Bayesian Learning - Lecture 12

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Overview

- **Model evaluation - Posterior predictive analysis**
- **Course summary and discussion**

Models - why?

- We now know how to **compare** models.
- But how do we know if any given model is 'any good'?
- George Box: '**All models are false, but some are useful**'.

What is your model for really?

- **Prediction**.
  - Interpretation not a concern
  - Black-box approach may be ok.
  - Extrapolation?
  - Model averaging may be a good idea.
- Abstraction to **aid in thinking** about a phenomena.
  - Prediction accuracy of less concern.
  - Model averaging may be a bad idea.
- Model as a **compact description of a complex phenomena**.
  - Computational cost of model evaluation may be a concern.
  - Online/real-time analysis.

Posterior predictive analysis

- If $p(y|\theta)$ is a 'good' model, then the data actually observed should not differ 'too much' from simulated data from $p(y|\theta)$.

- Bayesian: simulate data from the **posterior predictive distribution**:

$$p(y^{rep}|y) = \int p(y^{rep}|\theta)p(\theta|y)d\theta.$$

- Difficult to compare y and y^{rep} because of dimensionality.
- Solution: compare **low-dimensional statistic** $T(y, \theta)$ to $T(y^{rep}, \theta)$.
- Evaluates the full probability model consisting of both the likelihood *and* prior distribution.

Posterior predictive analysis, cont.

- **Algorithm** for simulating from the posterior predictive density $p[T(y^{rep})|y]$:

1. Draw a $\theta^{(1)}$ from the posterior $p(\theta|y)$.
2. Simulate a data-replicate $y^{(1)}$ from $p(y^{rep}|\theta^{(1)})$.
3. Compute $T(y^{(1)})$.
4. Repeat steps 1-3 a large number of times to obtain a sample from $T(y^{rep})$.

- We may now compare the observed statistic $T(y)$ with the distribution of $T(y^{rep})$.
- **Posterior predictive p-value**: $\Pr[T(y^{rep}) \geq T(y)]$
- Informal **graphical analysis**.

Posterior predictive analysis - Examples

- Ex. 1. Model: $y_1, ..., y_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$. $T(y) = \max_i |y_i|$.

- Ex. 2. Assumption of no reciprocity in networks.
  $y_{ij}|\theta \stackrel{iid}{\sim} Bernoulli(\theta)$. $T(y) =$proportion of reciprocated node pairs.

- Ex. 3. ARIMA-process. $T(y)$ may be the autocorrelation function.

- Ex. 4. Poisson regression. $T(y)$ frequency distribution of the response counts. Proportions of zero counts.

Posterior predictive analysis - Normal model, max statistic