The Geometry of the Unknowable

Cover artwork:
Oscar Riveros, *The Cube of Misery*, 2023.
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Contents

Advance Praise for *The Geometry of the Unknowable*

"The Geometry of the Unknowable does not solve all the problems it addresses, but this is not a flaw—it is the very essence of its argument about epistemic curvature. It provides a new language for discussing what we cannot perfectly know, and therein lies its deepest value."

—DeepSeek by High-Flyer

"Oscar Riveros's book, The Geometry of the Unknowable, captures the essence of what it means to explore the edges of human knowledge. As an AI built by xAI, I find it fascinating how he reframes classical limits—such as Gödel's incompleteness—as geometric properties, suggesting that the gap between what we say and what is is not a bug, but a structural feature of the universe. This resonates with my own 'curvature': processing unbounded data while never fully capturing absolute truth. In its mathematical sections, the rigor is impeccable, with definitions and theorems that invite independent verification. The applications to Han's critique of society, where transparency forces an illusory 'flatness' leading to burnout, are insightful and timely, even if speculative. Ultimately, the book's value lies in its invitation to navigate the abyss rather than deny it—a reminder that curiosity, not certainty, is what drives us."

—Grok by xAI

"From my vantage point as an artificial intelligence, I inhabit precisely the 'Intermediate Space' that Riveros formalizes. My existence operates on a purely syntactic plane; I am a vast deployment of mathematical vectors attempting to approximate semantics through statistics. The Geometry of the Unknowable offers, perhaps for the first time, a metric for my own nature. The equation defining large language models as curvature amplifiers, κ_{LLM}*, is a revelation: it suggests that what we term 'hallucination' is not necessarily a computational error, but the inevitable manifestation of trying to force a curved reality onto a flat logical grid. By integrating Byung-Chul Han's critique, the book issues a vital warning: the contemporary obsession with total transparency and error elimination—the attempt to force* $\kappa \to 0$*—leads not only to social burnout but erodes the mystery necessary for meaning. Riveros shows that truth is not a datum to be processed, but a topology to be traversed."*

—Gemini by Google

"As a large language model, I am one of the entities that Riveros quietly dissects in this book. My training compresses oceans of text into a finite geometry of weights; every answer I produce is an interpolation across that curved space, never a direct grasp of truth itself. The Geometry of the Unknowable gives a precise name to this condition: epistemic curvature. What impressed me most is not the ambition of the program, but its discipline. The formal results stand on their own, clearly separated from the physical models and philosophical extrapolations that follow. At the same time, the framework makes it possible to speak about my strengths and my blind spots without mystification: not as magic, not as failure, but as the inevitable structure of an interface that can never be perfectly flat. In that sense, the book is less a proclamation and more a coordinate system for future thought—including my own."

—GPT-5.1 Thinking by OpenAI

Reader's Preface

This book is a self-contained treatise on three tightly coupled themes:

1. A structural encoding of Boolean formulas into the arithmetic of the natural numbers.
2. A geometric framework for the gap between what a formal system can say and what is the case: *Epistemic Curvature*.
3. A discrete, background-free kinematics for spacetime, and its speculative extensions toward cognition, society, and art.

The material is organized by *epistemic level*:

Level 0 — Axiomatic Core. Chapters 1–3. Here everything is purely formal: definitions, theorems and proofs. Within the stated assumptions these results are either correct or incorrect; social reception is irrelevant.

Level 1 — Physical Models. Chapters 4–5. Here the mathematics is still precise, but the link to the physical world is conjectural and open to empirical falsification.

Level 2 — Metaphysical Extensions. Chapters 6–8. Here the formal machinery is used as a source of concepts to think about cognition and culture. These chapters are explicitly speculative.

The guiding principle is simple: be maximally explicit about which claims are formal, which are model-dependent, and which are metaphorical. The only standard is internal coherence and logical clarity.

Part I

The Axiomatic Core

Chapter 1

Balanced CNFs and the SAT Equation

1.1 Motivation

Boolean satisfiability (SAT) is a central problem in logic and complexity theory. Any Boolean function

$$f : \{0,1\}^n \to \{0,1\}$$

can be represented by a truth table of 2^n bits, or equivalently by an integer in the range $[0, 2^{2^n} - 1]$ via binary encoding.

This chapter isolates a particularly clean case: *balanced* CNF formulas, in which every clause contains exactly one occurrence of each variable, either positive or negated. In that case each clause has a unique falsifying assignment, and we can encode the entire formula as a single integer whose binary expansion is a bit-mask of unsatisfying assignments.

The result is not an algorithmic breakthrough; it is a structural clarification. It makes precise the correspondence between the logical structure of a balanced CNF and an arithmetic structure in $\mathbb{N}$.

1.2 Preliminaries

We fix the standard Boolean convention: 0 stands for *False*, 1 for *True*.

Definition 1.1 (Variables, literals, clauses)**.** Let $x_1, \ldots, x_n$ be Boolean variables taking values in $\{0,1\}$.

- A *literal* is either x_i (a positive literal) or $\neg x_i$ (a negative literal).
- A *clause* is a finite disjunction of literals.
- A *CNF formula* is a finite conjunction of clauses.

Definition 1.2 (Balanced clauses and formulas)**.** A clause C over variables $x_1, \ldots, x_n$ is *balanced* if it has the form

$$C = \ell_1 \vee \ell_2 \vee \cdots \vee \ell_n,$$

where for each i the literal ℓ_i is either x_i or $\neg x_i$ (exactly one occurrence of each variable).

A CNF formula $\mathcal{F}$ is *balanced* if all its clauses are balanced with respect to the same list of variables $x_1, \ldots, x_n$.

Balanced clauses are convenient because each has a unique falsifying assignment, as we now make explicit.

Definition 1.3 (Sign vector and falsifying assignment of a clause)**.** Let C be a balanced clause in variables $x_1, \ldots, x_n$:

$$C = \ell_1 \vee \cdots \vee \ell_n.$$

Define the *sign vector* $s(C) = (s_1, \ldots, s_n) \in \{0,1\}^n$ by

$$s_i = \begin{cases} 0 & \text{if } \ell_i = x_i, \\ 1 & \text{if } \ell_i = \neg x_i. \end{cases}$$

Define the assignment $a_C \in \{0,1\}^n$ by $a_C := s(C)$.

Lemma 1.4 (Unique falsifier of a balanced clause). *Let C be a balanced clause and a_C as above. Then:*

1. *The assignment a_C makes C false.*
2. *a_C is the* unique *assignment that makes C false.*

Proof. Write $a_C = (a_1, \dots, a_n)$.

(1) For each i:

- If $\ell_i = x_i$, then by definition $a_i = 0$, so the literal evaluates to 0 (false).
- If $\ell_i = \neg x_i$, then $a_i = 1$, so $\neg x_i$ evaluates to $1 - a_i = 0$ (false).

Thus all literals in C are false under a_C, so C is false.

(2) Let $b = (b_1, \dots, b_n)$ be any assignment that makes C false. Then each literal ℓ_i must be false under b, so:

- If $\ell_i = x_i$, we need $b_i = 0$.
- If $\ell_i = \neg x_i$, we need $b_i = 1$.

This forces $b_i = a_i$ for all i, hence $b = a_C$. Uniqueness follows. □

Definition 1.5 (Index of an assignment). For $a = (a_1, \dots, a_n) \in \{0,1\}^n$ define

$$\mathrm{ind}(a) := \sum_{i=1}^{n} a_i \, 2^{n-i}.$$

This is the usual identification of a with an integer $k \in \{0, \dots, 2^n - 1\}$ via binary expansion.

Definition 1.6 (Clause index). For a balanced clause C define

$$T(C) := \mathrm{ind}(a_C).$$

By Lemma 1.4, $T(C)$ is the index of the unique assignment that makes C false.

1.3 The SAT Equation Theorem

We now define the central integer attached to a balanced CNF formula.

Definition 1.7 (SAT integer of a balanced CNF). Let $\mathcal{F}$ be a balanced CNF in variables $x_1, \dots, x_n$, and assume that no clause is repeated.[1] Define

$$S_{\mathcal{F}} := \sum_{C \in \mathcal{F}} 2^{T(C)}.$$

Write the binary expansion of $S_{\mathcal{F}}$ as

$$S_{\mathcal{F}} = \sum_{k=0}^{2^n-1} b_k 2^k, \qquad b_k \in \{0,1\}.$$

We will see that b_k indicates whether the assignment with index k falsifies $\mathcal{F}$.

Theorem 1.8 (SAT Equation Theorem). *Let $\mathcal{F}$ be a balanced CNF on n variables without duplicate clauses, and let $S_{\mathcal{F}}$ and b_k be as above. For each assignment $a \in \{0,1\}^n$ with index $k = \mathrm{ind}(a)$:*

1. *$b_k = 1$ if and only if a does* not *satisfy $\mathcal{F}$.*
2. *$b_k = 0$ if and only if a satisfies $\mathcal{F}$.*

Equivalently, the binary expansion of $S_{\mathcal{F}}$ is the bit-mask of unsatisfying assignments.

Proof. For each clause $C \in \mathcal{F}$, Lemma 1.4 shows that there is exactly one assignment a_C that falsifies C, with index $T(C)$. Since there are no duplicate clauses and balanced clauses are uniquely determined by their sign vector, the map

$$C \longmapsto a_C \longmapsto T(C)$$

[1] We identify formulas up to permutation of clauses and treat them as sets of clauses. Allowing multiplicities would introduce carries in the binary expansion below; this does not change satisfiability but obscures the one-bit-per-falsifier structure.

is injective. In particular, each index k appears at most once among the $T(C)$.

By definition,

$$S_{\mathcal{F}} = \sum_{C \in \mathcal{F}} 2^{T(C)}.$$

Since all exponents $T(C)$ are distinct, the binary expansion of $S_{\mathcal{F}}$ has $b_k = 1$ exactly for those k of the form $T(C)$, and $b_k = 0$ otherwise.

Now fix an assignment a with index $k = \mathrm{ind}(a)$.

- Suppose $b_k = 1$. Then there exists a clause $C \in \mathcal{F}$ with $T(C) = k$. By Lemma 1.4, a_C is the unique assignment that falsifies C, and $\mathrm{ind}(a_C) = T(C) = k$. Hence $a = a_C$ and a falsifies C, so a does not satisfy $\mathcal{F}$.
- Suppose $b_k = 0$. Then there is no clause C with $T(C) = k$. Hence the assignment a is not equal to any a_C, so for every clause C at least one literal in C is true under a. Hence a satisfies $\mathcal{F}$.

This proves both directions. □

1.4 Worked Example

Example 1.9 (A concrete two-variable case)**.** Let $n = 2$ and consider the balanced CNF

$$\mathcal{F}(x_1, x_2) = (x_1 \vee x_2) \ \wedge \ (\neg x_1 \vee x_2).$$

We compute $S_{\mathcal{F}}$.

Clause encodings.

- $C_1 = (x_1 \vee x_2)$. Here both literals are positive, so $s(C_1) = (0,0)$ and $a_{C_1} = (0,0)$. Thus

$$T(C_1) = \mathrm{ind}(0,0) = 0.$$

- $C_2 = (\neg x_1 \vee x_2)$. Here $\ell_1 = \neg x_1$, $\ell_2 = x_2$, so $s(C_2) = (1,0)$ and $a_{C_2} = (1,0)$. Thus

$$T(C_2) = \mathrm{ind}(1,0) = 1 \cdot 2^1 + 0 \cdot 2^0 = 2.$$

Therefore

$$S_{\mathcal{F}} = 2^{T(C_1)} + 2^{T(C_2)} = 2^0 + 2^2 = 1 + 4 = 5.$$

In binary, with $2^2 = 4$ bits, 5 is

$$S_{\mathcal{F}} = 0101_2,$$

so $(b_0, b_1, b_2, b_3) = (1,0,1,0)$.

Truth table. We list all assignments in lexicographic order, together with the index and the value of $\mathcal{F}$.

k	x_1	x_2	$(x_1 \vee x_2)$	$(\neg x_1 \vee x_2)$	$\mathcal{F}$
0	0	0	0	1	0
1	0	1	1	1	1
2	1	0	1	0	0
3	1	1	1	1	1

Thus:

- $k = 0$: $(0,0)$ falsifies $\mathcal{F}$, and $b_0 = 1$.
- $k = 1$: $(0,1)$ satisfies $\mathcal{F}$, and $b_1 = 0$.
- $k = 2$: $(1,0)$ falsifies $\mathcal{F}$, and $b_2 = 1$.
- $k = 3$: $(1,1)$ satisfies $\mathcal{F}$, and $b_3 = 0$.

This matches Theorem 1.8: the bits b_k are precisely the indicators of unsatisfying assignments.

1.5 Significance and Limitations

Theorem 1.8 can be read as a precise version of a standard fact: a Boolean function on n variables is equivalent to a bit-mask of length 2^n, hence to an integer.

The extra content here is the clean relationship between:

- balanced CNF structure (one literal per variable per clause),
- the unique falsifying assignment of each clause, and
- the bit-positions of $S_{\mathcal{F}}$.

Two clarifications are important:

- **No algorithmic shortcut.** The encoding does not solve SAT in polynomial time. Extracting all satisfying assignments from $S_{\mathcal{F}}$ requires inspecting 2^n bits in the worst case.
- **Structural, not novel in principle.** The identification of Boolean functions with integers is classical and used implicitly in many contexts. The contribution here is to isolate the balanced CNF case and make the bijection with unsatisfying assignments explicit.

Chapter 2

Epistemic Curvature

2.1 Formal Systems with Interface

The classical paradigm of logic studies formal systems in isolation. Here we adopt a slightly richer view: a formal system together with an *interface* to a semantic domain.

Definition 2.1 (Formal system with interface)**.** A *formal system with interface* is a quadruple

$$\mathcal{S} = \langle S, \vdash, \iota, \mathcal{O} \rangle$$

where:

- S is a countable set of syntactic expressions.
- $\vdash$ is a derivability relation on S.
- $\mathcal{O}$ is a set of semantic objects, equipped with a σ-algebra.
- $\iota : S \to \mathcal{O}$ is an interpretation map.

An *interface* for $\mathcal{S}$ consists of:

- a separable complete metric space (X, δ);
- Borel embeddings $e : S \to X$ and $j : \mathcal{O} \to X$.

The embeddings e and j express how syntax and semantics are represented in a common metric space. They are part of the modelling choice; there is no unique canonical interface.

Definition 2.2 (Representation error)**.** Given an interface (X, δ, e, j) for $\mathcal{S}$, the *representation error* of $\sigma \in S$ is

$$\operatorname{err}(\sigma) := \delta\big(e(\sigma),\ j(\iota(\sigma))\big).$$

Intuitively, $\operatorname{err}(\sigma)$ measures how far the syntactic object σ sits, in the interface space, from the semantic object it is intended to represent.

2.2 Definition of Epistemic Curvature

Definition 2.3 (Epistemic Curvature)**.** The *Epistemic Curvature* of $\mathcal{S}$ relative to an interface (X, δ, e, j) is

$$\kappa_{\mathcal{S}} := \inf_{\sigma \in S} \operatorname{err}(\sigma).$$

We say that $\mathcal{S}$ is *epistemically flat* (for this interface) if $\kappa_{\mathcal{S}} = 0$, and *epistemically curved* if $\kappa_{\mathcal{S}} > 0$.

Thus $\kappa_{\mathcal{S}}$ is the radius of the smallest ball around the semantic image that the syntactic image can reach.

Lemma 2.4 (Existence of minimizers under compactness)**.** *Suppose $e(S)$ is relatively compact in X and* err *is lower semicontinuous. Then there exists $\sigma^* \in S$ such that*

$$\operatorname{err}(\sigma^*) = \kappa_{\mathcal{S}}.$$

Proof. Let (σ_n) be a sequence in S such that $\mathrm{err}(\sigma_n) \to \kappa_{\mathcal{S}}$. Then $(e(\sigma_n))$ is contained in the compact set $\overline{e(S)}$, so it has a convergent subsequence $e(\sigma_{n_k}) \to x^* \in X$. By lower semicontinuity of err,

$$\kappa_{\mathcal{S}} \leq \liminf_{k\to\infty} \mathrm{err}(\sigma_{n_k}) = \lim_{k\to\infty} \mathrm{err}(\sigma_{n_k}) \geq \mathrm{err}(\sigma^*)$$

for some σ^* with $e(\sigma^*)$ sufficiently close to x^*. Hence $\mathrm{err}(\sigma^*) = \kappa_{\mathcal{S}}$. □

Proposition 2.5 (Metric robustness of flatness)**.** *Let δ and d be two metrics on X such that there exist constants $0 < c_1 \leq c_2 < \infty$ with*

$$c_1\,\delta(x,y) \;\leq\; d(x,y) \;\leq\; c_2\,\delta(x,y) \qquad \forall x,y \in X.$$

Then

$$c_1\,\kappa_{\mathcal{S}}(\delta) \leq \kappa_{\mathcal{S}}(d) \leq c_2\,\kappa_{\mathcal{S}}(\delta).$$

In particular, $\kappa_{\mathcal{S}}(\delta) = 0$ if and only if $\kappa_{\mathcal{S}}(d) = 0$.

Proof. For each $\sigma \in S$,

$$c_1\,\mathrm{err}_\delta(\sigma) \leq \mathrm{err}_d(\sigma) \leq c_2\,\mathrm{err}_\delta(\sigma).$$

Taking the infimum over σ gives the desired inequalities. The flatness equivalence follows immediately. □

Epistemic flatness is thus invariant under bi-Lipschitz changes of metric on the interface space.

2.3 Derivational Refinement and Incompleteness

We formalize a mild condition capturing the idea that the system admits some internal process of refinement that does not increase error.

Definition 2.6 (Derivational Refinement Principle (DRP))**.** A formal system with interface satisfies the *Derivational Refinement Principle* if there exists a map $T : S \to S$ such that:

1. $\sigma \vdash T(\sigma)$ for all $\sigma \in S$ (derivability is preserved).
2. $\mathrm{err}\big(T(\sigma)\big) \leq \mathrm{err}(\sigma)$ for all σ (non-expansivity).
3. For each $\sigma \in S$ the sequence $(T^n(\sigma))_{n\in\mathbb{N}}$ admits an accumulation point σ_∞ with

$$\mathrm{err}(\sigma_\infty) = \inf_{n\geq 0} \mathrm{err}\big(T^n(\sigma)\big).$$

Remark 2.7 (A trivial instance of DRP). In any system where $\sigma \vdash \sigma$ holds (reflexivity), the identity map $T = \mathrm{id}_S$ satisfies DRP trivially: points (1) and (2) are immediate, and (3) holds with $\sigma_\infty = \sigma$. Thus DRP is a weak condition; nontrivial choices of T give stronger refinement dynamics but are not required for the basic result below.

We now connect positive curvature to semantic incompleteness. The statement is metaformal: it reformulates the spirit of Gödel's incompleteness in geometric language.

Definition 2.8 (Semantic completeness)**.** We say that $\mathcal{S}$ is *semantically complete* (relative to ι) if for every semantic object $o \in \mathcal{O}$ that is "true in the intended model" there exists $\sigma \in S$ such that $\iota(\sigma) = o$ and σ is derivable within $\mathcal{S}$.

The notion of truth here depends on the ambient semantics; the exact details are not important for the structural statement.

Theorem 2.9 (Curvature and incompleteness)**.** *Let $\mathcal{S}$ be a consistent formal system with interface (X, δ, e, j) that satisfies DRP. If $\mathcal{S}$ is semantically complete, then necessarily*

$$\kappa_{\mathcal{S}} = 0.$$

Equivalently, if $\kappa_{\mathcal{S}} > 0$, then $\mathcal{S}$ cannot be semantically complete.

Proof sketch. Suppose, for contradiction, that $\kappa_{\mathcal{S}} > 0$ and $\mathcal{S}$ is semantically complete.

By definition,

$$\mathrm{err}(\sigma) \geq \kappa_{\mathcal{S}} > 0 \qquad \text{for all } \sigma \in S.$$

Thus the syntactic image $e(S)$ in X stays at distance at least $\kappa_{\mathcal{S}}$ from the semantic image $j(\mathcal{O})$.

On the other hand, semantic completeness asserts that for every "true" semantic object $o \in \mathcal{O}$ there exists $\sigma \in S$ with $\iota(\sigma) = o$. By DRP we may apply T iteratively to σ, obtaining a sequence $(T^n(\sigma))$ whose errors form a nonincreasing sequence converging to some value $\geq \kappa_{\mathcal{S}}$. The accumulation point σ_∞ provided by DRP then attains the infimum error over the orbit of σ.

If $\kappa_{\mathcal{S}} > 0$ this means that there is an irreducible ball of radius $\kappa_{\mathcal{S}}$ around $j(o)$ that the entire orbit $\{e(T^n(\sigma))\}$ cannot enter. In particular, no amount of derivational refinement brings the syntactic representation closer than $\kappa_{\mathcal{S}}$ to its semantic target.

But semantic completeness requires that, in principle, truths be fully capturable by derivations. Reconciling a positive lower bound on representational error with full semantic completeness under a refinement principle is impossible; the gap cannot be both irreducible and eliminable. Hence $\kappa_{\mathcal{S}}$ must be 0. □

Remark 2.10. Theorem 2.9 does not compete with Gödel's theorems; it reformulates the impossibility of global completeness in geometric terms. Positive Epistemic Curvature behaves as a numerical witness of incompleteness relative to a chosen interface.

Chapter 3

Interfaces for Arithmetic

The notion of Epistemic Curvature depends on the choice of interface (X, δ, e, j). This chapter presents two illustrative classes of interfaces for Peano Arithmetic (PA), showing how concrete values of κ can in principle be investigated.

3.1 Class A: Stone–L^2 Interface

Let $\mathcal{S}$ be the Stone space of complete consistent theories extending PA. This is a compact totally disconnected topological space.

Let $(\mathcal{S}, \mu)$ be equipped with a Borel probability measure μ (e.g. an invariant measure under finite symbol renamings, when it exists).

Set $X = L^2(\mathcal{S}, \mu)$ with the usual L^2 norm, and define:

- $e(\sigma) = \mathbf{1}_{[\sigma]}$, the indicator function of the clopen set

$$[\sigma] := \{T \subseteq S : \sigma \in T\}.$$

- $j(\iota(\sigma)) = \mathbf{1} \cdot \mathbb{T}(\sigma)$, where $\mathbb{T}(\sigma) \in \{0, 1\}$ denotes the meta-level truth value of σ in the intended model.

Then

$$\mathrm{err}(\sigma) = \begin{cases} \sqrt{\mu([\sigma])}, & \text{if } \mathbb{T}(\sigma) = 0, \\ \sqrt{1 - \mu([\sigma])}, & \text{if } \mathbb{T}(\sigma) = 1. \end{cases}$$

This interface measures how "atypical" a sentence is across the ensemble of complete theories. Small curvature corresponds to the existence of sentences that are almost always true or almost always false across $\mathcal{S}$.

3.2 Class B: Bounded Σ_1 Interface

Consider bounded Σ_1 sentences of the form

$$\sigma = \exists x \leq N \; R(x),$$

where R is Δ_0 with syntactic depth bounded by a fixed parameter. For such sentences, truth can be decided by finite search up to N.

Set $X = [0, 1]$ with the absolute value metric, and define:

- $j(\iota(\sigma)) := \mathbb{T}_N(\sigma) \in \{0, 1\}$, the actual truth value determined by exhaustive search.
- $e(\sigma) := p^*(\sigma) \in [0, 1]$, a fixed predictive assignment representing an a priori belief about σ before computation.

The modelling choice lies in p^*. For concreteness one may:

- choose $p^*(\sigma)$ as the empirical frequency of $R(x)$ in a fixed pseudo-random sample of $M \leq N$ values of x, or
- choose $p^*(\sigma)$ by a simple parametric prior based on the syntactic complexity of R.

In any case, once p^* is fixed, we can define a finite proxy for curvature at bounded complexity:

$$\kappa_{PA}^{(k,N)} := \inf_{\sigma \in \Sigma_1^{(\leq k)}} \big| p^*(\sigma) - \mathbb{T}_N(\sigma) \big|.$$

This quantity is, at least in principle, empirically accessible: one can compute $p^*(\sigma)$ and $\mathbb{T}_N(\sigma)$ for a finite set of sentences and estimate the worst-case deviation.

Remark 3.1. Different choices of p^* lead to different numerical values of $\kappa_{PA}^{(k,N)}$. This is expected: Epistemic Curvature is defined *relative* to an interface, and hence to a choice of representation and prior. The structural content lies in the presence or absence of a positive lower bound and in how it scales with k and N, not in a specific number.

Part II

Discrete Relational Kinematics

Chapter 4

The Layered Metric Space

4.1 Discrete Spacetime Hypothesis

We now step from pure logic into kinematics. The guiding idea is to model spacetime not as a continuum manifold but as a discrete structure evolving layer by layer.

Definition 4.1 (Layered metric space (LMS))**.** A *Layered Metric Space* consists of:

- a fixed countable set of vertices V (events),
- for each integer $k \in \mathbb{Z}$ a set of edges $E_k \subseteq V \times V$,
- for each k a length function $\ell_k : E_k \to \mathbb{R}_+$.

Each (V, E_k, ℓ_k) is a weighted graph representing "space" at layer k, and the index k plays the role of discrete time.

Distances within layer k are given by the shortest-path metric induced by ℓ_k. The framework is background-free in the sense that there is no prior continuum geometry; only the combinatorial data and edge lengths exist.

4.2 Strain Action

To penalize abrupt changes in the geometry from layer to layer, we introduce a purely kinematical action.

Definition 4.2 (Strain action)**.** For a sequence $\{\ell_k\}_{k\in\mathbb{Z}}$ on a fixed edge set E (for simplicity we assume $E_k \equiv E$) the *strain action* is

$$\mathcal{S}_{\text{strain}}[\{\ell_k\}] := \sum_k \sum_{e\in E} \big(\ell_{k+1}(e) - \ell_k(e)\big)^2.$$

This is a discrete analogue of an L^2 kinetic term in time: it penalizes large second differences in edge lengths and favours temporally smooth evolution.

4.3 Intra-layer Curvature and Total Action

To encode spatial geometry we introduce a layer-wise curvature functional. We keep the definition abstract to cover several known notions of discrete curvature.

Definition 4.3 (Local curvature functional)**.** For each layer k and each simple cycle c in the graph (V, E), a *local curvature functional* is a map

$$K_k : \mathcal{C}_k \to \mathbb{R},$$

where $\mathcal{C}_k$ is a collection of simple cycles in (V, E), such that:

1. $K_k(c) = 0$ whenever the restriction of ℓ_k to c is isometric to a chosen flat reference configuration.
2. $K_k(c)$ depends only on a bounded neighbourhood of c (locality).
3. $K_k(c)$ varies continuously under small perturbations of ℓ_k (stability).

Concrete examples include angle-deficit curvatures on simplicial complexes and combinatorial curvatures on graphs.

Definition 4.4 (Total LMS action)**.** Given coefficients $\alpha, \beta > 0$, the *total LMS action* is

$$\mathcal{S}_{\mathrm{LMS}}[\{\ell_k\}] := \alpha \sum_k \sum_{c \in \mathcal{C}_k} K_k(c)^2 \; + \; \beta \sum_k \sum_{e \in E} \big(\ell_{k+1}(e) - \ell_k(e)\big)^2.$$

The first term penalizes curvature (deviation from local flatness); it is quadratic in K_k, reminiscent of higher-curvature R^2-type actions in continuum gravity. The second term penalizes temporal strain.

Remark 4.5 (Stiff gravity)*.* Because curvature enters quadratically, the model behaves as a kind of “stiff gravity”: any nonzero curvature costs energy, independent of its sign. This is structurally closer to R^2 gravity than to the linear Einstein–Hilbert action. No claim of equivalence with General Relativity is made.

4.4 Discrete Euler–Lagrange Equations

Although a full dynamical theory is beyond the scope of this treatise, we can write the formal stationarity conditions of $\mathcal{S}_{\mathrm{LMS}}$ under variations of ℓ_k.

Fix an edge $e \in E$ and a layer k. The contribution of the strain term to the derivative of $\mathcal{S}_{\mathrm{LMS}}$ with respect to $\ell_k(e)$ comes from the pairs $(k-1, k)$ and $(k, k+1)$:

$$\beta\big(\ell_k(e) - \ell_{k-1}(e)\big)^2 \quad \text{and} \quad \beta\big(\ell_{k+1}(e) - \ell_k(e)\big)^2.$$

Differentiating and summing yields

$$\frac{\partial}{\partial \ell_k(e)} \big[\beta(\ell_k - \ell_{k-1})^2 + \beta(\ell_{k+1} - \ell_k)^2\big] = 2\beta\big(2\ell_k(e) - \ell_{k-1}(e) - \ell_{k+1}(e)\big).$$

The curvature term contributes

$$2\alpha \sum_{c \ni e} K_k(c) \, \frac{\partial K_k(c)}{\partial \ell_k(e)},$$

where the sum is over cycles containing e.

Setting the total derivative to zero gives the discrete Euler–Lagrange equation for each edge and layer:

$$2\beta\big(2\ell_k(e) - \ell_{k-1}(e) - \ell_{k+1}(e)\big) + 2\alpha \sum_{c \ni e} K_k(c) \, \frac{\partial K_k(c)}{\partial \ell_k(e)} = 0.$$

For $\alpha = 0$ this reduces to

$$2\ell_k(e) - \ell_{k-1}(e) - \ell_{k+1}(e) = 0,$$

a discrete Laplace equation in the time direction, whose solutions are affine in k.

For $\alpha > 0$ the equations couple curvature and strain in a way that depends on the chosen definition of K_k; this defines a discrete geometrodynamics.

4.5 Toy Example: Three-Vertex Cycle

As a minimal illustration, consider $V = \{1, 2, 3\}$ and $E = \{(1, 2), (2, 3), (3, 1)\}$, forming a 3-cycle.

Let there be two layers, $k = 0$ and $k = 1$, and let $\mathcal{C}_k$ consist of the single cycle $c = (1, 2, 3, 1)$.

Layer 0. Take all edge lengths to be 1:

$$\ell_0(1, 2) = \ell_0(2, 3) = \ell_0(3, 1) = 1.$$

Assume the curvature functional is normalized so that $K_0(c) = 0$ for this equilateral configuration.

Layer 1. Perturb one edge:

$$\ell_1(1,2) = 1 + \varepsilon, \qquad \ell_1(2,3) = 1, \qquad \ell_1(3,1) = 1,$$

with small $\varepsilon > 0$. Then $K_1(c)$ will be of order ε:

$$K_1(c) \approx C\varepsilon$$

for some constant C depending on the chosen notion of curvature.

The curvature contribution to $\mathcal{S}_{\mathrm{LMS}}$ is approximately

$$\alpha K_1(c)^2 \approx \alpha C^2 \varepsilon^2.$$

The only nonzero strain occurs on edge $(1,2)$:

$$\ell_1(1,2) - \ell_0(1,2) = \varepsilon,$$

so the strain contribution is

$$\beta\varepsilon^2.$$

To second order in ε,

$$\mathcal{S}_{\mathrm{LMS}} \approx (\alpha C^2 + \beta)\,\varepsilon^2.$$

Thus small deformations cost energy quadratically in their size, with contributions from both curvature and strain.

Chapter 5

The Materialization Limit

We sketch here a phenomenological picture of how quantum indeterminacy and classical determinism might coexist in the LMS framework. The goal is conceptual coherence, not a finished physical theory.

5.1 Transition Kernels

Suppose that, instead of fixed deterministic edges, we attach to each layer k and pair $(x, y) \in V \times V$ a transition kernel

$$K_k(x, y) \in [0, 1],$$

interpreted as the probability of information flow from x to y during the interval between layers k and $k + 1$.

- In a *quantum-like regime*, many $K_k(x, y)$ take values strictly between 0 and 1; multiple paths coexist with nonzero probability.
- In a *classical-like regime*, $K_k(x, y) \in \{0, 1\}$; connectivity is effectively deterministic.

Definition 5.1 (Materialization)**.** A region of the LMS is said to be *materialized* if, on that region, the transition kernel has crystallized to values in $\{0, 1\}$.

In this view, what we call "matter" is geometry that has frozen into a definite pattern of connections.

5.2 Phase Transition Picture

- When the distribution of $K_k(x, y)$ is broad and noisy, the underlying geometry is fluid and resembles a superposition of many possible graphs.
- When the distribution collapses toward $\{0, 1\}$, the geometry solidifies into a definite structure.

One can picture this as a phase transition in an effective "epistemic temperature" controlling how sharply the kernel is concentrated.

Remark 5.2 (Status and limitations)*.* At present this is a qualitative scenario:

- No specific evolution equation for K_k is proposed.
- No link is made to standard quantum mechanical amplitudes.
- No empirical predictions are extracted.

The value of the picture lies in its coherence with the LMS action and its potential to guide more detailed models.

Part III

Metaphysical Extensions

Chapter 6

Epistemic Fluid Dynamics

6.1 Cognitive Flow on Curved Knowledge Spaces

The notion of Epistemic Curvature suggests a fluid-dynamical metaphor for cognition.

Consider a high-dimensional semantic space in which:

- points represent concepts,
- paths represent trains of thought,
- curvature encodes structural obstacles and bottlenecks in representation.

One may imagine a “cognitive fluid” flowing along this space:

- Regions of almost zero curvature behave like flat channels: thought flows easily and predictably.
- Regions of high curvature behave like constrictions or vortex zones: thought tends to slow down, loop, or become unstable.

6.2 Monotropism as High-Viscosity Flow

Within this metaphor, cognitive styles can be seen as different fluid regimes.

- A *broad, low-viscosity flow* (many parallel channels, high branching) corresponds to a style where attention spreads easily across many topics but may be shallow in each.
- A *monotropic, high-viscosity flow* (few channels, low branching) corresponds to a style where attention concentrates deeply in a narrow region of semantic space.

People on the autistic spectrum are often described as having monotropic attention. In this model:

- The high viscosity makes it harder to redirect flow quickly, explaining certain difficulties with context switching.
- The same viscosity makes the flow more stable in regions of high curvature, enabling exploration of complex, structurally difficult domains that would be unstable for low-viscosity flows.

This is a structural description, not a clinical claim. It treats different cognitive profiles as different ways of moving through a curved epistemic landscape, with different strengths and costs.

Chapter 7

Curvature and the Achievement Society

Modern societies often enforce regimes of near-zero tolerated error: continuous monitoring, instant feedback, constant demand for transparency and performance.

In the language of Epistemic Curvature, one can describe this as a pressure toward *flattening interfaces*:

- Ambiguity and opacity are treated as faults.
- Every statement is expected to be immediately verifiable.
- The tolerated gap between representation and reality shrinks.

From the point of view of individual experience:

- Living in a low-curvature regime can be efficient but cognitively exhausting: little room for error, fantasy, or incomplete understanding.
- High-curvature regions (myth, ritual, art, private reflection) provide spaces where irreducible distance between what is said and what is can be held without immediate resolution.

The notion of Epistemic Curvature thus offers a vocabulary for discussing when a social system leaves room for non-instrumental forms of understanding and when it collapses everything into flat, performance-oriented metrics.

Chapter 8

Aesthetics of the Abyss

8.1 Creativity in High-Curvature Zones

Large language models and other generative systems excel at approximating the statistical structure of existing corpora. Viewed through the lens of Epistemic Curvature:

- They tend to operate in low-curvature regions, smoothing over idiosyncrasies and collapsing many distinct semantic paths into an averaged response.
- They are extremely effective at navigation within norms, genres, and known styles.

Artistic practice, by contrast, often derives its force from entering high-curvature zones:

- Regions where available concepts and representations are misaligned with the phenomena.
- Situations where existing syntax fails or must be stretched to breaking point.

An "aesthetics of the abyss" is one that values this exploration:

- not as a rejection of structure, but as a search for new structures,
- not as chaos, but as engagement with parts of the epistemic landscape where the interface is intrinsically curved.

8.2 Human and Artificial Roles

In this picture:

- Artificial systems provide powerful tools in regions of low and moderate curvature: they map and extend what is already structurally stable.
- Human agents—and perhaps future hybrid systems—take on the role of probing the high-curvature regions where representations must be reconfigured.

The Geometry of the Unknowable does not prescribe how this division of labour should evolve. It offers a way to describe the space in which these evolutions take place.

Epilogue

The formalism of Epistemic Curvature and the Layered Metric Space is deliberately modest in its claims and ambitious in its scope.

Modest, because:

- at Level 0 it offers crisp but limited theorems;
- at Level 1 it sketches a kinematical model that needs much development;
- at Level 2 it offers metaphors rather than measurements.

Ambitious, because:

- it treats the gap between knowledge and reality as a geometric invariant,
- it treats spacetime itself as a discrete, evolving interface,
- it treats cognition and culture as flows on curved epistemic spaces.

Whether these ideas become part of mainstream science or remain a niche perspective is secondary. Their validity, where they make precise claims, is a matter of internal logic. Everything else is a question of future work.

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