Communication Complexity

Jake Kinsella and Max von Hippel

Introducti

Methods

Facility Car

. -----

Tiling Method

2-Party Discrepe

Multi-Party

Generalization

Multi-Party Discrepency Metho

Other Variani

Non-Determinis

Communication Complexity

Jake Kinsella and Max von Hippel

Northeastern University

April 9, 2021

Communication Complexity

Communication Complexity

Jake Kinsella and Max vor Hippel

Introduction

Methods

Tiling Method

2-Party Discrepency

Multi-Party

Generalization

Multi-Party Discrepency Metho

Discrepency Metho

Non-Determin

If Alice knows x, and Bob knows y, how many bits of information must they communicate, in order for both Alice and Bob to know f(x,y)?

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Examples

Fooling Set Method

Method

Generalization

Methods

Discrepency Metho

Non-Deterministic

1 Introduction

- Examples
- Methods
 - Fooling Set Method
 - Tiling Method
 - 2-Party Discrepency Method

2 Multi-Party Generalization

- Methods
 - Multi-Party Discrepency Method

3 Other Variants

- Non-Deterministic
- Randomized

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Fooling Set Meth

Tooling Set Weti

Tiling Method

2-Party Discrepend Method

Multi-Party Generalization

Multi-Party Discrepency Method

Non Determin

Randomized

Consider a two-party communication problem, in which the participants



(a) Alice

and



u

(b) Bob

participate to compute a function:

$$f: \underline{\mathbb{B}''} \times \underline{\mathbb{B}''} \to \underline{\mathbb{B}}$$
Alice's Bob's global input input output

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Methods Fooling Set Meth

Tiling Method

2-Party Discrepency
Method

Multi-Party

Generalization

Methods

Multi-Party Discrepency Method

Non-Deterministic

The players can come up with a protocol $\Pi = (p_1, ..., p_t)$, namely, for some natural $t \in \mathbb{N}$, a sequence of t-many functions $p_i : \mathbb{B}^* \to \mathbb{B}^*$ such that the communication between the players looks like this ...

Communication Complexity

Introduction

Alice is given input x.

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Methods

Fooling Set Meth

7-Party Discren

Multi-Party

Generalizatio

Multi-Party Discrepency Method

Other Varian

Non-Determi

Alice is given input x.

Hello Bob. I can't reveal x, but $p_1(x)$ is p1.

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Methods

Fooling Set Meth

Tiling Method 2-Party Discreper

Multi-Party

Generalizatio

N. D.

Randomized

Alice is given input x.

Hello Bob. I can't reveal x, but $p_1(x)$ is p1.

Bob is given input y.

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Methods

Fooling Set Met

Tiling Method

2-Party Discrepe

Multi-Party

Methods

.

Non-Determinis

Randomized

Alice is given input x.

Hello Bob. I can't reveal x, but $p_1(x)$ is p1.

Bob is given input y.

Thanks Alice. I can't reveal y, but $p_2(y, p1)$ is p2

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Methods

Fooling Set Me

Tiling Method

Tilling Method

Method

Muiti-Party Generalization

Methods

Discrepency Metho

Non-Determin

Randomized

Alice is given input x.

Hello Bob. I can't reveal x, but $p_1(x)$ is p1.

Bob is given input y.

Thanks Alice. I can't reveal y, but $p_2(y, p1)$ is p2

... yada yada yada ...

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Methods

Fooling Set Meth

Tiling Method

2-Party Discrepenc

Multi-Party Generalization

Methods
Multi-Party
Discrepency Method

Non-Deter

Randomized

Alice is given input x.

Hello Bob. I can't reveal x, but $p_1(x)$ is p1.

Bob is given input y.

Thanks Alice. I can't reveal y, but $p_2(y,p1)$ is p2.

... yada yada yada ...

Pleasure doing business with you Bob. My final clue for you is that $p_{n-1}(x, p1, ..., pn-2)$ is pn-1.

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Methods

Fooling Set Meth

Title - Markey

7-Party Discrepend

Multi-Party

Methods

Multi-Party Discrepency Method

Non-Deterministic

Alice is given input x.

Hello Bob. I can't reveal x, but $p_1(x)$ is p1.

Bob is given input y.

Thanks Alice. I can't reveal y, but $p_2(y, p1)$ is p2.

... yada yada yada ...

Pleasure doing business with you Bob. My final clue for you is that $p_{n-1}(x, p_1, ..., p_{n-2})$ is $p_{n-1}(x, p_1, ..., p_{n-2})$

Rad. Then $p_n(y, p1, ..., pn-1)$ is pn. TTFN!

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Methods

Fooling Set Metho

Tiling Method

Multi-Party

Generalizatio

Multi-Party Discrepency Metho

Other Variant

Randomized

- The functions p_i can be anything so long as they are well-defined. E.g., could solve the Halting Problem.
- After the final message, both parties must know f(x, y).

Communication Complexity

Introduction

■ The functions p_i can be anything so long as they are well-defined. E.g., could solve the Halting Problem.

• After the final message, both parties must know f(x,y).

Definition (Communication Complexity)

Suppose Π is a protocol for f in which at most t bits are communicated between Alice and Bob. Then the communication complexity of Π , denoted $C(\Pi)$, is t.

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Examples
Methods
Fooling Set Method
Tiling Method
2-Party Discrepency

Multi-Party Generalization

Multi-Party
Discrepency Method

Non-Deterministic

■ The functions p_i can be anything so long as they are well-defined. E.g., could solve the Halting Problem.

■ After the final message, both parties must know f(x, y).

Definition (Communication Complexity)

Suppose Π is a protocol for f in which at most t bits are communicated between Alice and Bob. Then the communication complexity of Π , denoted $C(\Pi)$, is t.

Definition (C(f))

The communication complexity of f, denoted C(f), is the minimum communication complexity achieved by any protocol for f.

Communication Complexity

Jake Kinsella and Max von Hippel

Introduct

Examples

Fooling Set Metho

2-Party Discrepe Method

Generalizati

Methods

Discrepency Metho

Non-Determ

Randomized

Example (Are the number of 1s in xy even (0), or odd (1)?)

 $f: \mathbb{B}^n \times \mathbb{B}^n \to \mathbb{B}$ is the function $(x, y) \mapsto \bigoplus xy$

Communication Complexity

Jake Kinsella and Max von Hippel

Introduct

Examples

Fooling Set Method
Tiling Method
2-Party Discrepency

Multi-Party

Generalizati

Multi-Party Discrepency Metho

Non-Determ

Randomized

Example (Are the number of 1s in xy even (0), or odd (1)?)

$$f: \mathbb{B}^n \times \mathbb{B}^n \to \mathbb{B}$$
 is the function $(x, y) \mapsto \bigoplus xy$

Example protocol Π :

P1 =
$$parity(x)$$
.

Communication Complexity

Jake Kinsella and Max von Hippel

Introduct

Examples

Fooling Set Method
Tiling Method
2-Party Discrepency

Multi-Party

Generalization

Multi-Party Discrepency Method

Other Var

Randomized

Example (Are the number of 1s in xy even (0), or odd (1)?)

$$f: \mathbb{B}^n \times \mathbb{B}^n \to \mathbb{B}$$
 is the function $(x, y) \mapsto \bigoplus xy$

Example protocol Π :

P1 =
$$parity(x)$$
.

 $P2 = parity(y) \oplus P1$

Communication Complexity

Jake Kinsella and Max von Hippel

Introduction

Examples

Fooling Set Method
Tiling Method
2-Party Discrepency

Multi-Party

Methods Multi-Party Discrepency Metho

Other Variants

Non-Deterministic

Example (Are the number of 1s in xy even (0), or odd (1)?)

$$f: \mathbb{B}^n \times \mathbb{B}^n \to \mathbb{B}$$
 is the function $(x, y) \mapsto \bigoplus xy$

Example protocol Π :

$$P1 = parity(x).$$

 $P2 = parity(y) \oplus P1$

Now both Alice and Bob know f(x,y) = P2. $C(f) \le 2$ because $C(\Pi) = 2$ and Π implements f. But $C(f) \ge 2$ because f depends on x and y. Hence C(f) = 2.

Halting (Jake)

Communication Complexity

Examples

Example 2: Halting

Function $H: \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$

 $x = 1^n$ and $y = \langle M \rangle$

H returns 1 if M halts on x

Protocol

Player 1	Player 2
$x = 1^{10}$	$y = \langle M_{accept} \rangle$
$P_1(1^{10})=1\longrightarrow$	
	$p_1 = 1$
	$p_1 = 1$ $P_2(y, p_1) = M_{accept}(1^{ < M_{acc} })$ $\longleftarrow M(1^{10}) = 1$
1	$\longleftarrow M(1^{10}) = 1$
$p_2 = 1$	

In communication complexity problems, both players have unlimited computation power. This allows Player 2 to solve the

Methods

Communication Complexity

Jake Kinsella and Max von Hippel

Introduct

Examples Methods

Fooling Set Method
Tiling Method
2-Party Discrepency

Aulti-Party

Methods

Other Variants

Non-Deterministic

- 1 Introduction
 - Examples
 - Methods
 - Fooling Set Method
 - Tiling Method
 - 2-Party Discrepency Method
- 2 Multi-Party Generalization
 - Methods
 - Multi-Party Discrepency Method
- 3 Other Variants
 - Non-Deterministic
 - Randomized

Fooling Set Method (Jake)

Communication Complexity

ake Kinsella nd Max von Hippel

Introduction

Examples

Fooling Set Method

Tiling Method 2-Party Discrepency Method

Multi-Party Generalization

Methods

Multi-Party Discrepency Method

Other Varian

Non-Determini

Tiling Method (Max)

Communication Complexity

ake Kinsella nd Max von Hippel

Introduction

meroduction

Methods

Fooling Set Metl

rooming set weth

Tiling Method

2-Party Discrepenc Method

Method

Generalizatio

Methods

Multi-Party Discrepency Metho

Other Varian

Non-Determinis

2-Party Discrepency Method (Max)

Communication Complexity

ake Kinsella nd Max von Hippel

Introduction

IIILIOGUCCIOI

Evamples

Methods

Fooling Set M

Tiling Method

2-Party Discrepen

Method

Generalization

Markada

Multi-Party Discrepency Metho

Other Variant

Non-Determinist

Randomized

Multi-Party Generalization (Jake)

Communication Complexity

ake Kinsella nd Max von Hippel

Introduction

200

Methods

Fooling Set Metl

Tiling Metho

2-Party Discr

Multi-Party Generalization

Methods

Multi-Party Discrepency Metho

Other varian

Non-Determi

Randomized

Methods

Communication Complexity

Jake Kinsella and Max von Hippel

Introductio

. .

Examples

Fooling Set Meth

Tiling Method

2-Party Discreper

Multi-Party

Generalization

Methods

Discrepency Weti

Other Variant

Non-Deterministi Randomized

- 1 Introduction
 - Examples
 - Methods
 - Fooling Set Method
 - Tiling Method
 - 2-Party Discrepency Method
- 2 Multi-Party Generalization
 - Methods
 - Multi-Party Discrepency Method
- 3 Other Variants
 - Non-Deterministic
 - Randomized

Multi-Party Discrepency Method (Max)

Communication Complexity

ake Kinsella nd Max von Hippel

Introduction

IIItioductioi

Mask - d-

Fooling Set Metho

Tiling Method

2-Party Discrepend Method

Aulti-Party

Generalizati

Multi-Party Discrepency Method

Other Variant

Non-Determin

Non-Deterministic (Jake)

Communication Complexity

Non-Deterministic

Randomized (Max)

Communication Complexity

ake Kinsella nd Max von Hippel

Introduction

introduction

Methods

Fooling Set Met

Tillian Mashaul

I iling ivietno

2-Party Discrepent

Generalizatio

NA ...

Multi-Party Discrepency Metho

Other Varian

Non-Determi

Randomized