DESIGN A FUNCTION OF THE H FAMILY THAT SOLVES THE NP-COMPLETE SUM SUBSET PROBLEM IN O(n)

P = NP

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This *Open Paper* aims to provide an irrefutable and easy understanding proof of the Millennium problem P vs NP, in addition to introducing the necessary tools for such purpose, the technologies derived from this paper, should strictly and explicitly mentioned the author being the only condition to be used for purposes appropriate, but either way without any restriction, I reserve the rights, patents and marketing this knowledge and directly derived from this work.

1 DESIGN AND DISCUSSION

Let be $\Phi = \{a, b, c, d\}$, an arbitrary set (without loss of generality), with $a, b, c, d \in \mathbb{N}$, consider all binary numbers of length $|\Phi|$:

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

and if we consider Φ as a vector, then

$$\mathcal{M}\Phi = (0,\ a,\ b,\ a+b,\ c,\ a+c,\ b+c,\ a+b+c,\ d,\ a+d,\ b+d,\ a+b+d,\ c+d,\ a+c+d,\ b+c+d,\ a+b+c+d)$$

Which is precisely, all sums of subsets of Φ.

$$[], [a], [b], [a,b], [c], [a,c], [b,c], [a,b,c], [d], [a,d], [b,d], [a,b,d], [c,d], [a,c,d], [b,c,d], [a,b,c,d]]$$

But the complexity of this method is complete, i,e. the complexity its $|\Phi|2^{|\Phi|}$, But with the help of the universal theory of numbers, build a function that allows write the same operation, but as a linear combination of elements of the set.

now consider this:

$$1010101010101010a + 1100110011001100b + 1111000011110000c + 1111111100000000d$$

This linear combination, from the elements and column of $\mathcal{M}$ considering these as integers. its the same operation is the equivalent of the matrix operation, but on universal theory of numbers, to see clearly, consider the following operation:

$$\phi = \sum_{t=0}^{|\Phi|} \Phi[t]10^t$$

Where $\Phi[n]$ correspond to n-th element of Φ in the natural order. it is a number $\phi \in \mathbb{N}$, now if $\Phi = [[], [a], [b], [a,b], [c], [a,c], [b,c], [a,b,c], [d], [a,d], [b,d], [a,b,d], [c,d], [a,c,d], [b,c,d], [a,b,c,d]]$

then ϕ its a number whose digits are the sum of subsets of {a,b,c,d,e} in the base 10, now to prevent numbers overlap with each other, rewrite

$$\phi = \sum_{t=0}^{|\Phi|} \Phi[t](10^m)^t$$

with appropriate m the numbers con be coexist without overlap, in the natural way. So the problem comes down to knowing how to generalize a formula for the columns of the matrix $\mathcal{M}$. The symmetries are evident, and to exploit it to the maximum, to reduce the combinatorial explosion, a simple mathematical formula:

$$K(n,m) = \sum_{t=0}^{n-1} [(10^m)^t(10^m)]^n$$

$$J(n,k,m) = \sum_{t=0}^{k} K(n,m)(10^m)^{2nt}$$

$$R(k,n,m) = J(2^k, 2^{(n-k-1)} - 1, m)$$

$$\mathcal{R}(k,n,m) = \frac{10^{2^k m}\left(10^{2^n m} - 1\right)}{\left(10^{2^k m} + 1\right)\left(10^m - 1\right)}$$

$$0 \le k < n$$

$$1 < m \le \max\{|\phi| : \phi \in \Phi\}$$

then

$$1010101010101010a + 1100110011001100b + 1111000011110000c + 1111111100000000d = R(0, |\Phi|, 1)a + R(1, |\Phi|, 1)b + R(2, |\Phi|, 1)c + R(3, |\Phi|, 1)d$$

Now in terms of computational complexity, since the formula $R(k,n,m)$ consists only of basic operations can be considered $O(1)$ for small sets $|\Phi| < 20$, but exist very efficient algorithms for computing large exponentials "exponentiation by squaring" is an example, for some fixed k the complexity is $\sum_{i=0}^{O(log(n))} (2^i O(log(x)))^k = O((nlog(x))^k)$, there are variety of algorithms for large exponentiation, and possibly exist more efficient in the future, for more information this is an excellent compendium and starting point to know these algorithms. http://en.wikipedia.org/wiki/Exponentiation_by_squaring

Example 1.

$$\Phi = [31, 554, 523, 5]$$

$$\sum_{t=0}^{|\Phi|} R(t, |\Phi|, 5)\Phi[t] = 1113010820055900528005900055900036000050110801077005540052300585005540003100000$$

and the sum os subsets in traditional way:

$$[1113, 1082, 559, 528, 590, 559, 36, 5, 1108, 1077, 554, 523, 585, 554, 31, 0]$$

Now, find the target can take many forms, such as a simple text search for example.

In practice, although the algorithm remains linear for sets of more than 20 elements, since the algorithm delivers the full details of the problem, it tends to push the boundaries of home computers, but with proper machines such as the large companies, this limit should be large. permitting, to do things that were thought impossible until now.

Suppose that $|\Phi|$ is the length of the set Φ, then the length of output of $\mathcal{R}$ is $< m|\Phi|2^{|\Phi|}$, and by the binary numbers technique used $\mathcal{R}$ is in PSAPCE-complete, like QSAT.

2 INDEPENDENCE OF THE BASIS AND SYMBOLIC COMPUTATION OF R

If we rewrite R, so that the base is represented by a variable z, we eliminate the parameter m, and obtain a polynomial of z:

$$\mathcal{R}(k,z)\frac{z^{\left(2^k\right)}\left(z^{\left(2^n\right)}-1\right)}{\left(z^{\left(2^k\right)}+1\right)(z-1)}$$

Example 2. So for the set $\Phi = \{11, 7, 5, 3, 2\}$:

$$\sum_{k=0}^{|\Phi|}\mathcal{R}(k,z)\Phi[k] = \frac{11\left(z^{32}-1\right)z^{16}}{(z^{16}+1)(z-1)} + \frac{7\left(z^{32}-1\right)z^{8}}{(z^{8}+1)(z-1)} + \frac{5\left(z^{32}-1\right)z^{4}}{(z^{4}+1)(z-1)} + \frac{3\left(z^{32}-1\right)z^{2}}{(z^{2}+1)(z-1)} + \frac{2\left(z^{32}-1\right)z}{(z+1)(z-1)}$$

and simplified to full

$$28\,z^{31}+26\,z^{30}+25\,z^{29}+23\,z^{28}+23\,z^{27}+21\,z^{26}+20\,z^{25}+18\,z^{24}+21\,z^{23}+19\,z^{22}+18\,z^{21}+16\,z^{20}+16\,z^{19}+14\,z^{18}+13\,z^{17}+11\,z^{16}+17\,z^{15}+15\,z^{14}+14\,z^{13}+12\,z^{12}+12\,z^{11}+10\,z^{10}+9\,z^{9}+7\,z^{8}+10\,z^{7}+8\,z^{6}+7\,z^{5}+5\,z^{4}+5\,z^{3}+3\,z^{2}+2\,z$$

an the sum of all subsets of Φ in traditional way is:

$$\{28, 26, 25, 23, 23, 21, 20, 18, 21, 19, 18, 16, 16, 14, 13, 11, 17, 15, 14, 12, 12, 10, 9, 7, 10, 8, 7, 5, 5, 3, 2, 0\}$$

which are the coefficients of above polynomial.

3 ON THE SET OF NUMBERS SATISFYING THE TARGET

Suppose our target is $21 = 3 + 7 + 11$ in the above example, is is easy to see that 21 is coefficient of the power 26 and 23 of z, then for construction:

$$26_2 = 11010 \rightarrow \Phi[4] + \Phi[3] + \Phi[1] = 11 + 7 + 3 = 21$$

$$23_2 = 10111 \rightarrow \Phi[4] + \Phi[2] + \Phi[1] + \Phi[0] = 11 + 5 + 3 + 2 = 21$$

4 References

1. [1]All my research has led me to this point, and all the material is free, read my work, for details on the universal theory of numbers. [2]https://independent.academia.edu/oarr
2. An implementation https://github.com/maxtuno/SSP-Riveros
3. Remember that there are more efficient algorithms to compute elementary functions, such as exponentiation, this is an excellent resource. http://www.amazon.com/Elementary-Functions-Implementation-Jean-Michel-Muller/dp/0817643729

The code for SAGE

```
k, m, n, t = var('k, m, n, t')
K(n, m)    = sum(((10^m)^(t) * (10^m)^n), t, 0, n - 1)
J(n, k, m) = sum(K(n, m)*(10^m)^((2*n)*t), t, 0, k)
R(k, n, m) = J(2^k,  2^(n - k - 1) - 1, m)

S = [31, 554, 523, 5]
sum([R(k, len(S), 5)*S[k] for k in range(len(S))])
# 1113010820055900528005900055900036000050110801077005540052300585005540003100000

[sum(s) for s in subsets(S)][::-1]
# [1113, 1082, 559, 528, 590, 559, 36, 5, 1108, 1077, 554, 523, 585, 554, 31, 0]
```

The code for Python 3

```
def R(k, n, m):
    return (10**((2**k)*m))*(10**((2**n)*m) - 1)//((10**((2**k)*m) + 1)*(10**m - 1))

S = [31, 554, 523, 5]

print(int(sum([R(k, len(S), 5)*S[k] for k in range(len(S))])))

# 1113010820055900528005900055900036000050110801077005540052300585005540003100000
# [1113, 1082, 559, 528, 590, 559, 36, 5, 1108, 1077, 554, 523, 585, 554, 31, 0]
```


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