Communication Complexity

Jake Kinsella and Max von Hippel

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April 9, 2021

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If Alice knows x, and Bob knows y, how many bits of information must they communicate, in order for both Alice and Bob to know f(x,y)?

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Consider a two-party communication problem, in which the participants



(a) Alice

and



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(b) Bob

participate to compute a function:

$$f: \underline{\mathbb{B}''} \times \underline{\mathbb{B}''} \to \underline{\mathbb{B}}$$
Alice's Bob's global input input output

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The players can come up with a protocol $\Pi = (p_1, ..., p_t)$, namely, for some natural $t \in \mathbb{N}$, a sequence of t-many functions $p_i : \mathbb{B}^* \to \mathbb{B}^*$ such that the communication between the players looks like this ...

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Alice is given input x.

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Alice is given input x.

Hello Bob. I can't reveal x, but $p_1(x)$ is p1.

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Bob is given input y.

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Alice is given input x.

Hello Bob. I can't reveal x, but $p_1(x)$ is p1.

Bob is given input y.

Thanks Alice. I can't reveal y, but $p_2(y, p1)$ is p2

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Thanks Alice. I can't reveal y, but $p_2(y,p1)$ is p2.

... yada yada yada ...

Pleasure doing business with you Bob. My final clue for you is that $p_{n-1}(x, p1, ..., pn-2)$ is pn-1.

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Alice is given input x.

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... yada yada yada ...

Pleasure doing business with you Bob. My final clue for you is that $p_{n-1}(x, p_1, ..., p_{n-2})$ is $p_{n-1}(x, p_1, ..., p_{n-2})$

Rad. Then $p_n(y, p1, ..., pn-1)$ is pn. TTFN!

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- The functions p_i can be anything so long as they are well-defined. E.g., could solve the Halting Problem.
- After the final message, both parties must know f(x, y).

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Definition (Communication Complexity)

Suppose Π is a protocol for f in which at most t bits are communicated between Alice and Bob. Then the communication complexity of Π , denoted $C(\Pi)$, is t.

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Definition (C(f))

The communication complexity of f, denoted C(f), is the minimum communication complexity achieved by any protocol for f.

Examples (Jake)

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