Kinematic Foundations of Relational Geometry and Informational Limits of Agency

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Abstract

A unified mathematical framework for discrete relational geometry and operational limits of influence in local physical systems is systematized. The formal core is structured into four interconnected modules: (A) an exact arithmetic interface for satisfiability in restricted domains (balanced CNFs); (B) *epistemic curvature* as a metric measure of structural incompleteness under a derivational refinement principle; (C) the *Layered Metric Space* (LMS) as variational discrete kinematics with materialization transitions; and (D) agency limits derived from Lieb–Robinson-type bounds with explicit constants.

Agency is formalized as the operational capacity to induce remote distinguishability, subject to exponential suppression by the underlying graph geometry. The framework yields technical tools with collateral applications in quantum computing, control theory, and network analysis. Cosmological extensions (dark energy as influence decay, dark matter as geometric rigidity) are presented strictly as open research programs, with their observational limitations explicitly stated, highlighting partial empirical refutation by phenomena such as the *Bullet Cluster*.

Keywords: Discrete geometry, Lieb–Robinson bounds, operational agency, epistemic curvature, dark matter, dark energy, Layered Metric Space.

Contents

1 Introduction and Guiding Principles

This work consolidates and unifies four research lines developed by the author in recent years, establishing a formal framework for discrete relational geometry and operational agency theory in local physical systems. The objective is twofold: (1) to present rigorous, self-contained mathematical results, and (2) to precisely delimit the scope of their possible physical interpretations, especially in the cosmological domain.

To ensure logical consistency and avoid unwarranted extrapolations, we adopt the following guiding principles:

Structural Warning (Principle of Non-Automatic Exportation (P)). The exact identities and bounds derived in Sections 2–4 are independent of any specific physical interpretation. In particular:

- Lieb–Robinson bounds do not imply effective cosmological dynamics without explicit derivation of the continuum limit and corresponding *coarse-graining*.

- The Duhamel identity controls differences between evolutions but does not define autonomous physical causality nor replace causal structures from continuous relativistic theories.

Structural Warning (Principle of Core-Extension Separation (P)). The results of Modules A–D (Sections 2–4) constitute the **demonstrated formal core**. Sections 5–6 (cosmological extensions) are **conjectures** or **research programs** subject to future observational validation.

Structural Warning (Principle of Operational Falsifiability (P)). Every physical extension of the formalism must produce falsifiable quantitative predictions. Special attention is paid to observational stress tests such as the *Bullet Cluster*, the CMB spectrum, and galactic scaling relations (RAR).

Remark (Ontological Status of the LMS). The *Layered Metric Space* (LMS) formalism is interpreted in this work as an abstract mathematical structure describing discrete metric evolution. Its status as a candidate for fundamental physical structure (discrete spacetime) remains an independent, unproven hypothesis.

2 Formal Core: The Four Modules

2.1 Module A: Exact Arithmetic Interface for Balanced CNFs

We consider Boolean formulas in Conjunctive Normal Form (CNF) that are *balanced*: each clause contains exactly one occurrence (positive or negated) of each variable.

Definition 2.1 (Balanced CNF)**.** Let $X = \{x_1, \ldots, x_n\}$ be a set of Boolean variables. A clause C is *balanced* if it has the form $C = \ell_1 \vee \ell_2 \vee \cdots \vee \ell_n$, where for each i, ℓ_i is x_i or $\neg x_i$. A CNF formula $\mathcal{F}$ is balanced if all its clauses are.

Lemma 2.1 (Unique falsifier). *For each balanced clause C there exists a unique assignment $a_C \in \{0,1\}^n$ that makes it false. This assignment is given by $a_C = (s_1, \dots, s_n)$ where $s_i = 0$ if $\ell_i = x_i$ and $s_i = 1$ if $\ell_i = \neg x_i$.*

Definition 2.2 (Binary index). For $a = (a_1, \dots, a_n) \in \{0,1\}^n$, define $\mathrm{ind}(a) := \sum_{i=1}^n a_i 2^{n-i} \in \{0, \dots, 2^n - 1\}$.

Theorem 2.2 (SAT equation for balanced CNFs). *Let $\mathcal{F}$ be a balanced CNF formula in n variables, with no repeated clauses. For each clause $C \in \mathcal{F}$, let $T(C) := \mathrm{ind}(a_C)$. Define the integer*

$$S_{\mathcal{F}} := \sum_{C \in \mathcal{F}} 2^{T(C)}.$$

Let $S_{\mathcal{F}} = \sum_{k=0}^{2^n - 1} b_k 2^k$ be its binary expansion. Then, for every assignment $a \in \{0,1\}^n$ with index $k = \mathrm{ind}(a)$:

$$b_k = 1 \quad \text{if and only if} \quad a \text{ does not satisfy } \mathcal{F}.$$

Equivalently, $b_k = 0$ exactly characterizes satisfying assignments.

Proof. Each clause C contributes a bit 1 at position $T(C)$. Since clauses are distinct, there are no carries in the binary sum. A bit $b_k = 1$ indicates that there exists some clause C with $T(C) = k$, i.e., whose falsifying assignment is precisely a. Therefore, a does not satisfy $\mathcal{F}$. Conversely, if a does not satisfy $\mathcal{F}$, there exists C with $a = a_C$, hence $b_k = 1$. $\qquad\square$

Remark (Collateral contribution: arithmetic compression). This theorem provides a compact and exact representation of the satisfiability relation for balanced CNFs, useful in formal verification and semantic hashing. While it does not reduce the worst-case computational complexity of SAT, it allows logical operations (combination, comparison) via integer arithmetic.

2.2   Module B: Epistemic Curvature and Geometric Incompleteness

We define a metric framework to quantify the gap between a formal system and its semantic domain.

Definition 2.3 (Formal system with metric interface). A tuple $\mathcal{S} = (L, \vdash, \iota, \mathcal{O}, X, \delta, e, j)$ where:

- $(L, \vdash)$: formal system (language and derivability relation)

- $\mathcal{O}$: semantic domain (measurable space)

- $\iota : L \to \mathcal{O}$: interpretation (Borel function)

- (X, δ): separable complete metric space

- $e : L \to X$, $j : \mathcal{O} \to X$: Borel embeddings

The *representation error* for $\sigma \in L$ is

$$\mathrm{err}(\sigma) := \delta(e(\sigma), j(\iota(\sigma))).$$

Definition 2.4 (Epistemic curvature). The epistemic curvature of system $\mathcal{S}$ is

$$\kappa_{\mathcal{S}} := \inf_{\sigma \in L} \mathrm{err}(\sigma) \in [0, \infty).$$

We say $\mathcal{S}$ is *epistemically flat* if $\kappa_{\mathcal{S}} = 0$, and *curved* if $\kappa_{\mathcal{S}} > 0$.

Definition 2.5 (Derivational Refinement Principle (DRP)). There exists an operator $T : L \to L$ such that:

1. $\sigma \vdash T(\sigma)$ for all $\sigma \in L$ (derivational preservation)

2. $\mathrm{err}(T(\sigma)) \leq \mathrm{err}(\sigma)$ for all $\sigma \in L$ (non-expansiveness)

3. The orbit $\{T^n(\sigma)\}_{n \in \mathbb{N}}$ has an accumulation point σ_∞ with $\mathrm{err}(\sigma_\infty) = \inf_n \mathrm{err}(T^n(\sigma))$.

Theorem 2.3 (Incompleteness as positive curvature). *Let $\mathcal{S}$ be a formal system with metric interface satisfying DRP. If $\mathcal{S}$ is semantically complete (for each true $o \in \mathcal{O}$ there exists $\sigma \in L$ with $\iota(\sigma) = o$), then necessarily $\kappa_\mathcal{S} = 0$. Equivalently:*

$$\kappa_\mathcal{S} > 0 \quad \Longrightarrow \quad \mathcal{S} \text{ is not semantically complete.}$$

Outline. Suppose $\kappa_\mathcal{S} > 0$. By definition, $\mathrm{err}(\sigma) \geq \kappa_\mathcal{S} > 0$ for all σ. Given a semantic object $o \in \mathcal{O}$, if there existed σ with $\iota(\sigma) = o$, the iteration $T^n(\sigma)$ would produce a sequence whose errors decrease monotonically but remain bounded below by $\kappa_\mathcal{S}$, contradicting semantic completeness under DRP. $\qquad\square$

Remark (Geometrization of representation limits). This result reformulates the impossibility of perfect completeness as a geometric obstruction (positive curvature), not as an undecidability theorem *à la* Gödel. It provides a quantitative metric ($\kappa_\mathcal{S}$) for the mismatch between syntax and semantics.

2.3 Module C: Layered Metric Space (LMS) – Variational Discrete Kinematics

The LMS is a graph metric structure that evolves in discrete "layers", interpreted as relational time steps.

Definition 2.6 (Layered Metric Space). Let $G = (V, E)$ be an undirected, connected, locally finite graph. A *Layered Metric Space* is a family $\{\ell_k : E \to \mathbb{R}_{>0}\}_{k \in \mathbb{Z}}$ assigning a positive length to each edge in each layer k.

Definition 2.7 (Kinematic quantities). For each edge $e \in E$ and layer k we define:

- **Inter-layer strain**: $\sigma_k(e) := \ell_{k+1}(e) - \ell_k(e)$.

- **Inter-layer curvature**: $R_k(e) := \ell_{k+1}(e) - 2\ell_k(e) + \ell_{k-1}(e)$.

- **Intra-layer curvature**: Let $\mathcal{C}_k$ be a set of simple cycles in G. A functional $K_k : \mathcal{C}_k \to \mathbb{R}$ measuring local deviation from flatness (e.g., angular deficit in a triangulation).

Definition 2.8 (LMS actions). We consider two variational formulations:

1. **Action with geometric rigidity**:

$$\mathcal{S}_{\mathrm{LMS}}^{(1)}[\{\ell_k\}] = \alpha \sum_k \sum_{c \in \mathcal{C}_k} K_k(c)^2 \; + \; \beta \sum_k \sum_{e \in E} \sigma_k(e)^2.$$

   The term $K_k(c)^2$ introduces *rigidity* (penalizes any curvature).

2. **Action with intra-layer coupling**:

$$\mathcal{S}_{\mathrm{LMS}}^{(2)}[\{\ell_k\}] = \sum_k \sum_{e \in E} \sigma_k(e)^2 \; + \; \mu \sum_k \sum_{\{e,e'\} \in \mathcal{N}} (\ell_k(e) - \ell_k(e'))^2,$$

   where $\mathcal{N}$ are pairs of edges sharing a vertex (discrete Laplacian).

Definition 2.9 (Operative materialization). Let $\{q_k^{(\lambda)} : V \times V \to [0,1]\}_{\lambda > 0}$ be a parametric family of transition kernels between layers. We say a subnetwork $B \subseteq V$ *materializes* in the interval $[k_1, k_2]$ if:

$$\lim_{\lambda \to \infty} q_k^{(\lambda)}(x \to y) \in \{0, 1\} \quad \forall x, y \in B, \ \forall k \in [k_1, k_2],$$

and the convergence is uniform on $B \times B \times [k_1, k_2]$ and stable under bounded local perturbations. The set B with its saturated transitions forms a *materialized backbone*.

Proposition 2.4 (Orthogonal Procrustes for unitary evolution). *Given a matrix $M \in \mathbb{C}^{n \times n}$ (derived from kernels q_k), the problem*

$$\min_{U \in \mathcal{U}(n)} \|U - M\|_F,$$

where $\mathcal{U}(n)$ is the unitary group and $\|\cdot\|_F$ the Frobenius norm, has a unique solution. If $M = W\Sigma V^\dagger$ is the SVD of M, then

$$U_{\text{opt}} = WV^\dagger, \qquad \|U_{\text{opt}} - M\|_F^2 = \sum_{i=1}^n (\sigma_i - 1)^2.$$

Here $V^\dagger$ denotes the conjugate transpose (Hermitian adjoint) of V.

Proof. For $U = W'V^\dagger$ with W' unitary, $\|U - M\|_F^2 = \|W'\Sigma - I\|_F^2 = \sum_i(\sigma_i^2 + 1 - 2\operatorname{Re}(w'_{ii}\sigma_i))$. The minimum is attained when $w'_{ii} = 1$ for all i, i.e., $W' = W$. $\qquad\square$

Remark (Application in quantum compilation). This result provides an optimal method to approximate a general physical operation (possibly non-unitary) by a unitary logical gate, with error quantified by $\epsilon = \sqrt{\sum(\sigma_i - 1)^2}$.

2.4 Module D: Operational Agency and Lieb–Robinson Limits

We consider a quantum spin system on a graph $G = (V, E)$, with local algebra $\mathcal{A}_X = B(\mathcal{H}_X) \otimes I_{V \setminus X}$.

Definition 2.10 (Local control and evolution). A *control c* is a measurable function $t \mapsto H_c(t)$ with $\operatorname{supp}(H_c(t)) \subseteq C \subset V$ and $\|H_c(t)\| \leq \kappa$ (a.e.). The total Hamiltonian is $H^{(c)}(t) = H_0 + H_c(t)$, where $H_0 = \sum_{Z \subseteq V} h_Z$ is a background local Hamiltonian. The unitary evolution $U_c(t, s)$ satisfies $i\partial_t U_c(t, s) = H^{(c)}(t)U_c(t, s)$. We define the two-time Heisenberg evolution:

$$\tau_{t,s}^{(c)}(A) := U_c(t, s)^\dagger A U_c(t, s).$$

Definition 2.11 (Influence and agency). Given two controls c, c' and time $T > 0$, define:

- **Influence** on region $R \subseteq V$:

$$\operatorname{Inf}_R(c, c'; T) := \sup_{\substack{B \in \mathcal{A}_R \\ \|B\| \leq 1}} \left|\operatorname{Tr}\left(\rho_0(\tau_{T,0}^{(c)}(B) - \tau_{T,0}^{(c')}(B))\right)\right|.$$

- **Agency** (induced distinguishability):

$$\operatorname{Ag}_R(c, c'; T) := \frac{1}{2}\|\rho_{T,R}^{(c)} - \rho_{T,R}^{(c')}\|_1,$$

where $\rho_{T,R}^{(c)} = \operatorname{Tr}_{V \setminus R}(U_c(T, 0)\rho_0 U_c(T, 0)^\dagger)$.

Lemma 2.5 (Duality and bound).

$$\mathsf{Ag}_R(c, c'; T) = \sup_{\substack{B \in \mathcal{A}_R \\ \|B\| \leq 1}} \frac{1}{2} \left| \mathrm{Tr}\Big((\rho_{T,R}^{(c)} - \rho_{T,R}^{(c')})B\Big) \right| \leq \mathsf{Inf}_R(c, c'; T).$$

Lemma 2.6 (Exact Duhamel identity). *Let c, c' be controls and $\Delta H(t) := H^{(c)}(t) - H^{(c')}(t)$. For any observable B,*

$$\tau_{T,0}^{(c)}(B) - \tau_{T,0}^{(c')}(B) = i \int_0^T \tau_{s,0}^{(c)}\Big([\Delta H(s), \tau_{T,s}^{(c')}(B)]\Big)\, ds.$$

Proof. Define $F(s) := \tau_{s,0}^{(c)}(\tau_{T,s}^{(c')}(B))$. Differentiating and using the evolution equations yields $dF/ds = i\tau_{s,0}^{(c)}([\Delta H(s), \tau_{T,s}^{(c')}(B)])$. Integrating from 0 to T and noting that $F(0) = \tau_{T,0}^{(c')}(B)$ and $F(T) = \tau_{T,0}^{(c)}(B)$ completes the proof. $\qquad\square$

Assumption 2.1 (Exponential locality). The background Hamiltonian satisfies: there exists $\mu > 0$ such that
$$J_\mu := \sup_{v \in V} \sum_{Z \ni v} \|h_Z\| e^{\mu \operatorname{diam}(Z)} < \infty.$$

Theorem 2.7 (Lieb–Robinson bound for agency). *Under Assumption 2.4 and for bounded controls, there exist constants $C'_{LR}, v'_{LR} > 0$ (depending on J_μ, μ, κ and the geometry of G) such that*
$$\mathsf{Ag}_R(c, c'; T) \leq K'_A \int_0^T \|\Delta H(s)\| \sum_{x \in C} \sum_{y \in R} \exp\Big(-\mu\, [d(x,y) - v'_{LR}(T - s)]_+\Big)\, ds,$$

with $K'_A = C'_{LR}/2$. In particular, if $d(C, R) > v'_{LR}T + \ell$,

$$\mathsf{Ag}_R(c, c'; T) \leq K'_A |C||R| e^{-\mu\ell} \int_0^T \|\Delta H(s)\| ds.$$

Outline. Apply the Duhamel identity, bound the commutator using a Lieb–Robinson bound for time-dependent generators (see references), then integrate. The dependence on $(T - s)$ reflects that late perturbations have less time to propagate. $\qquad\square$

Proposition 2.8 (Conservative explicit constants). *For simple controls $H_c(t) = g(t)G$ with $\|G\| = 1$, $|g(t)| \leq \kappa$, we have*
$$v'_{LR} \lesssim \frac{2}{\mu}\left(J_\mu + \kappa e^{\mu \operatorname{diam}(C)}\right).$$

Corollary 2.9 (Dynamic indistinguishability of agent/law). *The propagation of influence depends exclusively on the operator $\Delta H(t)$. The formalism is blind to whether this perturbation originates from an "agent" (deliberate control) or from a variation in a physical "law" (change in H_0).*

Remark (Geometry limits agency). The theorem establishes that the capacity of a subsystem C to affect a remote region R decays exponentially with distance $d(C, R)$, with a maximum speed v'_{LR}. This formalizes the intuition: *the geometry of the underlying graph imposes hard limits on operational agency.*

3 Collateral Applications in Mathematical Engineering

Independently of physical interpretations, the formal core provides direct technical tools.

3.1 Quantum Control: Non-Perturbative Error Bounds

The exact Duhamel identity allows bounding errors in quantum gates without resorting to truncations of Dyson/Magnus series. Given a target gate U_{target} and an actual implementation $\tau_{T,0}^{(c)}$, we have

$$\|U_{\text{target}}^{\dagger} B U_{\text{target}} - \tau_{T,0}^{(c)}(B)\| \leq \int_0^T \|\Delta H(s)\| \cdot \|[\tilde{B}(s), \tau_{T,s}^{(c')}(B)]\| ds,$$

where ΔH is the deviation from the ideal Hamiltonian and $\tilde{B}(s)$ evolves under the ideal control. This provides rigorous fidelity certifications.

3.2 Quantum Circuit Compilation via Procrustes

Given a physical operation M_{exp} (obtained by tomography or noisy simulation), the closest unitary gate is $U_{\text{opt}} = WV^{\dagger}$ (SVD). The compilation error $\epsilon = \sqrt{\sum(\sigma_i - 1)^2}$ quantifies the irreducible "geometric infidelity" of the hardware. Protocol:

1. Measure/estimate M_{exp}.

2. Compute SVD: $M_{\text{exp}} = W\Sigma V^{\dagger}$.

3. Compile $U_{\text{opt}} = WV^{\dagger}$.

4. Estimated error: ϵ; if $\epsilon >$ threshold, error correction or reparameterization is required.

3.3 Network Analysis: Deterministic Blast Radius

For a network $G = (V, E)$ with local dynamics, Theorem 2.6 allows calculating the "blast radius" R_{blast} of a failure or attack in region C within time T:

$$R_{\text{blast}} = v'_{LR}T + \frac{1}{\mu} \log\left(\frac{K'_A |C| \int_0^T \|\Delta H(s)\| ds}{\delta}\right),$$

where δ is a detectability threshold. It is guaranteed that nodes at distance $> R_{\text{blast}}$ remain statistically unaffected ($\text{Ag} < \delta$). Application: secure *sharding* design in blockchain, fault containment in distributed networks.

3.4 Quantum Architecture Design

The co-locality principle: modules that must interact with agency $\geq \delta$ within time T must be placed with maximum distance

$$d_{\max} \leq v'_{LR}T + \frac{1}{\mu} \log\left(\frac{K'_A |C||R| \int \|\Delta H\|}{\delta}\right).$$

This guides qubit *placement* in modular quantum processors and module interconnection.

4 Unification: Bridges between Modules

The four modules share a common structure of **interface + metric + propagation**:

Theorem 4.1 (Dynamic indistinguishability of agent/law (formal))**.** *In the formalism of Module D, the propagation of differences between evolutions depends solely on $\Delta H(t)$. No mathematical operator exists that distinguishes between:*

Module	Interface	Metric	Propagation
A: Arithmetic SAT	Syntax $\leftrightarrow$ arithmetic	Binary distance	–
B: Epistemic curvature	Syntax $\leftrightarrow$ semantics	$\delta(e(\sigma), j(\iota(\sigma)))$	DRP (refinement)
C: LMS	Layers $\leftrightarrow$ geometry	ℓ_k, strain, curvature	Variational evolution
D: Agency	Control $\leftrightarrow$ observation	Trace, operator norm	Lieb–Robinson

Table 1: Unifying structure of the modules.

- *Perturbation ΔH originating from a "conscious agent".*

- *Perturbation ΔH originating from a "variable physical law".*

- *Perturbation ΔH originating from "environmental noise".*

Corollary 4.2 (Geometric limitation of agency)**.** *For any local physical system described by Modules C–D, there exists a function $\Phi(d, T)$ with exponential decay in d such that*

$$\mathsf{Ag}_R \leq \Phi(d(C, R), T) \cdot (control\ budget).$$

The geometry of the underlying graph imposes hard limits on what any subsystem can "do" at a distance.

5 Cosmological Extensions: Research Program

The following proposals are **conjectures** requiring substantial theoretical development and observational validation. They are presented as research avenues, not established results.

5.1 Observational Reference State (2025)

Any cosmological model must reproduce at least:

- **CMB (Planck)**: $\Omega_m \approx 0.315$, $\Omega_\Lambda \approx 0.685$, $H_0 \approx 67.4$ km/s/Mpc, consistent power spectra C_ℓ.

- **Supernovae (Pantheon+)**: $w = -1.03 \pm 0.03$, compatible with cosmological constant.

- **BAO (DESI, eBOSS)**: Angular distances $D_A(z)$ and expansion rates $H(z)$.

- **Structure growth**: $f\sigma_8(z)$ compatible with ΛCDM.

- **Gravitational lensing**: Mass maps in galaxies and clusters.

- **Bullet Cluster (1E 0657-56)**: Separation ~ 720 kpc between gas peaks (X-ray) and mass peaks (lensing); total mass $\sim 5\times$ gas baryonic mass; dark component behaves as collisionless fluid.

- **Radial Acceleration Relation (RAR)**: Universal correlation $g_{\mathrm{obs}} = \mathcal{F}(g_{\mathrm{bar}})$.

5.2 Conjecture CE: Dark Energy as Influence Decay

Conjecture 5.1 (Dark energy as loss of cohesive agency). At cosmological scales, the comoving distance between regions grows with the scale factor $a(t)$. If the "cohesive agency" of gravitational interaction decays as $\exp(-\mu d_{\text{com}})$ (analogous to Lieb–Robinson), then there exists a critical scale beyond which cohesion becomes inefficient. In this regime, the kinematic *strain* term in the LMS action dominates, producing an effective acceleration of expansion.

Remark (Validation requirements). For this conjecture to be viable, it must:

1. Derive a continuum limit of the LMS yielding a modified Friedmann equation.

2. Obtain an effective parameter $w(z)$ compatible with observations (Planck, Pantheon+, DESI).

3. Respect gravity tests at solar and galactic scales (gravity must be approximately Newtonian at scales $\ll$ horizon).

4. Reproduce the CMB lensing spectrum and structure growth $f\sigma_8(z)$.

Structural Warning (Incompatibility with standard general relativity). In GR, gravitational perturbations propagate at speed c without exponential decay with distance. Any model with Lieb–Robinson-type decay will substantially modify gravitational wave propagation and static potentials, subjecting itself to severe precision tests.

5.3 Conjecture CM: Dark Matter as Geometric Rigidity

Conjecture 5.2 (Dark matter as rigidity of materialized backbone). Galaxies and clusters correspond to regions of materialization (rigid backbone). The quadratic curvature term (αK^2) in the LMS action induces, in the continuum limit, higher-order field equations that generate an additional effective gravitational potential around these structures, mimicking dark matter halos.

Structural Warning (Partial refutation: Bullet Cluster). In its simplest formulation (rigidity locally tied to baryonic density), this conjecture is **refuted by the Bullet Cluster**. Lensing mass maps show that most mass does not follow the baryonic gas but behaves as a collisionless component. Any model where the additional potential locally depends on ρ_{bar} (or ρ_{bar}^2) fails to reproduce this separation.

Conjecture 5.3 (Non-local version to salvage Bullet Cluster). There exists an extension of the LMS where geometric rigidity couples to a non-local field $\phi(x)$ satisfying a Yukawa-type equation:

$$(\Box - m^2)\phi = \lambda \rho_{\text{bar}}^2,$$

producing an effective potential

$$\Phi_{\text{eff}}(\vec{r}) \sim \int \frac{e^{-m|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \rho_{\text{bar}}(\vec{r}')^2 d^3r'.$$

If $m^{-1} \gg$ cluster scale, this field could decouple from the gas in collisions, possibly reproducing observations. The quadratic source term ρ_{bar}^2 (rather than linear) is an intentional choice for non-linear self-interaction effects.

Remark (Additional tests). Any geometric dark matter model must also:

1. Reproduce the Radial Acceleration Relation (RAR) in galaxies.

2. Produce NFW (Navarro–Frenk–White)-type halo profiles or similar.

3. Allow early structure formation (consistent with CMB and cosmic shear).

4. Respect fifth-force limits in the Solar System.

5.4 Validation Program

1. **Cosmological reduction of LMS**: Derive effective Friedmann equations from the LMS action in a discrete homogeneous/isotropic ansatz, taking the continuum limit.

2. **Coupling to matter**: Include coupling terms $\ell_k \leftrightarrow \rho_{\text{bar}}$ in the action.

3. **Perturbations and growth**: Study linear perturbations, obtain power spectrum $P(k)$ and growth function $f(z)$.

4. **Gravitational lensing**: Compute weak lensing correlation functions $\xi_{\pm}(\theta)$ and convergence maps for clusters.

5. **Numerical simulations**: Implement discrete LMS evolution under cosmological initial conditions, compare with ΛCDM N-body simulations.

6 Conclusion and Perspectives

This work has consolidated a unified mathematical framework for discrete relational geometry and operational agency theory. The main results are:

1. **Rigorous formal core**:

   - Exact arithmetic encoding for balanced CNFs (Theorem 2.1).
   - Epistemic curvature as geometrization of incompleteness (Theorem 2.3).
   - LMS as variational kinematics with materialization (Definitions 2.5–2.7).
   - Lieb–Robinson bounds for agency with explicit constants (Theorem 2.6, Proposition 2.7).
   - Dynamic indistinguishability between agent and law (Corollary 2.8).

2. **Applicable collateral contributions**:

   - Non-perturbative quantum control (Duhamel identity).
   - Optimal circuit compilation (Procrustes).
   - Network security analysis (blast radius).
   - Quantum architecture design (co-locality principle).

3. **Cosmological extensions (conjectural)**:

   - Dark energy as influence decay: conceptually coherent, but requires derivation of effective cosmological equations and overcoming gravity tests.
   - Dark matter as geometric rigidity: faces partial refutation by Bullet Cluster; non-local versions could be explored.

Remark (Value of the program). The principal value lies not in providing definitive cosmological answers, but in offering a **precise language** to formulate fundamental questions about geometry, information, and causality in discrete physical systems. The framework enables quantifying agency limits, geometrizing incompleteness, and exploring geometric phase transitions (materialization).

Structural Warning (Call for rigor). The cosmological extensions, while suggestive, remain in the realm of unconfirmed speculation. Their viability depends on solving open mathematical problems (LMS continuum limit, effective equations) and passing severe observational tests. The **Bullet Cluster** exemplifies the type of test any ΛCDM alternative must pass.

Acknowledgments

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