Critical Thinking Questions

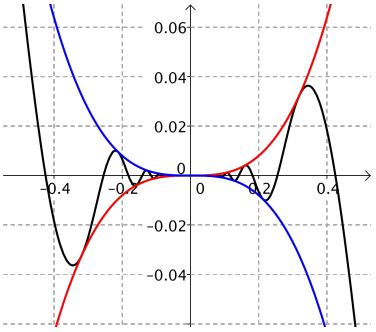

Figure 1: The Squeeze Theorem

1. Let's examine the function $y = \dfrac{x}{3x^2 + x + 1}$.

2. This is the symbol for the set of all real numbers: $\mathbb{R}$.

3. This is the symbol for the set of integers: $\mathbb{Z}$.

4. This is the symbol for the set of rationals: $\mathbb{Q}$.

5. Is it possible for a sequence to converge to two different numbers? If so, give an example. If not, explain why not.

6. Explain how to use partial sums to determine if a series converges or diverges. Give an example

7. Explain why $\int_1^\infty f(x)\,dx$ and $\sum_{n=1}^{\infty} a_n$ need not converge to the same value, even if they are both convergent.

8. In your own words, explain the Alternating Series Remainder Theorem. How is this theorem useful?

9. Explain the difference between absolute and conditional convergence. Give an example of each.

10. The Ratio Test is inconclusive if $\lim_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = 1$. Give an example of one convergent series and one divergent series for which $\lim_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = 1$.

Explain how you determined your examples.