Discrete Differential Geometry

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January 2, 2023

Introduction

This is an interactive blueprint to help with the formalisation of several definitions and results from *discrete differential geometry*, using Keenan Crane's textbook as a general template.

The actual Lean code can be found at (`https://github.com/maxwell-thum/DDG_Lean3`).

This blueprint is adapted from the blueprint of Thomas F. Bloom and Bhavik Mehta's Unit Fractions project (`https://github.com/b-mehta/unit-fractions`), which was itself based on the blueprint created by Patrick Massot for the Sphere Eversion project (`https://github.com/leanprover-community/sphere-eversion`).

This blueprint uses Patrick Massot's leanblueprint plugin (`https://github.com/PatrickMassot/leanblueprint`) for plasTeX (`http://plastex.github.io/plastex/`).

Chapter 1

Combinatorial Surfaces

Definition 1.1. *An* (finite) abstract simplicial complex *is a pair* $(V, \mathcal{K})$, *where* V *is a finite set,* $\mathcal{K} \subseteq \mathcal{P}(V)$ *is a set of subsets of* V, *every* $\sigma \in \mathcal{K}$ *is finite, and for all* $\sigma \in \mathcal{K}$, $\sigma' \subseteq \sigma$ *implies* $\sigma' \in \mathcal{K}$. V *is called the set of* vertices *and elements of* $\mathcal{K}$ *are called* simplices.

Definition 1.2. *Let* $(V, \mathcal{K})$ *be a finite simplicial complex. For* $k \in \mathbb{N}$, *an* abstract k-simplex *is a simplex* $\sigma \in \mathcal{K}$ *consisting of exactly* $k + 1$ *vertices.*