

Multiple linear regression

Grading the professor

Many college courses conclude by giving students the opportunity to evaluate the course and the instructor anonymously. However, the use of these student evaluations as an indicator of course quality and teaching effectiveness is often criticized because these measures may reflect the influence of non-teaching related characteristics, such as the physical appearance of the instructor. The article titled, “Beauty in the classroom: instructors’ pulchritude and putative pedagogical productivity” (Hamermesh and Parker, 2005) found that instructors who are viewed to be better looking receive higher instructional ratings. (Daniel S. Hamermesh, Amy Parker, Beauty in the classroom: instructors pulchritude and putative pedagogical productivity, *Economics of Education Review*, Volume 24, Issue 4, August 2005, Pages 369-376, ISSN 0272-7757, 10.1016/j.econedurev.2004.07.013. <http://www.sciencedirect.com/science/article/pii/S0272775704001165>.)

In this lab we will analyze the data from this study in order to learn what goes into a positive professor evaluation.

The data

The data were gathered from end of semester student evaluations for a large sample of professors from the University of Texas at Austin. In addition, six students rated the professors’ physical appearance. (This is a slightly modified version of the original data set that was released as part of the replication data for *Data Analysis Using Regression and Multilevel/Hierarchical Models* (Gelman and Hill, 2007).) The result is a data frame where each row contains a different course and columns represent variables about the courses and professors.

```
load("more/evals.RData")
```

variable	description
score	average professor evaluation score: (1) very unsatisfactory - (5) excellent.
rank	rank of professor: teaching, tenure track, tenured.
ethnicity	ethnicity of professor: not minority, minority.
gender	gender of professor: female, male.
language	language of school where professor received education: english or non-english.
age	age of professor.
cls_perc_eval	percent of students in class who completed evaluation.
cls_did_eval	number of students in class who completed evaluation.
cls_students	total number of students in class.
cls_level	class level: lower, upper.
cls_profs	number of professors teaching sections in course in sample: single, multiple.
cls_credits	number of credits of class: one credit (lab, PE, etc.), multi credit.
bty_f1lower	beauty rating of professor from lower level female: (1) lowest - (10) highest.
bty_f1upper	beauty rating of professor from upper level female: (1) lowest - (10) highest.
bty_f2upper	beauty rating of professor from second upper level female: (1) lowest - (10) highest.
bty_m1lower	beauty rating of professor from lower level male: (1) lowest - (10) highest.
bty_m1upper	beauty rating of professor from upper level male: (1) lowest - (10) highest.
bty_m2upper	beauty rating of professor from second upper level male: (1) lowest - (10) highest.
bty_avg	average beauty rating of professor.
pic_outfit	outfit of professor in picture: not formal, formal.
pic_color	color of professor’s picture: color, black & white.

Exploring the data

1. Is this an observational study or an experiment? The original research question posed in the paper is whether beauty leads directly to the differences in course evaluations. Given the study design, is it possible to answer this question as it is phrased? If not, rephrase the question.

It is a bit of both. There are experimental aspects but includes subjective assessments from students. The question could be posed as something like, "Is there a relationship between beauty and course evaluation?"

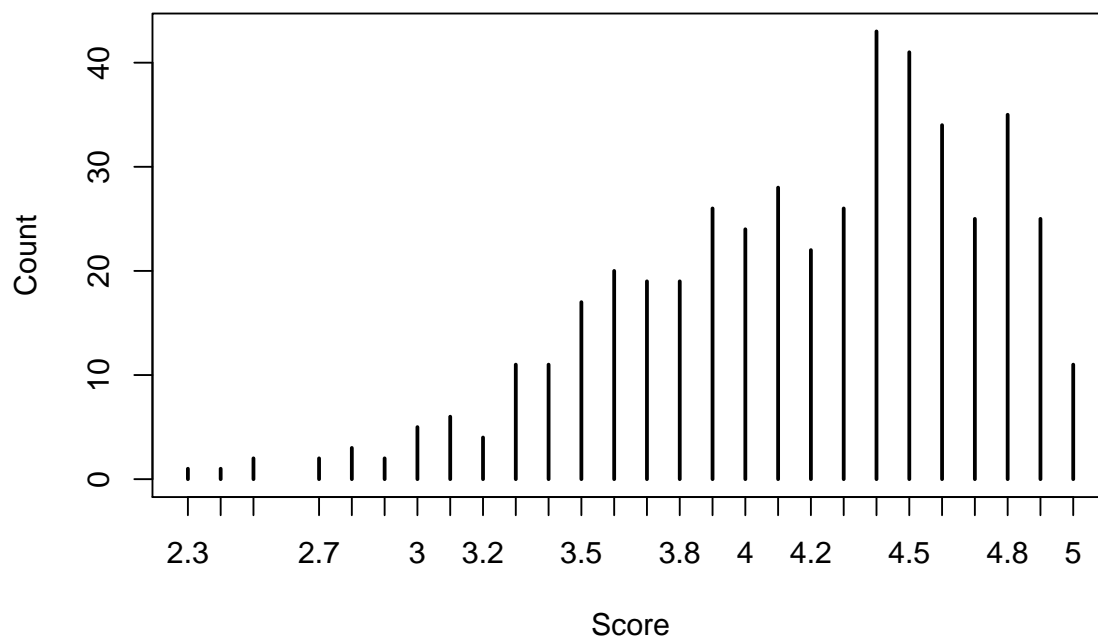
2. Describe the distribution of **score**. Is the distribution skewed? What does that tell you about how students rate courses? Is this what you expected to see? Why, or why not?

Generally students do not give extremely low scores to professors, and give a median score of 4.17473, which is relatively high considering it is a 5 point scale. From my own experience with evaluation, I think people generally tend to give high scores, but not too high. So, the average and distribution make sense.

```
table(evals$score)
```

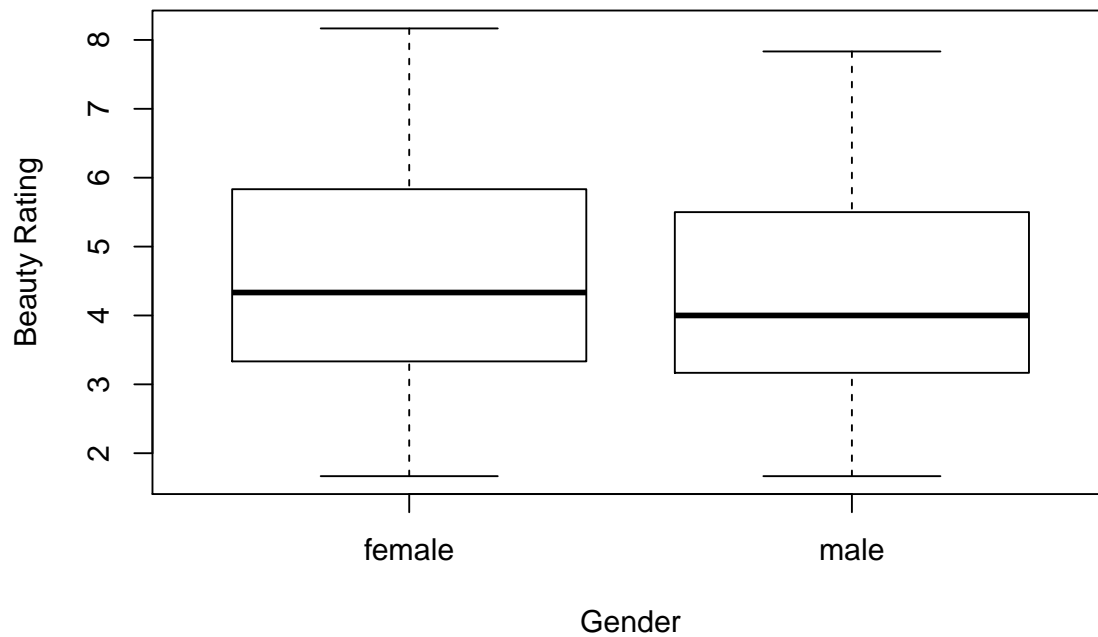
```
##
## 2.3 2.4 2.5 2.7 2.8 2.9 3 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 4 4.1
## 1 1 2 2 3 2 5 6 4 11 11 17 20 19 19 26 24 28
## 4.2 4.3 4.4 4.5 4.6 4.7 4.8 4.9 5
## 22 26 43 41 34 25 35 25 11
```

```
plot(table(evals$score), type = "h", xlab = "Score", ylab = "Count")
```



3. Excluding `score`, select two other variables and describe their relationship using an appropriate visualization (scatterplot, side-by-side boxplots, or mosaic plot).

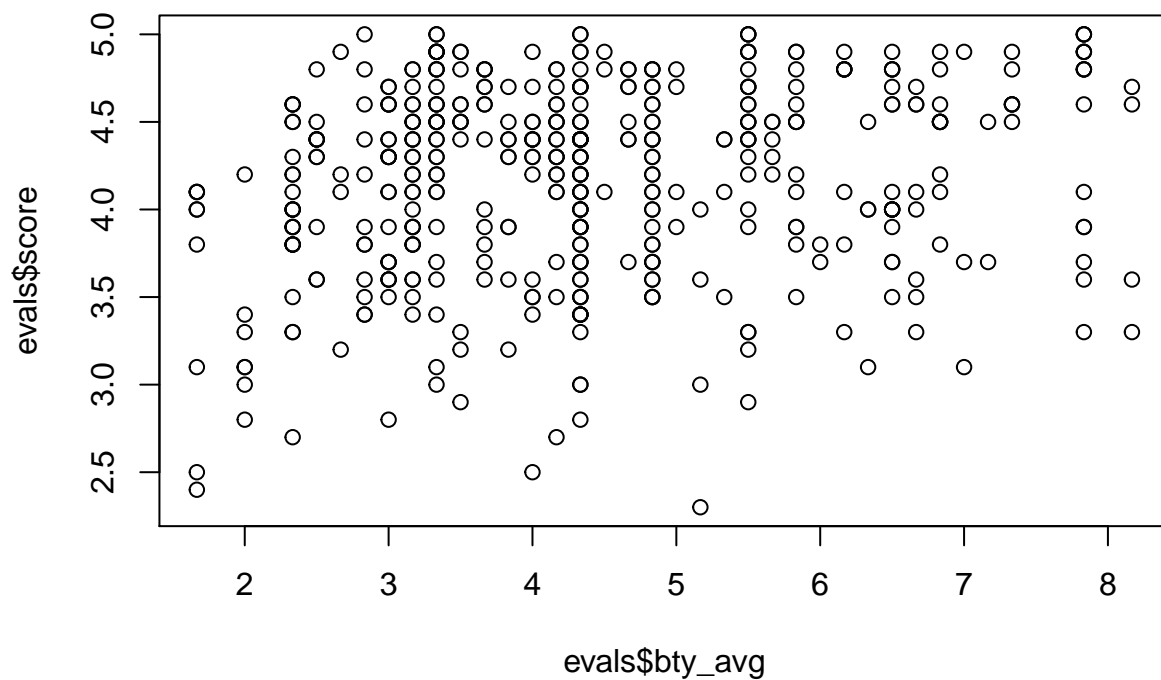
```
plot(x = evals$gender, y = evals$bty_avg, xlab = "Gender", ylab = "Beauty Rating")
```



Simple linear regression

The fundamental phenomenon suggested by the study is that better looking teachers are evaluated more favorably. Let's create a scatterplot to see if this appears to be the case:

```
plot(evals$score ~ evals$bty_avg)
```

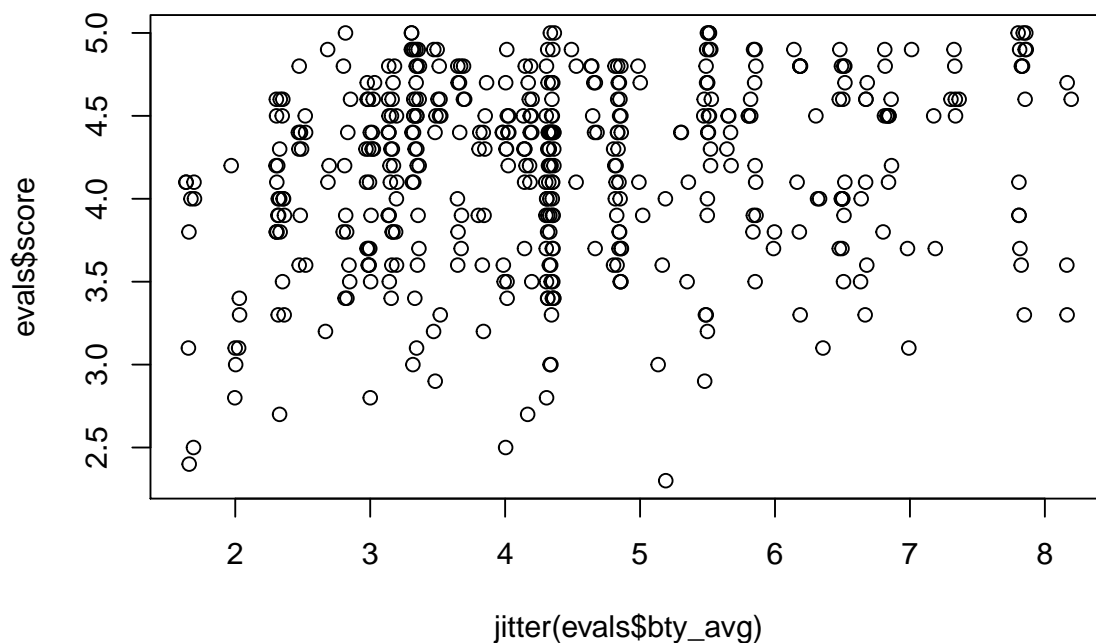


Before we draw conclusions about the trend, compare the number of observations in the data frame with the approximate number of points on the scatterplot. Is anything awry?

4. Replot the scatterplot, but this time use the function `jitter()` on the y - or the x -coordinate. (Use `?jitter` to learn more.) What was misleading about the initial scatterplot?

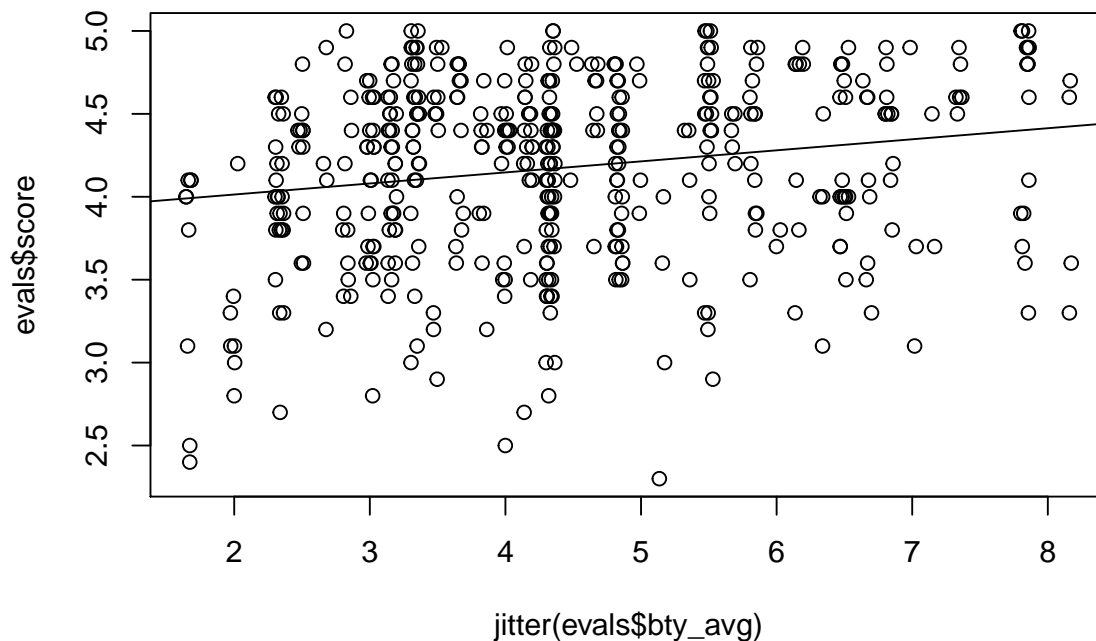
It appears that overlapping points do not visual well with plot.

```
plot(evals$score ~ jitter(evals$bty_avg))
```



5. Let's see if the apparent trend in the plot is something more than natural variation. Fit a linear model called `m_bty` to predict average professor score by average beauty rating and add the line to your plot using `abline(m_bty)`. Write out the equation for the linear model and interpret the slope. Is average beauty score a statistically significant predictor? Does it appear to be a practically significant predictor?

```
m_bty <- lm(eval$score ~ eval$bty_avg)
plot(eval$score ~ jitter(eval$bty_avg))
abline(m_bty)
```



```
summary(m_bty)
```

```
##
## Call:
## lm(formula = evals$score ~ evals$btty_avg)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.9246 -0.3690  0.1420  0.3977  0.9309
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.88034    0.07614   50.96 < 2e-16 ***
## evals$btty_avg 0.06664    0.01629    4.09 5.08e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5348 on 461 degrees of freedom
## Multiple R-squared:  0.03502,    Adjusted R-squared:  0.03293
## F-statistic: 16.73 on 1 and 461 DF,  p-value: 5.083e-05
```

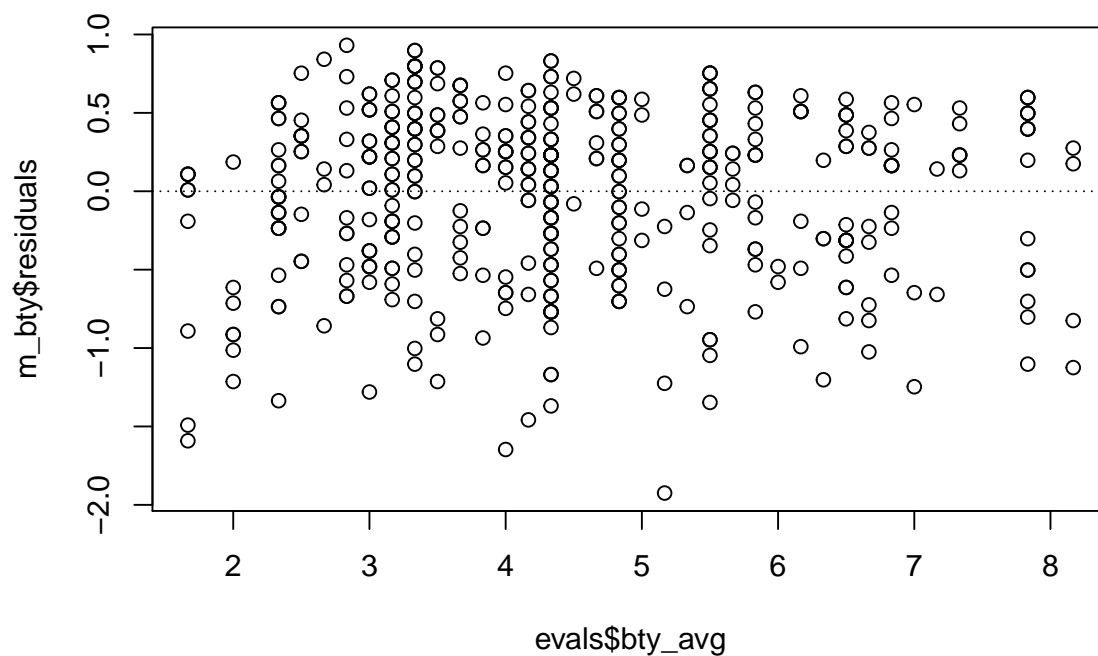
$$y = 3.88031 + 0.06664x$$

Everytime beauty increases by 1 point, score increases by 0.06664 points.

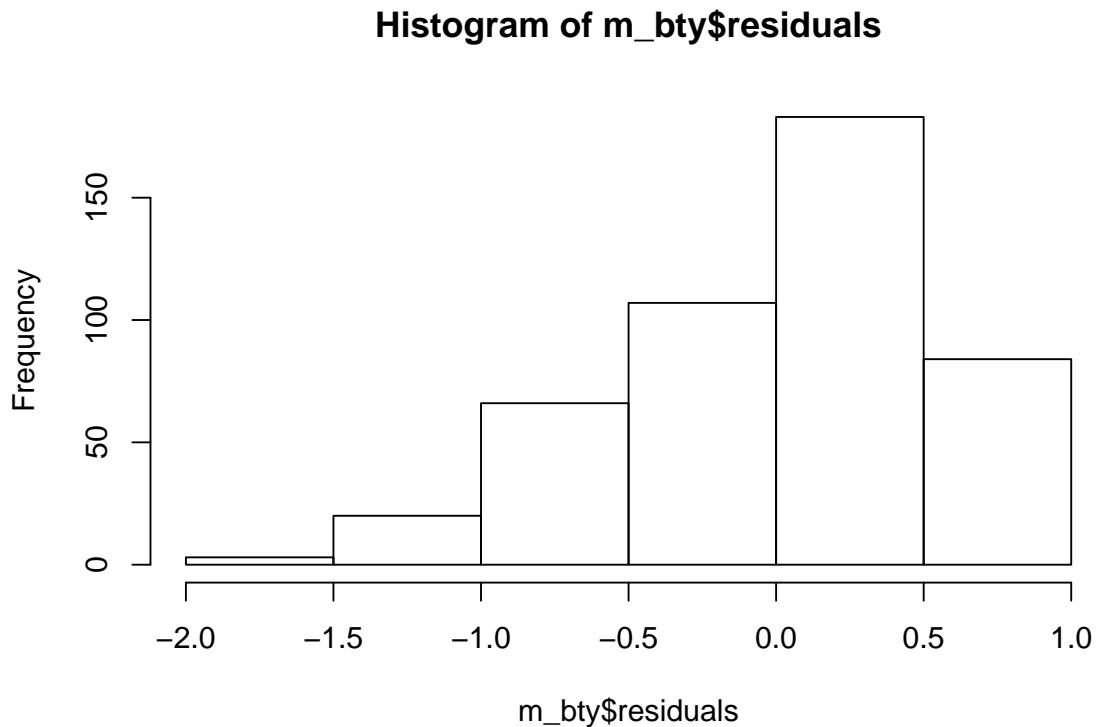
It is statistically predictive because of the low p-value. However the low R^2 means that it is not practically significant.

6. Use residual plots to evaluate whether the conditions of least squares regression are reasonable. Provide plots and comments for each one (see the Simple Regression Lab for a reminder of how to make these).

```
plot(m_bty$residuals ~ evals$bty_avg)
abline(h = 0, lty = 3)
```



```
hist(m_bty$residuals)
```



The residuals are not really normal. They definitely show a peak with declining numbers on either side, but are skewed fairly strongly. The wide spread in the first graph does provide evidence that the data is not tightly linear.

Multiple linear regression

The data set contains several variables on the beauty score of the professor: individual ratings from each of the six students who were asked to score the physical appearance of the professors and the average of these six scores. Let's take a look at the relationship between one of these scores and the average beauty score.

```
plot(evals$bty_avg ~ evals$bty_follower)
cor(evals$bty_avg, evals$bty_follower)
```

As expected the relationship is quite strong - after all, the average score is calculated using the individual scores. We can actually take a look at the relationships between all beauty variables (columns 13 through 19) using the following command:

```
plot(evals[,13:19])
```

These variables are collinear (correlated), and adding more than one of these variables to the model would not add much value to the model. In this application and with these highly-correlated predictors, it is reasonable to use the average beauty score as the single representative of these variables.

In order to see if beauty is still a significant predictor of professor score after we've accounted for the gender of the professor, we can add the gender term into the model.


```
m_bty_gen <- lm(score ~ bty_avg + gender, data = evals)
summary(m_bty_gen)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + gender, data = evals)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-1.8305	-0.3625	0.1055	0.4213	0.9314

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	3.74734	0.08466	44.266	< 2e-16 ***
bty_avg	0.07416	0.01625	4.563	6.48e-06 ***
gendermale	0.17239	0.05022	3.433	0.000652 ***

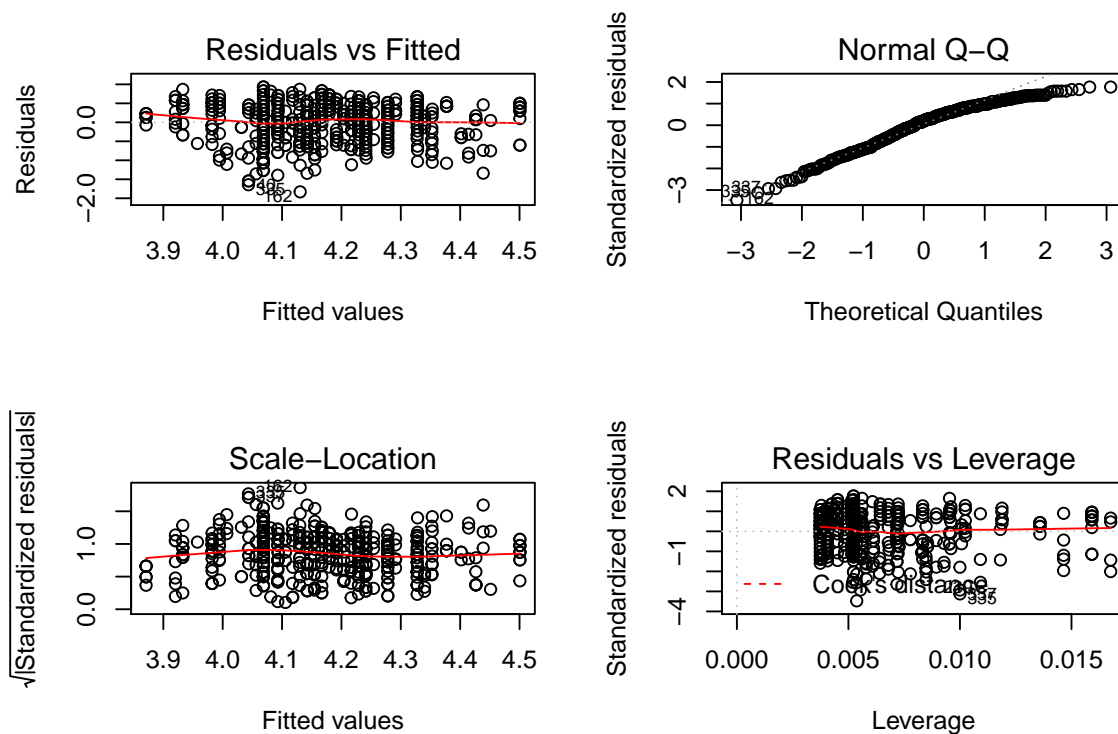
```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5287 on 460 degrees of freedom
## Multiple R-squared:  0.05912,    Adjusted R-squared:  0.05503
## F-statistic: 14.45 on 2 and 460 DF,  p-value: 8.177e-07
```

7. P-values and parameter estimates should only be trusted if the conditions for the regression are reasonable. Verify that the conditions for this model are reasonable using diagnostic plots.

We need to check for a few things:

- Independance of each data points
- Correct distribution of the residuals
- Correct specification of the variance structure
- Linear relationship between the response and the linear predictor

```
par(mfrow = c(2, 2))
plot(m_bty_gen)
```



```
par(mfrow = c(1, 1))
```

Each graph above serves its own purpose. The top left looks random, and the top right is nearly straight with two tails on either end. The check passes.

8. Is `bty_avg` still a significant predictor of `score`? Has the addition of `gender` to the model changed the parameter estimate for `bty_avg`?

```
summary(m_bty_gen)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + gender, data = evals)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.8305 -0.3625  0.1055  0.4213  0.9314
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.74734    0.08466  44.266 < 2e-16 ***
## bty_avg        0.07416    0.01625   4.563 6.48e-06 ***
## gendermale     0.17239    0.05022   3.433 0.000652 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5287 on 460 degrees of freedom
```

```
## Multiple R-squared:  0.05912,    Adjusted R-squared:  0.05503
## F-statistic: 14.45 on 2 and 460 DF,  p-value: 8.177e-07
```

The parameter estimate has changed with the addition of gender. It changed from 0.06664 to 0.07416. The R^2 value increase from 0.03293 to 0.05503 which indicates a better fitting model.

Note that the estimate for **gender** is now called **gendermale**. You'll see this name change whenever you introduce a categorical variable. The reason is that R recodes **gender** from having the values of **female** and **male** to being an indicator variable called **gendermale** that takes a value of 0 for females and a value of 1 for males. (Such variables are often referred to as “dummy” variables.)

As a result, for females, the parameter estimate is multiplied by zero, leaving the intercept and slope form familiar from simple regression.

$$\begin{aligned}\widehat{score} &= \hat{\beta}_0 + \hat{\beta}_1 \times bty_avg + \hat{\beta}_2 \times (0) \\ &= \hat{\beta}_0 + \hat{\beta}_1 \times bty_avg\end{aligned}$$

We can plot this line and the line corresponding to males with the following custom function.

```
multiLines(m_bty_gen)
```

9. What is the equation of the line corresponding to males? (*Hint:* For males, the parameter estimate is multiplied by 1.) For two professors who received the same beauty rating, which gender tends to have the higher course evaluation score?

$$\widehat{score} = 3.7473382 + 0.0741554 \times bty_avg + 0.0741554 \times (1)$$

The slope section of the equation is positive, so males will have a higher average eval score.

The decision to call the indicator variable **gendermale** instead of **genderfemale** has no deeper meaning. R simply codes the category that comes first alphabetically as a 0. (You can change the reference level of a categorical variable, which is the level that is coded as a 0, using **relevel** function. Use **?relevel** to learn more.)

10. Create a new model called **m_bty_rank** with **gender** removed and **rank** added in. How does R appear to handle categorical variables that have more than two levels? Note that the rank variable has three levels: **teaching**, **tenure track**, **tenured**.

```
m_bty_rank <- lm(evals$score ~ evals$bty_avg + evals$rank)
summary(m_bty_rank)
```

```
##
## Call:
## lm(formula = evals$score ~ evals$bty_avg + evals$rank)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.8713 -0.3642  0.1489  0.4103  0.9525
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3.98155    0.09078  43.860  < 2e-16 ***
```

```
## evals$btty_avg          0.06783    0.01655    4.098 4.92e-05 ***
## evals$ranktenure track -0.16070    0.07395   -2.173  0.0303 *
## evals$ranktenured      -0.12623    0.06266   -2.014  0.0445 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5328 on 459 degrees of freedom
## Multiple R-squared:  0.04652,    Adjusted R-squared:  0.04029
## F-statistic: 7.465 on 3 and 459 DF,  p-value: 6.88e-05
```

It seems to add an additional variable similar to the gendermale one from before. So a combination of variables is used instead of just one.

The interpretation of the coefficients in multiple regression is slightly different from that of simple regression. The estimate for `btty_avg` reflects how much higher a group of professors is expected to score if they have a beauty rating that is one point higher *while holding all other variables constant*. In this case, that translates into considering only professors of the same rank with `btty_avg` scores that are one point apart.

The search for the best model

We will start with a full model that predicts professor score based on rank, ethnicity, gender, language of the university where they got their degree, age, proportion of students that filled out evaluations, class size, course level, number of professors, number of credits, average beauty rating, outfit, and picture color.

11. Which variable would you expect to have the highest p-value in this model? Why? *Hint:* Think about which variable would you expect to not have any association with the professor score.

I would guess that the number of students who filled out evals would likely not have much to do with overall score.

Let's run the model...

```
m_full <- lm(score ~ rank + ethnicity + gender + language + age + cls_perc_eval
             + cls_students + cls_level + cls_profs + cls_credits + btty_avg
             + pic_outfit + pic_color, data = evals)
summary(m_full)
```

```
##
## Call:
## lm(formula = score ~ rank + ethnicity + gender + language + age +
##     cls_perc_eval + cls_students + cls_level + cls_profs + cls_credits +
##     btty_avg + pic_outfit + pic_color, data = evals)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.77397 -0.32432  0.09067  0.35183  0.95036
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    4.0952141  0.2905277   14.096 < 2e-16 ***
## ranktenure track -0.1475932  0.0820671   -1.798  0.07278 .
## ranktenured     -0.0973378  0.0663296   -1.467  0.14295
```

```
## ethnicitynot minority 0.1234929 0.0786273 1.571 0.11698
## gendermale 0.2109481 0.0518230 4.071 5.54e-05 ***
## languagenon-english -0.2298112 0.1113754 -2.063 0.03965 *
## age -0.0090072 0.0031359 -2.872 0.00427 **
## cls_perc_eval 0.0053272 0.0015393 3.461 0.00059 ***
## cls_students 0.0004546 0.0003774 1.205 0.22896
## cls_levelupper 0.0605140 0.0575617 1.051 0.29369
## cls_profssingle -0.0146619 0.0519885 -0.282 0.77806
## cls_creditsone credit 0.5020432 0.1159388 4.330 1.84e-05 ***
## bty_avg 0.0400333 0.0175064 2.287 0.02267 *
## pic_outfitnot formal -0.1126817 0.0738800 -1.525 0.12792
## pic_colorcolor -0.2172630 0.0715021 -3.039 0.00252 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.498 on 448 degrees of freedom
## Multiple R-squared: 0.1871, Adjusted R-squared: 0.1617
## F-statistic: 7.366 on 14 and 448 DF, p-value: 6.552e-14
```

12. Check your suspicions from the previous exercise. Include the model output in your response.

Above we can see that while my guess was accurate, the least amount of influence came from having a single professor or not.

13. Interpret the coefficient associated with the ethnicity variable.

When the professor is not a minority, the score increases by 0.1234929.

14. Drop the variable with the highest p-value and re-fit the model. Did the coefficients and significance of the other explanatory variables change? (One of the things that makes multiple regression interesting is that coefficient estimates depend on the other variables that are included in the model.) If not, what does this say about whether or not the dropped variable was collinear with the other explanatory variables?

```
m_full <- lm(score ~ rank + ethnicity + gender + language + age + cls_perc_eval
             + cls_students + cls_level + cls_credits + bty_avg
             + pic_outfit + pic_color, data = evals)
summary(m_full)
```

```
##
## Call:
## lm(formula = score ~ rank + ethnicity + gender + language + age +
##     cls_perc_eval + cls_students + cls_level + cls_credits +
##     bty_avg + pic_outfit + pic_color, data = evals)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.7836 -0.3257  0.0859  0.3513  0.9551
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    4.0872523   0.2888562   14.150 < 2e-16 ***
## ranktenure track -0.1476746   0.0819824   -1.801  0.072327 .
## ranktenured     -0.0973829   0.0662614   -1.470  0.142349
## ethnicitynot minority 0.1274458   0.0772887    1.649  0.099856 .
```

```
## gendermale          0.2101231  0.0516873   4.065 5.66e-05 ***
## languagenon-english -0.2282894  0.1111305  -2.054 0.040530 *
## age                 -0.0089992  0.0031326  -2.873 0.004262 **
## cls_perc_eval       0.0052888  0.0015317   3.453 0.000607 ***
## cls_students        0.0004687  0.0003737   1.254 0.210384
## cls_levelupper      0.0606374  0.0575010   1.055 0.292200
## cls_creditsone credit 0.5061196  0.1149163   4.404 1.33e-05 ***
## bty_avg             0.0398629  0.0174780   2.281 0.023032 *
## pic_outfitnot formal -0.1083227  0.0721711  -1.501 0.134080
## pic_colorcolor      -0.2190527  0.0711469  -3.079 0.002205 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4974 on 449 degrees of freedom
## Multiple R-squared:  0.187, Adjusted R-squared:  0.1634
## F-statistic: 7.943 on 13 and 449 DF,  p-value: 2.336e-14
```

Some values in the table changed, which indicates at least a small amount of colinearity with cls_profs.

15. Using backward-selection and p-value as the selection criterion, determine the best model. You do not need to show all steps in your answer, just the output for the final model. Also, write out the linear model for predicting score based on the final model you settle on.

If we are going purely by R^2 values, then the model excluding only number of professors seems to be the best. We could remove all the high p-value variables, but the R^2 value decreases. The best approach might be to remove all the high p-value variables even if R^2 decreases. The formula is just combining all the estimates from the above chart like we did earlier.

score = 3.77 + 0.167872ethnicity + 0.207112gender - 0.206178language - 0.006046age + 0.004656cls_perc_eval + 0.505306cls_credits + 0.051069bty_avg - 0.190579pic_color

```
m_best <- lm(score ~ ethnicity + gender + language + age + cls_perc_eval + cls_credits + bty_avg +
summary(m_best)
```

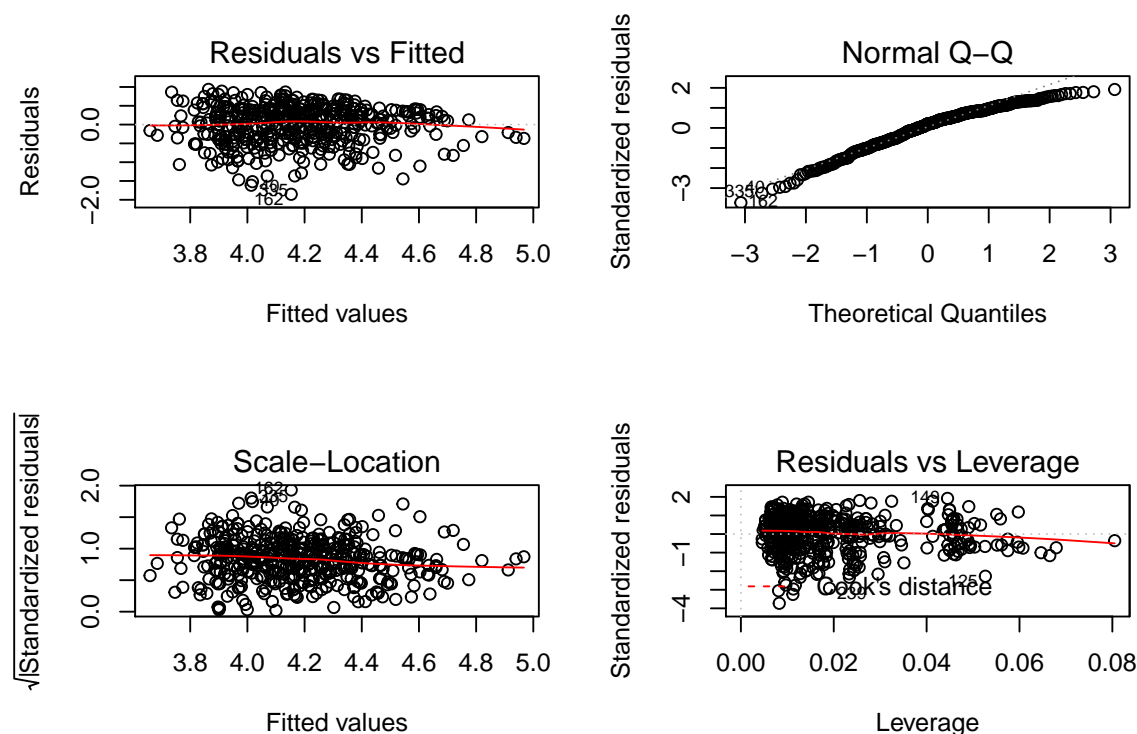
```
##
## Call:
## lm(formula = score ~ ethnicity + gender + language + age + cls_perc_eval +
##     cls_credits + bty_avg + pic_color, data = evals)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.85320 -0.32394  0.09984  0.37930  0.93610
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    3.771922   0.232053  16.255 < 2e-16 ***
## ethnicitynot minority 0.167872   0.075275   2.230  0.02623 *
## gendermale      0.207112   0.050135   4.131 4.30e-05 ***
## languagenon-english -0.206178   0.103639  -1.989  0.04726 *
## age            -0.006046   0.002612  -2.315  0.02108 *
## cls_perc_eval    0.004656   0.001435   3.244  0.00127 **
## cls_creditsone credit 0.505306   0.104119   4.853 1.67e-06 ***
## bty_avg          0.051069   0.016934   3.016  0.00271 **
## pic_colorcolor  -0.190579   0.067351  -2.830  0.00487 **
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4992 on 454 degrees of freedom
## Multiple R-squared:  0.1722, Adjusted R-squared:  0.1576
## F-statistic: 11.8 on 8 and 454 DF,  p-value: 2.58e-15
```

16. Verify that the conditions for this model are reasonable using diagnostic plots.

The conditions seem to pass given the graphs below.

```
par(mfrow = c(2, 2))
plot(m_best)
```



```
par(mfrow = c(1, 1))
```

17. The original paper describes how these data were gathered by taking a sample of professors from the University of Texas at Austin and including all courses that they have taught. Considering that each row represents a course, could this new information have an impact on any of the conditions of linear regression?

It's not exactly a random sample if every single class is used.

18. Based on your final model, describe the characteristics of a professor and course at University of Texas at Austin that would be associated with a high evaluation score.

We would want the following:

variable	ideal
ethnicity	not minority

variable	ideal
gender	male
language	english
age	younger
cls_perc_eval	high percentage
cls_credits	1 credit class
bty_avg	beautiful
pic_color	color photo

19. Would you be comfortable generalizing your conclusions to apply to professors generally (at any university)? Why or why not?

I would be comfortable applying this to U of Texas Austin professors but not to other schools. To be comfortable applying to other schools, the sample would need to include professors from those schools.

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