Communication Complexity

Jake Kinsella and Max von Hippel

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If Alice knows x, and Bob knows y, how many bits of information must they communicate, in order for both Alice and Bob to know f(x,y)?

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Consider a two-party communication problem, in which the participants



(a) Alice

and



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(b) Bob

participate to compute a function:

$$f: \underline{\mathbb{B}''} \times \underline{\mathbb{B}''} \to \underline{\mathbb{B}}$$
Alice's Bob's global input input output

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The players can come up with a protocol $\Pi = (p_1, ..., p_t)$, namely, for some natural $t \in \mathbb{N}$, a sequence of t-many functions $p_i : \mathbb{B}^* \to \mathbb{B}^*$ such that the communication between the players looks like this ...

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Alice is given input x.

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Alice is given input x.

Hello Bob. I can't reveal x, but $p_1(x)$ is p1.

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Bob is given input y.

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Alice is given input x.

Hello Bob. I can't reveal x, but $p_1(x)$ is p1.

Bob is given input y.

Thanks Alice. I can't reveal y, but $p_2(y, p1)$ is p2

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... yada yada yada ...

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Thanks Alice. I can't reveal y, but $p_2(y,p1)$ is p2.

... yada yada yada ...

Pleasure doing business with you Bob. My final clue for you is that $p_{n-1}(x, p1, ..., pn-2)$ is pn-1.

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Alice is given input x.

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Thanks Alice. I can't reveal y, but $p_2(y, p1)$ is p2.

... yada yada yada ...

Pleasure doing business with you Bob. My final clue for you is that $p_{n-1}(x, p_1, ..., p_{n-2})$ is $p_{n-1}(x, p_1, ..., p_{n-2})$

Rad. Then $p_n(y, p1, ..., pn-1)$ is pn. TTFN!

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- The functions p_i can be anything so long as they are well-defined. E.g., could solve the Halting Problem.
- After the final message, both parties must know f(x, y).

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■ The functions p_i can be anything so long as they are well-defined. E.g., could solve the Halting Problem.

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Definition (Communication Complexity)

Suppose Π is a protocol for f in which at most t bits are communicated between Alice and Bob. Then the communication complexity of Π , denoted $C(\Pi)$, is t.

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Definition (Communication Complexity)

Suppose Π is a protocol for f in which at most t bits are communicated between Alice and Bob. Then the communication complexity of Π , denoted $C(\Pi)$, is t.

Definition (C(f))

The communication complexity of f, denoted C(f), is the minimum communication complexity achieved by any protocol for f.

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Example (Are the number of 1s in xy even (0), or odd (1)?)

 $f: \mathbb{B}^n \times \mathbb{B}^n \to \mathbb{B}$ is precisely $(x, y) \mapsto \bigoplus xy$.

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Example protocol Π :

$$P1 = parity(x)$$
.

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Example protocol Π :

P1 =
$$parity(x)$$
.

 $P2 = parity(y) \oplus P1$

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Example (Are the number of 1s in xy even (0), or odd (1)?)

 $f: \mathbb{B}^n \times \mathbb{B}^n \to \mathbb{B}$ is precisely $(x, y) \mapsto \bigoplus xy$.

Example protocol Π :

$$P1 = parity(x)$$
.

 $P2 = parity(y) \oplus P1$

Now both Alice and Bob know f(x,y) = P2. $C(f) \le 2$ because $C(\Pi) = 2$ and Π implements f. But $C(f) \ge 2$ because f depends on x and y. Hence C(f) = 2.

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Example (A_{TM})

 $H: \mathbb{B}^n \times \mathbb{B}^n \to \mathbb{B}$ is precisely $\langle M, x \rangle \mapsto 1$ if M halts on x else 0.

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Example (A_{TM})

 $H: \mathbb{B}^n \times \mathbb{B}^n \to \mathbb{B}$ is precisely $\langle M, x \rangle \mapsto 1$ if M halts on x else 0.

Example protocol Π :

P1 = y.

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Example (A_{TM})

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Example protocol Π :

$$y_1 = y$$
.

P2=
$$(M \text{ does/doesn't accept } y)$$
.

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Example (A_{TM})

 $H: \mathbb{B}^n \times \mathbb{B}^n \to \mathbb{B}$ is precisely $\langle M, x \rangle \mapsto 1$ if M halts on x else 0.

Example protocol Π :

$$P1 = y$$
.

P2=
$$(M \text{ does/doesn't accept } y)$$
.

Both players have unlimited computation power. We are only interest in communication complexity.

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If we find a protocol Π , then we know C(f) is at most $C(\Pi)$.

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If we find a protocol Π , then we know C(f) is at most $C(\Pi)$. What if we don't know any protocol Π ?

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If we find a protocol Π , then we know C(f) is at most $C(\Pi)$. What if we don't know any protocol Π ?

■ Could we upper-bound C(f) without knowing Π ?

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If we find a protocol Π , then we know C(f) is at most $C(\Pi)$. What if we don't know any protocol Π ?

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What if the only protocols we find seem really lousy?

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If we find a protocol Π , then we know C(f) is at most $C(\Pi)$. What if we don't know any protocol Π ?

- Could we upper-bound C(f) without knowing Π ?
- What if the only protocols we find seem really lousy?
 - Could we lower-bound C(f) without finding a better protocol?

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If we find a protocol Π , then we know C(f) is at most $C(\Pi)$. What if we don't know any protocol Π ?

■ Could we upper-bound C(f) without knowing Π ?

What if the only protocols we find seem really lousy?

■ Could we lower-bound C(f) without finding a better protocol?

TL;DR: yup.

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