

Modeling Techniques for Increasing Traffic Flow at an Intersection

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Introduction

We wish to model the flow of traffic through an intersection using a linear approximation to solve the partial differential equation $\partial p / \partial t + \partial f(p) / \partial x = 0$, representing the conservation of flow in a system. We detail how we construct this model. We then propose a scenario for traffic flow into an intersection and potential solutions to help improve the flow of traffic through the intersection.

Modeling a Traffic Light

Goal

We begin by aiming to construct a model to represent the flow of traffic for a basic intersection. The intersection consists of four roads entering a junction. As they enter the junction, they split into two lanes which we will consider as separate roads. The road on the left represents a left turn lane in which cars can only turn left. The road on the right represents a straight or right turn in which cars can continue straight or turn right.

Methodology

Setup of Situational Parameters

Our methodology is borrowed largely from Göttlich et al¹. We start by considering I roads. In the described situation, $I=16$. We label the roads as shown in Figure 1.

¹ Göttlich, S., Herty, M., & Ziegler, U. (2015). Modeling and optimizing traffic light settings in road networks. *Computers & Operations Research*, 55, 36–51. doi: 10.1016/j.cor.2014.10.001

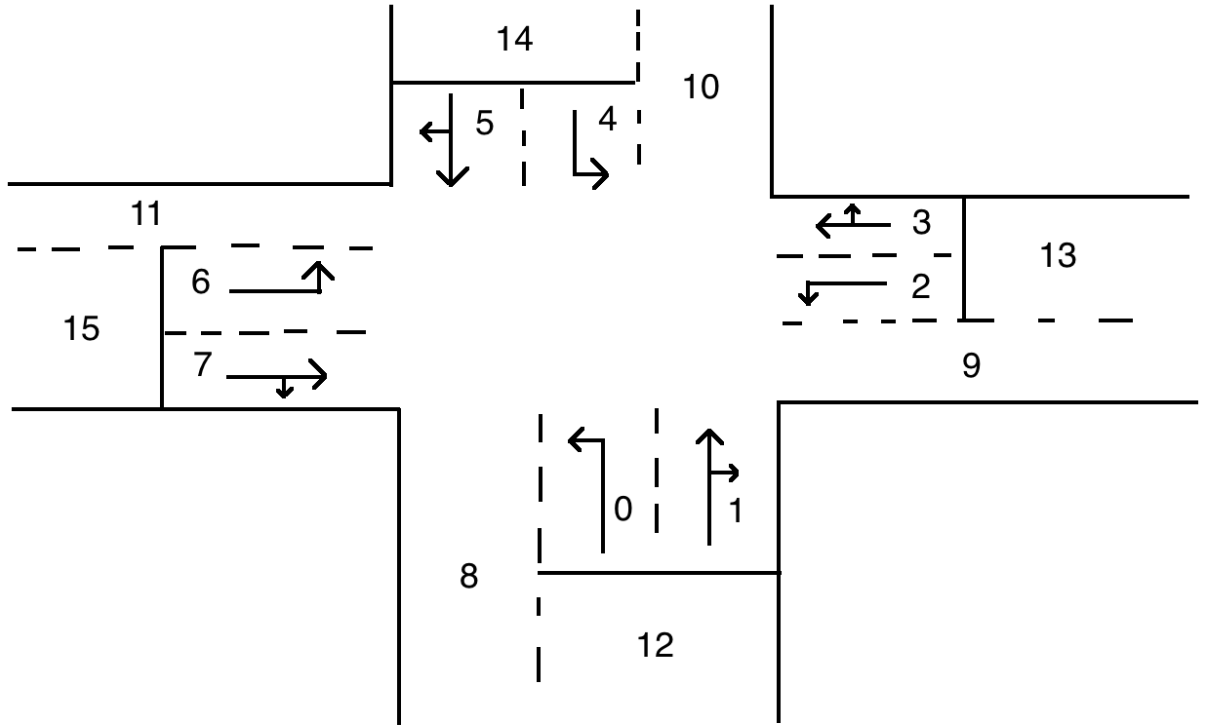


Figure 1

The roads are labeled with $i=0,1,\dots,15$. Roads $\{0,2,4,6\}$ are left turn roads, roads $\{1,3,5,7\}$ are straight or right turn roads, roads $\{8,9,10,11\}$ are outward bound roads, and roads $\{12,13,14,15\}$ are inward bound roads. We will use the following example to illustrate how the road map works. If a car enters the system at the beginning of road 12, they would travel to the end of that road and then decide to enter either the left turn lane or the straight and right turn lane. If they choose to turn left, they would move from road 12 to road 0, then from road 0 to road 11, where they would then exit the system.

We start by introducing an $I \times I$ matrix called D . An entry $D_{i,j}$ represents the proportion of the road i that moves into road j at the end of its road. Using the previous example, $D_{12,0}$ would represent the proportion of drivers on road 12 that decide to turn left. $D_{0,11}$ would be 1 as all drivers in the left turn lane turn left.

Each road i has a length L_i . We divide this road into points where the distance between point k and point $k+1$ is Δx . This will give the points $k=0,1,\dots, L_i/\Delta x=n$ where $k=0$ represents the beginning of the road and $k=n$ represents the end of the road.

We now consider our overall time interval T . We divide T into segments so that the length of time from t to $t+1$ is Δt . This will give us the moments in time $t=0,2,\dots,T-1$ where $t=1$ represents the beginning of our time interval and $t=T-1$ represents the end.

Density Function

The primary function in our model is our density function $p(i,k,t)$. This function represents the density of the i th road at its k th point at time t . We assume that $0 \leq p \leq p_{\max}$ where $p=0$ represents an entirely empty road and $p=p_{\max}$ represents bumper to bumper traffic.

Flow Function

We consider a secondary function for the flow of traffic which will aid in our calculation of the density. The flow of traffic is equal to the density*velocity. We assume there is an optimal density of p_{opt} at which the flow is maximized. For our purposes, we assumed $p_{\text{opt}}=.5$ and $p_{\max}=1$. We also initially assume a constant velocity of β . We use these values to create the following triangular piecewise function for the flow $f(p)$.

$$f(p) = \begin{cases} \beta * p & \text{if } 0 \leq p \leq p_{\text{opt}} \\ \beta * (2 * p_{\text{opt}} - p) & \text{if } p_{\text{opt}} < p \leq p_{\max} \end{cases} \quad (1)$$

The function is continuous, although not differentiable, and given our assumptions for p_{opt} and p_{\max} is equal to 0 if $p=0$ or if $p=p_{\max}$. This is ideal because in bumper to bumper traffic, there is no possible flow.

Lax-Friedrichs Scheme

To model the change in density over time, we look to solve the partial differential equation $\partial p / \partial t + \partial f(p) / \partial x = 0$ with some initial density $p(i,k,0)$. This equation guarantees the conservation of flow of traffic. There are many ways to attempt to solve this equation. We chose to use a first-order Lax-Friedrichs (LxF) scheme². This is a linear approximation of the solution to the partial differential equation that is solved iteratively, similar to Euler's Method.

For the center points along the road i ($k \neq 0, n$) the LxF scheme is given by:

$$p(i,k,t+1) = \frac{1}{4} * \{p(i,k-1,t) + 2 * p(i,k,t) + p(i,k+1,t)\} - \frac{1}{2} * (\Delta t / \Delta x) * \{f(p(i,k+1,t)) - f(p(i,k-1,t))\} \quad (2)$$

Detailed derivations of this model are available^{1,2} but we will attempt to summarize the function's workings in relation to traffic. The left hand side of the equation averages the densities of the points in the neighborhood of k at the previous time step to estimate the point's density. This could be achieved through simply using $p(i,k,t)$, but the average accounts for a "sharing" of density and is more representative of a situation in which density flows.

² Jiang, G. S., Levy, D., Lin, C. T., Osher, S., & Tadmor, E. (1998). High-Resolution Nonoscillatory Central Schemes with Nonstaggered Grids for Hyperbolic Conservation Laws. *SIAM Journal on Numerical Analysis*, 35(6), 2147–2168. doi: 10.1137/s0036142997317560

The right hand side of the equation estimates the change in density. The function looks at the amount of density that flowed out of the point $k+1$ and counts that negatively towards the current density while the density that flowed out of the point $k-1$ counts positively.. In terms of traffic, this would represent cars moving out of the point $k+1$, allowing for cars at point k to move into the point $k+1$, and cars at point $k-1$ moving into point k .

We will use an example to illustrate why this usage of the flow function requires us to include the density at the points $k-1$ and $k+1$ in our model.

For this example, we assume $\beta=1$, $\Delta t=.1$ and $\Delta x=.2$ so that $\frac{1}{2}*(\Delta t/\Delta x)=1$. This also satisfies the smoothness condition given by Jiang et al.² which states that the approximation function for the density is smooth if $\Delta t*\max(|f'(p)|)\leq\Delta x/2$. For our flow function, $\max(|f'(p)|)$ occurs at p_{opt} and has a value of 1. Substituting these values into our equation gives a value of .1 on both sides, satisfying the smoothness condition.

We now consider the initial condition $p(0,0,0)=0$, $p(0,1,0)=.3$ and $p(0,2,0)=0$. We wish to find the value of $p(0,1,1)$. We will first use equation (2), and then use equation (2) without the averaging used in the left hand side.

Using equation (2), we would find that $p(0,1,1)=.15$. Using equation (2) without the averaging term would give $p(0,1,1)=.3$. Despite an empty road in front and behind the point $k=1$, without the averaging term the density would not change. The sharing of density property that is gained through the averaging term is a strength of this model.

Handling Boundaries

It is important to note that equation (2) is undetermined for $k=0$ and $k=n$, as the points $k=-1$ and $k=K$ do not exist. The authors Göttlich et al.¹ derive the following modification to (2) so that the equation is determined.

If $k=0$:

$$p(i,0,t+1) = \frac{1}{4} * \{3 * p(i,0,t) + p(i,1,t)\} - \frac{1}{2} * (\Delta t / \Delta x) * \{f(p(i,0,t)) + f(p(i,1,t)) - 2 * \gamma_{in}(p,i,t)\} \quad (3)$$

If $k=n$:

$$p(i,n,t+1) = \frac{1}{4} * \{3 * p(i,n,t) + p(i,n-1,t)\} - \frac{1}{2} * (\Delta t / \Delta x) * \{2 * \gamma_{out}(p,i,t) - f(p(i,n,t)) - f(p(i,n-1,t))\} \quad (4)$$

We will now define the functions $\gamma_{in}(i,t)$ and $\gamma_{out}(i,t)$. These functions represent the flow into the beginning of road i and out of road i respectively. We start by considering the boundaries of the road we are modeling. For $i=0$, the point $k=0$ represents a boundary. We cannot know the flow into that point from the state of our system. This allows us to make an assumption about what that flow is. For road $i=1$, the point $k=n$ represents a boundary and likewise motivates us to make an assumption. We make the following assumptions:

$$\gamma_{in}(p,i,t) = \text{some function } h(t) \text{ with range } [0, f(p_{opt})] \quad \text{if } i \text{ has an inward boundary} \quad (5)$$

$$\gamma_{\text{out}}(p,i,t) = f(p(i,n,t)) \quad \text{if } i \text{ has an outward boundary (6)}$$

The assumption in equation (5) allows us to introduce different traffic patterns into our model. The assumption in equation (6) assumes that the traffic will continue to flow as it otherwise would.

Flow at the Traffic Light

In our prescribed situation, there are still 8 points at which the Lax-Friedrichs scheme is undefined. We need to define the γ_{in} and γ_{out} functions when the road i 's endpoint(s) do not occur at the boundary. These are the points that are at which one road enters another.

We start by considering a reasonable configuration of a traffic light. Let S be a set of all possible two road combinations of our roads $i \in \{(0,1),(0,2),\dots,(1,0),\dots,(15,14)\}$. Let S' be a subset of S representing all i,j combinations for which $D_{i,j} > 0$. Let S'' be a subset of S' whose combinations are given by the combinations in S' that do not occur at the intersection. This is a combination like $(12,0)$, which is not actually at the traffic light, it is a natural progression from one road to another. For our scenario, S'' is the following:

$$S'' = \{(12,0), (12,1), (13,2), (13,3), (14,4), (14,5), (15,6), (15,7)\}.$$

Finally, we have the set S''' , which requires a more careful definition. S''' is composed of subsets of S' that do not appear in S'' . Each subset will be a set of points whose lights can be turned on at the same time t . We use Figures 2 and 3 to help illustrate these points.

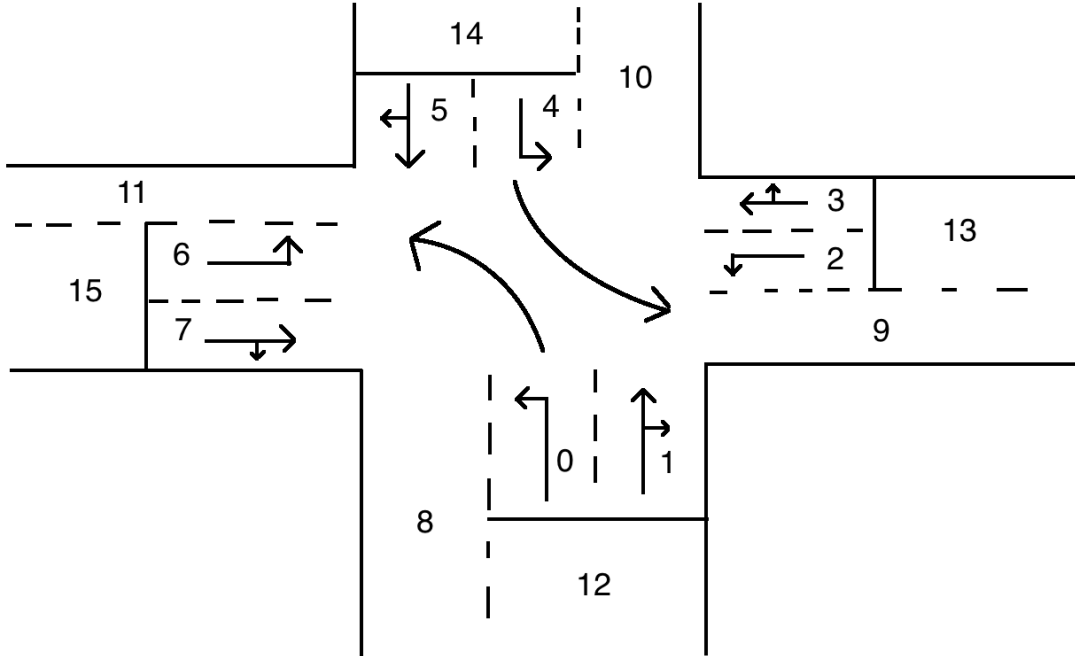


Figure 2

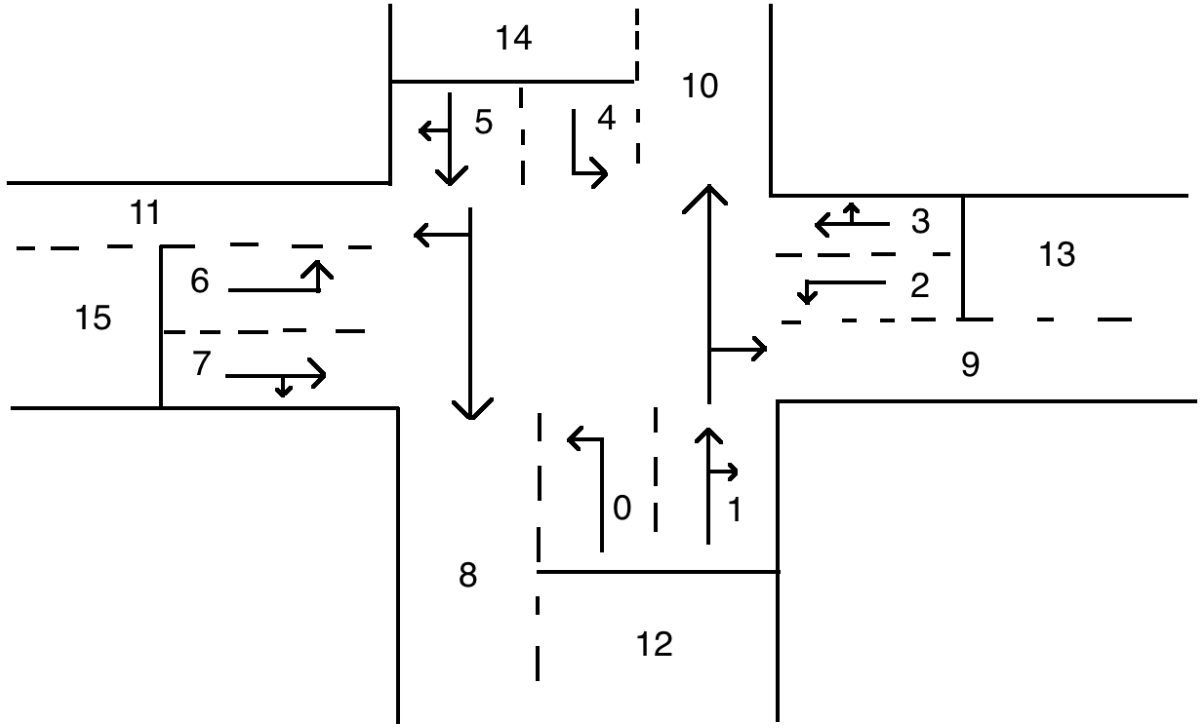


Figure 3

S''' contains the traffic light pairings that do not lead to an accident. Two left turns can be made simultaneously as shown in Figure 2, but no other lights can be green. In Figure 3, straight and right turn lights can both be green for opposing roads. This gives us the following for S''' :

$$S''' = \{ \{ (0,11), (4,9) \}, \{ (1,9), (1,10), (5,8), (5,11) \}, \{ (2,8), (6,10) \}, \{ (3,10), (3,11), (7,8), (7,9) \} \}$$

We use our sets S'' and S''' to help create a function $A((i,j),t)$. This will represent the traffic light and take a value of 1 for a green light and 0 for a red light. This is a function that can be controlled, so we will present methods of controlling this function to increase traffic flow later in the paper, but we start with these base constraints:

$$A((i,j),t) = 1 \quad \text{for all } 0 \leq t \leq T \text{ for all } (i,j) \text{ in } S'' \quad (7)$$

$$A((i,j),t) = 1 \quad \text{for } (i,j) \text{ in } S''' \text{ if and only if } A((x,y),t)=0 \text{ for all points } (x,y) \text{ not in the subset of } S''' \text{ containing } (i,j) \quad (8)$$

This constraint ensures there is no collision in the flow of traffic.

We now use $A((i,j),t)$ to construct γ_{out} . We start this process by looking at the density at the point $p(i,n,t)$ to consider the maximum possible flow that could “leave” the point n , which we will call F_{leave} . We define F_{leave} as the following:

$$F_{\text{leave}} = \begin{cases} A(t) * f(p(i,n,t)) & \text{if } p \leq p_{\text{opt}} \\ A(t) * f(p_{\text{opt}}) & \text{if } p > p_{\text{opt}} \end{cases} \quad (9)$$

The assumption is that if there is a greater than optimal amount of density at the end of the road, such as a high density of cars waiting at a stop light, then the optimal flow of cars will “try” to leave the end of that road. We multiple this flow by the current state of the traffic light. If the traffic light is green, the F_{leave} is the flow described. If the traffic light is red, F_{leave} is 0 since no density is “trying” to leave the road.

We now consider the density at all points $p(j,0,t)$ where j is a road for which $D_{i,j} > 0$. We define a function F_{enter} as the following:

$$F_{\text{enter}} = \begin{cases} f(p_{\text{opt}}) & \text{if } p \leq p_{\text{opt}} \\ f(p(j,0,t)) & \text{if } p > p_{\text{opt}} \end{cases} \quad (10)$$

This considers the maximum flow that the road could “accept” at its entry. An empty or suboptimal density at the road’s beginning means the road can “accept” an optimal flow, while a denser entry point cannot “accept” an optimal flow.

We use the functions F_{leave} and F_{enter} to define γ_{out} .

$$\gamma_{\text{out}}(p(i,n,t),i,t) = \min\{F_{\text{leave}}, 1/D_{i,j} * F_{\text{enter}}\} \text{ for all points } j \text{ where } D_{i,j} > 0 \quad (11)$$

We now highlight how γ_{out} works. If the traffic light is red, $F_{\text{leave}}=0$ and is thus the minimum and no traffic flows. If there is a backup on a road j that road i connects with, that will contribute negatively towards F_{enter} , decreasing the minimum and flow out. This would represent somebody waiting to turn onto a busy road j , causing a backup on the original road i . But if a very small proportion of drivers turn onto this busy road, it is less likely to cause a backup on the original road. That is why we divide by $1/D_{i,j}$.

We now use γ_{out} to define γ_{in} , with the idea that we wish to preserve the conservation of flow:

$$\gamma_{\text{in}}(i,0,t) = \sum \gamma_{\text{out}}(p(j,0,t),j,t) * D_{j,i} \quad \text{for all points } j \text{ where } D_{j,i} > 0 \quad (12)$$

This examines the flow out of each road that is flowing into road i . It then multiplies that flow by $D_{j,i}$, which represents the proportion of the flow that would leave road j for road i .

With this function, our Lax-Friedrichs Scheme is now well defined and can be used for simulation.

Default Simulation

Scenario

We construct a traffic flow situation to model how different techniques can be used to increase the flow of traffic through the intersection. We wish to represent a situation in which there is a commonly used road that other roads have a preference towards connecting to. The road that makes a left turn to the preferred road has a heavier flow of traffic than the others.

Parameters

T	Time Interval	400
K	# of road sections	10
I	# of roads	16
L_i	Length of each road	All equal to 20
Δt	Time step	1
Δx	Distance between points along road	2
p_{\max}	Maximum density	1
p_{opt}	Optimal density	.5
β	Velocity	1
γ_{out} at the boundary	Traffic flow out of the system	$f(p(i,n,t))$
$p(i,k,0)$	Initial density	.2 at all points
D	Directional Matrix	Defined below
$A(i,j,t)$	Traffic Light Setting	Defined below
C_{left}	# of time steps for a left turn traffic light combination (see definition below)	10

C_{straight}	# of time steps for a straight traffic light combination (see definition below)	10
γ_{in} at the boundary	Traffic flow into the system	Defined below

Default Setting of D

We first display only the rows 0 through 7 and columns 8 through 11 for our matrix D, to represent decisions at the traffic light.

Road	8	9	10	11
0	0	0	0	1
1	0	.7	.3	0
2	1	0	0	0
3	0	0	.1	.9
4	0	1	0	0
5	.5	0	0	.5
6	0	0	1	0
7	.3	.7	0	0

We now display rows 12 through 15 and columns 0 through 7 to display decisions entering each lane.

Road	0	1	2	3	4	5	6	7
12	.5	.5	0	0	0	0	0	0
13	0	0	.3	.7	0	0	0	0
14	0	0	0	0	.3	.7	0	0
15	0	0	0	0	0	0	.3	.7

All other entries in the matrix D are 0. D has been constructed to reflect the scenario described in which road 11 is the road most likely to be preferred.

Default Setting of the Traffic Light

We first set all $A(i,j,t)=1$ for all pairs (i,j) found in S'' . There is not an actual traffic light at these points, it is the point at which a driver decides to turn either left or right. Since there is no time a driver cannot do this, this light is always green.

We consider a complete cycle of the traffic light to be a loop through the subsets in S''' . The first entry in S''' , $\{(0,11),(4,9)\}$ is a combination of permissible simultaneous left turns. We will call this S'''^1 . The next entry $S'''^2, \{(1,9),(1,10),(5,8),(5,11)\}$, is a combination of permissible simultaneous straight and right turns. Looping through the four entries will allow each road a chance to release its traffic. We define the parameters C_{left} and C_{straight} to represent the length of a cycle for the two types of entries in S''' . We use this to set the values of $A(i,j,t)$ for one cycle.

$$\begin{aligned}
 &\text{For } (i,j) \text{ in } S'''^1 \text{ if } m < C_{\text{left}} \\
 &\text{For } (i,j) \text{ in } S'''^2 \text{ if } C_{\text{left}} \leq m < C_{\text{left}} + C_{\text{straight}} \\
 &\text{For } (i,j) \text{ in } S'''^3 \text{ if } C_{\text{left}} + C_{\text{straight}} \leq m < 2 * C_{\text{left}} + C_{\text{straight}} \\
 &\text{For } (i,j) \text{ in } S'''^4 \text{ if } 2 * C_{\text{left}} + C_{\text{straight}} \leq m < 2(C_{\text{left}} + C_{\text{straight}})
 \end{aligned}$$

$$A(i,j,t) = 1 \tag{13}$$

Where m is an integer that starts at 0, increases with each time step, and resets to 0 when $m > 2(C_{\text{left}} + C_{\text{straight}})$.

For our default case, we set both C_{left} and C_{straight} equal to 10, meaning that each set of permissible lights are green for 10 timesteps.

Default Setting of γ_{in} at the Boundaries

We construct the value of γ_{in} at the boundaries to match the scenario described. Road 12 leads to road 0, the road that turns left onto road 11, which is the preferred road. As the situation describes, road 12 will have the greatest value of γ_{in} . We use the following for γ_{in} .

$$\gamma_{\text{in}}(p(i,0,t),i,t)= \begin{cases} .5 & \text{for all } t \text{ if } i=12 \\ .2 & \text{for all } t \text{ if } i \neq 12 \end{cases} \quad (14)$$

Results

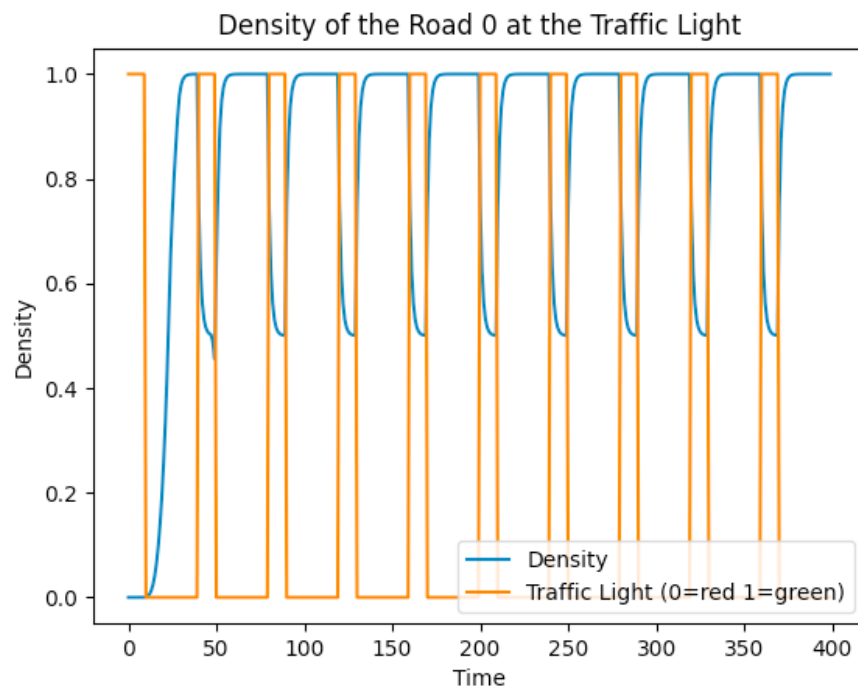
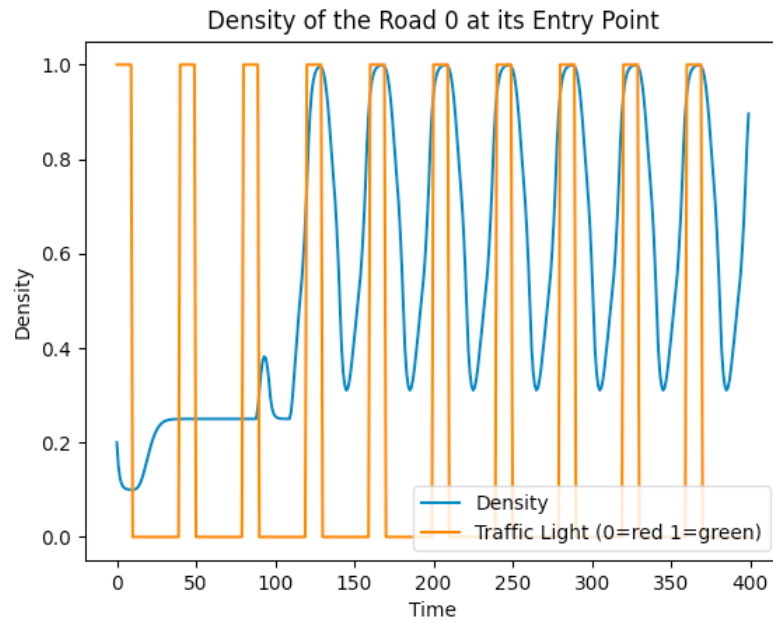


Figure 4



With Figures 4 and 5 we see how traffic builds backwards from the traffic to the beginning of the road. While the traffic light is red, the density at the traffic light immediately reaches the maximum density. This build up moves down the road until the entry point, and is then relieved after the green light when traffic can resume flow.

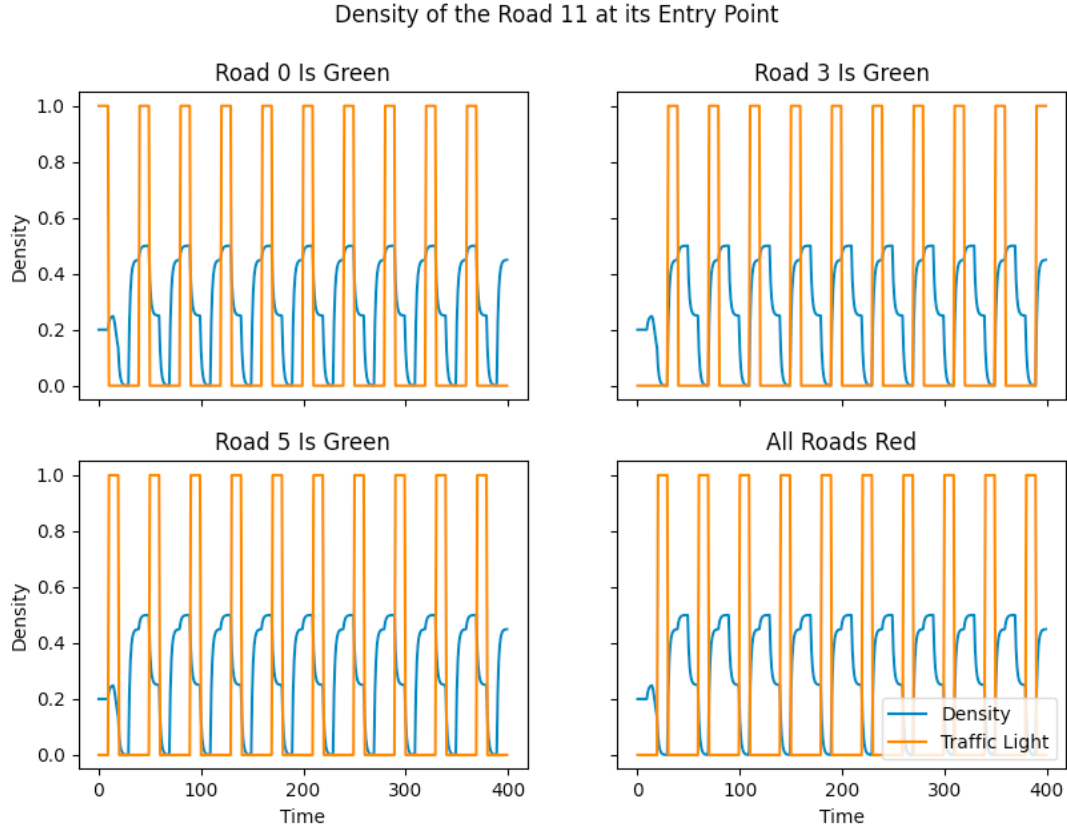


Figure 6 shows the density at the entry point to Road 11, the preferred destination road of the scenario. Based off of our construction of γ_{in} and D , we would expect Road 11 to be densest when Road 0 and Road 3 are green, less dense when Road 5 is green, and have no cars entering the system when Roads 2 and 6 are green, representing a left turn pair where neither left turn enters Road 11. These expectations are confirmed with the graphs.

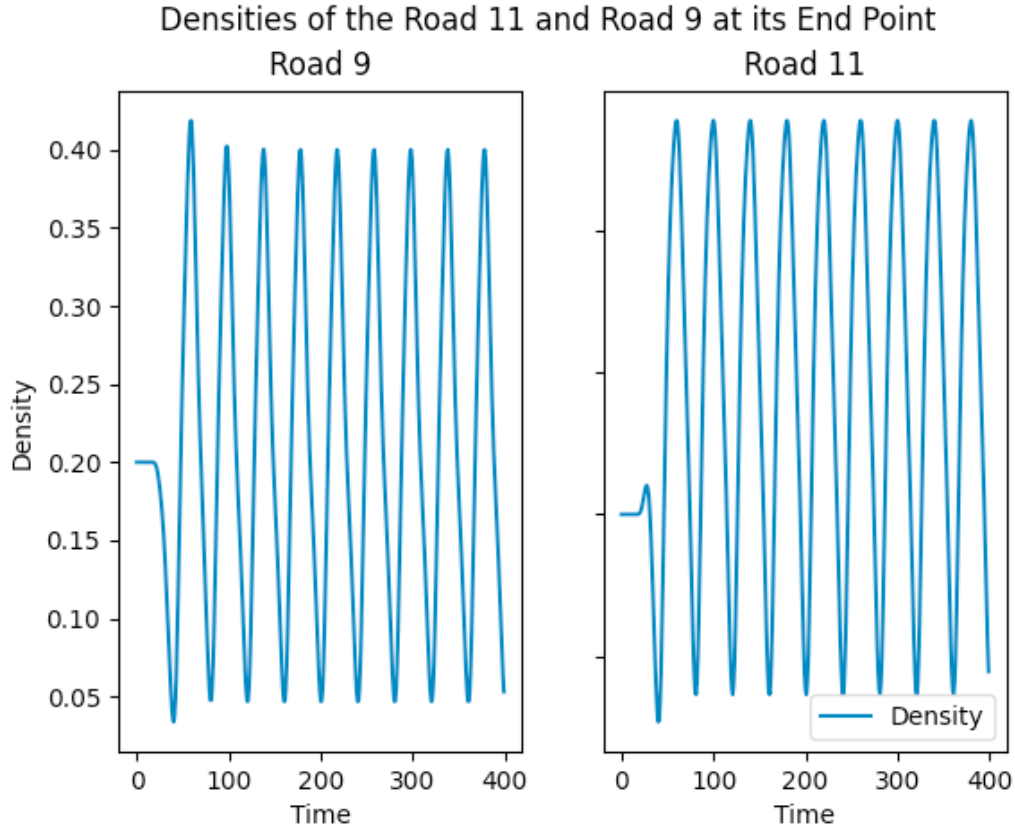


Figure 7

We see with Figure 7 that in general Road 11 has a higher average density at its endpoint, but the difference is very small. This does not match up with our scenario as much as we would like, but showcases a powerful aspect of our model.

Consider our assumption of γ_{out} at the boundary. We assume that traffic continues to flow out of the system using the function f in (1). Traffic cannot flow into the outward boundary roads at a value greater than $f(p_{\text{opt}})$ due to the construction of γ_{in} . If we consider an outward boundary road to be a single system, a density less than $f(p_{\text{opt}})$ is entering the system, and using (1) that density $f(p)$ would flow out of the system. If conservation of flow is being used correctly, the density would never be greater than p_{opt} . This is the result of Figure 7, where the density is never greater than $p_{\text{opt}} = .5$.

This also highlights that our model would be improved by considering a network of intersections. If road 11 led to a traffic light, allowing density to build, we would have a new scenario to model. Our model could easily be adapted to handle this network, all that is required is to add the necessary roads to I and follow the procedure for creating D , our S sets, and our traffic light assignment.

Evaluating the Model

To evaluate the performance of our model, we find the average value of γ_{out} at the boundary, which we will call Γ_{out} . Running the simulation with our default settings resulted in a Γ_{out} of .186.

Increasing Traffic Flow

Goal

We will now attempt to simulate methods of traffic flow improvement to test if they improve flow through our intersection. We will compare Γ_{out} values to our default setting to test improvement.

Adding a Right-Turn Lane

Methodology

From our base case, we added a right-turn lane by splitting lanes 1, 3, 5, 7 into two, keeping the numbers to denote the now exclusively straight lanes, and renaming the right-turn lanes 16, 17, 18, 19.

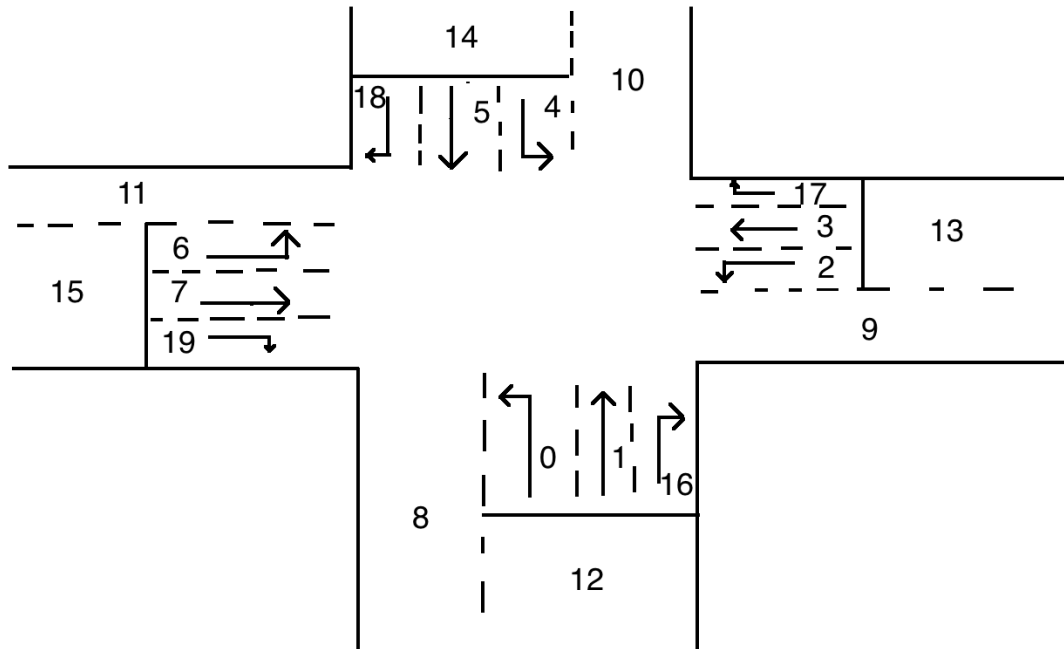


Figure 8

With this new configuration, we allow for new right turns in our construction of S''' , which will flow on the corners the left-turning cars don't touch. This is shown in Figure 9 below.

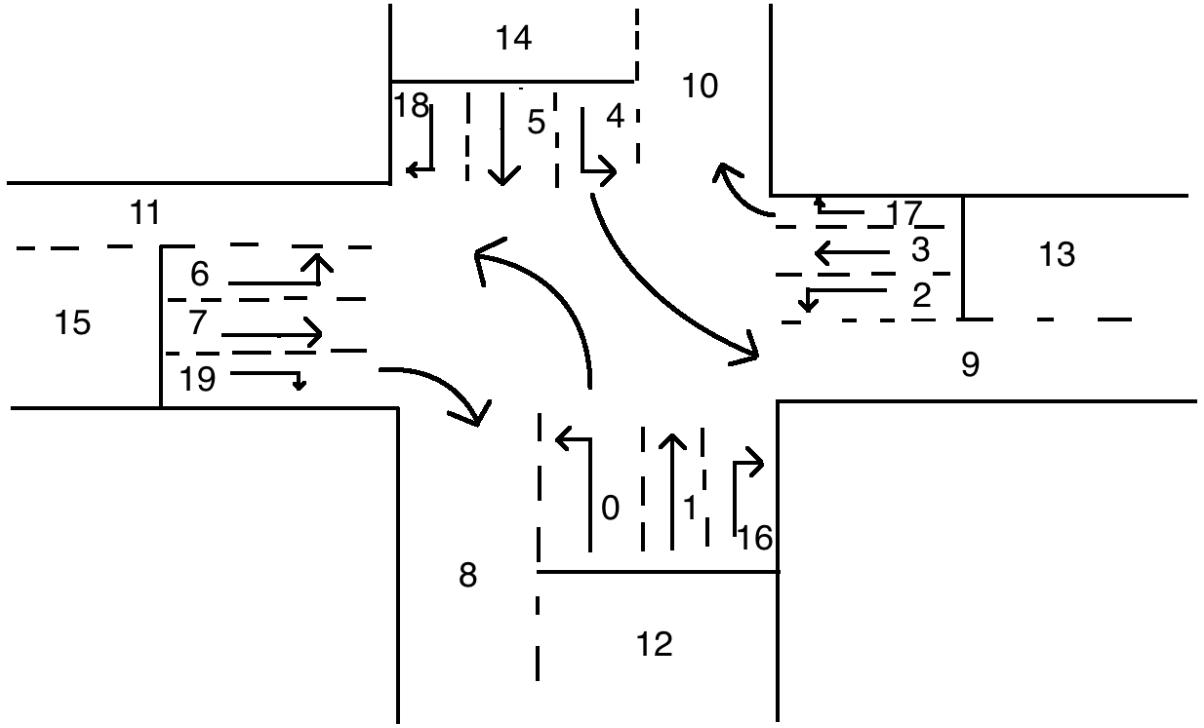


Figure 9

All other parameters were held constant and we preserved the proportions going from inward boundary roads to outward boundary roads in our new matrix D.

Results

Γ_{out} increased to .260 with this configuration. We propose that the larger set entries in S'''^1 and S'''^3 contribute to the majority of this improvement. We see in Figure 10 below that outward boundary roads such as 11 no longer have a cycle where no traffic is flowing into the road, which we saw was the case for our default setting in Figure 6.

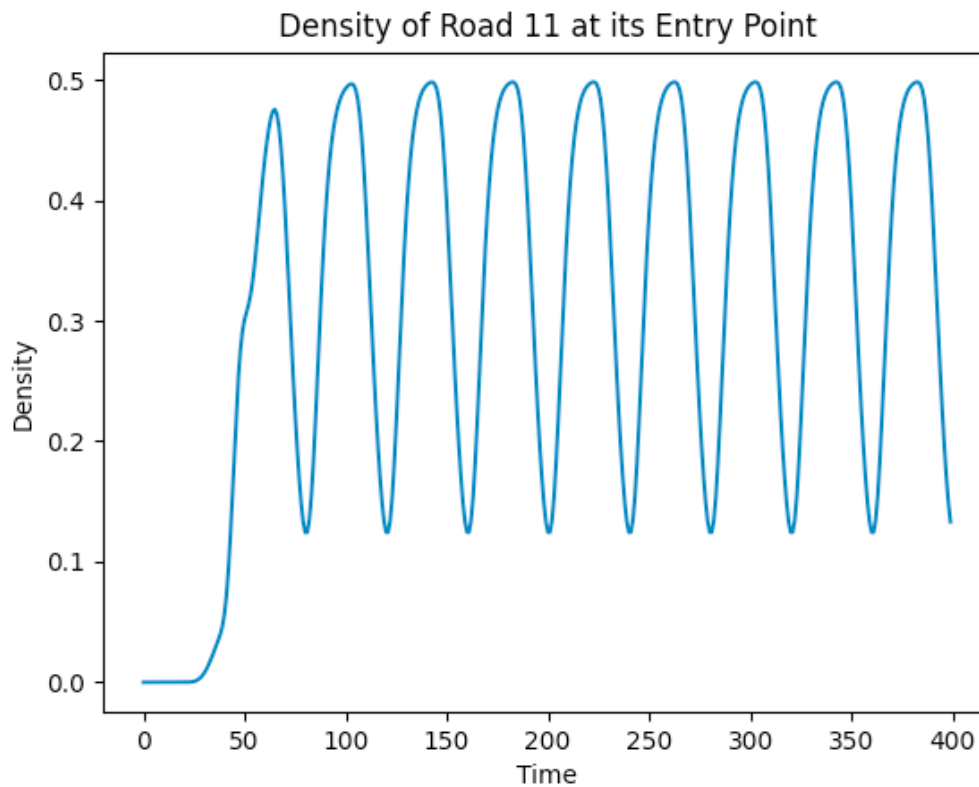


Figure 10

Optimizing Cycle Length to Maximize Traffic Flow

Methodology

We now reset back to our original intersection without the extra right turn lane and wish to find the optimal values for C_{left} and C_{straight} . By simulating the model with many different cycle lengths, we can find what combination of cycle lengths maximizes outflow.

Results

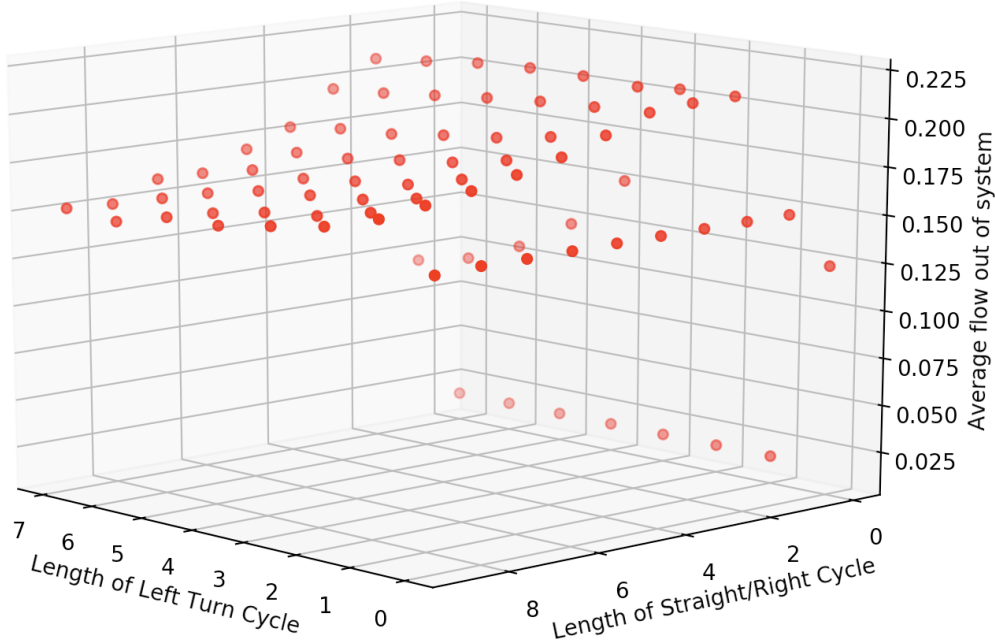


Figure 11

We obtained a maximum flow, Γ_{out} , of 0.2158, which was better than our base case of 0.186. The optimal left turn cycle is 4 time steps and the optimal straight/right cycle is 2 time steps. This particular street had a very busy left turn lane which we input into the model, for example, a left turn onto a highway.

A cycle of 1 for all lanes creates a simulation similar to a stop sign intersection and since we do not account for deceleration and acceleration, Γ_{out} is still efficient. Looking at the graph we can also see that when the cycle length for the straight and right lane is equal to zero there is very little flow as only cars in the left turn lane get to flow out. If either cycle length is equal to zero Γ_{out} will approach zero as traffic builds up stopping flow all together. When we set the straight and right cycle to zero due to the heavier traffic, Γ_{out} approaches zero much faster than when we set the left turn cycle to zero.

Potential added aspects to further optimize Γ_{out} would be to break down the cycle for each lane and their corresponding partner lane. In our model, we change the cycle length for all left turn lanes and all straight/right turn lanes. Varying the densities and traffic volume on each road would further allow us to optimize the cycle length to the specific case, however we optimize the cycle with our base case properties. Furthermore, we can implement cycle lengths for if the densities change according to a time variable such as a rush hour simulation where the cycle would change due to the new dynamic densities. However, our base case model uses a constant density inflow, γ_{in} .

Using a Traffic Sensor to Assign Green Lights

Methodology

In order to emulate a traffic sensor, our model determined which road i has the maximum at a point k every ten time steps. Our model would then turn that light green for all entries in S'' that contain the road i . If multiple roads have the same maximum density, one of those roads is chosen at random to be the road i for which the traffic light is applied. We simulated this with different values of k to determine the optimal placement of our sensor.

Results

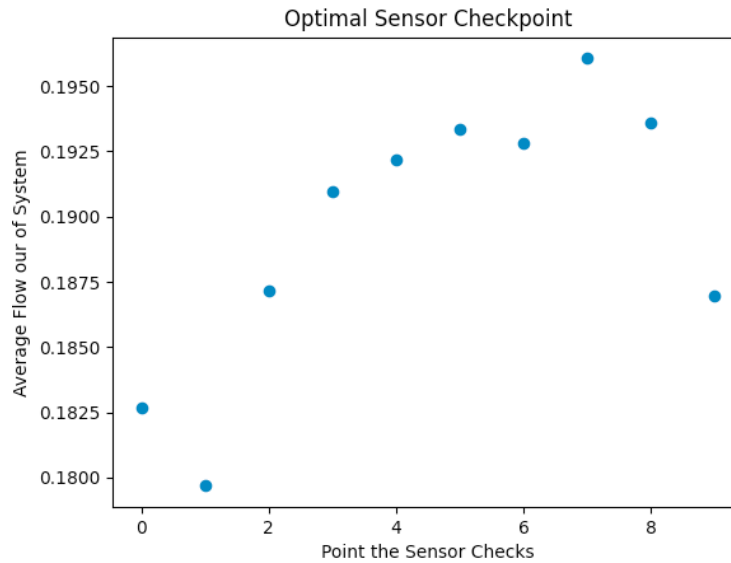


Figure 12

Figure 12 shows how average traffic flow responds to various placements of the sensor system. As clearly shown, the relationship is not perfectly linear but we do see a trend of stable increase outside of the endpoints on each side. Nevertheless, the data isolated segment 7 as the ideal placement for the sensor, in regards to effectiveness in increasing overall traffic flow in our system. A sensor in this position produces a Γ_{out} of roughly .196, which is better than our base case. Interestingly enough, we posited beforehand that the optimal sensor checkpoint would be at the traffic light, however this is clearly not the case. In most cases a sensor increases the quality of traffic flow, but still it is surprising to see the limitations of a sensor system directly at the intersection - which is fairly common in quotidian life. These findings indicate an amelioration of traffic density is achievable with sensor technology but in more removed checkpoints in order to mitigate density prior to the actual junction.

Application: Dynamically Optimizing Rush Hour Traffic

In order to demonstrate the effectiveness of such an extension, we wanted to employ our optimal sensor system to mitigate the overwhelming traffic density of something like rush hour traffic. This is similar to leaving and entering a city such as Los Angeles. We held all parameters constant from our base setting, but increased the overall time interval T to 2000 and modified the γ_{in} at the boundaries to match this scenario as described below:

$$\gamma_{in}(p(i,0,t),i,t) = \begin{array}{ll} \text{a random value less than .4} & \text{for } t < 400 \text{ if } i=12 \\ \text{a random value less than .1} & \text{for } t < 400 \text{ if } i \neq 12 \\ \text{a random value less than .1} & \text{for } 400 \leq t < 1600 \text{ for all } i \\ \text{a random value less than .1} & \text{for } 1600 \leq t \leq T \text{ if } i \neq 14 \\ \text{a random value less than .4} & \text{for } 1600 \leq t \leq T \text{ if } i=14 \end{array}$$

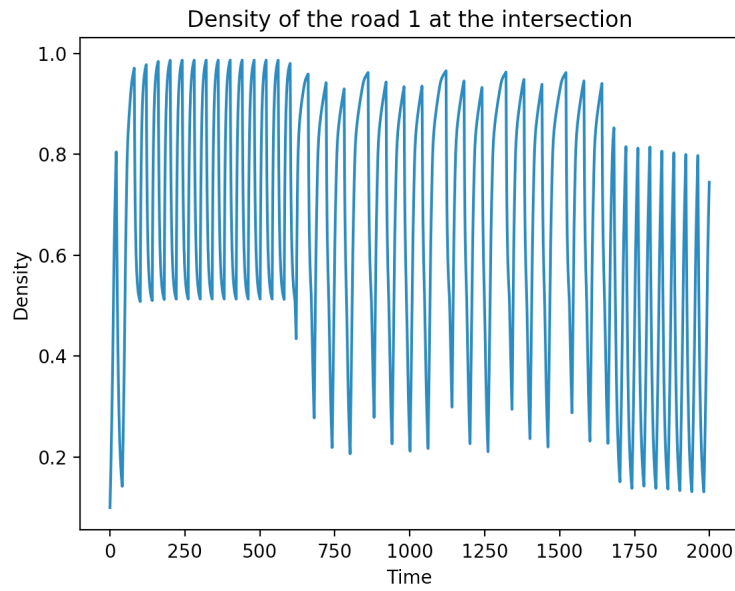


Figure 13

Figure 11 shows the variation of traffic density for road 1 throughout the simulation. As previously mentioned, this road receives more traffic in the morning than evening, as the outer boundary road 12 leading into it has a greater γ_{in} .

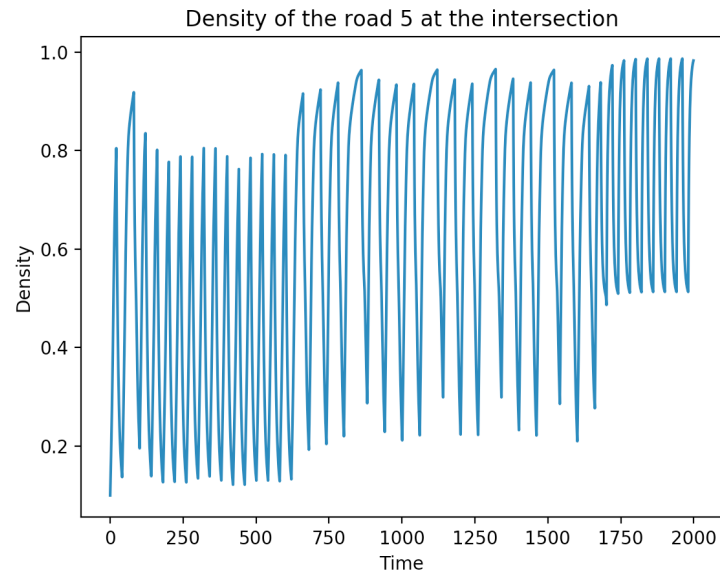


Figure 14

Figure 14 depicts the density of road 5, the afternoon road, throughout the simulation. As expected, we see more intense traffic at the end of the simulation. Nevertheless, the lower bound is almost the optimal density indicating no light is wasted on a suboptimal density on road 5 near the tail end of the model.

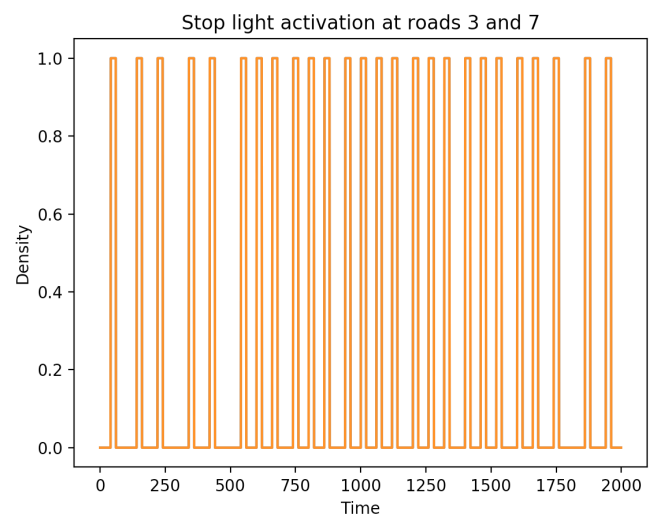
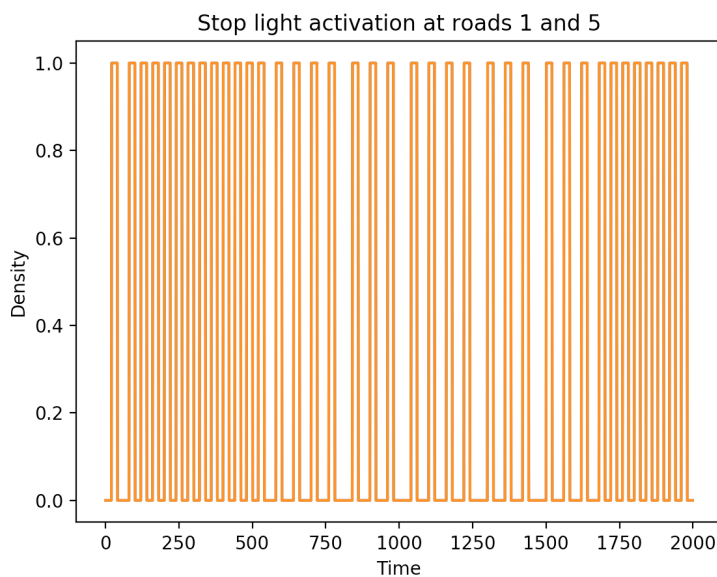


Figure 15

Finally, Figure 15 shows the true power of the sensor. In the morning and afternoons, the roads experiencing rush hour traffic have their stop lights activated far more frequently. We believe technology like this could be incredibly useful to optimize the flow of traffic.

Conclusion

We believe we have created a mathematical model that represents a real-world scenario of a traffic light quite powerfully. There are improvements to the model to be made, such as incorporating acceleration to our velocity function. However, the model preserves conservation of flow in an interesting and compelling way.

While we assumed many hypothetical parameters, these parameters could be constructed in a way to fit real world observations, improving the accuracy of our model and allowing the model to be applied to a real intersection. If this is done, we have provided a way to test solutions to improve the flow through that intersection. These results could then help shape traffic policy and technology initiatives to improve flow through grid-locked cities.