Communication Complexity Examples

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Example 1: Parity

$f(x,y) = \oplus xy$

"Are the number of ones in xy even or odd?"

Protocol Π

Player 1	Player 2
$x = 101$ $P_1(x) = parity(101) = 0 \longrightarrow$ $p_2 = 0$	$y = 110$ $p_1 = 0$ $\longleftarrow P_2(y, p_1) = parity(110) \oplus 1 = 0$

$C(\Pi) = 2$

$C(f)$ is at least 2 because the function f depends on both x and y so there must be at least one bit of communication from both players.

Thus $C(f) = 2$ via Π

Example 2: Halting

Function $H : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$

$x = 1^n$ and $y =< M >$

H returns 1 if M halts on x

Protocol Π

Player 1	Player 2
$x = 1^{10}$ $P_1(1^{10}) = 1 \longrightarrow$ $p_2 = 1$	$y =< M_{accept} >$ $p_1 = 1$ $P_2(y, p_1) = M_{accept}(1^{\lvert <M_{accept}> \rvert})$ $\longleftarrow M(1^{10}) = 1$

In communication complexity problems, both players have unlimited computation power. This allows Player 2 to solve the halting problem. Computational power and time is ignored to focus on communication between players.