

# A $O(n)$ UNT ALGORITHM FOR THE UNION-FIND PROBLEM

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[http://github.com/maxtuno/On\\_UNT\\_Union\\_Find\\_Algorithm](http://github.com/maxtuno/On_UNT_Union_Find_Algorithm)

## 1 THE UNION-FIND PROBLEM

The general setting for the union-find problem is that we are maintaining a collection of disjoint sets  $\{S_0, S_1 \dots S_{k-1}\}$  over some universe, with the following operations:

**union**( $x, y$ ): replace the set  $\{x\}$  is in (let's call it  $S$ ) and the set  $\{y\}$  is in (let's call it  $S'$ ) with the single set  $S \cup S'$ .

**connected**( $x, y$ ): test if exist a subset  $S$  with  $x$  and  $y \in S$ .

## 2 DATA REPRESENTATION

Suppose there is an arbitrary but finite number of elements  $\{x_0, x_1 \dots x_{k-1}\}$  then can represent the entire structure with,  $\{1, 2, 4, \dots, 2^{(k-1)}\}$  and the subsets how  $\{x_{i_0}, x_{i_1} \dots x_{i_l}\} = \sum_{i \in \{i_0, \dots, i_l\}} 2^i$ .

**Example 1.** Let be  $U = \{P, NP, =, \neq\}$  then the subset  $\{P, =, NP\}$  is writting like  $2^0 + 2^1 + 2^3 = 11$ , and  $\{P, \neq, NP\}$  is writting like  $2^0 + 2^1 + 2^4 = 19$ .

### 3 ALGORITHM DESIGN & IMPLEMENTATION

With all generality can only work with integers forget the particular given set, then implementation is as follows:

#### 3.0.1 PYTHON IMPLEMENTATION

```
universe = []

def union(a, b):
    element = (1 << a) | (1 << b)
    idx = 0
    while idx != len(universe):
        if universe[idx] & element:
            universe[idx] |= element
            element = universe[idx]
            del universe[idx]
            idx -= 1
        idx += 1
    universe.append(element)

def connected(a, b):
    for item in universe:
        if (item & (1 << a)) and (item & (1 << b)):
            return item
    return 0
```

## 4 EXPLANATION

### 4.1 UNION

1. There is the simple unsorted list for storage the elements and fusion them

```
universe = []
```

2. The definition of function, this receives two arguments that are the indices of the elements of the original set.

```
def union(a, b):
```

3. This is equivalent to  $2^a + 2^b$  but it is more efficient for that is a bit shifting and not an exponencicion

```
element = (1 << a) | (1 << b)
```

4. It is to merge elements if necessary, the loop is executed  $|U|$  times, that is equal to number of subsets or paths.

```
idx = 0
while idx != len(universe):
```

5. If there are elements in common the merges and eliminates the excess if not exit the loop, this operation is executed each time.

```
if universe[idx] & element:
    universe[idx] |= element
    element = universe[idx]
    del universe[idx]
    idx -= 1
idx += 1
```

6. Finally append the new element.

```
universe.append(element)
```

### 4.2 CONNECTED

The explanation its the same, the diference is when it finds a match, it returns the subset, if not it returns 0 ( $\emptyset$ ).

## 5 COMPLEXITY

Suppose the base set has  $n$  elements, the minimal amount of information for representing this on a Turing Machine (binary) its the sequence  $\{0, 1, 2, \dots n\}$ , there is

$$\text{tape} = \{\underbrace{00 \dots 001}_0 - \underbrace{00 \dots 010}_1 - \underbrace{00 \dots 010}_2 - \dots - \underbrace{10 \dots 000}_n - \}$$

then on the working tape, on the a-th and b-th bit can be on  $n$  steps. with the following turing machine.

Turing Machine:

OR GATE (010, 101)

tape: 010101

1)010[1]01

2)01[1]\_01 (0,1=1)

3)011\_[0]1

4)0[1]1\_\_1 (0,1=1)

5)011\_\_[1]

6)[1]11\_\_\_\_ (0,1=1)

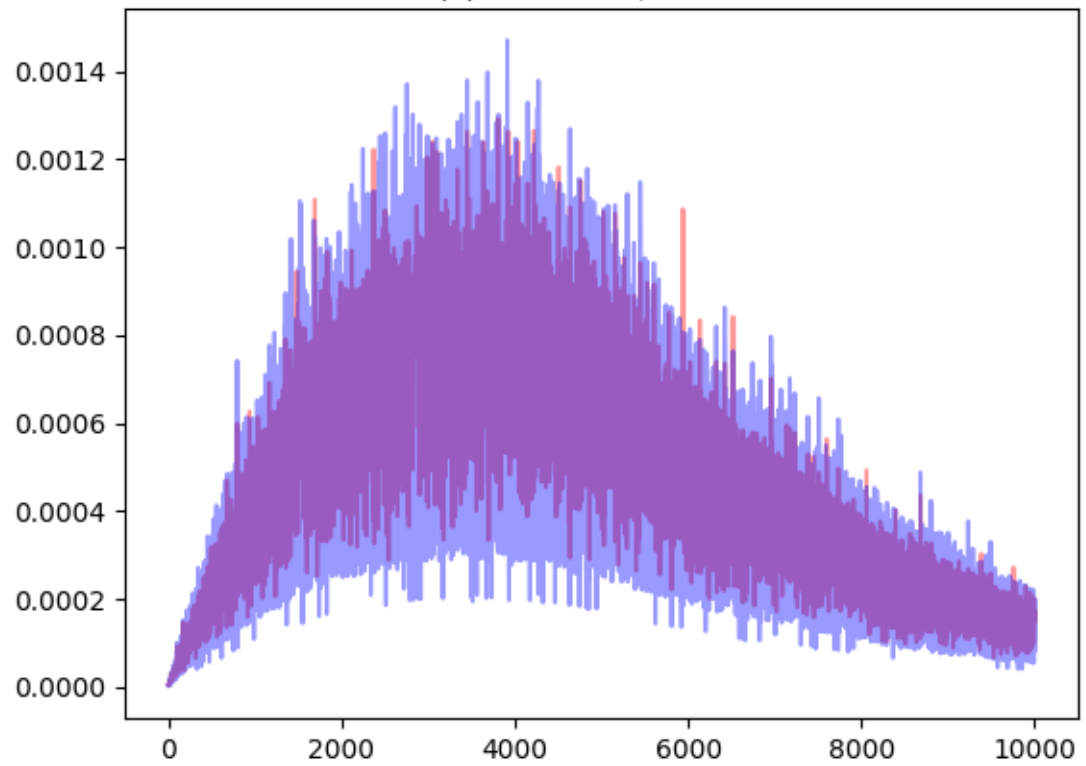
$\Rightarrow O(n)$

For the & gate its the same, and for representing sets, not needed do  $(1 \ll a) \mid (1 \ll b)$  if already take for  $n$  (3) 001, 010, 100 = 1,2,4, then input tape is for  $m$  elements:

$$\text{tape} = \{\underbrace{00 \dots 001}_0 - \underbrace{00 \dots 010}_1 - \underbrace{00 \dots 010}_2 - \dots - \underbrace{10 \dots 000}_n - \} = n + n^2 \text{ this take } n \text{ for}$$

OR +  $n$  times AND =  $n + n^2$ , this is  $O(n)$  (same of input size).

UNION CONNECTED UNT ALGORITHM  
TIME (s) vs UNION/CONNECTED



## 6 EXAMPLE FROM STANDARD PUBLICATIONS

```
universe = []

def union(a, b):
    element = (1 << a) | (1 << b)
    idx = 0
    while idx != len(universe):
        if universe[idx] & element:
            universe[idx] |= element
            element = universe[idx]
            del universe[idx]
            idx -= 1
        idx += 1
    universe.append(element)

def connected(a, b):
    for item in universe:
        if (item & (1 << a)) and (item & (1 << b)):
            return item
    return 0

def print_universe():
    print(', '.join([bin(element)[2:] for element in universe]))

if __name__ == '__main__':
    union(3, 4)
    print_universe()
    union(4, 9)
    print_universe()
    union(8, 0)
    print_universe()
    union(2, 3)
    print_universe()
    union(5, 6)
    print_universe()
    union(5, 9)
    print_universe()
    union(7, 3)
    print_universe()
    union(4, 8)
    print_universe()
    union(6, 1)
    print_universe()
```

```

11000
1000011000
1000011000, 1000000001
1000000001, 1000011100
1000000001, 1000011100, 1100000
1000000001, 1001111100
1000000001, 1011111100
1111111101
1111111111

```

3-4 0 1 2 4 4 5 6 7 8 9

4-9 0 1 2 9 9 5 6 7 8 9

8-0 0 1 2 9 9 5 6 7 0 9

2-3 0 1 9 9 9 5 6 7 0 9

5-6 0 1 9 9 9 6 6 7 0 9

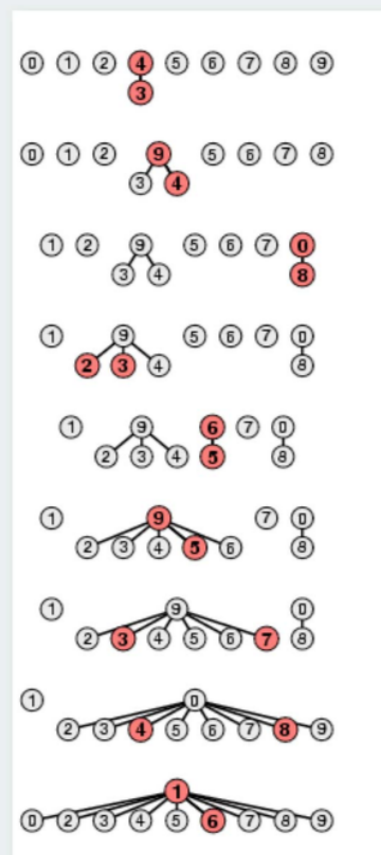
5-9 0 1 9 9 9 9 9 7 0 9

7-3 0 1 9 9 9 9 9 9 0 9

4-8 0 1 0 0 0 0 0 0 0 0

6-1 1 1 1 1 1 1 1 1 1

problem: many values can change



## 7 DISCLAIMER

All content is original and of my authorship, there are no partners or affiliations, only used basic concepts of the theory of computational complexity, and standard definitions, giving references is impossible, are part of the general culture. the UNT is acronym for the UNIVERSAL NUMBER THEORY, look at my other papers.

## 8 LICENCE

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