Discrete Differential Geometry

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Introduction

This is an interactive blueprint to help with the formalisation of several definitions and results from *discrete differential geometry*, using Keenan Crane's textbook as a general template.

The actual Lean code can be found at (`https://github.com/maxwell-thum/DDG_Lean3`).

This blueprint is adapted from the blueprint of Thomas F. Bloom and Bhavik Mehta's Unit Fractions project (`https://github.com/b-mehta/unit-fractions`), which was itself based on the blueprint created by Patrick Massot for the Sphere Eversion project (`https://github.com/leanprover-community/sphere-eversion`).

This blueprint uses Patrick Massot's leanblueprint plugin (`https://github.com/PatrickMassot/leanblueprint`) for plasTeX (`http://plastex.github.io/plastex/`).

Chapter 1

Combinatorial Surfaces

Definition 1.1. An *abstract simplicial complex* is a pair $(V, \mathcal{K})$, where V is a set, $\mathcal{K} \subseteq \mathcal{P}(V)$ is a set of finite, non-empty subsets of V, and for all $\sigma \in \mathcal{K}$, if $\sigma' \subseteq \sigma$ and $\sigma' \neq \varnothing$, then $\sigma' \in \mathcal{K}$. V is called the set of *vertices* and elements of $\mathcal{K}$ are called *simplices*.

Definition 1.2. Let $(V, \mathcal{K})$ be a finite simplicial complex. For $k \in \mathbb{N}$, an *abstract k-simplex* is a simplex $\sigma \in \mathcal{K}$ consisting of exactly $k+1$ vertices.

Definition 1.3. The *degree* of an abstract simplex of size k is $k-1$. For instance, 0-simplices are vertices, 1-simplices are edges, 2-simplices are faces, and 3-simplices are tetrahedra. The empty set is the only -1-simplex.