Maxwell Maia

21236277

Number Systems

Table of Contents

Positional notation

Positional notation refers to a system of writing numbers where the position of each digit has a place value.

The order of the digits in a number matter. The least significant digit is in the rightmost position and the most significant digit is in the leftmost position.

We number positions starting from 0. So, we count positions like this: 0, 1, 2, 3 … etc
The least significant digit (the rightmost) is position 0 and we count positions from right to left.

--------> decreasing significance

Example number: 640362 (decimal number)

Position: 5 4 3 2 1 0

2 is the rightmost digit. So, it is the least significant digit. This also means that it is position 0.

The difference between binary, octal and hexadecimal

The main difference between these number systems is that they have a different base.

Binary is to the base 2.
Octal is to the base 8.
Hexadecimal is to the base 16.
Decimal, which is our normal numbering system, is to the base 10.

The base determines how many digits can be used to represent a number using a particular number system. Since 0 is always included, the highest digit will have a value of one less than the base. The systems will use these digits:

Binary – 0 1

Octal – 0 1 2 3 4 5 6 7

Hexadecimal – 0 1 2 3 4 5 6 7 8 9 A B C D E F

Note: The A B C D E and F letters in hexadecimal represent 10 11 12 13 14 and 15 respectively.

In the evaluation of this whole number 640362: the “2” will be multiplied by the base and the base will be raised to the position of the digit. Which in this case will be 0. This process will be repeated for each digit and the sum of all of the results will give you the decimal number.

Convert decimal to binary

1149

Step 1

Lay out bases of binary to the power of 0,1,2,3..etc Like this:

2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
2048	1024	512	256	128	64	32	16	8	4	2	1

Step 2

Divide the 1149 by the highest of these values that is not bigger than 1149. Divide so that you get a whole number and a remainder.

Step 3

Divide the remainder by the next value down the 2^{power} list. Remember: divide so that you get a whole number and a remainder. Repeat this step until after you divide by 2^0.

$\frac{1149}{1024} = 1$ remainder (125)

$\frac{125}{512} = 0$ remainder (125)

$\frac{125}{256} = 0$ r (125)

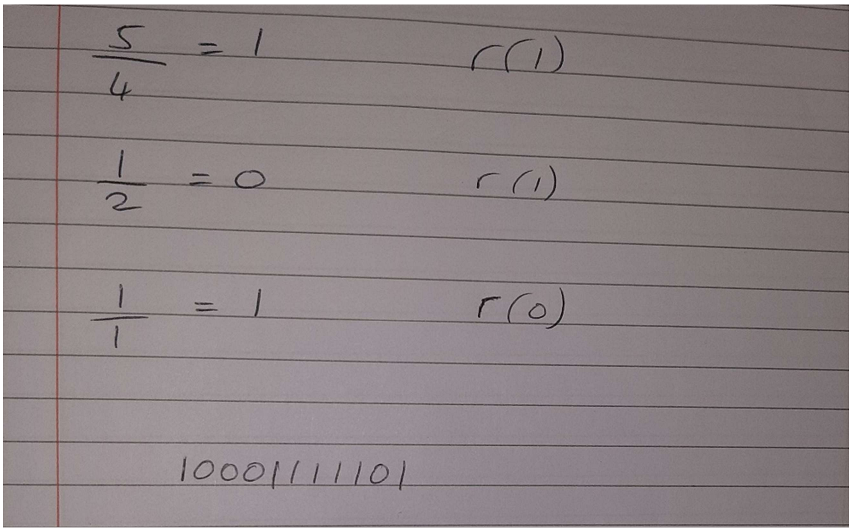
$\frac{125}{128} = 0$ r (125)

$\frac{125}{64} = 1$ r (61)

$\frac{61}{32} = 1$ r (29)

$\frac{29}{16} = 1$ r (13)

$\frac{13}{8} = 1$ r (5)



Step 4

The whole number answers are the digits to the binary version of the decimal number. Write out the digits starting from the answer of the division from the base$^{\text{highest power}}$. Write left to right.

So 1149 in binary is 10001111101.

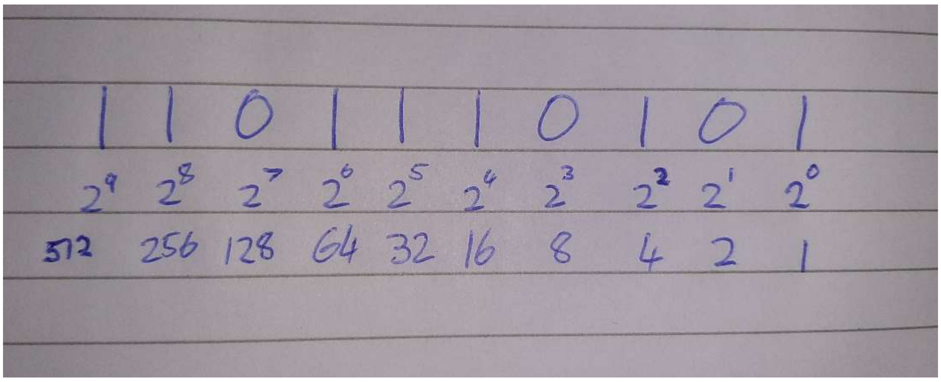
<u>Convert binary to decimal</u>

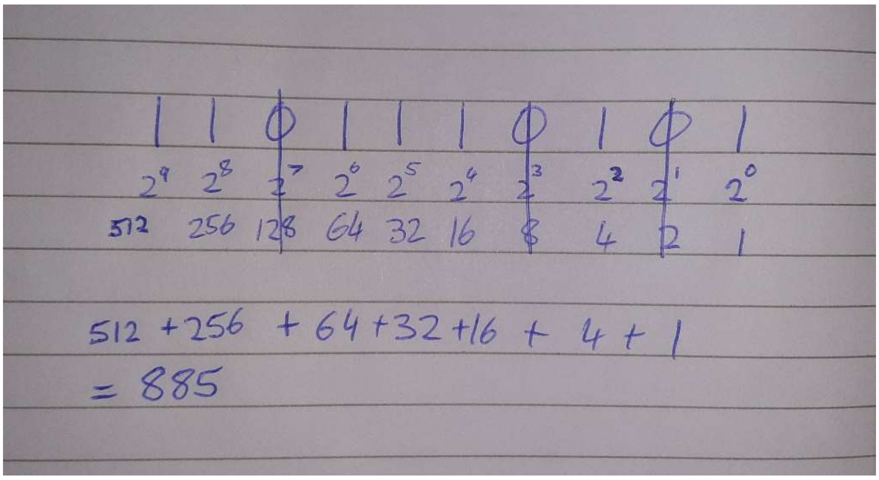
1101110101_2

-> Starting from position 0. (the rightmost position) assign each digit a value of 2^n. Where n is the position.

-> Add the values for each digit together EXCLUDING the values assigned to a 0.

-> The result is the decimal translation.





Convert decimal to octal

9856

Step 1

Lay out bases of octal to the power of 0,1,2,3..etc Like this:

8^5	8^4	8^3	8^2	8^1	8^0
32768	4096	512	64	8	1

Step 2

Divide the 9856 by the highest of these values that is not bigger than 9856. Divide so that you get a whole number and a remainder.

Step 3

Divide the remainder by the next value down the 8^{power} list. Remember: divide so that you get a whole number and a remainder. Repeat this step until after you divide by 8^0.

8^5	8^4	8^3	8^2	8^1	8^0
32768	4096	512	64	8	1

$\frac{9856}{4096} = \cancel{1}2$ r(1664)

$\frac{1664}{512} = 3$ r(128)

$\frac{128}{64} = 2$ r(0)

$\frac{0}{8} = 0$ r(0)

$\frac{0}{1} = 0$ r(0)

Step 4

The whole number answers are the digits of the octal version of the decimal number. Write out the digits starting from the answer of the division from the base$^{\text{highest power}}$. Write left to right.

So 9856 in octal is 23200.

Convert octal to decimal

7543_8

= 3939

Step 1

-> For each digit calculate a value according to the following formula. The “base” being 8 because we are converting from an octal number.

Digit * base^{position}

Step 2

-> Add all of the values together. The sum is the decimal conversion.

7543_8 digit x base^{position}

Step 1

position	digit.	formula		result
0.	3	3×8^0	=	3
1.	4	4×8^1	=	32
2.	5	5×8^2	=	320
3.	7	7×8^3	=	3584

Step 2

3 + 32 + 320 + 3584
= 3939

<u>Convert decimal to hexadecimal</u>

9999

Step 1

Lay out bases of hexadecimal to the power of 0,1,2,3..etc Like this:

16^4	16^3	16^2	16^1	16^0
65536	4096	256	16	1

Step 2

Divide the 9999 by the highest of these values that is not bigger than 9999. Divide so that you get a whole number and a remainder.

Step 3

Divide the remainder by the next value down the 16^{power} list. Remember: divide so that you get a whole number and a remainder. Repeat this step until after you divide by 16^0.

16^4	16^3	16^2	16^1	16^0
65536	4096	256	16	1

$\frac{9999}{4096} = 2 \quad r(1807)$

$\frac{1807}{256} = 7 \quad r(15)$

$\frac{15}{16} = 0 \quad r(15)$

$\frac{15}{1} = 15\ (F) \quad r(0)$

$\cancel{2711_{16}} \quad 270F_{16}$

Step 4

The whole number answers are the digits of the hexadecimal version of the decimal number. Write out the digits starting from the answer of the division from the base$^{\text{highest power}}$. Write left to right.

So 9999 in hexadecimal is 270F.

Convert hexadecimal to decimal

$F6AD_{16}$ = 63149

Step 1

-> For each digit calculate a value according to the following formula. The “base” being 16 because we are converting from a hexadecimal number.

A = 10

B = 11

C = 12

D = 13

E = 14

F = 15

Digit * base^{position}

Step 2

-> Add all of the values together. The sum is the decimal conversion.

Step 1

position	digit	formula		result
0.	D (13)	13×16^0	=	13
1.	A (10)	10×16^1	=	160
2.	6	6×16^2	=	1536
3.	~~D(13)~~ F (15)	15×16^3	=	61440

Step 2

13 + 160 + 1536 + 61 440

= 63149

Convert binary to octal

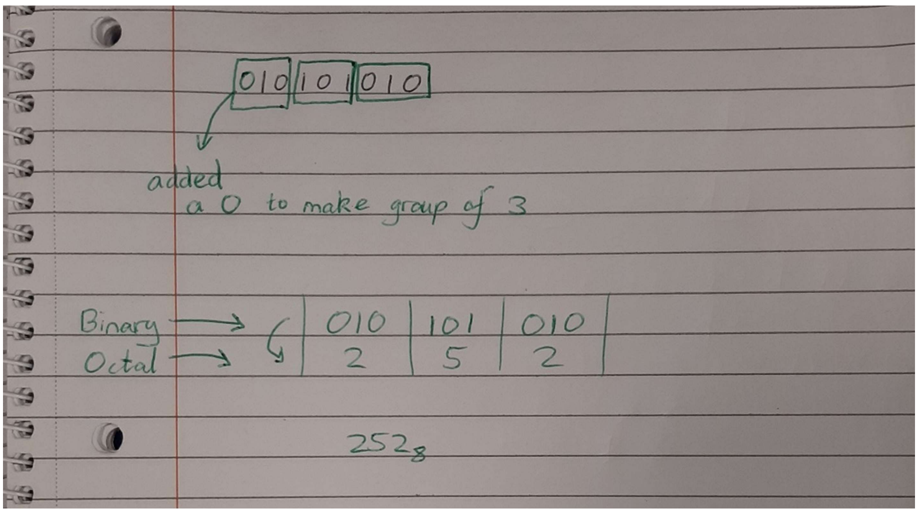
10101010_2

Step 1

Group the digits in groups of 3 starting from the right. Keep grouping until all the digits have a group. If there aren't enough digits to make a group of 3 add 0's to the left of the digits.

Step 2

Translate each group of 3 binary numbers into decimal. These are the octal digits.



Convert octal to binary

72242164_8

In an octal number, each digit has 8 possibilities. 2^3 is 8. 2^3 is the same as 3 bits. So a 3 digit binary number can represent one octal digit.

Write each digit as binary. Put all the binary digits together.

72242164_8

2^3 2^1 2^0

4 2 1

7	2	2	4	2	1	6	4
111	010	010	100	010	001	110	100

72242164_8 is:

111010010100010001110100 in binary.

Addition of 2 binary numbers

The table below shows a summary of the possible operations that will take place when adding 2 binary numbers.

Cases:		Sum	Carry
1.	0 + 0	0	0
2.	0 + 1	1	0
3.	1 + 0	1	0
4.	1 + 1	0	1

Special case: you have a carry from a previous operation and the case is 1 + 1.
Sum = 1 and Carry = 1

Step 1

Starting from the rightmost digit of both numbers (position 0), compare the numbers and apply the operations from the above table.

Step 2

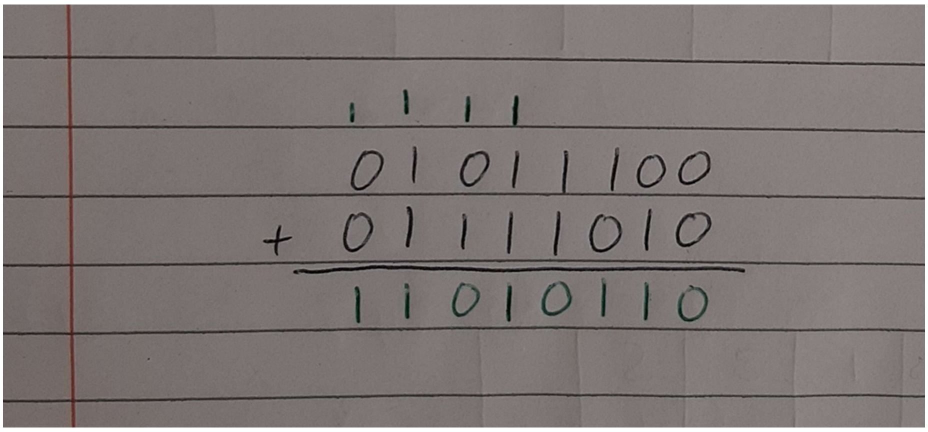
Record the "sum".

Step 3

If there is a carry, write a "1" above the digits one position to the left.

Step 4

Repeat these steps for the next position to the left.



Subtraction of 2 binary numbers

The table below shows a summary of the possible operations that will take place when subtracting 2 binary numbers.

Cases:	Subtract	Borrow
1. 0 - 0	0	0
2. 1 - 0	1	0
3. 1 - 1	0	0
4. 0 - 1	0	1

In case 4 we need to borrow one power of the base (2) from the next digit to the left.

Step 1

Starting from the rightmost digit of both numbers (position 0), compare the numbers and apply the operations from the above table.

Step 2

Record the “subtract”.

Step 3

If a borrow is necessary, reduce the digit to the left by 1 and place a 2 above the digit you want to borrow to. You can keep track of changes to numbers when borrowing by crossing digits out and writing their new value above them.

Step 4

Repeat these steps for the next position to the left.