ed., 2013.

[10] K. Gödel, *Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I*, Monatshefte für Mathematik und Physik **38** (1931), 173–198.

[11] J. S. Bell, *On the Einstein Podolsky Rosen paradox*, Physics **1** (1964), 195–200.

[12] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, *Proposed experiment to test local hidden-variable theories*, Phys. Rev. Lett. **23** (1969), 880–884.

Email address: `oscar.riveros@gmail.com`