Master's Thesis Proposal

Aplicação de um Modelo Híbrido de Seleção de Características no Prognóstico do Câncer de Mama (Provisório)

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Maceió
3 de Março, 2021

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A thesis submitted by Alfredo Lima Moura Silva in partial fulfillment of the requirements for the degree of Master of Science in Informatics at the Federal University of Alagoas, Computing Institute.

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Abstract

The MINIMUM BROADCAST TIME (MBT) texto [...] ex abstract

Keywords: Aprendizagem de Máquina, Cardiologia, Diagnóstico, Isquemia, Arritmia.

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Introduction

No cenário de Machine Learning (ML), dados podem ser definidos como um fato, texto, imagem ou som que não foi processado. São considerados partes essenciais no contexto de ML, pois sem estes não é possível treinar o modelo, e consequentemente inferir alguma informação do objeto de estudo. Um modelo de aprendizagem de máquina é caracterizado como um algoritmo que tem a capacidade de reconhecer padrões sobre um conjunto de dados aplicados a este.

Com os sucessivos avanços que houveram na Inteligência Artificial (IA) e ML, estas tecnologias passaram a ser consideradas fundamentais no papel do cuidado à saúde da sociedade. Sendo amplamente utilizadas nos diversos ramos da medicina. Nesse universo, os dados do paciente utilizados em modelos de ML, podem ser divididos em dois tipos: (i) dados clínicos e (ii) dados moleculares. Dados clínicos são aqueles coletados a partir de diagnóstico, testes laboratoriais e dados hereditários do paciente. Já os dados moleculares, também chamados de microarranjos ou dados genômicos, podem ser definidos como um conjunto de dados que contêm informações das células do indivíduo, por exemplo, a sequência de RNA.

Uma das principais aplicações das tecnologias acima citadas, se dá no contexto de prognóstico e predição da neoplasia câncer. O câncer é considerado como a segunda maior causa de mortes decorrentes de doenças no mundo, de acordo com Organização Mundial de Saúde (OMS) [1], segundo senso realizado no ano de 2018. Neste contexto, destaca-se o câncer de mama. Este, por sua vez, é considerado a maior ocorrência de câncer entre as mulheres em todo o mundo [2].

Um dos principais desafios da medicina neste cenário, é utilizar as tecnologias disponíveis para conseguir fazer o prognóstico ou predição do câncer de mama com a melhor assertividade possível. Sendo assim, já existem diversos modelos apresentados na literatura com o objetivo de ajudar neste tipo de predição. Alguns destes utilizam dados clínicos dos pacientes, outros utilizam dados moleculares dos mesmos.

Quando trata-se da utilização de dados moleculares dos pacientes, a utilização de uma ferramenta capaz de reduzir a dimensionalidade dos dados passou a ser fundamental devido ao tamanho das características contidas nestes tipos de dados. Como mencionado anteriormente, os avanços tecnológicos possibilitaram obter informações cada vez mais detalhadas acerca das células dos indivíduos, de tal forma que a quantidade de informações disponíveis chegam à casa das milhares. Portanto, é necessário utilizar algum mecanismo apto a reduzir a quantidade de informações, pois nem todas elas são relevantes para a predição da doença em questão, ou até mesmo algumas delas podem ser duplicadas. Neste cenário, surge o conceito de modelos de seleção de características, que pode ser denominada como um processamento dos dados analisados, a fim de reduzir a quantidade de características que serão utilizadas em um modelo de aprendizado de máquina, com o principal objetivo de encontrar quais são as características mais significativas dentre todas as disponíveis na análise [3].

Segundo levantamento feito na literatura acerca de quais são os métodos de seleção de características mais utilizados neste tipo de prognóstico, destacam-se os métodos de seleção de característica: (i) ReliefF; (ii) Information Gain; e (iii) Gain Ratio [Adicionar referência da nossa revisão].

Portanto, este trabalho apresenta um método híbrido de seleção de características, utilizando dados clínicos e moleculares dos pacientes com susceptibilidade a desenvolverem o câncer de mama. Analisando o desempenho dos dados resultantes da seleção sobre os modelos de aprendizagem de máquina aplicados no contexto de predição da neoplasia em análise.

1.1 Trabalhos Relacionados

A utilização de técnicas de seleção de características na detecção de propriedades determinantes no prognóstico do câncer vêm sendo estudadas ao longo dos anos, muitos trabalhos sobre este tema foram publicados. Neste contexto, foi realizada uma revisão sistemática de literatura, com o objetivo de identificar trabalhos que tratam da utilização de técnicas de seleção de características no contexto do prognóstico da doença de câncer, durante a execução do protocolo definido na revisão, foi possível identificar que não existem muitos trabalhos que tratam deste tema, no final da revisão foram selecionados 21 trabalhos para ser feita uma análise estatística sobre eles a fim de identificar aspectos relevantes sobre o tema proposto.

Um revisão sistemática é uma revisão da literatura realizada a partir de uma pergunta de pesquisa definida, por meio da qual se busca identificar, avaliar, selecionar e sintetizar evidências de estudos empíricos que atendam a critérios de elegibilidade predefinidos.

Através da revisão foi possível identificar que o principal tipo de dados utilizados no prognóstico de câncer são dados moleculares da doença. Um dado molecular pode ser definido como um dado obtido a partir da utilização de técnicas de biotecnologia a fim de extrair informações mais detalhadas acerca da doença analisada, por exemplo, uma sequência de DNA ou um conjunto de microarranjos das células. Também foi identificado que as técnicas mais utilizadas são: ReliefF, Information Gain, Gain Ratio, Random Forest e T-Test.

1.2 Objectives

Our work is centralized in solving Minimum Broadcast Time Problem and variants using Biased Random-Key Genetic Algorithm (BRKGA). We preferably will work with the large and sparse networks, since they are the hardest instances to find the optimal solution (Hasson and Sipper, 2004). Our proposal can be summarized as follows:

- We have done:
  - (i) An algorithm to calculate a lower bound of the classical MBT;
  - (ii) 5 BRKGA metaheuristics for the MBT;
  - (iii) 4 BRKGA metaheuristics coupled with a refinement method;
  - (iv) A hybrid algorithm (BRKGA + ILP);
  - (v) Method to create instances with known-optima to the classical MBT.
- We are currently working on:
  - (i) Adapting the these algorithms for MBT variants;
  - (ii) Testing the hybrid algorithm;
- We will do:
  - (i) Exact algorithms for the MBT variants;
  - (ii) Create benchmark instances for the MBT variants.

Our contribution to the MBT and variants include:

- Lower bound algorithms;
- Metaheuristics and matheuristics algorithms;
- New benchmarks instances.

1.3 Structure of the Thesis

This work is structured as follows: Chapter 2 presents a lower bound algorithm for MBT, a greedy algorithm for the forest, an explanation about BRKGA and some decoding way, and describes a method to build synthetic instances with known-optima. In Chapter 3, we present the results of the comparison between a state-of-art algorithm and our first algorithm. Also, we compare our various methods. Some results of our proposal have already been published in the *Simpósio Brasileiro de Pesquisa Operacional* (SBPO) 2020 (Lima et al., 2020), and another is being evaluated (under review). The final considerations and the next activities on the working schedule are presented in Chapter 4.

2

Proposed Algorithms

This chapter presents our contributions for the MBT problem. First, Section 2.1 presents the simple lower bound for the MBT. Second, we show a polynomial algorithm to find MBT in a forest. In Section 2.3, we describe the Biased Random-Key Genetic Algorithms and some decoders that can be employed to solve the MBT problem. Finally, we present an approach to build new instances with a know optimal solution.

2.1 New Lower Bound

This section presents a simple and effective lower bound for the classical MBT. As the MBT uses a discrete-time model, a good estimate about the planning horizon length is very important for the performance of a mathematical formulation. A tighter lower bound can reduce the number of columns in the coefficient matrix and, thus, reduce the computational demand and the total amount of used memory. Moreover, a lower bound can be used by a heuristic algorithm to prove the optimality of a feasible solution, i.e., if a heuristic finds a solution whose value meets the lower bound, then this solution is proved optimal.

Clearly, $|V| - |V_0|$ is an upper bound for the optimal broadcast time. Furthermore, the value

$$TLB = \left\lceil \log_2 \frac{|V|}{|V_0|} \right\rceil \tag{2.1}$$

defines a theoretical lower bound for the MBT (Ivanova, 2019). It is easy to see that a complete graph needs exactly $\left\lceil \log_2 \frac{|V|}{|V_0|} \right\rceil$ steps to broadcast.

Algorithm 1 presents the pseudo-code for the proposed lower bound calculation. The proposed lower bound, called LBB-BFS, is based on a multisource breadth-first search (BFS), starting at every vertex of V_0. Its main objective is to find the maximum shortest path between a target vertex and its nearest source. Note a straightforward implementation of LBB-BFS, i.e., applying a breadth-first search on each $v \in V_0$, would require O($|V_0| \cdot |E|$). This issue can be addressed by labeling every vertex in V_0 as discovered, and placing them at the beginning of the known vertices queue Q_0 (see lines 5–7). Overall, the worst-case running time of the LBB-BFS function is $O(|E|)$. A similar approach based on BFS has also been used by de Sousa et al. (2018) to reduce the number of variables in their proposed ILP formulation. Note also that LBB-BFS can be applied in the k-broadcast variants.

Algorithm 1: Lower Bound to Broadcast

Input : Connected undirected graph: $G = (V, E)$,
Source set: V_0
Output: Lower bound to broadcast: $lowerBound$

1 LBB-BFS (G, V_0)
2   **for** $v \in V$ **do**
3         $dist[v] \leftarrow \infty$
4     $Q \leftarrow \emptyset$ // let Q be an empty queue
5   **for** $v \in V_0$ **do**
6         $dist[v] \leftarrow 0$
7         $Q.enqueue(v)$
8   **while** $Q \neq \emptyset$ **do**
9         $v \leftarrow Q.dequeue()$
10   **for** *each* $u \in G.adj[v]$ **do**
11   **if** $dist[u] > dist[v] + 1$ **then**
12                 $dist[u] \leftarrow dist[v] + 1$
13                 $Q.enqueue(u)$
14     $lowerBound \leftarrow \max_{v \in V}\{dist[v]\}$
15   **return** $lowerBound$

Theorem 1. *Algorithm 1 return a lower bound for the optimum of the MBT with input* $G = (V, E)$ *and* V_0.

Proof. Let b^* be the optimum of MBT for an instance (G, V_0) and z be the value returned by the LBB-BFS algorithm. Let $v \in V_0$ be the closest vertex of V_0 to a vertex $\ell \in V$, such that the distance between them is exactly z. We need at least z steps to reach ℓ from any vertex in V_0. Therefore, $z \leq b^*$, i.e., z is a lower bound for the optimum MBT value. □

Finally, given an instance (G, V_0), we compute the lower bound as follows:

$$LB(G, V_0) = \max(TLB(G, V_0), LBB\text{-}BFS(G, V_0)), \tag{2.2}$$

where $TLB(G, V_0)$ is defined in Eq. (2.1) and $LBB\text{-}BFS(G, V_0)$ is calculated by Algorithm 1.

2.2 A Greedy Algorithm to Forest

Slater et al. (1981) showed an algorithm to find the broadcast centers in a tree (see Definition **??**). Based on this approach, Su et al. (2010) and Koh and Tcha (1991) develop a polynomial algorithm to find the MBT in a tree. We adapted the algorithm of Slater et al. (1981) to find the MBT on a forest in polynomial time (optima). It uses the depth-first search (DFS) for building the costs of each transmission. The Algorithm 2 finds the minimum broadcast time in each tree, and the answer is the greater value of each tree. We label the method as SCHA, because of the names of the authors of work (Slater et al., 1981). Overall, the worst-case running time of the SCHA is $O(|V|)$.

Algorithm 2: Algorithm for Minimum Broadcast Time to Forest (Slater et al., 1981)

Input : Forest undirected: $F = (V, E)$,
Source set: V_0
Output: Total step time to broadcast: $time$

MBT_Tree (T, v, π)
  $time \leftarrow 0$
  $L_c \leftarrow \emptyset$
  for each $u \in N(v) \setminus \pi$ **do**
    $L_c \leftarrow L_c \cup \{MBT\_Tree(T, u, v)\}$
  $Sort(L_c)$
  $t \leftarrow 1$
  for each $c \in L_c$ **do**
    $time \leftarrow max(time, c + t)$
    $t \leftarrow t + 1$
  return $time$
SCHA (F, V_0)
  $time \leftarrow 0$
  for each $v \in V_0$ **do**
    $time \leftarrow max(time, MBT\_Tree(F, v, \emptyset))$
  return $time$

Note that in Fig. 2.1 is illustrated the execution of SCHA (Algorithm 2). When the vertex has an arrow indicates that the DFS is processing its. Our algorithm starts at the root of the tree, queries the children its costs, and each child does the same. The vertices that are leaves have a cost is 0, and they reply to the parents the value. Each vertex builds a list of cost's children, L_c. Subsequently, the vertex ordering the list L_c, and compute the minimum broadcast time each of the branch. See that the vertices 4 and 3 are leaves, so they have cost 0. The vertex 2 has cost 1 $(0+1)$, and the vertex 0 has cost 2 – $\max(0+2, 1+1)$. That is, the MBT of this example is 2.

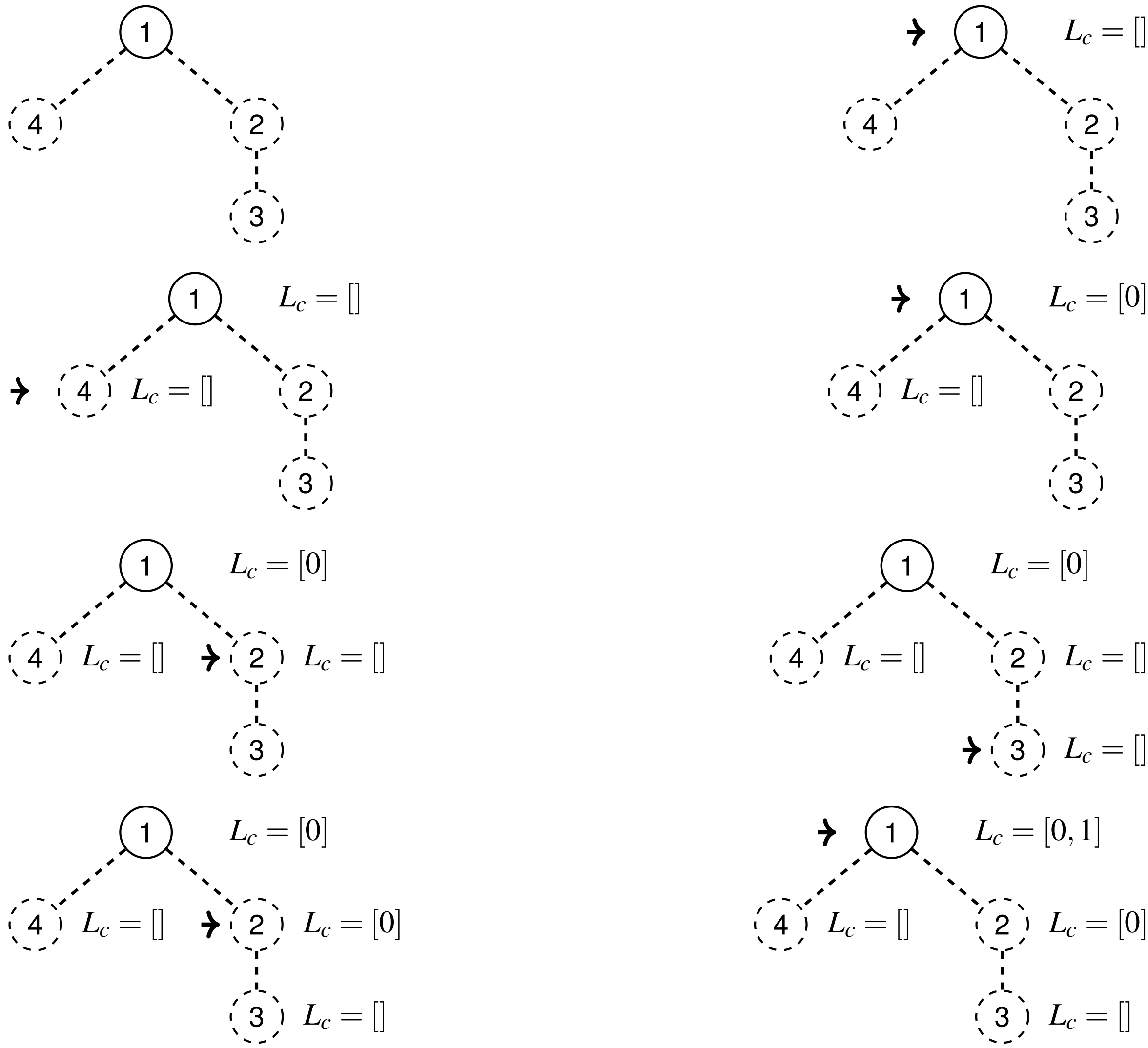


Figure 2.1: SCHA runs in a tree.

2.3 Biased Random-Key Genetic Algorithms

Genetic algorithms (GA) is a population-based metaheuristic that relies on the principles of natural selection. It can be viewed as an iterative improvement in a population of solutions. A GA usually has seven steps (Holland, 1975): (i) Initial population, (ii) Evaluation, (iii) Selection, (iv) Recombination, (v) Mutation, (vi) Replacement, and (vii) Repeat (ii)—(vi). First, the population is initialized and evaluated. Then, a new population of solutions is generated using some selection, followed by recombination and mutation procedures. Finally, this new population is integrated into the current one.

In a GA, each individual is referred as a chromosome, which encodes a candidate solution. A chromosome consists of a string of genes whose values are called alleles. An objective function associates a fitness value with each individual indicating its fitting to the problem. In the evolving process, individuals are selected to reproduce, and a crossover operator is used on the selected individuals to produce a new offspring that make up the next generation. In addition, mutation operators are involved in this process. Finally, a replacement step is applied to determine which individuals of the population (parents and offspring) will survive.

The proposed GA is not similar to classical Genetic Algorithms (Holland, 1975), where individuals are represented using a binary array and modified by some mutation operator. Our algorithm is based on more recent implementations using random-keys encodings. In random-key genetic algorithms (RKGA), developed by Bean (1994), the chromosomes consist of vectors of real numbers generated randomly in the range $[0,1)$. Besides, a deterministic algorithm called *decoder* processes the vector to calculate the fitness of the solution. RKGA does not make use of the standard mutation operator, where some alleles are changed with a given probability. Instead, new (mutant) solutions are randomly generated and introduced in the current population in each generation, in the same way as the initial population is created.

A biased random-key genetic algorithm (BRKGA) (Gonçalves and Resende, 2011) differs from an RKGA in the way parents are selected for crossover. As shown in Fig. 2.2, at each generation, the population is partitioned into two parts: elite and non-elite. The elite part contains the best solutions in the total population and is simply copied to the next population generation. The remaining elements are created by crossover or randomly generated (mutants). In the recombination phase, each new individual is generated by combining one individual selected from the elite set and another individual selected from the non-elite population. The elite parent has a higher probability of passing its genes to the offspring generation, i.e., a biased selection via the parameterized uniform crossover operator proposed by Spears and Jong (1991). This process is illustrated in Fig. 2.2, where $\rho_e = 0.7$ (70%) is the probability that the offspring will inherit each of its alleles from the best fit of the two parents. Gonçalves et al. (2014) made an empirical comparison of biased and unbiased random-keys genetic algorithms in four types of covering problems and has shown that the biased variant is faster than the original Bean's algorithm.

In the following subsections, we present some decoder procedures for the MBT.

2.3.1 Based on Priority (BP)

The Based on Priority (BP) decoder describe a solution by a set of $|V|$ random keys (allele). Each random key represent the priority of a vertex receive and transmit the message, that is, the vertex with the lowest allele value in the transmitters list has the highest sending priority, and so on. Once defined the transmitter, the same principle is applied to define which vertex will receive the message (receive priority). Overall, the worst-case running time of the decoder function is $O(|V| \cdot |E|)$.

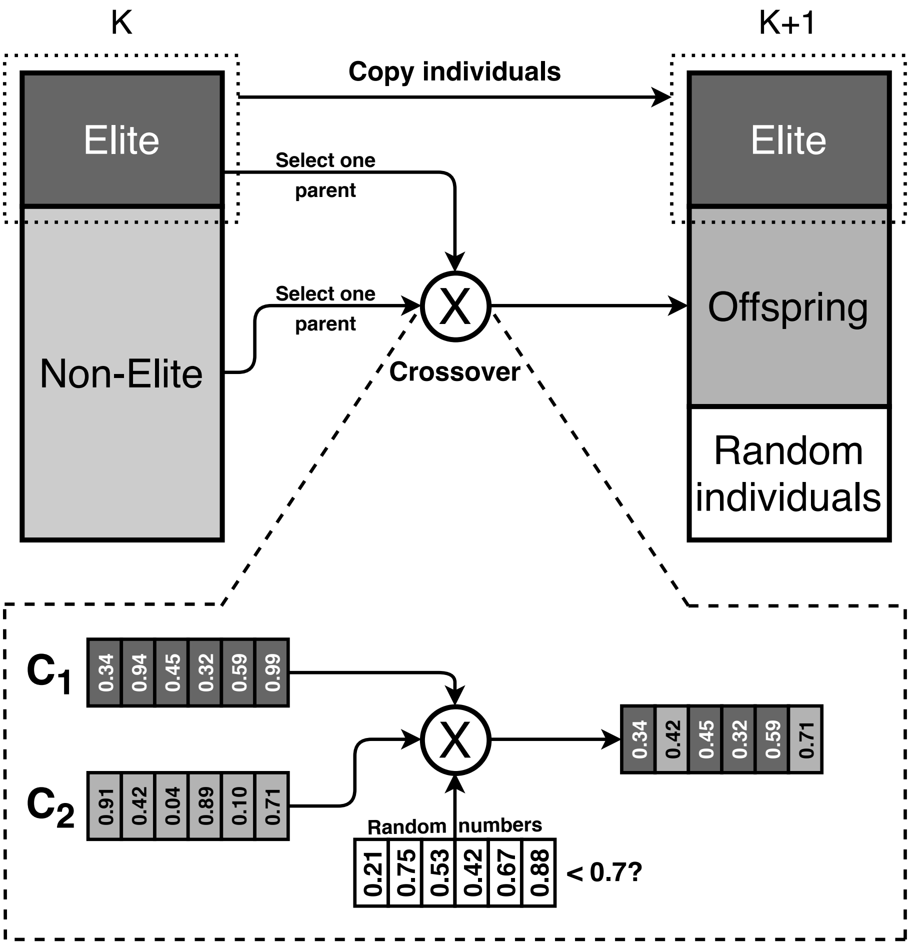


Figure 2.2: BRKGA

Figure 2.3: Input graph

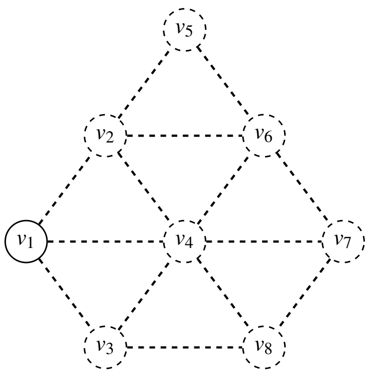


Table 2.1: Chromosomes of input

Vertices	**Value(*Cr*)**
v_1	0.1
v_2	0.5
v_3	0.6
v_4	0.4
v_5	0.7
v_6	0.8
v_7	0.9
v_8	0.3

Figure 2.3 presents an instance example with 8 vertices, whose source vertex set is composed of a single gateway v_1. Table 2.1 shows the values of a random-key vector example. The complete execution of Algorithm 3 is depicted in Figure 2.4. Initially, vertex v_1 is designated as the unique source (see Fig. 2.4-a). Among the neighbors of v_1, vertex v_4 has the highest priority value (according to his chromosome value), hence vertex v_1 sends the message to v_4 and we

Algorithm 3: Based on Priority Decoder

Input : Connected undirected graph: $G = (V, E)$,
Source set: V_0,
Chromosomes vector: Cr

Output: Total step time to broadcast: $time$

`BestCandidate` (G, T, v, Cr)
  if $(N(v) \setminus T) \neq \emptyset$ **then**
    return $min_{u \in N(v) \setminus T} \{(Cr[u], u)\}$
  return $\emptyset$

`BP` (G, V_0, Cr)
  $time \leftarrow 0$
  $Ranking \leftarrow \emptyset$
  $Transmitters \leftarrow V_0$
  for *each* $v \in V_0$ **do**
    $Ranking \leftarrow Ranking \cup \{(Cr[v], v)\}$
  while $Transmitters \neq V$ **do**
    $V_{time} \leftarrow \emptyset$
    $Sort(Ranking)$ // Ordering in ascending order of allele value
    for *each* $(r, v) \in Ranking$ **do**
      $(c, u) \leftarrow BestCandidate(G, Transmitters \cup V_{time}, v, Cr)$
      if $(c, u) \neq \emptyset$ **then**
        $V_{time} \leftarrow V_{time} \cup \{u\}$
    for *each* $v \in V_{time}$ **do**
      $Transmitters \leftarrow Transmitters \cup \{v\}$
      $Ranking \leftarrow Ranking \cup \{(Cr[v], v)\}$
    $time \leftarrow time + 1$ // Increment in time
  return $time$

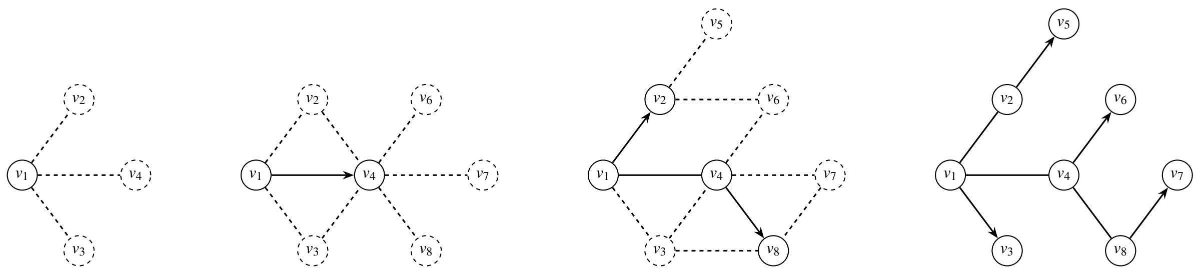
perform another iteration (Fig.2.4-b). In time 1, only vertices v_1 and v_4 have received the message. Vertex v_1, with the highest priority value, will transmit first to vertex v_2 (its second highest priority neighbor), and v_4 transmits to v_8 (see Fig. 2.4-c). Finally, in Fig. 2.4-d, the other vertices receive the messages in the following order: $v_1 \to v_3$; $v_8 \to v_7$; $v_4 \to v_6$ and $v_2 \to v_5$.

We remark some characteristics of vertices that should preferably receive the message in order to minimize transmission time:

vertices with a high degree, when the vertex with a high degree receives data in a short time, it can do more transmission over time.

distant vertices of the gateway, if a distant vertices receive the data more quickly, it is possible to minimize the transmission time.

Based on this characteristics, we introduce a new way to compute the preference of vertices. We label as Based on Priority With Heuristics (see Algorithm 4). This approach has the same



(a) $t = 0$ (b) $t = 1$ (c) $t = 2$ (d) $t = 3$

Figure 2.4: Time horizon.

decoding principle of Algorithm 3, although the priority each vertex is calculated with a combination of random key value, degree, and gateway distance. The parameters α and β (defined in $[0,1]$) indicate percent contribution of each characteristic — degree and gateway distance, respectively. Overall, the worst-case running time of the decoder function is $O(|V| \cdot |E|)$.

2.3.2 First Receive First Send (FRFS)

In this approach, the first vertex to receive the message is the first vertex to transmit the message in the next step. Differently from the BD decoder that employ two kinds of priority (send and receive), the FRFS decoder adopted only the receive priority. In each step, the transmission sequence is defined in a FIFO fashion. The First Receive First Send (FRFS) Decoder is described on Algorithm 5, its worst-case running time is $O(|V| \cdot |E|)$.

We implement the same heuristics of Algorithm 4 on Algorithm 5, which the result is the Algorithm 6. Overall, the worst-case running time of the decoder function is $O(|V| \cdot |E|)$.

2.3.3 Previous Decoders with refinement approach

The previous decoders (Algorithms 3, 4, 5, and 6) perform poorly when the graph is sparse or forest. Thus, we propose the use of a refinement method at the end of decoding a random key. We use the SCHA (see Algorithm 2), which solves the MBT in polynomial time when the instance is a tree. This approach is illustrated in Figure 2.5. Firstly, we apply the decoder in order to obtain a initial solution — represented by a spanning forest. Finally, we find the optimal transmission scheduling using only the edges of the forest. Overall, the worst-case running time of the decoder functions are $O(|V| \cdot |E|)$.

2.3.4 Based on Minimum Spanning Forest (MSF)

We development a decoder based on Minimum Spanning Forest (MSF). In this approach, each random key is associated with an edge of the graph and its value indicates the weight of this edge. Next, we execute an algorithm for MSF, e.g., Kruskal (Kruskal, 1956), and we compute

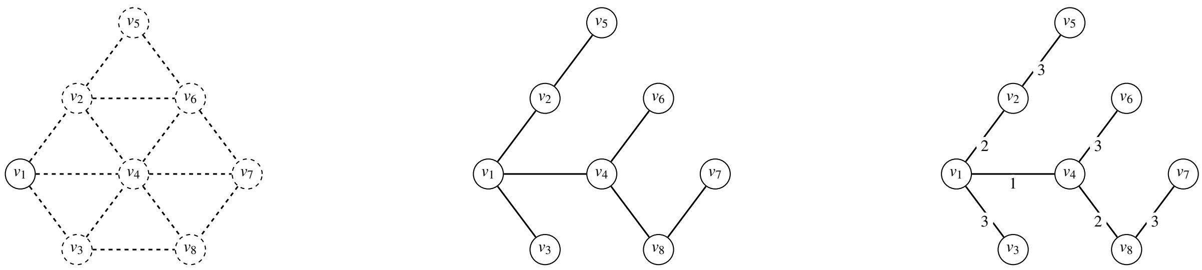
Algorithm 4: Based on Priority With Heuristics Decoder

Input : Connected undirected graph: $G = (V, E)$,
Source set: V_0,
Chromosomes vector: Cr,
Alpha: α,
Beta: β,
Distances vector: $Dist$,
Degree vector: $Degree$

Output: Total step time to broadcast: $time$

`BestCandidate` $(G, T, v, \alpha, \beta, Cr, Dist, Degree)$
  if $(N(v) \setminus T) \neq \emptyset$ **then**
    return $min_{u \in N(v) \setminus T}\{(Cr[u] + \frac{\alpha}{Dist[u]} + \frac{\beta}{Degree[u]}, u)\}$
  return $\emptyset$

`BPH` $(G, V_0, Cr, \alpha, \beta, Dist, Degree)$
  $time \leftarrow 0$
  $Ranking \leftarrow \emptyset$
  $Transmitters \leftarrow V_0$
  for *each* $v \in V_0$ **do**
    $Ranking \leftarrow Ranking \cup \{(Cr[v] + \frac{\beta}{Degree[v]}, v)\}$
  while $Transmitters \neq V$ **do**
    $V_{time} \leftarrow \emptyset$
    $Sort(Ranking)$ // Ordering in ascending order of allele value
    for *each* $(r, v) \in Ranking$ **do**
      $(c, u) \leftarrow BestCandidate(G, Transmitters \cup V_{time}, v, \alpha, \beta, Cr, Dist, Degree)$
      if $(c, u) \neq \emptyset$ **then**
        $V_{time} \leftarrow V_{time} \cup \{u\}$
    for *each* $v \in V_{time}$ **do**
      $Transmitters \leftarrow Transmitters \cup \{v\}$
      $Ranking \leftarrow Ranking \cup \{(Cr[v] + \frac{\alpha}{Dist[v]} + \frac{\beta}{Degree[v]}, v)\}$
    $time \leftarrow time + 1$ // Increment in time
  return $time$



(a) Graph (b) Spanning Forest (c) Schedule

Figure 2.5: Process of Improvement

the MBT over this forest using the SCHA. Unfortunately, the results of this approach were not

Algorithm 5: First Receive First Send Decoder

```
Input  : Connected undirected graph: G = (V, E),
         Source set: V_0,
         Chromosomes vector: Cr
Output: Total step time to broadcast: time

BestCandidate(G, T, v, Cr)
    if (N(v) \ T) ≠ ∅ then
        return min_{u ∈ N(v)\T} {(Cr[u], u)}
    return ∅

FRFS(G, V_0, Cr)
    time ← 0
    Ranking ← ∅
    Transmitters ← V_0
    for each v ∈ V_0 do
        Ranking ← Ranking ∪ {(Cr[v], v)}
    Sort(Ranking)            // Ordering in ascending order of allele value
    while Transmitters ≠ V do
        V_time ← ∅
        for each (r, v) ∈ Ranking do
            (c, u) ← BestCandidate(G, Transmitters ∪ V_time, v, Cr)
            if (c, u) ≠ ∅ then
                V_time ← V_time ∪ {u}
        for each v ∈ V_time do
            Transmitters ← Transmitters ∪ {v}
            Ranking ← Ranking ∪ {(Cr[v], v)}
        time ← time + 1                            // Increment in time
    return time
```

satisfactory. Overall, the worst-case running time of the decoder function is $O(|E| \cdot \log |E|)$.

Figure 2.6: Input graph

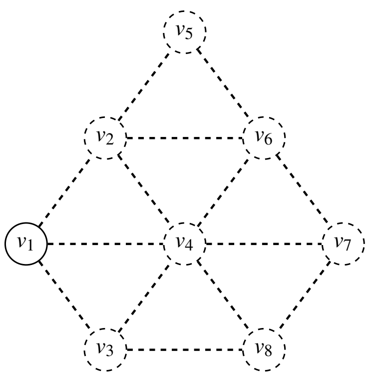


Table 2.2: Chromosomes of input

Edges	Value(*Cr*)	Edges	Value(*Cr*)
$v_1 - v_2$	0.15	$v_3 - v_8$	0.40
$v_1 - v_3$	0.20	$v_4 - v_6$	0.25
$v_1 - v_4$	0.30	$v_4 - v_7$	0.50
$v_2 - v_4$	0.55	$v_4 - v_8$	0.45
$v_2 - v_5$	0.60	$v_5 - v_6$	0.35
$v_2 - v_6$	0.65	$v_6 - v_7$	0.70
$v_3 - v_4$	0.75	$v_7 - v_8$	0.05

Algorithm 6: First Receive First Send With Heuristics Decoder

Input : Connected undirected graph: $G = (V, E)$,
Source set: V_0,
Chromosomes vector: Cr,
Alpha: α,
Beta: β,
Distances vector: $Dist$,
Degree vector: $Degree$

Output: Total step time to broadcast: $time$

```
BestCandidate(G, T, v, α, β, Cr, Dist, Degree)
    if (N(v) \ T) ≠ ∅ then
        return min_{u ∈ N(v)\T} {(Cr[u] + α/Dist[u] + β/Degree[u], u)}
    return ∅
FRFSH(G, V_0, Cr, α, β, Dist, Degree)
    time ← 0
    Ranking ← ∅
    Transmitters ← V_0
    for each v ∈ V_0 do
        Ranking ← Ranking ∪ {(Cr[v] + β/Degree[v], v)}
    Sort(Ranking)                 // Ordering in ascending order of allele value
    while Transmitters ≠ V do
        V_time ← ∅
        for each (r, v) ∈ Ranking do
            (c, u) ← BestCandidate(G, Transmitters ∪ V_time, v, α, β, Cr, Dist, Degree)
            if (c, u) ≠ ∅ then
                V_time ← V_time ∪ {u}
        for each v ∈ V_time do
            Transmitters ← Transmitters ∪ {v}
            Ranking ← Ranking ∪ {(Cr[v] + α/Dist[v] + β/Degree[v], v)}
        time ← time + 1                                  // Increment in time
    return time
```

2.4 Hybrid Algorithm

A hybrid algorithm is an algorithm that combines two or more algorithms that solve the same problem. Our idea is described in Algorithm 10, which combines the BRKGA and ILP. We use some best elements of the population of BRKGA, and we join each solution (broadcast forest) to generates a subgraph. We apply ILP in this subgraph with an initial solution based on the best chromosome of BRKGA. Figures 2.7–2.8 show an example, which is selected 3 solution of population P_K for build a subgraph, after it's apply the ILP on subgraph. Note that if the ILP obtains a better solution (see 22–24 lines), we insert this solution in the population of BRKGA. In the ILP_MBT function, we compute the lower bound of the subgraph, if the lower bound is equal to the BRKGA solution (see 2–4 lines), we did not run the ILP.

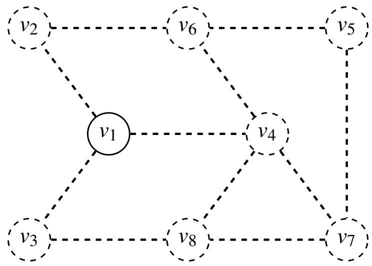


Figure 2.7: Input graph for Hybrid Algorithm

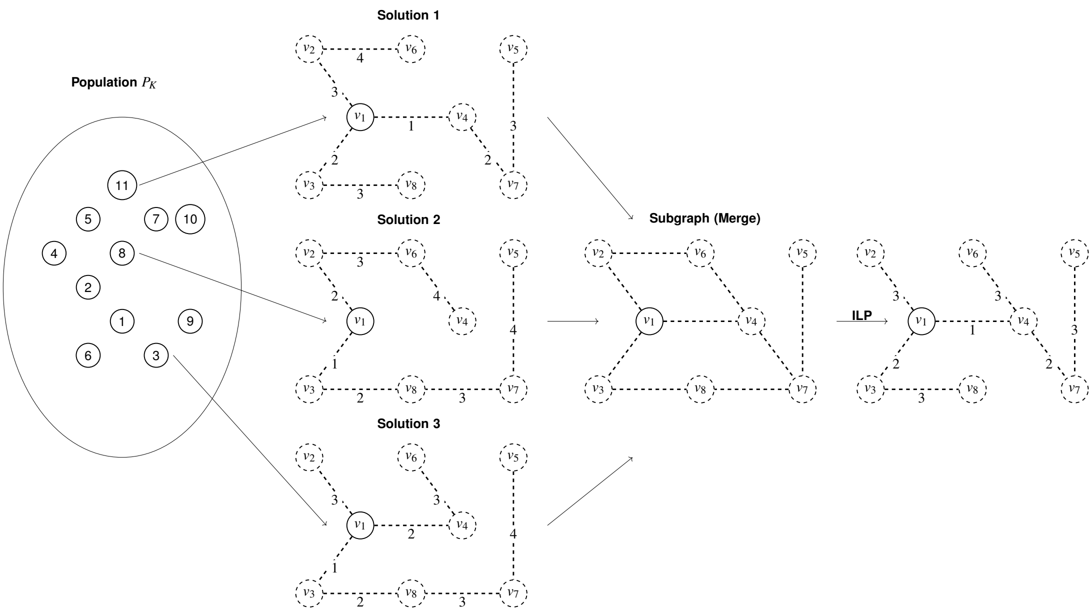


Figure 2.8: Hybrid process

Algorithm 7: Based on Priority With Heuristics and SCHA Decoder

Input : Connected undirected graph: $G = (V, E)$,
Source set: V_0,
Chromosomes vector: Cr,
Alpha: α,
Beta: β,
Distances vector: $Dist$,
Degree vector: $Degree$

Output: Total step time to broadcast: $time$

```
BestCandidate(G, T, v, α, β, Cr, Dist, Degree)
    if (N(v) \ T) ≠ ∅ then
        return min_{u∈N(v)\T} {(Cr[u] + α/Dist[u] + β/Degree[u], u)}
    return ∅

BPHI(G, V0, Cr, α, β, Dist, Degree)
    Gb ← ∅
    time ← 0
    Ranking ← ∅
    Transmitters ← V0
    for each v ∈ V0 do
        Ranking ← Ranking ∪ {(Cr[v], v)}
    while Transmitters ≠ V do
        Vtime ← ∅
        Sort(Ranking)            // Ordering in ascending order of allele value
        for each (r, v) ∈ Ranking do
            (c, u) ← BestCandidate(G, Transmitters ∪ Vtime, v, α, β, Cr, Dist, Degree)
            if (c, u) ≠ ∅ then
                Vtime ← Vtime ∪ {u}
                Gb ← Gb ∪ {(v, u)}
        for each v ∈ Vtime do
            Transmitters ← Transmitters ∪ {v}
            Ranking ← Ranking ∪ {(Cr[v], v)}
    return MBT_Forest(Gb, V0)
```

2.5 Building The Synthetic Instances

Given that the optimal solution for large MBT instances is often unknown, we generated a new benchmark with known optimal solutions using the following procedure. Let the instance $G = (V, E)$ be defined by the union of a binomial tree $B_k = (V_B, E_B)$ and a random graph $G_r = (V_r, E_r)$, where $V = V_B = V_r$ and $E = E_B \cup E_r$. The binomial tree B_k is an ordered tree defined recursively as follows: (i) B_0 is a trivial graph, (ii) B_k is constructed from two binomial trees B_{k-1} by attaching one of them as the rightmost (can be leftmost) child of the root of the other. Figure 2.9 shows the binomial trees B_0 through B_3. Note that the MBT of a binomial tree B_k is equal to k if the root of B_k is in V_0. The random graph G_r is based on the $\mathbb{G}(n, p)$ model, also known as binomial model

Algorithm 8: First Receive First Send With Heuristics and SCHA Decoder

Input : Connected undirected graph: $G = (V,E)$,
Source set: V_0,
Chromosomes vector: Cr,
Alpha: α,
Beta: β,
Distances vector: $Dist$,
Degree vector: $Degree$

Output: Total step time to broadcast: $time$

`BestCandidate` $(G,T,v,\alpha,\beta,Cr,Dist,Degree)$
    if $(N(v) \setminus T) \neq \emptyset$ **then**
        return $min_{u \in N(v) \setminus T}\{(Cr[u] + \frac{\alpha}{Dist[u]} + \frac{\beta}{Degree[u]}, u)\}$
    return $\emptyset$

`FRFSHI` $(G,V_0,Cr,\alpha,\beta,Dist,Degree)$
    $G_b \leftarrow \emptyset$
    $time \leftarrow 0$
    $Ranking \leftarrow \emptyset$
    $Transmitters \leftarrow V_0$
    for *each* $v \in V_0$ **do**
        $Ranking \leftarrow Ranking \cup \{(Cr[v], v)\}$
    $Sort(Ranking)$ `// Ordering in ascending order of allele value`
    while $Transmitters \neq V$ **do**
        $V_{time} \leftarrow \emptyset$
        for *each* $(r,v) \in Ranking$ **do**
            $(c,u) \leftarrow BestCandidate(G, Transmitters \cup V_{time}, v, \alpha, \beta, Cr, Dist, Degree)$
            if $(c,u) \neq \emptyset$ **then**
                $V_{time} \leftarrow V_{time} \cup \{u\}$
                $G_b \leftarrow G_b \cup \{(v,u)\}$
        for *each* $v \in V_{time}$ **do**
            $Transmitters \leftarrow Transmitters \cup \{v\}$
            $Ranking \leftarrow Ranking \cup \{(Cr[v], v)\}$
    return $MBT\_Forest(G_b, V_0)$

(Gilbert, 1959). Each graph $G_r = (V, E_r)$ is generated with n vertices and each potential edge in E_r is created with probability p. In Figure 2.10, we apply a union of graphs B_3 and G_r and we obtain a random graph G with 8 vertices.

This methodology can be applied to creates synthetic instances with multiple sources ($|V_0| > 1$). The idea is simple, we will build $|V_0|$ binomial trees and connected them with random graph. In Figure 2.11 we created a random instance with $V_0 = \{1, 5\}$.

Algorithm 9: MST and SCHA Decoder

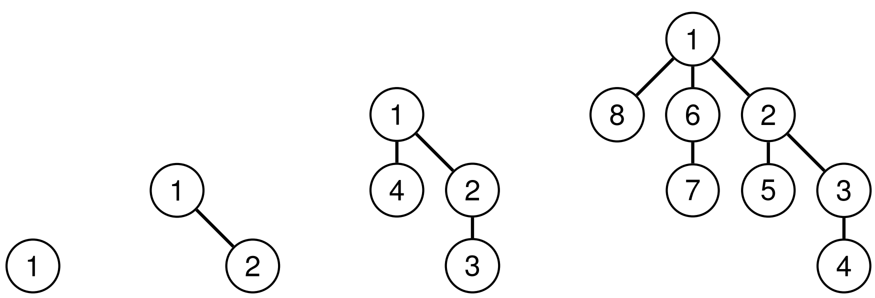
Input : Connected undirected graph: $G = (V, E)$,
Source set: V_0,
Chromosomes vector: Cr,
Alpha: α,
Beta: β,
Distances vector: $Dist$,
Degree vector: $Degree$

Output: Total step time to broadcast: $time$

Root $(v, parent)$
    if *parent[v] < 0* **then**
        return v
    $parent[v] \leftarrow Root(parent[v], parent)$
    return $parent[v]$

Merge $(u, v, parent)$
    $u \leftarrow Root(u, parent)$
    $v \leftarrow Root(v, parent)$
    $parent[v] \leftarrow parent[u]$

MSF (G, V_0, Cr)
    $G_b \leftarrow \emptyset$
    for *each* $v \in V(G)$ **do**
        $parent[v] \leftarrow -1$
    $r \leftarrow \forall v_0 \in V_0$
    for *each* $v_0 \in V_0 \; \{r\}$ **do**
        $Merge(r, v_0)$
    $Ranking \leftarrow \emptyset$
    for *each* $e \in E(G)$ **do**
        $Ranking \leftarrow Ranking \cup \{(Cr[e], e_v, e_u)\}$
    while $|E(G_b)| \neq |V(G)| - |V_0|$ **do**
        $e \leftarrow ExtractMin(Ranking)$
        if $Root(e_v, parent) \neq Root(e_u, parent)$ **then**
            $G_b \leftarrow G_b \cup \{(e_v, e_u)\}$
            $Merge(e_v, e_u)$
    return $MBT\_Forest(G_b, V_0)$



(a) B_0 (b) B_1 (c) B_2 (d) B_3

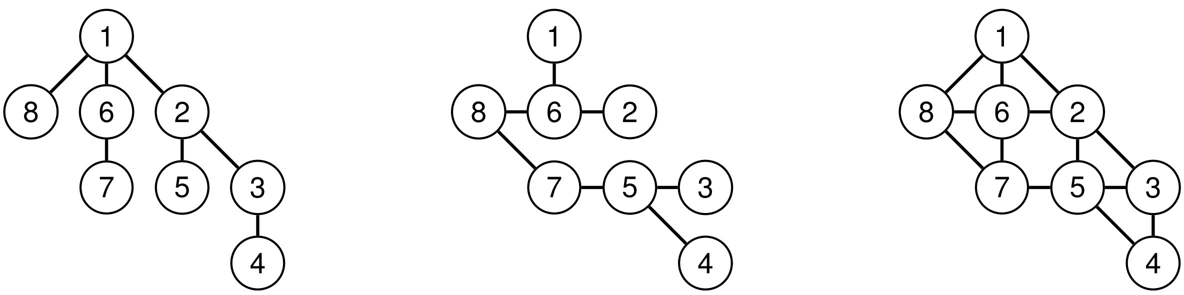
Figure 2.9: Examples of binomial trees.

Algorithm 10: Hybrid BRKGA-ILP

Input : Connected undirected graph: $G = (V, E)$,
Source set: V_0,
Initial solution: S_i,
Upper bound: T_{max},
Time limit for each running of ILP: T_{ILP},
Number of Evolution before to Run ILP: T_e,
Number of Element to build Residual Graph: B

Output: Schedule Broadcast: S

```
ILP_MBT(G, V_0, S_i, T_max, t_max)
    T_min ← LB(G, V_0)
    if lbb == T_max then
        return S_i
    model ← BuildModelILP(G, V_0, T_min, T_max)
    model.initialSolution(S_i)
    model.timeLimit(t_max)
    S_ILP ← model.solve()
    return S_ILP
Hybrid_BRKGA_ILP(G, V_0, E, B, T_max, T_ILP)
    A ← BRKGA(G, V_0)                                   // Instance of a BRKGA
    S ← ∅                                                // Best schedule
    while stopping criterion do
        A.evolve(T_e)                                    // Evolve T_e times
        S ← A.BestSolution()
        G_R ← ∅
        for each S_A ∈ A.getBestSolutions(B)             // Get the B best solutions
        do
            G_R ← G_R ∪ S_A.getEdges()
        T_max ← S.BT()            // S.BT() indicates the broadcast time of solution
        S_ILP ← ILP_MBT(G_R, V_0, S, T_max, T_ILP)
        if S_ILP.BT() < S.BT() then
            S ← S_ILP
            A.insertSolution(S)
    return S
```



(a) B_3 (b) G_r (c) $G = B_3 \cup G_r$

Figure 2.10: Building a synthetic instance.

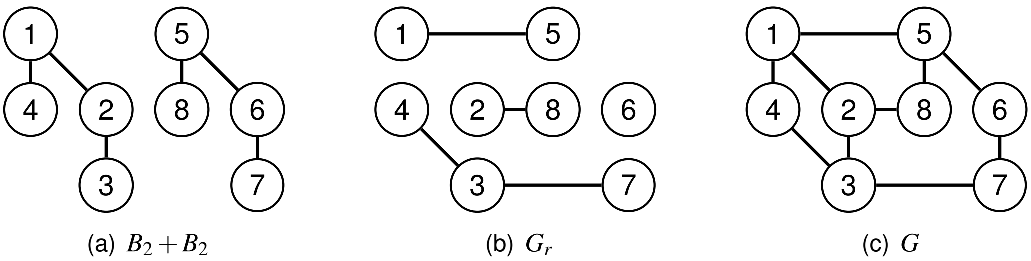


Figure 2.11: Building a synthetic instance with 2 sources.

3

Results and Discussion

This section shows the computational experiments conducted to evaluate the effectiveness of our proposed algorithms. We compare some proposed: (i) the ACS proposed by Hasson and Sipper (2004) with our BRKGA-BP method, (ii) all BRKGA proposed with each other, and (iii) the hybrid method with BRKGA and ILP.

Since we could not obtain the source code of ACS and ILP, we implemented our version of these algorithms based on their original definitions. We did not implement the heuristics Treeblock (de Sousa et al., 2018) and GA (Hoelting et al., 1996) since the results reported in Hasson and Sipper (2004) are sufficient to conclude that ACS outperforms them.

After the decoding process on BRKGA, we use the lower bound described in Eq. (2.2) to check if the optimal solution has been found. SOLTO

3.1 Instances

BRKGA was tested on a total of 111 instances, which include:

- **Harary graphs** (16 instances): These are the same instances used in the work of de Sousa et al. (2018). A Harary graph, denoted by $H_{k,n}$, is a k-connected graph with n vertices having the smallest possible number of edges.

- **Network Data Repository**[1] (59 instances): Because the MBT instances used in previous works (Scheuermann and Wu, 1984; Hoelting et al., 1996; Hasson and Sipper, 2004) are no longer available, we have chosen some instances from well-established benchmarks of complex network problems. To cover different industry scenarios, we consider instances based on connected small-world networks with 100 or 1000 vertices (Freitas et al., 2019). A communication strategy is small-world when the communication network presents a high

[1] http://www.networkrepository.com

clustering coefficient and a small shortest path length. We have chosen the small-world model because it is commonly used to represent communication networks in industrial scenarios, as suggested in Guidoni et al. (2010) and Cabral et al. (2013).

- **Large synthetic instances based on Binomial Tree** (36 instances): Given that the optimal solution for large MBT instances is often unknown, we generated a new benchmark with known optimal solutions using the following procedure (see Section 2.5).

For each instance, Table 3.1 gives the name (column 'Instance'), the number of vertices (column '$|V|$'), the number of edges (column '$|E|$'), the initial source vertex (column 'V_0'), the theoretical lower bound (column 'TLB', see Eq. (2.1)), the lower bound found by the proposed Algorithm 1 (column 'LBB'), and the edge density (column 'density').

Table 3.1: Description of the test instances.

Instance	$\lvert V\rvert$	$\lvert E\rvert$	V_0	TLB	LBB	density	Instance	$\lvert V\rvert$	$\lvert E\rvert$	V_0	TLB	LBB	density
Harary Graphs													
$H_{10,30}$	30	150	{30}	5	3	0.3448	$H_{11,50}$	50	275	{50}	6	3	0.2245
$H_{20,50}$	50	500	{50}	6	3	0.4082	$H_{21,50}$	50	525	{50}	6	2	0.4286
$H_{2,100}$	100	100	{100}	7	50	0.0202	$H_{2,17}$	17	17	{17}	4	8	0.125
$H_{2,30}$	30	30	{30}	5	15	0.069	$H_{2,50}$	50	50	{50}	6	25	0.0408
$H_{3,17}$	17	26	{6}	4	4	0.1912	$H_{3,30}$	30	45	{30}	5	8	0.1034
$H_{3,50}$	50	75	{50}	6	13	0.0612	$H_{5,17}$	17	43	{17}	4	3	0.3162
$H_{6,17}$	17	51	{17}	4	3	0.375	$H_{7,17}$	17	60	{17}	4	2	0.4412
$H_{8,30}$	30	120	{30}	5	4	0.2759	$H_{9,30}$	30	135	{30}	5	3	0.3103
Network Repository													
SW-100-3-0d1-trial1	100	100	{1}	7	61	0.0202	SW-100-3-0d2-trial1	100	100	{1}	7	31	0.0202
SW-100-3-0d2-trial3	100	100	{1}	7	31	0.0202	SW-100-4-0d1-trial1	100	200	{1}	7	7	0.0404
SW-100-4-0d1-trial2	100	200	{1}	7	7	0.0404	SW-100-4-0d1-trial3	100	200	{1}	7	9	0.0404
SW-100-4-0d2-trial1	100	200	{1}	7	7	0.0404	SW-100-4-0d2-trial2	100	200	{1}	7	7	0.0404
SW-100-4-0d2-trial3	100	200	{1}	7	7	0.0404	SW-100-4-0d3-trial1	100	200	{1}	7	6	0.0404
SW-100-4-0d3-trial2	100	200	{1}	7	6	0.0404	SW-100-4-0d3-trial3	100	200	{1}	7	7	0.0404
SW-100-5-0d1-trial1	100	200	{1}	7	8	0.0404	SW-100-5-0d1-trial2	100	200	{1}	7	9	0.0404
SW-100-5-0d1-trial3	100	200	{1}	7	11	0.0404	SW-100-5-0d2-trial1	100	200	{1}	7	8	0.0404
SW-100-5-0d2-trial2	100	200	{1}	7	9	0.0404	SW-100-5-0d2-trial3	100	200	{1}	7	7	0.0404
SW-100-5-0d3-trial1	100	200	{1}	7	6	0.0404	SW-100-5-0d3-trial2	100	200	{1}	7	6	0.0404
SW-100-5-0d3-trial3	100	200	{1}	7	6	0.0404	SW-100-6-0d1-trial1	100	300	{1}	7	5	0.0606
SW-100-6-0d1-trial2	100	300	{1}	7	6	0.0606	SW-100-6-0d1-trial3	100	300	{1}	7	6	0.0606
SW-100-6-0d2-trial1	100	300	{1}	7	6	0.0606	SW-100-6-0d2-trial2	100	300	{1}	7	4	0.0606
SW-100-6-0d2-trial3	100	300	{1}	7	4	0.0606	SW-100-6-0d3-trial1	100	300	{1}	7	4	0.0606
SW-100-6-0d3-trial2	100	300	{1}	7	5	0.0606	SW-100-6-0d3-trial3	100	300	{1}	7	5	0.0606
SW-1000-3-0d2-trial1	1000	1000	{1}	10	89	0.002	SW-1000-3-0d2-trial2	1000	1000	{1}	10	88	0.002
SW-1000-3-0d3-trial2	1000	1000	{1}	10	87	0.002	SW-1000-4-0d1-trial1	1000	2000	{1}	10	14	0.004
SW-1000-4-0d1-trial2	1000	2000	{1}	10	15	0.004	SW-1000-4-0d1-trial3	1000	2000	{1}	10	15	0.004
SW-1000-4-0d2-trial1	1000	2000	{1}	10	10	0.004	SW-1000-4-0d2-trial2	1000	2000	{1}	10	10	0.004
SW-1000-4-0d2-trial3	1000	2000	{1}	10	11	0.004	SW-1000-4-0d3-trial1	1000	2000	{1}	10	9	0.004
SW-1000-4-0d3-trial3	1000	2000	{1}	10	8	0.004	SW-1000-5-0d1-trial1	1000	2000	{1}	10	14	0.004
SW-1000-5-0d1-trial2	1000	2000	{1}	10	15	0.004	SW-1000-5-0d1-trial3	1000	2000	{1}	10	12	0.004
SW-1000-5-0d2-trial1	1000	2000	{1}	10	11	0.004	SW-1000-5-0d2-trial2	1000	2000	{1}	10	10	0.004
SW-1000-5-0d2-trial3	1000	2000	{1}	10	10	0.004	SW-1000-5-0d3-trial1	1000	2000	{1}	10	9	0.004
SW-1000-5-0d3-trial2	1000	2000	{1}	10	9	0.004	SW-1000-5-0d3-trial3	1000	2000	{1}	10	10	0.004

Continued on next page

Table 3.1 – continued from previous page

Instance	$\|V\|$	$\|E\|$	V_0	TLB	LBB	density	Instance	$\|V\|$	$\|E\|$	V_0	TLB	LBB	density
SW-1000-6-0d1-trial1	1000	3000	{1}	10	10	0.006	SW-1000-6-0d1-trial2	1000	3000	{1}	10	9	0.006
SW-1000-6-0d1-trial3	1000	3000	{1}	10	8	0.006	SW-1000-6-0d2-trial1	1000	3000	{1}	10	8	0.006
SW-1000-6-0d2-trial2	1000	3000	{1}	10	8	0.006	SW-1000-6-0d2-trial3	1000	3000	{1}	10	7	0.006
SW-1000-6-0d3-trial1	1000	3000	{1}	10	6	0.006	SW-1000-6-0d3-trial2	1000	3000	{1}	10	6	0.006
SW-1000-6-0d3-trial3	1000	3000	{1}	10	7	0.006							
Graphs based on binomial tree													
$B_5 \cup RG_{32,0.05}$	32	48	{1}	5	5	0.0968	$B_5 \cup RG_{32,0.075}$	32	64	{1}	5	4	0.129
$B_5 \cup RG_{32,0.1}$	32	83	{1}	5	3	0.1673	$B_5 \cup RG_{32,0.15}$	32	89	{1}	5	3	0.1794
$B_5 \cup RG_{32,0.2}$	32	142	{1}	5	2	0.2863	$B_5 \cup RG_{32,0.25}$	32	156	{1}	5	2	0.3145
$B_6 \cup RG_{64,0.05}$	64	159	{1}	6	3	0.0789	$B_6 \cup RG_{64,0.075}$	64	184	{1}	6	3	0.0913
$B_6 \cup RG_{64,0.1}$	64	243	{1}	6	3	0.1205	$B_6 \cup RG_{64,0.15}$	64	349	{1}	6	2	0.1731
$B_6 \cup RG_{64,0.2}$	64	461	{1}	6	2	0.2287	$B_6 \cup RG_{64,0.25}$	64	558	{1}	6	2	0.2768
$B_7 \cup RG_{128,0.05}$	128	560	{1}	7	3	0.0689	$B_7 \cup RG_{128,0.075}$	128	716	{1}	7	3	0.0881
$B_7 \cup RG_{128,0.1}$	128	923	{1}	7	3	0.1136	$B_7 \cup RG_{128,0.15}$	128	1313	{1}	7	3	0.1615
$B_7 \cup RG_{128,0.2}$	128	1742	{1}	7	2	0.2143	$B_7 \cup RG_{128,0.25}$	128	2140	{1}	7	2	0.2633
$B_8 \cup RG_{256,0.05}$	256	1863	{1}	8	3	0.0571	$B_8 \cup RG_{256,0.075}$	256	2657	{1}	8	3	0.0814
$B_8 \cup RG_{256,0.1}$	256	3450	{1}	8	2	0.1057	$B_8 \cup RG_{256,0.15}$	256	5168	{1}	8	2	0.1583
$B_8 \cup RG_{256,0.2}$	256	6691	{1}	8	2	0.205	$B_8 \cup RG_{256,0.25}$	256	8307	{1}	8	2	0.2545
$B_9 \cup RG_{512,0.05}$	512	6881	{1}	9	3	0.0526	$B_9 \cup RG_{512,0.075}$	512	10304	{1}	9	3	0.0788
$B_9 \cup RG_{512,0.1}$	512	13444	{1}	9	2	0.1028	$B_9 \cup RG_{512,0.15}$	512	20009	{1}	9	2	0.153
$B_9 \cup RG_{512,0.2}$	512	27012	{1}	9	2	0.2065	$B_9 \cup RG_{512,0.25}$	512	33313	{1}	9	2	0.2547
$B_{10} \cup RG_{1024,0.05}$	1024	27259	{1}	10	3	0.052	$B_{10} \cup RG_{1024,0.075}$	1024	40222	{1}	10	3	0.0768
$B_{10} \cup RG_{1024,0.1}$	1024	53480	{1}	10	2	0.1021	$B_{10} \cup RG_{1024,0.15}$	1024	79574	{1}	10	2	0.1519
$B_{10} \cup RG_{1024,0.2}$	1024	105448	{1}	10	2	0.2013	$B_{10} \cup RG_{1024,0.25}$	1024	131643	{1}	10	2	0.2513

3.2 Comparison between ACS and BP

The experiments presented in this section are part of an article under review — submitted to Computer & Operations Research. Here, we will show the parameter settings, experimental protocol, and the results obtained.

3.2.1 Parameter settings and experimental protocol

All experiments were conducted on an Intel Core i3-5005U with 2.00 GHz, 4 GB of RAM, running Ubuntu 18.04.1. The heuristic algorithms were coded in C++ and compiled with g++ 7.5 and '-O3' flag. The BRKGA C++ framework developed by Toso and Resende (2014) has been used to implement our BRKGA. Moreover, IBM Cplex 12.9 has been used to solve the ILP models proposed by de Sousa et al. (2018).

We have used the Irace (López-Ibáñez et al., 2016) tunning tool to configure the parameters of our BRKGA-BP. The best parameters values obtained through Irace were: (i) elite population: $p_e = 0.22$ (22%), (ii) mutant population: $p_m = 0.26$ (26%), (iii) biased: $\rho_e = 0.66(66\%)$, (iv) number of independent populations: $K = 9$. For the ACS algorithm, we use the same parameter settings indicated by their authors.

We have set a time limit of 3600 s (1 h) to Cplex for solving the ILP models. To assess the average performance of the heuristic algorithms, we have performed 10 runs of BRKGA-BP and ACS in each benchmark instance with different random seeds for each run. A time limit of 60 s has been used for these runs.

3.2.2 Experiments on Harary Graphs

Table 3.2 presents a comparison between the heuristic methods in Harary Graphs. The methods are compared using the following criteria: the best, the average and the worst solution obtained (columns 'Best', 'Avg.' ad 'Worst', respectively), as well as the average CPU time to find the best solution (column 't (s)'). If in a run the heuristic fails to attain the best solution, its CPU time to find the best in this run is considered to be the cutoff time. An asterisk means the algorithm proves the optimal solution. The bottom of Table 3.2 shows a summary that includes: the average of the average CPU time values to find the best solution, number of instances in which the method found the best broadcast time, and number of instances in which the algorithm determined the best average broadcast time. The results for TreeBlock are reproduced from its original paper.

The results in Table 3.2 show that BRKGA outperformed the other heuristic methods. It was able to prove the optimal solution for all instances. ACS and Treeblock missed the best solutions in 1 and 7 instances, respectively. Moreover, BRKGA-BP was, on average, 8.3 times faster than ACS.

Table 3.3 compares the proposed BRKGA-BP with the ILP model proposed in de Sousa et al. (2018). Four variants of this ILP were tested: (i) the original model without bounds, (ii) the model with the lower bound (see Eq (2.1)), (iii) the model with an upper bound determined by our BRKGA-BP, and (iv) the model with both bounds. A hyphen in Table 3.3 indicates that an ILP variant was not able to attain a feasible solution. To assess the performance improvement of the proposed lower bound $LBB\text{-}BFS$ in comparison with the theoretical lower bound TLB, for the instances in which $LBB\text{-}BFS > TLB$, column 'Lower bound' gives in parenthesis the corresponding CPU time considering that the lower bound is determined only by TLB.

Table 3.2: Comparative results on Harary graphs (BRKGA-BP and reference heuristics).

Instance	Treeblock	ACS				BRKGA-BP			
	Best	Best	Avg.	Worst	t (s)	Best	Avg.	Worst	t (s)
$H_{10,30}$	6	**5***	5	5	0.46	**5***	5	5	0.15
$H_{11,50}$	7	**6***	6	6	0.22	**6***	6	6	0.26
$H_{20,50}$	8	**6***	6	6	0.44	**6***	6	6	0.33
$H_{21,50}$	7	**6***	6	6	0.55	**6***	6	6	0.33
$H_{2,100}$	50	**50***	50	50	0.21	**50***	50	50	0.88
$H_{2,17}$	**9***	**9***	9	9	0.02	**9***	9	9	0.04
$H_{2,30}$	**15***	**15***	15	15	0.04	**15***	15	15	0.12
$H_{2,50}$	**25***	**25***	25	25	0.08	**25***	25	25	0.27
$H_{3,17}$	**5***	**5***	5	5	0.02	**5***	5	5	0.04
$H_{3,30}$	**9***	**9***	9	9	0.04	**9***	9	9	0.1

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Table 3.2 – continued from previous page

Instance	Treeblock	ACS				BRKGA-BP			
	Best	Best	Avg.	Worst	t (s)	Best	Avg.	Worst	t (s)
$H_{3,50}$	**14***	**14***	14	14	0.07	**14***	14	14	0.23
$H_{5,17}$	**5***	**5***	5	5	0.03	**5***	5	5	0.05
$H_{6,17}$	**5***	**5***	5	5	0.03	**5***	5	5	0.05
$H_{7,17}$	**5***	**5***	5	5	0.04	**5***	5	5	0.06
$H_{8,30}$	6	6	6	6	60	**5***	5	5	4.51
$H_{9,30}$	6	**5***	5	5	0.1	**5***	5	5	0.16
Avg. t (s)	-	3.90				0.47			
# Best	9	15				16			
# Best Avg.	-	15				16			

Table 3.3: Comparative results on Harary graphs (BRKGA-BP and ILP variants).

Instance	de Sousa et al's ILP								BRKGA-BP		
	Original		Lower Bound		Upper Bound		Both Bounds				
	Best	t (s)	Best	t (s)	Best	t (s)	Best	t (s)	Best	Avg.	t(s)
$H_{10,30}$	**5***	18.17	**5***	96.71	**5***	3.4	**5***	0.22	**5***	5	0.15
$H_{11,50}$	-	-	49	3600	**6**	3600	**6***	0.37	**6***	6	0.26
$H_{20,50}$	-	-	-	-	**6**	3600	**6***	0.01	**6***	6	0.33
$H_{21,50}$	-	-	-	-	**6**	3600	**6***	0.02	**6***	6	0.33
$H_{2,100}$	**50***	24.05	**50***	0.25 (17.53)	**50***	2.48	**50***	0.13	**50***	50	0.88
$H_{2,17}$	**9***	0.04	**9***	0.04 (0.05)	**9***	0.02	**9***	0.01	**9***	9	0.04
$H_{2,30}$	**15***	0.17	**15***	0.01 (0.17)	**15***	0.06	**15***	<0.01	**15***	15	0.12
$H_{2,50}$	**25***	2.24	**25***	0.03 (1.71)	**25***	0.29	**25***	0.01	**25***	25	0.27
$H_{3,17}$	**5***	0.24	**5***	0.07	**5***	0.02	**5***	0.01	**5***	5	0.04
$H_{3,30}$	**9***	1.53	**9***	0.75 (1.36)	**9***	0.09	**9***	0.05	**9***	9	0.1
$H_{3,50}$	**14***	7.05	**14***	1.39 (4.92)	**14***	0.25	**14***	0.14	**14***	14	0.23
$H_{5,17}$	**5***	1.64	**5***	< 0.01	**5***	1.5	**5***	< 0.01	**5***	5	0.05
$H_{6,17}$	**5***	1.26	**5***	< 0.01 (<0.01)	**5***	1.0	**5***	< 0.01	**5***	5	0.05
$H_{7,17}$	**5***	126.76	**5***	< 0.01 (<0.01)	**5***	285.81	**5***	< 0.01	**5***	5	0.06
$H_{8,30}$	**5***	8.32	**5***	4.24	**5***	0.63	**5***	0.2	**5***	5	4.51
$H_{9,30}$	**5***	33.18	**5***	105.38	**5***	24.92	**5***	0.16	**5***	5	0.16
Avg. t (s)	14.04		237.24 (238.85)		694.03		0.08		0.47		
# Best	13		13		16		16		16		

As can be seen in this table, the bounds helped the exact approach to rapidly prove the optimality of all instances. In particular, without the upper bounds provided by our BRKGA-BP, instances $H_{11,50}$, $H_{20,50}$, and $H_{21,50}$ could not be optimally solved. These results reveal that BRKGA-BP was able to prove more optimal solutions than the original ILP model. Additionally, the proposed LBB-BFS improved the CPU time of the ILP model in the following instances: $H_{2,100}$, $H_{2,30}$, $H_{2,50}$, $H_{3,30}$, and $H_{3,50}$.

3.2.3 Experiments on Network Repository (Small-world) graphs

We now test the reference methods on Small-world graphs (Table 3.4), which are commonly used to represent real-world industrial communication networks (Guidoni et al., 2010; Cabral et al., 2013). For this experiment and the next one, we adopt the ILP model variant with both bounds. For each instance in Table 3.4, column 'Lower Bound' gives the instance lower bound, as calculated by Eq. (2.2).

Table 3.4: Comparative results on Network Repository (Small-world) graphs.

Instance	Lower	de Sousa et al's ILP		ACS				BRKGA-BP			
	Bound	MBT	t (s)	Best	Avg.	Worst	t (s)	Best	Avg.	Worst	t (s)
SW-100-3-0d1-trial1	61	**61***	1.55	**61**	61	61	0.94	**61***	61	61	1.27
SW-100-3-0d2-trial1	31	**31***	0.67	**31**	31	31	0.21	**31***	31	31	0.85
SW-100-3-0d2-trial3	31	**31***	0.65	**31**	31	31	0.22	**31***	31	31	0.85
SW-100-4-0d1-trial1	7	**9***	2.22	**9**	9.9	10	54.72	**9***	9	9	22.87
SW-100-4-0d1-trial2	7	**8***	1.93	9	9	9	1.56	9	9	9	0.42
SW-100-4-0d1-trial3	9	**10***	1.44	11	11	11	60	**10***	10	10	6.32
SW-100-4-0d2-trial1	7	**8***	2.87	9	9	9	9.37	9	9	9	0.47
SW-100-4-0d2-trial2	7	**8***	8.68	9	9	9	0.91	9	9	9	0.39
SW-100-4-0d2-trial3	7	**9***	3.84	**9**	9	9	3.64	**9***	9	9	0.41
SW-100-4-0d3-trial1	7	**8***	2.34	9	9	9	60	**8***	8.3	9	38.52
SW-100-4-0d3-trial2	7	**8***	34.69	**8**	8	8	3.8	**8***	8	8	0.37
SW-100-4-0d3-trial3	7	**8**	2.13	9	9	9	60	**8***	8.5	9	49.05
SW-100-5-0d1-trial1	8	**9***	1.21	10	10	10	60	**9***	9.8	10	55.62
SW-100-5-0d1-trial2	9	**10***	1.46	11	11	11	60	**10***	10	10	8.27
SW-100-5-0d1-trial3	11	**12***	0.69	13	13	13	60	**12***	12	12	5.11
SW-100-5-0d2-trial1	8	**9***	1.8	10	10	10	4.08	10	10	10	0.49
SW-100-5-0d2-trial2	9	**9***	1.72	10	10.3	11	38.55	10	10	10	0.46
SW-100-5-0d2-trial3	7	**8***	5.22	9	9.5	10	44.42	9	9	9	0.64
SW-100-5-0d3-trial1	7	**8***	6.63	**8**	8.8	9	49.66	**8***	8	8	4.98
SW-100-5-0d3-trial2	7	**8**	3600	**8**	8	8	1.3	**8**	8	8	0.41
SW-100-5-0d3-trial3	7	**8***	23.69	**8**	8	8	8.59	**8***	8	8	0.44
SW-100-6-0d1-trial1	7	**7***	4.56	8	8	8	1.05	8	8	8	0.52
SW-100-6-0d1-trial2	7	**8***	2097.74	9	9	9	60	**8***	8	8	2.75
SW-100-6-0d1-trial3	7	**7***	40.01	8	8.5	9	43.63	8	8	8	0.63
SW-100-6-0d2-trial1	7	**7***	19.22	8	8	8	60	**7***	7.9	8	55.14
SW-100-6-0d2-trial2	7	**7***	0.22	8	8	8	60	**7***	7.3	8	29.23
SW-100-6-0d2-trial3	7	**7***	7.66	8	8	8	60	**7***	7.9	8	57.5
SW-100-6-0d3-trial1	7	**7***	3.41	8	8	8	60	**7***	7.5	8	46.0
SW-100-6-0d3-trial2	7	**7***	2.66	8	8	8	60	**7***	7	7	25.16
SW-100-6-0d3-trial3	7	**7***	1.72	8	8	8	60	**7***	7	7	8.18
SW-1000-3-0d2-trial1	89	-	-	97	97.5	99	60	**94**	95.9	97	60
SW-1000-3-0d2-trial2	88	-	-	93	93.5	94	60	**91**	92	93	59.34
SW-1000-3-0d3-trial2	87	-	-	94	95.3	96	60	**92**	93.5	94	59.41
SW-1000-4-0d1-trial1	14	-	-	19	19	19	60	**18**	18	18	15.52
SW-1000-4-0d1-trial2	15	-	-	20	20	20	60	**19**	19	19	19.74
SW-1000-4-0d1-trial3	15	-	-	20	20.3	21	60	**19**	19.8	20	55.05
SW-1000-4-0d2-trial1	10	-	-	**15**	15	15	2.61	**15**	15	15	8.2
SW-1000-4-0d2-trial2	10	-	-	**15**	15.7	16	51.15	**15**	15	15	8.04
SW-1000-4-0d2-trial3	11	-	-	**16**	16	16	19.86	**16**	16	16	8.0
SW-1000-4-0d3-trial1	10	-	-	14	14	14	60	**13**	13.4	14	43.51
SW-1000-4-0d3-trial3	10	-	-	14	14	14	60	**13**	13.1	14	32.65
SW-1000-5-0d1-trial1	14	-	-	**19**	19.9	20	59.65	**19**	19	19	14.02
SW-1000-5-0d1-trial2	15	-	-	**19**	19.5	20	42.87	**19**	19	19	10.9
SW-1000-5-0d1-trial3	12	-	-	**17**	17.5	18	46.84	**17**	17	17	9.71
SW-1000-5-0d2-trial1	11	-	-	16	16	16	60	**15**	15	15	21.44
SW-1000-5-0d2-trial2	10	-	-	16	16	16	60	**15**	15	15	8.69
SW-1000-5-0d2-trial3	10	-	-	15	15	15	60	**14**	14.7	15	51.31

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Table 3.4 – continued from previous page

Instance	Lower Bound	de Sousa et al's ILP Best	t (s)	ACS Best	Avg.	Worst	t (s)	BRKGA-BP Best	Avg.	Worst	t (s)
SW-1000-5-0d3-trial1	10	-	-	**14**	14	14	2.7	**14**	14	14	6.84
SW-1000-5-0d3-trial2	10	-	-	**14**	14	14	2.72	**14**	14	14	7.31
SW-1000-5-0d3-trial3	10	-	-	**14**	14.6	15	42.88	**14**	14	14	7.9
SW-1000-6-0d1-trial1	10	-	-	16	16	16	60	**15**	15	15	9.72
SW-1000-6-0d1-trial2	10	-	-	15	15	15	60	**14**	14	14	9.85
SW-1000-6-0d1-trial3	10	-	-	**14**	14	14	18.4	**14**	14	14	9.13
SW-1000-6-0d2-trial1	10	-	-	13	13	13	60	**12**	12.2	13	21.88
SW-1000-6-0d2-trial2	10	-	-	**13**	13.7	14	56.28	**13**	13	13	8.19
SW-1000-6-0d2-trial3	10	-	-	13	13	13	60	**12**	12.2	13	38.8
SW-1000-6-0d3-trial1	10	-	-	**12**	12	12	7.17	**12**	12	12	6.96
SW-1000-6-0d3-trial2	10	-	-	**12**	12	12	7.76	**12**	12	12	7.11
SW-1000-6-0d3-trial3	10	-	-	**12**	12	12	4.93	**12**	12	12	7.2
Avg. t (s)		99.7		39.21				17.79			
# Best		30		23				51			
# Best (only heuristics)		-		31				59			
# Best Avg.		-		20				59			

As can be seen from results in Table 3.4, even with an upper bound set by BRKGA-BP, the ILP model could not produce any feasible results for instances with 1000 vertices. BRKGA-BP proved the optimal solution in 21 instances, whereas this number was 29 for the ILP model. Moreover, our algorithm outperformed ACS in terms of both solution quality and CPU time. BRKGA-BP attained the best solution in 51 out of the 59 instances. Note that in the cases in which BRKGA-BP missed the best solution, the solution determined by BRKGA-BP is at most one unit time longer than the best one.

3.2.4 Experiments on binomial tree based graphs

Finally, in Table 3.5, we test the reference methods in synthetic instances. Recall that these instances were devised in such a way that we know the optimal MBT (see column 'MBT' in this table). The results reveal that proving the optimality of these instances is very challenging, as the ILP model with bounds could only solve instances with up to 128 vertices. Hence, we think this benchmark set is a very good one to evaluate future new exact methods. The results also confirm the robustness of BRKGA-BP and its superiority over ACS. The gap between the best solutions obtained by BRKGA-BP and the known optimal ones is at most one unit time. Moreover, BRKGA-BP was able to optimally solve some instances with more than 1000 vertices.

Table 3.5: Comparative results on binomial tree based graphs.

Instance	MBT	de Sousa et al's ILP Best	t (s)	ACS Best	Avg.	Worst	t (s)	BRKGA Best	Avg.	Worst	t (s)
$B_5 \cup G_r(32, 0.05)$	5	**5***	0.31	**5**	5.33	6	23.27	**5***	5	5	5.99
$B_5 \cup G_r(32, 0.075)$	5	**5***	0.11	**5**	5.9	6	57.51	**5***	5	5	18.44

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Table 3.5 – continued from previous page

Instance	MBT	de Sousa et al's ILP		ACS				BRKGA			
		Best	t (s)	Best	Avg.	Worst	t (s)	Best	Avg.	Worst	t (s)
$B_5 \cup G_r(32,0.1)$	5	**5***	0.03	**5**	5.2	6	29.27	**5***	5	5	0.52
$B_5 \cup G_r(32,0.15)$	5	**5***	0.03	**5**	5	5	9.57	**5***	5	5	0.13
$B_5 \cup G_r(32,0.2)$	5	**5***	0.22	**5**	5	5	0.17	**5***	5	5	0.15
$B_5 \cup G_r(32,0.25)$	5	**5***	0.31	**5**	5	5	0.22	**5***	5	5	0.15
$B_6 \cup G_r(64,0.05)$	6	**6***	4.79	**6**	6.57	7	37.39	**6***	6.33	7	20.31
$B_6 \cup G_r(64,0.075)$	6	**6***	16.79	7	7	7	0.22	7	7	7	0.24
$B_6 \cup G_r(64,0.1)$	6	**6***	1.44	7	7	7	60	**6***	6.3	7	29.35
$B_6 \cup G_r(64,0.15)$	6	**6***	2.52	**6**	6.7	7	50.9	**6***	6	6	0.42
$B_6 \cup G_r(64,0.2)$	6	**6***	3.94	**6**	6	6	9.33	**6***	6	6	0.44
$B_6 \cup G_r(64,0.25)$	6	**6***	4.79	**6**	6	6	1.25	**6***	6	6	0.51
$B_7 \cup G_r(128,0.05)$	7	**7***	432.82	**7**	7.67	8	41.5	**7***	7.33	8	21.17
$B_7 \cup G_r(128,0.075)$	7	**7***	3291.25	8	8	8	0.75	8	8	8	0.8
$B_7 \cup G_r(128,0.1)$	7	**7***	120.68	8	8	8	60	**7***	7.2	8	30.68
$B_7 \cup G_r(128,0.15)$	7	-	-	8	8	8	60	**7**	7	7	1.63
$B_7 \cup G_r(128,0.2)$	7	**7***	409.48	**7**	7.2	8	30.2	**7***	7	7	1.59
$B_7 \cup G_r(128,0.25)$	7	**7***	432.82	**7**	7	7	4.46	**7***	7	7	1.88
$B_8 \cup G_r(256,0.05)$	8	-	-	**8**	8.77	9	53.64	**8***	8.33	9	25.39
$B_8 \cup G_r(256,0.075)$	8	**9**	3600	**9**	9	9	3.65	**9**	9	9	2.81
$B_8 \cup G_r(256,0.1)$	8	**9**	3600	**9**	9	9	5.81	**9**	9	9	3.53
$B_8 \cup G_r(256,0.15)$	8	-	-	9	9	9	60	**8***	8	8	8.14
$B_8 \cup G_r(256,0.2)$	8	-	-	**8**	8.8	9	56.62	**8***	8	8	6.5
$B_8 \cup G_r(256,0.25)$	8	-	-	**8**	8.3	9	40.85	**8***	8	8	7.98
$B_9 \cup G_r(512,0.05)$	9	-	-	**9**	9.93	10	60	**9***	9.4	10	44.91
$B_9 \cup G_r(512,0.075)$	9	-	-	**10**	10	10	23.66	**10**	10	10	11.8
$B_9 \cup G_r(512,0.1)$	9	-	-	**10**	10	10	36.89	**10**	10	10	16.43
$B_9 \cup G_r(512,0.15)$	9	-	-	10	10	10	60	**9***	9.2	10	35.41
$B_9 \cup G_r(512,0.2)$	9	-	-	**9**	9.9	10	60	**9***	9	9	35.26
$B_9 \cup G_r(512,0.25)$	9	-	-	**9**	9.8	10	60	**9***	9	9	39.13
$B_{10} \cup G_r(1024,0.05)$	10	-	-	11	11	11	60	**10***	10.5	11	60
$B_{10} \cup G_r(1024,0.075)$	10	-	-	**11**	11	11	60	**11**	11	11	57.76
$B_{10} \cup G_r(1024,0.1)$	10	-	-	**11**	11	11	60	**11**	11	11	60
$B_{10} \cup G_r(1024,0.15)$	10	-	-	11	11	11	60	**10***	10.5	11	60
$B_{10} \cup G_r(1024,0.2)$	10	-	-	11	11	11	60	**10***	10	10	60
$B_{10} \cup G_r(1024,0.25)$	10	-	-	11	11	11	60	**10***	10	10	60
Avg. t (s)		330.02		37.76				20.28			
# Best		19		25				34			
# Best (only heuristics)		-		27				36			
# Best Avg.		-		14				36			

3.3 Comparison Our Various Decoders

In this section, we will show the results of the comparison between the decoders described. We did preliminary tests in each decoder or method. We tried development some way of pre-processing the input graph, but we did not have good results.

Here, we adopted that the names to variants of BP and FRFS decoders as `<DECODER>-<HEURISTIC>` according to decoder and heuristics used. The `<DECODER>` can be BP or FRFS and the `<HEURISTIC>` can be 0, 1, 2 or 3. The heuristic 0 is represented by of Algorithm 3 or Algorithm 4 with $\alpha = 0$ and $\beta = 0$. The heuristic 1 is represented by Algorithm 4 with $\alpha \neq 0$ and

$\beta = 0$. The heuristic 2 is represented of Algorithm 4 with $\alpha = 0$ and $\beta \neq 0$. Finally, the heuristic 3 is represented of Algorithm 4 with $\alpha \neq 0$ and $\beta \neq 0$. The similar idea was adopted to variants of BP-SCHA and FRFS-SCHA decoders, denoted by `<DECODER>-<HEURISTIC>`-SCHA.

3.3.1 Parameter settings and experimental protocol

All experiments were conducted on an Intel Core i7-6700 with 3.40 GHz, 32 GB of RAM, running Ubuntu 18.04.5. The heuristic algorithms were coded in C++ and compiled with g++ 7.5 and '-O3' flag.

We have used the Irace (López-Ibáñez et al., 2016) tunning tool to configure the parameters of each decoder. Since the goal is to find the best decoder, we set the value of the number of independent population equal to 1 ($K = 1$). All best parameters values identified by the tuning experiment are reported in Table 3.6.

Table 3.6: Parameters of decoders

Decoder	Parameters				
	p_e	p_m	ρ_e	α	β
BP-0	0.11	0.10	0.76	-	-
BP-0-SCHA	0.11	0.10	0.76	-	-
BP-1	0.23	0.27	0.71	0.09	-
BP-1-SCHA	0.23	0.27	0.71	0.09	-
BP-2	0.13	0.19	0.71	-	0.09
BP-2-SCHA	0.13	0.19	0.71	-	0.08
BP-3	0.12	0.22	0.80	0.08	0.18
BP-3-SCHA	0.12	0.22	0.80	0.08	0.18
FRFS-0	0.14	0.12	0.70	-	-
FRFS-SCHA	0.14	0.12	0.70	-	-
FRFS-1	0.24	0.25	0.71	0.06	-
FRFS-1-SCHA	0.24	0.25	0.71	0.06	-
FRFS-2	0.17	0.14	0.73	-	0.12
FRFS-2-SCHA	0.17	0.14	0.73	-	0.12
FRFS-3	0.12	0.22	0.80	0.08	0.18
FRFS-3-SCHA	0.12	0.22	0.80	0.08	0.18
MSF	0.16	0.10	0.74	-	-

3.3.2 Experiments for comparison of BRKGA-BPs

Table 3.7 presents a comparison between the BPs decoders in Harary Graphs, Small-world graphs and synthetic instances. These decoders are compared using the following criteria: the best and the average solution obtained (columns 'Best', ad 'Avg.', respectively), as well as the average CPU time to find the best solution (column 't (s)'). The bottom of Table 3.7 shows a summary that includes: the average of the average CPU time values to find the best solution, number of instances in which the method found the best broadcast time, and number of instances in which the algorithm determined the best average broadcast time. These results show that variants BP-0 and BP-2 consistently obtain the best performance over other variants in this experiment.

Table 3.7: BP Decoders

Instance	BP-0			BP-1			BP-2			BP-3		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
BT05-RG050	**5**	5.30	27.06	6	6.00	60.00	**5**	**5.00**	12.29	6	6.00	60.00
BT05-RG075	**5**	**5.00**	2.53	6	6.00	60.00	**5**	**5.00**	3.18	6	6.00	60.00
BT05-RG100	**5**	**5.00**	0.04	6	6.00	60.00	**5**	**5.00**	0.04	6	6.00	60.00
BT05-RG150	**5**	**5.00**	0.03	6	6.00	60.00	**5**	**5.00**	0.02	6	6.00	60.00
BT05-RG200	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
BT05-RG250	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
BT06-RG050	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01
BT06-RG075	7	7.00	60.00	7	7.00	60.00	**6**	**6.90**	58.52	7	7.00	60.00
BT06-RG100	**6**	**6.00**	4.81	7	7.00	60.00	**6**	**6.00**	4.32	7	7.00	60.00
BT06-RG150	**6**	**6.00**	0.02	**6**	**6.00**	0.25	**6**	**6.00**	0.02	**6**	**6.00**	0.25
BT06-RG200	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
BT06-RG250	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
BT07-RG050	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
BT07-RG075	8	8.00	60.00	8	8.00	60.00	**7**	**7.80**	55.17	8	8.00	60.00
BT07-RG100	**7**	**7.00**	2.78	**7**	7.10	22.13	**7**	**7.00**	2.30	**7**	**7.00**	20.70
BT07-RG150	**7**	**7.00**	0.07	**7**	**7.00**	0.27	**7**	**7.00**	0.08	**7**	**7.00**	0.51
BT07-RG200	**7**	**7.00**	0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	0.01	**7**	**7.00**	< 0.01
BT07-RG250	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01
BT08-RG050	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
BT08-RG075	9	9.00	60.00	9	9.00	60.00	**8**	**8.90**	56.17	9	9.00	60.00
BT08-RG100	**8**	**8.00**	8.44	**8**	8.50	44.95	**8**	8.10	10.46	**8**	8.90	59.74
BT08-RG150	**8**	**8.00**	0.31	**8**	**8.00**	0.26	**8**	**8.00**	0.26	**8**	**8.00**	0.32
BT08-RG200	**8**	**8.00**	0.02	**8**	**8.00**	0.02	**8**	**8.00**	0.01	**8**	**8.00**	0.02
BT08-RG250	**8**	**8.00**	< 0.01	**8**	**8.00**	0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	0.01
BT09-RG050	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01
BT09-RG075	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01
BT09-RG100	**9**	9.50	45.48	**9**	9.60	49.58	**9**	**9.40**	39.79	**9**	9.60	45.93
BT09-RG150	**9**	**9.00**	1.18	**9**	**9.00**	0.36	**9**	**9.00**	0.40	**9**	**9.00**	0.62
BT09-RG200	**9**	**9.00**	0.07	**9**	**9.00**	0.05	**9**	**9.00**	0.04	**9**	**9.00**	0.08
BT09-RG250	**9**	**9.00**	0.02	**9**	**9.00**	0.01	**9**	**9.00**	0.03	**9**	**9.00**	0.01
BT10-RG050	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01
BT10-RG075	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01
BT10-RG100	**10**	10.90	58.50	11	11.00	60.00	11	11.00	60.00	**10**	**10.80**	57.95
BT10-RG150	**10**	**10.00**	3.28	**10**	**10.00**	5.22	**10**	**10.00**	3.85	**10**	**10.00**	2.21
BT10-RG200	**10**	**10.00**	0.19	**10**	**10.00**	0.30	**10**	**10.00**	0.18	**10**	**10.00**	0.44
BT10-RG250	**10**	**10.00**	0.10	**10**	**10.00**	0.05	**10**	**10.00**	0.08	**10**	**10.00**	0.07
H10-30	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
H11-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H20-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H21-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H2-100	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01
H2-17	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H2-30	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01
H2-50	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01
H3-17	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
H3-30	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H3-50	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01
H5-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H6-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01

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Table 3.7 – continued from previous page

Instance	BP-0			BP-1			BP-2			BP-3		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
H7-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H8-30	**5**	**5.00**	0.26	6	6.00	60.00	**5**	**5.00**	0.18	6	6.00	60.00
H9-30	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
SW-100-3-0d1-trial1	**61**	**61.00**	0.03	67	67.00	60.00	**61**	**61.00**	0.02	67	67.00	60.00
SW-100-3-0d2-trial1	**31**	**31.00**	0.01	36	36.00	60.00	**31**	**31.00**	0.01	36	36.00	60.00
SW-100-3-0d2-trial3	**31**	**31.00**	0.01	34	34.00	60.00	**31**	**31.00**	0.01	34	34.00	60.00
SW-100-4-0d1-trial1	**9**	**9.00**	3.87	10	10.00	60.00	**9**	**9.00**	1.85	10	10.00	60.00
SW-100-4-0d1-trial2	**8**	**8.80**	51.08	10	10.00	60.00	9	9.00	60.00	10	10.00	60.00
SW-100-4-0d1-trial3	**10**	**10.00**	0.10	12	12.00	60.00	**10**	**10.00**	0.34	12	12.00	60.00
SW-100-4-0d2-trial1	**9**	**9.00**	0.01	11	11.00	60.00	**9**	**9.00**	0.01	11	11.00	60.00
SW-100-4-0d2-trial2	**9**	**9.00**	< 0.01	10	10.00	60.00	**9**	**9.00**	< 0.01	10	10.00	60.00
SW-100-4-0d2-trial3	**9**	**9.00**	0.01	10	10.00	60.00	**9**	**9.00**	0.01	10	10.00	60.00
SW-100-4-0d3-trial1	**8**	8.10	9.95	10	10.00	60.00	**8**	**8.00**	4.66	10	10.00	60.00
SW-100-4-0d3-trial2	**8**	**8.00**	0.02	9	9.00	60.00	**8**	**8.00**	0.02	9	9.00	60.00
SW-100-4-0d3-trial3	**8**	8.20	26.27	10	10.00	60.00	**8**	**8.00**	10.18	10	10.00	60.00
SW-100-5-0d1-trial1	**9**	**9.20**	15.47	11	11.00	60.00	**9**	**9.20**	18.43	11	11.00	60.00
SW-100-5-0d1-trial2	**10**	**10.00**	1.17	12	12.00	60.00	**10**	**10.00**	2.71	12	12.00	60.00
SW-100-5-0d1-trial3	**12**	**12.00**	0.24	14	14.00	60.00	**12**	**12.00**	0.20	14	14.00	60.00
SW-100-5-0d2-trial1	**10**	**10.00**	0.01	12	12.00	60.00	**10**	**10.00**	0.01	12	12.00	60.00
SW-100-5-0d2-trial2	**10**	**10.00**	0.02	11	11.00	60.00	**10**	**10.00**	0.02	11	11.00	60.00
SW-100-5-0d2-trial3	**9**	**9.00**	0.03	11	11.00	60.00	**9**	**9.00**	0.03	11	11.00	60.00
SW-100-5-0d3-trial1	**8**	**8.00**	0.43	9	9.00	60.00	**8**	**8.00**	0.26	9	9.00	60.00
SW-100-5-0d3-trial2	**8**	**8.00**	< 0.01	9	9.00	60.00	**8**	**8.00**	< 0.01	9	9.00	60.00
SW-100-5-0d3-trial3	**8**	**8.00**	0.01	9	9.00	60.00	**8**	**8.00**	0.02	9	9.00	60.00
SW-100-6-0d1-trial1	**8**	**8.00**	< 0.01	9	9.00	60.00	**8**	**8.00**	< 0.01	9	9.00	60.00
SW-100-6-0d1-trial2	**8**	**8.00**	0.16	9	9.00	60.00	**8**	**8.00**	0.24	9	9.00	60.00
SW-100-6-0d1-trial3	**8**	**8.00**	0.04	10	10.00	60.00	**8**	**8.00**	0.04	10	10.00	60.00
SW-100-6-0d2-trial1	**7**	7.80	52.07	10	10.00	60.00	**7**	**7.20**	40.67	10	10.00	60.00
SW-100-6-0d2-trial2	**7**	**7.00**	3.41	8	8.00	60.00	**7**	**7.00**	4.75	8	8.00	60.00
SW-100-6-0d2-trial3	**7**	**7.60**	45.90	8	8.00	60.00	**7**	**7.60**	45.02	8	8.00	60.00
SW-100-6-0d3-trial1	**7**	**7.00**	12.32	8	8.00	60.00	**7**	**7.00**	7.16	8	8.00	60.00
SW-100-6-0d3-trial2	**7**	**7.00**	4.01	8	8.00	60.00	**7**	**7.00**	3.14	8	8.00	60.00
SW-100-6-0d3-trial3	**7**	**7.00**	2.15	8	8.00	60.00	**7**	**7.00**	0.77	8	8.00	60.00
SW-1000-3-0d2-trial1	**89**	**89.00**	6.99	105	105.00	60.00	**89**	**89.00**	9.43	105	105.00	60.00
SW-1000-3-0d2-trial2	**88**	**88.00**	4.78	100	100.00	60.00	**88**	**88.00**	5.57	100	100.00	60.00
SW-1000-3-0d3-trial2	**87**	**87.00**	4.63	114	114.00	60.00	**87**	**87.00**	5.28	114	114.00	60.00
SW-1000-4-0d1-trial1	**17**	**17.70**	51.55	20	20.00	60.00	**17**	17.80	55.20	20	20.00	60.00
SW-1000-4-0d1-trial2	**18**	**18.00**	3.87	21	21.00	60.00	**18**	**18.00**	6.03	21	21.00	60.00
SW-1000-4-0d1-trial3	**18**	**18.60**	45.74	19	19.70	60.00	**18**	**18.60**	43.19	19	19.80	60.00
SW-1000-4-0d2-trial1	**14**	**14.90**	59.79	16	16.00	60.00	15	15.00	60.00	16	16.00	60.00
SW-1000-4-0d2-trial2	**15**	**15.00**	0.19	16	16.00	60.00	**15**	**15.00**	0.18	16	16.00	60.00
SW-1000-4-0d2-trial3	16	16.00	60.00	17	17.00	60.00	**15**	**15.80**	55.70	17	17.00	60.00
SW-1000-4-0d3-trial1	**13**	**13.00**	12.07	14	14.00	60.00	**13**	**13.00**	5.40	14	14.00	60.00
SW-1000-4-0d3-trial3	**13**	**13.00**	3.52	14	14.00	60.00	**13**	**13.00**	1.60	14	14.00	60.00
SW-1000-5-0d1-trial1	18	18.00	60.00	21	21.00	60.00	**17**	**17.90**	56.97	21	21.00	60.00
SW-1000-5-0d1-trial2	18	18.00	60.00	19	19.00	60.00	**17**	**17.90**	58.36	19	19.00	60.00
SW-1000-5-0d1-trial3	**16**	16.20	20.17	18	18.00	60.00	**16**	**16.10**	23.94	18	18.00	60.00
SW-1000-5-0d2-trial1	**15**	**15.00**	0.79	17	17.00	60.00	**15**	**15.00**	0.93	17	17.00	60.00
SW-1000-5-0d2-trial2	15	15.00	60.00	16	16.00	60.00	**14**	**14.90**	58.98	16	16.00	60.00
SW-1000-5-0d2-trial3	**14**	14.10	15.28	16	16.00	60.00	**14**	**14.00**	10.11	16	16.00	60.00
SW-1000-5-0d3-trial1	**14**	**14.00**	< 0.01	**14**	14.10	21.73	**14**	**14.00**	< 0.01	**14**	14.30	35.75
SW-1000-5-0d3-trial2	**14**	**14.00**	< 0.01	**14**	**14.00**	0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01
SW-1000-5-0d3-trial3	**14**	**14.00**	0.17	15	15.00	60.00	**14**	**14.00**	0.11	15	15.00	60.00

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Table 3.7 – continued from previous page

Instance	BP-0			BP-1			BP-2			BP-3		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
SW-1000-6-0d1-trial1	**14**	**14.90**	55.90	16	16.00	60.00	**14**	**14.90**	57.30	16	16.00	60.00
SW-1000-6-0d1-trial2	**14**	**14.00**	0.23	15	15.00	60.00	**14**	**14.00**	0.18	15	15.00	60.00
SW-1000-6-0d1-trial3	**13**	13.90	57.61	14	14.00	60.00	**13**	**13.80**	52.72	14	14.00	60.00
SW-1000-6-0d2-trial1	**12**	**12.00**	1.78	13	13.00	60.00	**12**	**12.00**	1.07	13	13.00	60.00
SW-1000-6-0d2-trial2	**13**	**13.00**	0.01	14	14.00	60.00	**13**	**13.00**	0.01	14	14.00	60.00
SW-1000-6-0d2-trial3	**12**	**12.00**	5.70	13	13.00	60.00	**12**	**12.00**	4.73	13	13.00	60.00
SW-1000-6-0d3-trial1	**12**	**12.00**	< 0.01	**12**	**12.00**	0.04	**12**	**12.00**	< 0.01	**12**	**12.00**	0.07
SW-1000-6-0d3-trial2	**12**	**12.00**	< 0.01	13	13.00	60.00	**12**	**12.00**	< 0.01	13	13.00	60.00
SW-1000-6-0d3-trial3	**12**	**12.00**	< 0.01	**12**	**12.00**	7.00	**12**	**12.00**	< 0.01	**12**	12.10	21.65
Avg. t (s)		10.40			37.59			9.74			37.90	
# Best (# Best Avg.)		104 (95)			44 (40)			108 (106)			45 (41)	

3.3.3 Experiments for comparison of BRKGA-FRFSs

Table 3.8 shows a comparison between the FRFS decoders in Harary Graphs, Small-world graphs and synthetic instances. The decoders are compared using the following same criteria of the previous section. The results on Table 3.8 show that FRFS-0 and FRFS-2 decoders are the best in this experiment.

Table 3.8: FRFS Decoders

Instance	BRKGA-FRFS			BRKGA-FRFS-1			BRKGA-FRFS-2			BRKGA-FRFS-3		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
BT05-RG050	**5**	**5.30**	26.97	6	6.00	60.00	**5**	5.40	29.81	6	6.00	60.00
BT05-RG075	**5**	**5.00**	2.33	6	6.00	60.00	**5**	**5.00**	1.85	6	6.00	60.00
BT05-RG100	**5**	**5.00**	0.03	6	6.00	60.00	**5**	**5.00**	0.04	6	6.00	60.00
BT05-RG150	**5**	**5.00**	0.02	6	6.00	60.00	**5**	**5.00**	0.01	6	6.00	60.00
BT05-RG200	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
BT05-RG250	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
BT06-RG050	**7**	**7.00**	< 0.01	8	8.00	60.00	**7**	**7.00**	< 0.01	8	8.00	60.00
BT06-RG075	**7**	**7.00**	< 0.01	8	8.00	60.00	**7**	**7.00**	< 0.01	8	8.00	60.00
BT06-RG100	**6**	**6.00**	3.45	7	7.00	60.00	**6**	**6.00**	3.22	7	7.00	60.00
BT06-RG150	**6**	**6.00**	0.01	7	7.00	60.00	**6**	**6.00**	0.01	7	7.00	60.00
BT06-RG200	**6**	**6.00**	< 0.01	7	7.00	60.00	**6**	**6.00**	< 0.01	7	7.00	60.00
BT06-RG250	**6**	**6.00**	< 0.01	7	7.00	60.00	**6**	**6.00**	< 0.01	7	7.00	60.00
BT07-RG050	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
BT07-RG075	**7**	**7.90**	58.00	8	8.00	60.00	**7**	**7.90**	55.73	8	8.00	60.00
BT07-RG100	**7**	**7.00**	2.48	8	8.00	60.00	**7**	**7.00**	1.80	8	8.00	60.00
BT07-RG150	**7**	**7.00**	0.05	8	8.00	60.00	**7**	**7.00**	0.07	8	8.00	60.00
BT07-RG200	**7**	**7.00**	0.01	8	8.00	60.00	**7**	**7.00**	0.01	8	8.00	60.00
BT07-RG250	**7**	**7.00**	< 0.01	8	8.00	60.00	**7**	**7.00**	< 0.01	8	8.00	60.00
BT08-RG050	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
BT08-RG075	**8**	**8.80**	54.27	9	9.00	60.00	9	9.00	60.00	9	9.00	60.00
BT08-RG100	**8**	8.10	27.02	9	9.00	60.00	**8**	**8.00**	27.70	9	9.00	60.00
BT08-RG150	**8**	**8.00**	0.12	9	9.00	60.00	**8**	**8.00**	0.18	9	9.00	60.00
BT08-RG200	**8**	**8.00**	0.03	9	9.00	60.00	**8**	**8.00**	0.02	9	9.00	60.00
BT08-RG250	**8**	**8.00**	0.01	9	9.00	60.00	**8**	**8.00**	0.01	9	9.00	60.00
BT09-RG050	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01

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Table 3.8 – continued from previous page

Instance	BRKGA-FRFS			BRKGA-FRFS-1			BRKGA-FRFS-2			BRKGA-FRFS-3		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
BT09-RG075	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01
BT09-RG100	**9**	9.70	44.03	10	10.00	60.00	**9**	**9.50**	47.94	10	10.00	60.00
BT09-RG150	**9**	**9.00**	1.13	10	10.00	60.00	**9**	**9.00**	0.72	10	10.00	60.00
BT09-RG200	**9**	**9.00**	0.08	10	10.00	60.00	**9**	**9.00**	0.09	10	10.00	60.00
BT09-RG250	**9**	**9.00**	0.02	10	10.00	60.00	**9**	**9.00**	0.03	10	10.00	60.00
BT10-RG050	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01
BT10-RG075	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01
BT10-RG100	11	11.00	60.00	11	11.00	60.00	**10**	**10.90**	59.73	11	11.00	60.00
BT10-RG150	**10**	**10.00**	4.18	11	11.00	60.00	**10**	**10.00**	2.29	11	11.00	60.00
BT10-RG200	**10**	**10.00**	0.19	11	11.00	60.00	**10**	**10.00**	0.35	11	11.00	60.00
BT10-RG250	**10**	**10.00**	0.08	11	11.00	60.00	**10**	**10.00**	0.05	11	11.00	60.00
H10-30	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
H11-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H20-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H21-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H2-100	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01
H2-17	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H2-30	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01
H2-50	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01
H3-17	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
H3-30	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H3-50	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01
H5-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H6-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H7-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H8-30	**5**	**5.00**	0.02	6	6.00	60.00	**5**	**5.00**	0.01	6	6.00	60.00
H9-30	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
SW-100-3-0d1-trial1	**61**	**61.00**	0.02	67	67.00	60.00	**61**	**61.00**	0.01	67	67.00	60.00
SW-100-3-0d2-trial1	**31**	**31.00**	0.01	36	36.00	60.00	**31**	**31.00**	< 0.01	36	36.00	60.00
SW-100-3-0d2-trial3	**31**	**31.00**	0.01	34	34.00	60.00	**31**	**31.00**	0.01	34	34.00	60.00
SW-100-4-0d1-trial1	**9**	**9.00**	0.15	11	11.00	60.00	**9**	**9.00**	0.11	11	11.00	60.00
SW-100-4-0d1-trial2	**8**	**8.20**	27.92	10	10.00	60.00	**8**	8.60	39.75	10	10.00	60.00
SW-100-4-0d1-trial3	**10**	**10.00**	0.10	13	13.00	60.00	**10**	**10.00**	0.06	13	13.00	60.00
SW-100-4-0d2-trial1	**8**	**8.60**	46.54	12	12.00	60.00	**8**	8.90	57.87	12	12.00	60.00
SW-100-4-0d2-trial2	**9**	**9.00**	< 0.01	10	10.00	60.00	**9**	**9.00**	< 0.01	10	10.00	60.00
SW-100-4-0d2-trial3	**9**	**9.00**	< 0.01	10	10.00	60.00	**9**	**9.00**	< 0.01	10	10.00	60.00
SW-100-4-0d3-trial1	**8**	**8.00**	2.16	10	10.00	60.00	**8**	**8.00**	1.57	10	10.00	60.00
SW-100-4-0d3-trial2	**8**	**8.00**	< 0.01	10	10.00	60.00	**8**	**8.00**	< 0.01	10	10.00	60.00
SW-100-4-0d3-trial3	**8**	**8.00**	1.61	10	10.00	60.00	**8**	**8.00**	2.21	10	10.00	60.00
SW-100-5-0d1-trial1	**9**	**9.00**	1.89	11	11.00	60.00	**9**	**9.00**	1.23	11	11.00	60.00
SW-100-5-0d1-trial2	**10**	**10.00**	0.04	13	13.00	60.00	**10**	**10.00**	0.05	13	13.00	60.00
SW-100-5-0d1-trial3	**12**	**12.00**	0.05	14	14.00	60.00	**12**	**12.00**	0.05	14	14.00	60.00
SW-100-5-0d2-trial1	**10**	**10.00**	< 0.01	12	12.00	60.00	**10**	**10.00**	0.01	12	12.00	60.00
SW-100-5-0d2-trial2	**10**	**10.00**	< 0.01	11	11.00	60.00	**10**	**10.00**	< 0.01	11	11.00	60.00
SW-100-5-0d2-trial3	**9**	**9.00**	0.01	11	11.00	60.00	**9**	**9.00**	0.01	11	11.00	60.00
SW-100-5-0d3-trial1	**8**	**8.00**	0.09	10	10.00	60.00	**8**	**8.00**	0.09	10	10.00	60.00
SW-100-5-0d3-trial2	**8**	**8.00**	< 0.01	9	9.00	60.00	**8**	**8.00**	< 0.01	9	9.00	60.00
SW-100-5-0d3-trial3	**8**	**8.00**	< 0.01	10	10.00	60.00	**8**	**8.00**	< 0.01	10	10.00	60.00
SW-100-6-0d1-trial1	**8**	**8.00**	< 0.01	10	10.00	60.00	**8**	**8.00**	< 0.01	10	10.00	60.00
SW-100-6-0d1-trial2	**8**	**8.00**	0.02	10	10.00	60.00	**8**	**8.00**	0.02	10	10.00	60.00
SW-100-6-0d1-trial3	**8**	**8.00**	0.01	10	10.00	60.00	**8**	**8.00**	0.01	10	10.00	60.00
SW-100-6-0d2-trial1	**7**	7.20	20.38	10	10.00	60.00	**7**	**7.00**	12.21	10	10.00	60.00
SW-100-6-0d2-trial2	**7**	**7.00**	0.30	9	9.00	60.00	**7**	**7.00**	0.45	9	9.00	60.00

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Table 3.8 – continued from previous page

Instance	BRKGA-FRFS			BRKGA-FRFS-1			BRKGA-FRFS-2			BRKGA-FRFS-3		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
SW-100-6-0d2-trial3	**7**	**7.00**	5.53	9	9.00	60.00	**7**	**7.00**	7.28	9	9.00	60.00
SW-100-6-0d3-trial1	**7**	**7.00**	1.13	9	9.00	60.00	**7**	**7.00**	1.22	9	9.00	60.00
SW-100-6-0d3-trial2	**7**	**7.00**	0.75	8	8.00	60.00	**7**	**7.00**	0.34	8	8.00	60.00
SW-100-6-0d3-trial3	**7**	**7.00**	0.07	8	8.00	60.00	**7**	**7.00**	0.12	8	8.00	60.00
SW-1000-3-0d2-trial1	**89**	**89.00**	3.98	105	105.00	60.00	**89**	**89.00**	3.59	105	105.00	60.00
SW-1000-3-0d2-trial2	**88**	**88.00**	2.40	100	100.00	60.00	**88**	**88.00**	2.57	100	100.00	60.00
SW-1000-3-0d3-trial2	**87**	**87.00**	2.24	114	114.00	60.00	**87**	**87.00**	2.42	114	114.00	60.00
SW-1000-4-0d1-trial1	**17**	**17.00**	10.96	21	21.00	60.00	**17**	**17.00**	7.08	21	21.00	60.00
SW-1000-4-0d1-trial2	**17**	**17.80**	55.45	21	21.00	60.00	18	18.00	60.00	21	21.00	60.00
SW-1000-4-0d1-trial3	**18**	**18.00**	5.26	20	20.00	60.00	**18**	**18.00**	6.26	20	20.00	60.00
SW-1000-4-0d2-trial1	**14**	**14.50**	38.67	16	16.00	60.00	**14**	**14.50**	44.56	16	16.00	60.00
SW-1000-4-0d2-trial2	**15**	**15.00**	0.06	17	17.00	60.00	**15**	**15.00**	0.05	17	17.00	60.00
SW-1000-4-0d2-trial3	**15**	**15.20**	25.90	18	18.00	60.00	**15**	15.30	26.95	18	18.00	60.00
SW-1000-4-0d3-trial1	**13**	**13.00**	1.20	14	14.00	60.00	**13**	**13.00**	0.36	14	14.00	60.00
SW-1000-4-0d3-trial3	**13**	**13.00**	0.15	15	15.00	60.00	**13**	**13.00**	0.28	15	15.00	60.00
SW-1000-5-0d1-trial1	**17**	17.50	35.43	22	22.00	60.00	**17**	**17.40**	35.97	22	22.00	60.00
SW-1000-5-0d1-trial2	**17**	**17.10**	29.33	20	20.00	60.00	**17**	17.30	31.95	20	20.00	60.00
SW-1000-5-0d1-trial3	**16**	**16.00**	0.92	18	18.00	60.00	**16**	**16.00**	1.84	18	18.00	60.00
SW-1000-5-0d2-trial1	**15**	**15.00**	0.10	17	17.00	60.00	**15**	**15.00**	0.05	17	17.00	60.00
SW-1000-5-0d2-trial2	**14**	**14.90**	56.68	16	16.00	60.00	**14**	**14.90**	57.70	16	16.00	60.00
SW-1000-5-0d2-trial3	**14**	**14.00**	1.38	17	17.00	60.00	**14**	**14.00**	0.72	17	17.00	60.00
SW-1000-5-0d3-trial1	**13**	13.90	59.48	15	15.00	60.00	**13**	**13.80**	49.43	15	15.00	60.00
SW-1000-5-0d3-trial2	**13**	13.20	28.20	15	15.00	60.00	**13**	**13.00**	29.11	15	15.00	60.00
SW-1000-5-0d3-trial3	**14**	**14.00**	0.03	15	15.00	60.00	**14**	**14.00**	0.02	15	15.00	60.00
SW-1000-6-0d1-trial1	**14**	**14.30**	34.73	17	17.00	60.00	**14**	14.40	36.50	17	17.00	60.00
SW-1000-6-0d1-trial2	**14**	**14.00**	0.03	16	16.00	60.00	**14**	**14.00**	0.04	16	16.00	60.00
SW-1000-6-0d1-trial3	**13**	**13.00**	15.11	14	14.00	60.00	**13**	**13.00**	7.39	14	14.00	60.00
SW-1000-6-0d2-trial1	**12**	**12.00**	0.09	13	13.00	60.00	**12**	**12.00**	0.07	13	13.00	60.00
SW-1000-6-0d2-trial2	**13**	**13.00**	< 0.01	14	14.00	60.00	**13**	**13.00**	< 0.01	14	14.00	60.00
SW-1000-6-0d2-trial3	**12**	**12.00**	0.35	13	13.00	60.00	**12**	**12.00**	0.31	13	13.00	60.00
SW-1000-6-0d3-trial1	**12**	**12.00**	< 0.01	13	13.00	60.00	**12**	**12.00**	< 0.01	13	13.00	60.00
SW-1000-6-0d3-trial2	**12**	**12.00**	< 0.01	13	13.00	60.00	**12**	**12.00**	< 0.01	13	13.00	60.00
SW-1000-6-0d3-trial3	**12**	**12.00**	< 0.01	13	13.00	60.00	**12**	**12.00**	< 0.01	13	13.00	60.00
Avg. t (s)	7.22			50.27			7.40			50.27		
# Best (# Best Avg.)	110 (104)			18 (18)			109 (103)			18 (18)		

3.3.4 Experiments for comparison of BP-0, BP-2, FRFS-0, and FRFS-2

Table 3.9 exhibits a comparison between the BP-0, BP-2, FRFS-0, and FRFS-2 decoders in Harary Graphs, Small-world graphs, and synthetic instances. The comparison of decoders using the following same criteria of previous sections. Table 3.9 indicates that FRFS decoders presented the best behavior when compared to BP decoders. Additionally, there is almost a technical tie between FRFS-0 and FRFS-2.

Table 3.9: Comparison BP-0, FRFS-0, BP-2, and FRFS-2

Instance	BP-0			FRFS-0			BP-2			FRFS-2		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
BT05-RG050	**5**	5.30	27.06	**5**	5.30	26.97	**5**	**5.00**	12.29	**5**	5.40	29.81
BT05-RG075	**5**	**5.00**	2.53	**5**	**5.00**	2.33	**5**	**5.00**	3.18	**5**	**5.00**	1.85
BT05-RG100	**5**	**5.00**	0.04	**5**	**5.00**	0.03	**5**	**5.00**	0.04	**5**	**5.00**	0.04
BT05-RG150	**5**	**5.00**	0.03	**5**	**5.00**	0.02	**5**	**5.00**	0.02	**5**	**5.00**	0.01
BT05-RG200	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01
BT05-RG250	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01
BT06-RG050	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01
BT06-RG075	7	7.00	60.00	7	7.00	60.00	**6**	**6.90**	58.52	7	7.00	60.00
BT06-RG100	**6**	**6.00**	4.81	**6**	**6.00**	3.45	**6**	**6.00**	4.32	**6**	**6.00**	3.22
BT06-RG150	**6**	**6.00**	0.02	**6**	**6.00**	0.01	**6**	**6.00**	0.02	**6**	**6.00**	0.01
BT06-RG200	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01
BT06-RG250	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01
BT07-RG050	**8**	**8.00**	<0.01	**8**	**8.00**	<0.01	**8**	**8.00**	<0.01	**8**	**8.00**	<0.01
BT07-RG075	8	8.00	60.00	**7**	7.90	58.00	**7**	**7.80**	55.17	**7**	7.90	55.73
BT07-RG100	**7**	**7.00**	2.78	**7**	**7.00**	2.48	**7**	**7.00**	2.30	**7**	**7.00**	1.80
BT07-RG150	**7**	**7.00**	0.07	**7**	**7.00**	0.05	**7**	**7.00**	0.08	**7**	**7.00**	0.07
BT07-RG200	**7**	**7.00**	0.01	**7**	**7.00**	0.01	**7**	**7.00**	0.01	**7**	**7.00**	0.01
BT07-RG250	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01
BT08-RG050	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01
BT08-RG075	9	9.00	60.00	**8**	**8.80**	54.27	**8**	8.90	56.17	9	9.00	60.00
BT08-RG100	**8**	**8.00**	8.44	**8**	8.10	27.02	**8**	8.10	10.46	**8**	**8.00**	27.70
BT08-RG150	**8**	**8.00**	0.31	**8**	**8.00**	0.12	**8**	**8.00**	0.26	**8**	**8.00**	0.18
BT08-RG200	**8**	**8.00**	0.02	**8**	**8.00**	0.03	**8**	**8.00**	0.01	**8**	**8.00**	0.02
BT08-RG250	**8**	**8.00**	<0.01	**8**	**8.00**	0.01	**8**	**8.00**	<0.01	**8**	**8.00**	0.01
BT09-RG050	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01
BT09-RG075	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01
BT09-RG100	**9**	9.50	45.48	**9**	9.70	44.03	**9**	**9.40**	39.79	**9**	9.50	47.94
BT09-RG150	**9**	**9.00**	1.18	**9**	**9.00**	1.13	**9**	**9.00**	0.40	**9**	**9.00**	0.72
BT09-RG200	**9**	**9.00**	0.07	**9**	**9.00**	0.08	**9**	**9.00**	0.04	**9**	**9.00**	0.09
BT09-RG250	**9**	**9.00**	0.02	**9**	**9.00**	0.02	**9**	**9.00**	0.03	**9**	**9.00**	0.03
BT10-RG050	**11**	**11.00**	<0.01	**11**	**11.00**	<0.01	**11**	**11.00**	<0.01	**11**	**11.00**	<0.01
BT10-RG075	**11**	**11.00**	<0.01	**11**	**11.00**	<0.01	**11**	**11.00**	<0.01	**11**	**11.00**	<0.01
BT10-RG100	**10**	**10.90**	58.50	11	11.00	60.00	11	11.00	60.00	**10**	**10.90**	59.73
BT10-RG150	**10**	**10.00**	3.28	**10**	**10.00**	4.18	**10**	**10.00**	3.85	**10**	**10.00**	2.29
BT10-RG200	**10**	**10.00**	0.19	**10**	**10.00**	0.19	**10**	**10.00**	0.18	**10**	**10.00**	0.35
BT10-RG250	**10**	**10.00**	0.10	**10**	**10.00**	0.08	**10**	**10.00**	0.08	**10**	**10.00**	0.05
H10-30	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01
H11-50	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01
H20-50	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01
H21-50	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01
H2-100	**50**	**50.00**	<0.01	**50**	**50.00**	<0.01	**50**	**50.00**	<0.01	**50**	**50.00**	<0.01
H2-17	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01
H2-30	**15**	**15.00**	<0.01	**15**	**15.00**	<0.01	**15**	**15.00**	<0.01	**15**	**15.00**	<0.01
H2-50	**25**	**25.00**	<0.01	**25**	**25.00**	<0.01	**25**	**25.00**	<0.01	**25**	**25.00**	<0.01
H3-17	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01
H3-30	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01
H3-50	**14**	**14.00**	<0.01	**14**	**14.00**	<0.01	**14**	**14.00**	<0.01	**14**	**14.00**	<0.01
H5-17	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01
H6-17	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01
H7-17	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01
H8-30	**5**	**5.00**	0.26	**5**	**5.00**	0.02	**5**	**5.00**	0.18	**5**	**5.00**	0.01
H9-30	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01

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Table 3.9 – continued from previous page

Instance	BP-0			FRFS-0			BP-2			FRFS-2		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
SW-100-3-0d1-trial1	**61**	**61.00**	0.03	**61**	**61.00**	0.02	**61**	**61.00**	0.02	**61**	**61.00**	0.01
SW-100-3-0d2-trial1	**31**	**31.00**	0.01	**31**	**31.00**	0.01	**31**	**31.00**	0.01	**31**	**31.00**	< 0.01
SW-100-3-0d2-trial3	**31**	**31.00**	0.01	**31**	**31.00**	0.01	**31**	**31.00**	0.01	**31**	**31.00**	0.01
SW-100-4-0d1-trial1	**9**	**9.00**	3.87	**9**	**9.00**	0.15	**9**	**9.00**	1.85	**9**	**9.00**	0.11
SW-100-4-0d1-trial2	**8**	8.80	51.08	**8**	**8.20**	27.92	9	9.00	60.00	**8**	8.60	39.75
SW-100-4-0d1-trial3	**10**	**10.00**	0.10	**10**	**10.00**	0.10	**10**	**10.00**	0.34	**10**	**10.00**	0.06
SW-100-4-0d2-trial1	9	9.00	60.00	**8**	**8.60**	46.54	9	9.00	60.00	**8**	8.90	57.87
SW-100-4-0d2-trial2	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
SW-100-4-0d2-trial3	**9**	**9.00**	0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	0.01	**9**	**9.00**	< 0.01
SW-100-4-0d3-trial1	**8**	8.10	9.95	**8**	**8.00**	2.16	**8**	**8.00**	4.66	**8**	**8.00**	1.57
SW-100-4-0d3-trial2	**8**	**8.00**	0.02	**8**	**8.00**	< 0.01	**8**	**8.00**	0.02	**8**	**8.00**	< 0.01
SW-100-4-0d3-trial3	**8**	8.20	26.27	**8**	**8.00**	1.61	**8**	**8.00**	10.18	**8**	**8.00**	2.21
SW-100-5-0d1-trial1	**9**	9.20	15.47	**9**	**9.00**	1.89	**9**	9.20	18.43	**9**	**9.00**	1.23
SW-100-5-0d1-trial2	**10**	**10.00**	1.17	**10**	**10.00**	0.04	**10**	**10.00**	2.71	**10**	**10.00**	0.05
SW-100-5-0d1-trial3	**12**	**12.00**	0.24	**12**	**12.00**	0.05	**12**	**12.00**	0.20	**12**	**12.00**	0.05
SW-100-5-0d2-trial1	**10**	**10.00**	0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	0.01	**10**	**10.00**	0.01
SW-100-5-0d2-trial2	**10**	**10.00**	0.02	**10**	**10.00**	< 0.01	**10**	**10.00**	0.02	**10**	**10.00**	< 0.01
SW-100-5-0d2-trial3	**9**	**9.00**	0.03	**9**	**9.00**	0.01	**9**	**9.00**	0.03	**9**	**9.00**	0.01
SW-100-5-0d3-trial1	**8**	**8.00**	0.43	**8**	**8.00**	0.09	**8**	**8.00**	0.26	**8**	**8.00**	0.09
SW-100-5-0d3-trial2	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
SW-100-5-0d3-trial3	**8**	**8.00**	0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	0.02	**8**	**8.00**	< 0.01
SW-100-6-0d1-trial1	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
SW-100-6-0d1-trial2	**8**	**8.00**	0.16	**8**	**8.00**	0.02	**8**	**8.00**	0.24	**8**	**8.00**	0.02
SW-100-6-0d1-trial3	**8**	**8.00**	0.04	**8**	**8.00**	0.01	**8**	**8.00**	0.04	**8**	**8.00**	0.01
SW-100-6-0d2-trial1	**7**	7.80	52.07	**7**	7.20	20.38	**7**	7.20	40.67	**7**	**7.00**	12.21
SW-100-6-0d2-trial2	**7**	**7.00**	3.41	**7**	**7.00**	0.30	**7**	**7.00**	4.75	**7**	**7.00**	0.45
SW-100-6-0d2-trial3	**7**	7.60	45.90	**7**	**7.00**	5.53	**7**	7.60	45.02	**7**	**7.00**	7.28
SW-100-6-0d3-trial1	**7**	**7.00**	12.32	**7**	**7.00**	1.13	**7**	**7.00**	7.16	**7**	**7.00**	1.22
SW-100-6-0d3-trial2	**7**	**7.00**	4.01	**7**	**7.00**	0.75	**7**	**7.00**	3.14	**7**	**7.00**	0.34
SW-100-6-0d3-trial3	**7**	**7.00**	2.15	**7**	**7.00**	0.07	**7**	**7.00**	0.77	**7**	**7.00**	0.12
SW-1000-3-0d2-trial1	**89**	**89.00**	6.99	**89**	**89.00**	3.98	**89**	**89.00**	9.43	**89**	**89.00**	3.59
SW-1000-3-0d2-trial2	**88**	**88.00**	4.78	**88**	**88.00**	2.40	**88**	**88.00**	5.57	**88**	**88.00**	2.57
SW-1000-3-0d3-trial2	**87**	**87.00**	4.63	**87**	**87.00**	2.24	**87**	**87.00**	5.28	**87**	**87.00**	2.42
SW-1000-4-0d1-trial1	**17**	17.70	51.55	**17**	**17.00**	10.96	**17**	17.80	55.20	**17**	**17.00**	7.08
SW-1000-4-0d1-trial2	18	18.00	60.00	**17**	**17.80**	55.45	18	18.00	60.00	18	18.00	60.00
SW-1000-4-0d1-trial3	**18**	18.60	45.74	**18**	**18.00**	5.26	**18**	18.60	43.19	**18**	**18.00**	6.26
SW-1000-4-0d2-trial1	**14**	14.90	59.79	**14**	**14.50**	38.67	15	15.00	60.00	**14**	**14.50**	44.56
SW-1000-4-0d2-trial2	**15**	**15.00**	0.19	**15**	**15.00**	0.06	**15**	**15.00**	0.18	**15**	**15.00**	0.05
SW-1000-4-0d2-trial3	16	16.00	60.00	**15**	**15.20**	25.90	**15**	15.80	55.70	**15**	15.30	26.95
SW-1000-4-0d3-trial1	**13**	**13.00**	12.07	**13**	**13.00**	1.20	**13**	**13.00**	5.40	**13**	**13.00**	0.36
SW-1000-4-0d3-trial3	**13**	**13.00**	3.52	**13**	**13.00**	0.15	**13**	**13.00**	1.60	**13**	**13.00**	0.28
SW-1000-5-0d1-trial1	18	18.00	60.00	**17**	17.50	35.43	**17**	17.90	56.97	**17**	**17.40**	35.97
SW-1000-5-0d1-trial2	18	18.00	60.00	**17**	**17.10**	29.33	**17**	17.90	58.36	**17**	17.30	31.95
SW-1000-5-0d1-trial3	**16**	16.20	20.17	**16**	**16.00**	0.92	**16**	16.10	23.94	**16**	**16.00**	1.84
SW-1000-5-0d2-trial1	**15**	**15.00**	0.79	**15**	**15.00**	0.10	**15**	**15.00**	0.93	**15**	**15.00**	0.05
SW-1000-5-0d2-trial2	15	15.00	60.00	**14**	**14.90**	56.68	**14**	**14.90**	58.98	**14**	**14.90**	57.70
SW-1000-5-0d2-trial3	**14**	14.10	15.28	**14**	**14.00**	1.38	**14**	**14.00**	10.11	**14**	**14.00**	0.72
SW-1000-5-0d3-trial1	14	14.00	60.00	**13**	13.90	59.48	14	14.00	60.00	**13**	**13.80**	49.43
SW-1000-5-0d3-trial2	14	14.00	60.00	**13**	13.20	28.20	14	14.00	60.00	**13**	**13.00**	29.11
SW-1000-5-0d3-trial3	**14**	**14.00**	0.17	**14**	**14.00**	0.03	**14**	**14.00**	0.11	**14**	**14.00**	0.02
SW-1000-6-0d1-trial1	**14**	14.90	55.90	**14**	**14.30**	34.73	**14**	14.90	57.30	**14**	14.40	36.50
SW-1000-6-0d1-trial2	**14**	**14.00**	0.23	**14**	**14.00**	0.03	**14**	**14.00**	0.18	**14**	**14.00**	0.04
SW-1000-6-0d1-trial3	**13**	13.90	57.61	**13**	**13.00**	15.11	**13**	13.80	52.72	**13**	**13.00**	7.39

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Table 3.9 – continued from previous page

Instance	BP-0			FRFS-0			BP-2			FRFS-2		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
SW-1000-6-0d2-trial1	**12**	**12.00**	1.78	**12**	**12.00**	0.09	**12**	**12.00**	1.07	**12**	**12.00**	0.07
SW-1000-6-0d2-trial2	**13**	**13.00**	0.01	**13**	**13.00**	<0.01	**13**	**13.00**	0.01	**13**	**13.00**	<0.01
SW-1000-6-0d2-trial3	**12**	**12.00**	5.70	**12**	**12.00**	0.35	**12**	**12.00**	4.73	**12**	**12.00**	0.31
SW-1000-6-0d3-trial1	**12**	**12.00**	<0.01	**12**	**12.00**	<0.01	**12**	**12.00**	<0.01	**12**	**12.00**	<0.01
SW-1000-6-0d3-trial2	**12**	**12.00**	<0.01	**12**	**12.00**	<0.01	**12**	**12.00**	<0.01	**12**	**12.00**	<0.01
SW-1000-6-0d3-trial3	**12**	**12.00**	<0.01	**12**	**12.00**	<0.01	**12**	**12.00**	<0.01	**12**	**12.00**	<0.01
Avg. t (s)		12.53			7.76			11.85			7.94	
# Best (# Best Avg.)		100 (85)			109 (101)			104 (91)			108 (100)	

3.3.5 Experiments for comparison of BRKGA-BP-SCHAs

Table 3.10 presents a comparison between the BP-SCHA decoders in Harary Graphs, Small-world graphs and synthetic instances. The decoders are compared using the following same criteria of previous section. Note that, the results of best solution, average solution and computational time have been improved compared to BP decoders without the SCHA refinement.

Table 3.10: Decoders BP-SCHAs

Instance	BP-0-SCHA			BP-1-SCHA			BP-2-SCHA			BP-3-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
BT05-RG050	**5**	**5.00**	5.16	6	6.00	60.00	**5**	**5.00**	3.32	6	6.00	60.00
BT05-RG075	**5**	**5.00**	1.13	6	6.00	60.00	**5**	**5.00**	1.92	6	6.00	60.00
BT05-RG100	**5**	**5.00**	0.07	**5**	**5.00**	2.15	**5**	**5.00**	0.04	**5**	**5.00**	1.56
BT05-RG150	**5**	**5.00**	0.02	**5**	**5.00**	1.35	**5**	**5.00**	0.02	**5**	**5.00**	1.86
BT05-RG200	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01
BT05-RG250	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01	**5**	**5.00**	<0.01
BT06-RG050	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01
BT06-RG075	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01
BT06-RG100	**6**	**6.00**	5.06	**6**	6.30	35.34	**6**	**6.00**	3.81	**6**	6.80	54.22
BT06-RG150	**6**	**6.00**	0.03	**6**	**6.00**	0.25	**6**	**6.00**	0.03	**6**	**6.00**	0.17
BT06-RG200	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01
BT06-RG250	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01	**6**	**6.00**	<0.01
BT07-RG050	**8**	**8.00**	<0.01	**8**	**8.00**	<0.01	**8**	**8.00**	<0.01	**8**	**8.00**	<0.01
BT07-RG075	8	8.00	60.00	8	8.00	60.00	**7**	**7.90**	57.97	8	8.00	60.00
BT07-RG100	**7**	**7.00**	4.87	**7**	7.20	26.56	**7**	**7.00**	3.89	**7**	7.10	26.03
BT07-RG150	**7**	**7.00**	0.11	**7**	**7.00**	0.49	**7**	**7.00**	0.13	**7**	**7.00**	0.81
BT07-RG200	**7**	**7.00**	0.01	**7**	**7.00**	0.01	**7**	**7.00**	0.01	**7**	**7.00**	0.01
BT07-RG250	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01	**7**	**7.00**	<0.01
BT08-RG050	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01	**9**	**9.00**	<0.01
BT08-RG075	9	9.00	60.00	9	9.00	60.00	**8**	**8.90**	57.47	9	9.00	60.00
BT08-RG100	**8**	**8.00**	13.36	**8**	8.60	47.11	**8**	8.10	12.97	9	9.00	60.00
BT08-RG150	**8**	**8.00**	0.45	**8**	**8.00**	0.38	**8**	**8.00**	0.38	**8**	**8.00**	0.55
BT08-RG200	**8**	**8.00**	0.03	**8**	**8.00**	0.03	**8**	**8.00**	0.02	**8**	**8.00**	0.04
BT08-RG250	**8**	**8.00**	0.01	**8**	**8.00**	0.02	**8**	**8.00**	0.01	**8**	**8.00**	0.02
BT09-RG050	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01
BT09-RG075	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01	**10**	**10.00**	<0.01
BT09-RG100	**9**	**9.60**	50.19	**9**	**9.60**	50.97	**9**	**9.60**	45.04	**9**	9.80	53.53
BT09-RG150	**9**	**9.00**	1.54	**9**	**9.00**	0.75	**9**	**9.00**	0.59	**9**	**9.00**	1.10

Continued on next page

Table 3.10 – continued from previous page

Instance	BP-0-SCHA			BP-1-SCHA			BP-2-SCHA			BP-3-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
BT09-RG200	**9**	**9.00**	0.09	**9**	**9.00**	0.11	**9**	**9.00**	0.06	**9**	**9.00**	0.14
BT09-RG250	**9**	**9.00**	0.02	**9**	**9.00**	0.02	**9**	**9.00**	0.04	**9**	**9.00**	0.02
BT10-RG050	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01
BT10-RG075	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01
BT10-RG100	**10**	**10.90**	59.53	11	11.00	60.00	11	11.00	60.00	11	11.00	60.00
BT10-RG150	**10**	**10.00**	3.82	**10**	**10.00**	11.24	**10**	**10.00**	5.34	**10**	**10.00**	2.20
BT10-RG200	**10**	**10.00**	0.21	**10**	**10.00**	0.67	**10**	**10.00**	0.24	**10**	**10.00**	0.75
BT10-RG250	**10**	**10.00**	0.11	**10**	**10.00**	0.10	**10**	**10.00**	0.11	**10**	**10.00**	0.11
H10-30	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
H11-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H20-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H21-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H2-100	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01
H2-17	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H2-30	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01
H2-50	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01
H3-17	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
H3-30	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H3-50	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01
H5-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H6-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H7-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H8-30	**5**	**5.00**	1.72	6	6.00	60.00	**5**	**5.00**	0.63	6	6.00	60.00
H9-30	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
SW-100-3-0d1-trial1	**61**	**61.00**	< 0.01	**61**	**61.00**	< 0.01	**61**	**61.00**	< 0.01	**61**	**61.00**	< 0.01
SW-100-3-0d2-trial1	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01
SW-100-3-0d2-trial3	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01
SW-100-4-0d1-trial1	**9**	**9.00**	0.13	10	10.00	60.00	**9**	**9.00**	0.20	10	10.00	60.00
SW-100-4-0d1-trial2	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
SW-100-4-0d1-trial3	**10**	**10.00**	0.23	11	11.00	60.00	**10**	**10.00**	0.18	11	11.00	60.00
SW-100-4-0d2-trial1	**8**	**8.90**	54.04	9	9.00	60.00	9	9.00	60.00	9	9.00	60.00
SW-100-4-0d2-trial2	**9**	**9.00**	< 0.01	**9**	**9.00**	0.04	**9**	**9.00**	< 0.01	**9**	**9.00**	0.02
SW-100-4-0d2-trial3	**9**	**9.00**	< 0.01	**9**	**9.00**	1.49	**9**	**9.00**	< 0.01	**9**	**9.00**	2.87
SW-100-4-0d3-trial1	**8**	**8.00**	8.08	9	9.00	60.00	**8**	**8.00**	11.43	9	9.00	60.00
SW-100-4-0d3-trial2	**8**	**8.00**	< 0.01	9	9.00	60.00	**8**	**8.00**	< 0.01	9	9.00	60.00
SW-100-4-0d3-trial3	**8**	**8.00**	7.51	9	9.00	60.00	**8**	**8.00**	14.29	9	9.00	60.00
SW-100-5-0d1-trial1	**9**	**9.00**	1.09	10	10.00	60.00	**9**	**9.00**	0.88	10	10.00	60.00
SW-100-5-0d1-trial2	**10**	**10.00**	0.07	11	11.00	60.00	**10**	**10.00**	0.09	11	11.00	60.00
SW-100-5-0d1-trial3	**12**	**12.00**	0.02	13	13.00	60.00	**12**	**12.00**	0.01	13	13.00	60.00
SW-100-5-0d2-trial1	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01
SW-100-5-0d2-trial2	**9**	**9.90**	56.23	10	10.00	60.00	10	10.00	60.00	10	10.00	60.00
SW-100-5-0d2-trial3	**9**	**9.00**	0.01	10	10.00	60.00	**9**	**9.00**	0.01	10	10.00	60.00
SW-100-5-0d3-trial1	**8**	**8.00**	0.18	**8**	8.50	36.66	**8**	**8.00**	0.16	**8**	8.40	37.51
SW-100-5-0d3-trial2	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
SW-100-5-0d3-trial3	**8**	**8.00**	< 0.01	**8**	**8.00**	0.08	**8**	**8.00**	< 0.01	**8**	**8.00**	0.07
SW-100-6-0d1-trial1	**8**	**8.00**	< 0.01	**8**	8.10	23.13	**8**	**8.00**	< 0.01	**8**	8.20	32.00
SW-100-6-0d1-trial2	**8**	**8.00**	0.12	**8**	8.90	59.29	**8**	**8.00**	0.15	9	9.00	60.00
SW-100-6-0d1-trial3	**8**	**8.00**	0.01	**8**	**8.00**	0.42	**8**	**8.00**	0.01	**8**	**8.00**	0.39
SW-100-6-0d2-trial1	**7**	**7.60**	47.60	8	8.00	60.00	**7**	7.90	58.18	8	8.00	60.00
SW-100-6-0d2-trial2	**7**	**7.00**	6.29	8	8.00	60.00	**7**	**7.00**	5.16	8	8.00	60.00
SW-100-6-0d2-trial3	**7**	7.70	54.48	8	8.00	60.00	**7**	**7.30**	38.60	8	8.00	60.00
SW-100-6-0d3-trial1	**7**	7.20	20.86	8	8.00	60.00	**7**	**7.00**	12.67	8	8.00	60.00
SW-100-6-0d3-trial2	**7**	**7.00**	8.34	8	8.00	60.00	**7**	**7.00**	3.61	8	8.00	60.00

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Table 3.10 – continued from previous page

Instance	BP-0-SCHA			BP-1-SCHA			BP-2-SCHA			BP-3-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
SW-100-6-0d3-trial3	**7**	**7.00**	1.95	8	8.00	60.00	**7**	**7.00**	1.07	8	8.00	60.00
SW-1000-3-0d2-trial1	**89**	**89.00**	0.01	**89**	**89.00**	0.01	**89**	**89.00**	0.01	**89**	**89.00**	< 0.01
SW-1000-3-0d2-trial2	**88**	**88.00**	0.01	**88**	**88.00**	< 0.01	**88**	**88.00**	0.01	**88**	**88.00**	0.01
SW-1000-3-0d3-trial2	**87**	**87.00**	0.01	**87**	**87.00**	0.01	**87**	**87.00**	0.01	**87**	**87.00**	0.01
SW-1000-4-0d1-trial1	**17**	**17.00**	0.55	18	18.00	60.00	**17**	**17.00**	0.40	18	18.00	60.00
SW-1000-4-0d1-trial2	**17**	**17.70**	49.42	19	19.00	60.00	**17**	**17.70**	47.98	19	19.00	60.00
SW-1000-4-0d1-trial3	**18**	**18.00**	0.50	**18**	**18.00**	2.13	**18**	**18.00**	0.56	**18**	**18.00**	5.45
SW-1000-4-0d2-trial1	**14**	**14.00**	0.60	15	15.00	60.00	**14**	**14.00**	0.65	15	15.00	60.00
SW-1000-4-0d2-trial2	**14**	**14.00**	12.72	15	15.00	60.00	**14**	14.10	27.06	15	15.00	60.00
SW-1000-4-0d2-trial3	**15**	**15.00**	0.15	16	16.00	60.00	**15**	**15.00**	0.23	16	16.00	60.00
SW-1000-4-0d3-trial1	**13**	**13.00**	0.02	**13**	**13.00**	0.49	**13**	**13.00**	0.02	**13**	**13.00**	0.38
SW-1000-4-0d3-trial3	**13**	**13.00**	0.03	14	14.00	60.00	**13**	**13.00**	0.03	14	14.00	60.00
SW-1000-5-0d1-trial1	**17**	**17.00**	3.88	18	18.00	60.00	**17**	**17.00**	4.06	18	18.00	60.00
SW-1000-5-0d1-trial2	**17**	**17.00**	3.21	18	18.00	60.00	**17**	**17.00**	3.53	18	18.00	60.00
SW-1000-5-0d1-trial3	**15**	**15.60**	46.39	16	16.00	60.00	**15**	15.80	52.09	16	16.00	60.00
SW-1000-5-0d2-trial1	**15**	**15.00**	0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	0.01	**15**	**15.00**	< 0.01
SW-1000-5-0d2-trial2	**14**	**14.00**	11.45	**14**	14.60	48.61	**14**	**14.00**	8.03	**14**	14.80	53.48
SW-1000-5-0d2-trial3	**14**	**14.00**	0.04	15	15.00	60.00	**14**	**14.00**	0.03	15	15.00	60.00
SW-1000-5-0d3-trial1	**13**	**13.00**	2.35	14	14.00	60.00	**13**	**13.00**	1.89	14	14.00	60.00
SW-1000-5-0d3-trial2	**13**	**13.00**	0.61	**13**	**13.00**	11.35	**13**	**13.00**	0.41	**13**	**13.00**	10.11
SW-1000-5-0d3-trial3	**13**	**13.90**	54.71	14	14.00	60.00	**13**	**13.90**	54.51	14	14.00	60.00
SW-1000-6-0d1-trial1	**14**	**14.00**	10.82	15	15.00	60.00	**14**	**14.00**	8.05	15	15.10	60.00
SW-1000-6-0d1-trial2	14	14.00	60.00	14	14.00	60.00	**13**	**13.90**	54.22	14	14.00	60.00
SW-1000-6-0d1-trial3	**13**	**13.00**	4.35	14	14.00	60.00	**13**	**13.00**	1.82	14	14.00	60.00
SW-1000-6-0d2-trial1	**12**	**12.00**	0.17	13	13.00	60.00	**12**	**12.00**	0.15	13	13.00	60.00
SW-1000-6-0d2-trial2	**13**	**13.00**	0.01	**13**	**13.00**	0.40	**13**	**13.00**	0.01	**13**	**13.00**	0.71
SW-1000-6-0d2-trial3	**12**	**12.00**	0.58	13	13.00	60.00	**12**	**12.00**	0.40	13	13.00	60.00
SW-1000-6-0d3-trial1	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01
SW-1000-6-0d3-trial2	**12**	**12.00**	< 0.01	**12**	**12.00**	8.46	**12**	**12.00**	< 0.01	**12**	**12.00**	3.65
SW-1000-6-0d3-trial3	**12**	**12.00**	< 0.01	**12**	**12.00**	0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	0.01
Avg. t (s)	7.18			26.04			7.09			26.39		
# Best (# Best Avg.)	108 (106)			69 (62)			108 (104)			67 (61)		

3.3.6 Experiments for comparison of BRKGA-FRFS-SCHAs

Table 3.10 presents a comparison between the BP-SCHA decoders in Harary Graphs, Small-world graphs and synthetic instances. The decoders are compared using the following same criteria of previous secitons. Note that the results of best solution, average solution and computational time for FRFS-0-SCHA and FRFS-2-SCHA have been improved compared to FRFS decoders without the SCHA refinement.

Table 3.11: Decoders FRFS-SCHAs

Instance	FRFS-0-SCHA			FRFS-1-SCHA			FRFS-2-SCHA			FRFS-3-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
BT05-RG050	**5**	5.10	12.35	6	6.00	60.00	**5**	**5.00**	21.43	6	6.00	60.00
BT05-RG075	**5**	**5.00**	2.60	6	6.00	60.00	**5**	**5.00**	2.22	6	6.00	60.00
BT05-RG100	**5**	**5.00**	0.02	6	6.00	60.00	**5**	**5.00**	0.02	6	6.00	60.00

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Table 3.11 – continued from previous page

Instance	FRFS-0-SCHA			FRFS-1-SCHA			FRFS-2-SCHA			FRFS-3-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
BT05-RG150	**5**	**5.00**	0.01	6	6.00	60.00	**5**	**5.00**	0.02	6	6.00	60.00
BT05-RG200	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
BT05-RG250	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
BT06-RG050	**7**	**7.00**	< 0.01	8	8.00	60.00	**7**	**7.00**	< 0.01	8	8.00	60.00
BT06-RG075	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01
BT06-RG100	**6**	**6.00**	4.23	7	7.00	60.00	**6**	**6.00**	4.64	7	7.00	60.00
BT06-RG150	**6**	**6.00**	0.02	7	7.00	60.00	**6**	**6.00**	0.01	7	7.00	60.00
BT06-RG200	**6**	**6.00**	0.01	7	7.00	60.00	**6**	**6.00**	< 0.01	7	7.00	60.00
BT06-RG250	**6**	**6.00**	< 0.01	7	7.00	60.00	**6**	**6.00**	< 0.01	7	7.00	60.00
BT07-RG050	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
BT07-RG075	8	8.00	60.00	8	8.00	60.00	**7**	**7.90**	57.36	8	8.00	60.00
BT07-RG100	**7**	**7.00**	4.76	8	8.00	60.00	**7**	**7.00**	3.36	8	8.00	60.00
BT07-RG150	**7**	**7.00**	0.09	8	8.00	60.00	**7**	**7.00**	0.11	8	8.00	60.00
BT07-RG200	**7**	**7.00**	0.01	8	8.00	60.00	**7**	**7.00**	0.01	8	8.00	60.00
BT07-RG250	**7**	**7.00**	< 0.01	8	8.00	60.00	**7**	**7.00**	< 0.01	8	8.00	60.00
BT08-RG050	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
BT08-RG075	**8**	**8.90**	56.97	9	9.00	60.00	9	9.00	60.00	9	9.00	60.00
BT08-RG100	**8**	**8.30**	35.68	9	9.00	60.00	**8**	8.40	39.86	9	9.00	60.00
BT08-RG150	**8**	**8.00**	0.18	9	9.00	60.00	**8**	**8.00**	0.29	9	9.00	60.00
BT08-RG200	**8**	**8.00**	0.04	9	9.00	60.00	**8**	**8.00**	0.02	9	9.00	60.00
BT08-RG250	**8**	**8.00**	0.01	9	9.00	60.00	**8**	**8.00**	0.01	9	9.00	60.00
BT09-RG050	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01
BT09-RG075	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01
BT09-RG100	**9**	**9.70**	44.93	10	10.00	60.00	**9**	9.80	53.50	10	10.00	60.00
BT09-RG150	**9**	**9.00**	1.49	10	10.00	60.00	**9**	**9.00**	1.05	10	10.00	60.00
BT09-RG200	**9**	**9.00**	0.09	10	10.00	60.00	**9**	**9.00**	0.13	10	10.00	60.00
BT09-RG250	**9**	**9.00**	0.03	10	10.00	60.00	**9**	**9.00**	0.04	10	10.00	60.00
BT10-RG050	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01
BT10-RG075	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01
BT10-RG100	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01
BT10-RG150	**10**	**10.00**	4.90	11	11.00	60.00	**10**	**10.00**	3.19	11	11.00	60.00
BT10-RG200	**10**	**10.00**	0.22	11	11.00	60.00	**10**	**10.00**	0.48	11	11.00	60.00
BT10-RG250	**10**	**10.00**	0.08	11	11.00	60.00	**10**	**10.00**	0.06	11	11.00	60.00
H10-30	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
H11-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H20-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H21-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H2-100	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01
H2-17	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H2-30	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01
H2-50	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01
H3-17	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
H3-30	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H3-50	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01
H5-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H6-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H7-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H8-30	**5**	**5.00**	0.03	6	6.00	60.00	**5**	**5.00**	0.01	6	6.00	60.00
H9-30	**5**	**5.00**	< 0.01	6	6.00	60.00	**5**	**5.00**	< 0.01	6	6.00	60.00
SW-100-3-0d1-trial1	**61**	**61.00**	< 0.01	**61**	**61.00**	< 0.01	**61**	**61.00**	< 0.01	**61**	**61.00**	< 0.01
SW-100-3-0d2-trial1	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01
SW-100-3-0d2-trial3	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01
SW-100-4-0d1-trial1	**9**	**9.00**	0.01	11	11.00	60.00	**9**	**9.00**	0.01	11	11.00	60.00

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Table 3.11 – continued from previous page

Instance	FRFS-0-SCHA			FRFS-1-SCHA			FRFS-2-SCHA			FRFS-3-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
SW-100-4-0d1-trial2	**8**	**8.00**	2.62	10	10.00	60.00	**8**	**8.00**	2.08	10	10.00	60.00
SW-100-4-0d1-trial3	**10**	**10.00**	0.01	11	11.00	60.00	**10**	**10.00**	0.01	11	11.00	60.00
SW-100-4-0d2-trial1	**8**	8.50	43.92	10	10.00	60.00	**8**	**8.30**	33.73	10	10.00	60.00
SW-100-4-0d2-trial2	**9**	**9.00**	< 0.01	10	10.00	60.00	**9**	**9.00**	< 0.01	10	10.00	60.00
SW-100-4-0d2-trial3	**9**	**9.00**	< 0.01	10	10.00	60.00	**9**	**9.00**	< 0.01	10	10.00	60.00
SW-100-4-0d3-trial1	**8**	**8.00**	0.95	10	10.00	60.00	**8**	**8.00**	0.82	10	10.00	60.00
SW-100-4-0d3-trial2	**8**	**8.00**	< 0.01	9	9.00	60.00	**8**	**8.00**	< 0.01	9	9.00	60.00
SW-100-4-0d3-trial3	**8**	**8.00**	0.25	9	9.00	60.00	**8**	**8.00**	0.36	9	9.00	60.00
SW-100-5-0d1-trial1	**9**	**9.00**	0.11	11	11.00	60.00	**9**	**9.00**	0.06	11	11.00	60.00
SW-100-5-0d1-trial2	**10**	**10.00**	< 0.01	12	12.00	60.00	**10**	**10.00**	< 0.01	12	12.00	60.00
SW-100-5-0d1-trial3	**12**	**12.00**	< 0.01	13	13.00	60.00	**12**	**12.00**	< 0.01	13	13.00	60.00
SW-100-5-0d2-trial1	**10**	**10.00**	< 0.01	11	11.00	60.00	**10**	**10.00**	< 0.01	11	11.00	60.00
SW-100-5-0d2-trial2	**9**	**9.90**	55.94	10	10.00	60.00	10	10.00	60.00	10	10.00	60.00
SW-100-5-0d2-trial3	**9**	**9.00**	< 0.01	10	10.00	60.00	**9**	**9.00**	< 0.01	10	10.00	60.00
SW-100-5-0d3-trial1	**8**	**8.00**	0.02	9	9.00	60.00	**8**	**8.00**	0.01	9	9.00	60.00
SW-100-5-0d3-trial2	**8**	**8.00**	< 0.01	9	9.00	60.00	**8**	**8.00**	< 0.01	9	9.00	60.00
SW-100-5-0d3-trial3	**8**	**8.00**	< 0.01	10	10.00	60.00	**8**	**8.00**	< 0.01	10	10.00	60.00
SW-100-6-0d1-trial1	**8**	**8.00**	< 0.01	10	10.00	60.00	**8**	**8.00**	< 0.01	10	10.00	60.00
SW-100-6-0d1-trial2	**8**	**8.00**	0.01	10	10.00	60.00	**8**	**8.00**	< 0.01	10	10.00	60.00
SW-100-6-0d1-trial3	**8**	**8.00**	< 0.01	10	10.00	60.00	**8**	**8.00**	< 0.01	10	10.00	60.00
SW-100-6-0d2-trial1	**7**	**7.10**	16.68	9	9.00	60.00	**7**	**7.10**	20.26	9	9.00	60.00
SW-100-6-0d2-trial2	**7**	**7.00**	0.31	9	9.00	60.00	**7**	**7.00**	0.22	9	9.00	60.00
SW-100-6-0d2-trial3	**7**	**7.00**	4.93	9	9.00	60.00	**7**	**7.00**	8.78	9	9.00	60.00
SW-100-6-0d3-trial1	**7**	**7.00**	1.35	9	9.00	60.00	**7**	**7.00**	1.85	9	9.00	60.00
SW-100-6-0d3-trial2	**7**	**7.00**	0.49	8	8.00	60.00	**7**	**7.00**	0.43	8	8.00	60.00
SW-100-6-0d3-trial3	**7**	**7.00**	0.05	8	8.00	60.00	**7**	**7.00**	0.08	8	8.00	60.00
SW-1000-3-0d2-trial1	**89**	**89.00**	0.01	**89**	**89.00**	< 0.01	**89**	**89.00**	< 0.01	**89**	**89.00**	< 0.01
SW-1000-3-0d2-trial2	**88**	**88.00**	< 0.01	**88**	**88.00**	< 0.01	**88**	**88.00**	< 0.01	**88**	**88.00**	< 0.01
SW-1000-3-0d3-trial2	**87**	**87.00**	< 0.01	**87**	**87.00**	< 0.01	**87**	**87.00**	< 0.01	**87**	**87.00**	< 0.01
SW-1000-4-0d1-trial1	**16**	**16.60**	46.55	18	18.00	60.00	**16**	16.80	51.20	18	18.00	60.00
SW-1000-4-0d1-trial2	**17**	**17.00**	0.38	19	19.00	60.00	**17**	**17.00**	0.19	19	19.00	60.00
SW-1000-4-0d1-trial3	**17**	17.50	43.10	19	19.00	60.00	**17**	**17.40**	43.24	19	19.00	60.00
SW-1000-4-0d2-trial1	**14**	**14.00**	0.01	15	15.00	60.00	**14**	**14.00**	0.02	15	15.00	60.00
SW-1000-4-0d2-trial2	**14**	**14.00**	0.27	15	15.00	60.00	**14**	**14.00**	0.36	15	15.00	60.00
SW-1000-4-0d2-trial3	**15**	**15.00**	0.01	16	16.00	60.00	**15**	**15.00**	0.01	16	16.00	60.00
SW-1000-4-0d3-trial1	**13**	**13.00**	< 0.01	14	14.00	60.00	**13**	**13.00**	< 0.01	14	14.00	60.00
SW-1000-4-0d3-trial3	**13**	**13.00**	< 0.01	14	14.00	60.00	**13**	**13.00**	0.01	14	14.00	60.00
SW-1000-5-0d1-trial1	17	17.00	60.00	19	19.00	60.00	**16**	**16.90**	55.37	19	19.00	60.00
SW-1000-5-0d1-trial2	**17**	**17.00**	0.12	18	18.00	60.00	**17**	**17.00**	0.06	18	18.00	60.00
SW-1000-5-0d1-trial3	**15**	**15.00**	0.72	16	16.00	60.00	**15**	**15.00**	0.36	16	16.00	60.00
SW-1000-5-0d2-trial1	**14**	**14.00**	1.57	15	15.00	60.00	**14**	**14.00**	1.41	15	15.00	60.00
SW-1000-5-0d2-trial2	**14**	**14.00**	0.06	15	15.00	60.00	**14**	**14.00**	0.07	15	15.00	60.00
SW-1000-5-0d2-trial3	**14**	**14.00**	0.01	15	15.00	60.00	**14**	**14.00**	0.01	15	15.00	60.00
SW-1000-5-0d3-trial1	**13**	**13.00**	0.04	14	14.00	60.00	**13**	**13.00**	0.04	14	14.00	60.00
SW-1000-5-0d3-trial2	**13**	**13.00**	0.02	14	14.00	60.00	**13**	**13.00**	0.02	14	14.00	60.00
SW-1000-5-0d3-trial3	**13**	**13.00**	4.67	14	14.00	60.00	**13**	**13.00**	1.85	14	14.00	60.00
SW-1000-6-0d1-trial1	**14**	**14.00**	0.36	17	17.00	60.00	**14**	**14.00**	0.28	17	17.00	60.00
SW-1000-6-0d1-trial2	**13**	13.20	18.83	15	15.00	60.00	**13**	**13.00**	22.81	15	15.00	60.00
SW-1000-6-0d1-trial3	**13**	**13.00**	0.07	14	14.00	60.00	**13**	**13.00**	0.04	14	14.00	60.00
SW-1000-6-0d2-trial1	**12**	**12.00**	0.01	13	13.00	60.00	**12**	**12.00**	0.01	13	13.00	60.00
SW-1000-6-0d2-trial2	**13**	**13.00**	< 0.01	14	14.00	60.00	**13**	**13.00**	< 0.01	14	14.00	60.00
SW-1000-6-0d2-trial3	**12**	**12.00**	0.02	13	13.00	60.00	**12**	**12.00**	0.02	13	13.00	60.00
SW-1000-6-0d3-trial1	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01

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Table 3.11 – continued from previous page

Instance	FRFS-0-SCHA			FRFS-1-SCHA			FRFS-2-SCHA			FRFS-3-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
SW-1000-6-0d3-trial2	**12**	**12.00**	< 0.01	13	13.00	60.00	**12**	**12.00**	< 0.01	13	13.00	60.00
SW-1000-6-0d3-trial3	**12**	**12.00**	< 0.01	13	13.00	60.00	**12**	**12.00**	< 0.01	13	13.00	60.00
Avg. t (s)	4.80			45.41			4.99			45.41		
# Best (# Best Avg.)	109 (105)			27 (27)			109 (106)			67 (61)		

3.3.7 Experiments for comparison of BP-0-SCHA, BP-2-SCHA and FRFS-0-SCHA, and FRFS-2-SCHA

Table 3.12 presents a comparison between the BP and FRFSs decoders with the SCHA refinement in Harary Graphs, Small-world graphs, and synthetic instances. The results show in Table 3.12 indicate that FRFS-0-SCHA outperforms the BP-0-SCHA decoder. Additionally, FRFS-2-SCHA outperforms the BP-2-SCHA decoder. We remark that the FRFS-SCHAs decoder found 5% better solutions, and it was almost twice as fast.

Table 3.12: Comparison BP-0-SCHA, FRFS-0-SCHA, BP-2-SCHA and FRFS-2-SCHA

Instance	BP-0-SHCA			FRFS-0-SCHA			BP-2-SCHA			FRFS-2-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
BT05-RG050	**5**	**5.00**	5.16	**5**	5.10	12.35	**5**	**5.00**	3.32	**5**	**5.00**	21.43
BT05-RG075	**5**	**5.00**	1.13	**5**	**5.00**	2.60	**5**	**5.00**	1.92	**5**	**5.00**	2.22
BT05-RG100	**5**	**5.00**	0.07	**5**	**5.00**	0.02	**5**	**5.00**	0.04	**5**	**5.00**	0.02
BT05-RG150	**5**	**5.00**	0.02	**5**	**5.00**	0.01	**5**	**5.00**	0.02	**5**	**5.00**	0.02
BT05-RG200	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
BT05-RG250	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
BT06-RG050	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01
BT06-RG075	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01
BT06-RG100	**6**	**6.00**	5.06	**6**	**6.00**	4.23	**6**	**6.00**	3.81	**6**	**6.00**	4.64
BT06-RG150	**6**	**6.00**	0.03	**6**	**6.00**	0.02	**6**	**6.00**	0.03	**6**	**6.00**	0.01
BT06-RG200	**6**	**6.00**	< 0.01	**6**	**6.00**	0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
BT06-RG250	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
BT07-RG050	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
BT07-RG075	8	8.00	60.00	8	8.00	60.00	**7**	**7.90**	57.97	**7**	**7.90**	57.36
BT07-RG100	**7**	**7.00**	4.87	**7**	**7.00**	4.76	**7**	**7.00**	3.89	**7**	**7.00**	3.36
BT07-RG150	**7**	**7.00**	0.11	**7**	**7.00**	0.09	**7**	**7.00**	0.13	**7**	**7.00**	0.11
BT07-RG200	**7**	**7.00**	0.01	**7**	**7.00**	0.01	**7**	**7.00**	0.01	**7**	**7.00**	0.01
BT07-RG250	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01
BT08-RG050	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
BT08-RG075	9	9.00	60.00	**8**	**8.90**	56.97	**8**	**8.90**	57.47	9	9.00	60.00
BT08-RG100	**8**	**8.00**	13.36	**8**	8.30	35.68	**8**	8.10	12.97	**8**	8.40	39.86
BT08-RG150	**8**	**8.00**	0.45	**8**	**8.00**	0.18	**8**	**8.00**	0.38	**8**	**8.00**	0.29
BT08-RG200	**8**	**8.00**	0.03	**8**	**8.00**	0.04	**8**	**8.00**	0.02	**8**	**8.00**	0.02
BT08-RG250	**8**	**8.00**	0.01	**8**	**8.00**	0.01	**8**	**8.00**	0.01	**8**	**8.00**	0.01
BT09-RG050	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01
BT09-RG075	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01
BT09-RG100	**9**	**9.60**	50.19	**9**	9.70	44.93	**9**	**9.60**	45.04	**9**	9.80	53.50
BT09-RG150	**9**	**9.00**	1.54	**9**	**9.00**	1.49	**9**	**9.00**	0.59	**9**	**9.00**	1.05
BT09-RG200	**9**	**9.00**	0.09	**9**	**9.00**	0.09	**9**	**9.00**	0.06	**9**	**9.00**	0.13

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Table 3.12 – continued from previous page

Instance	BP-0-SHCA			FRFS-0-SCHA			BP-2-SCHA			FRFS-2-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
BT09-RG250	**9**	**9.00**	0.02	**9**	**9.00**	0.03	**9**	**9.00**	0.04	**9**	**9.00**	0.04
BT10-RG050	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01
BT10-RG075	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01
BT10-RG100	**10**	**10.90**	59.53	11	11.00	60.00	11	11.00	60.00	11	11.00	60.00
BT10-RG150	**10**	**10.00**	3.82	**10**	**10.00**	4.90	**10**	**10.00**	5.34	**10**	**10.00**	3.19
BT10-RG200	**10**	**10.00**	0.21	**10**	**10.00**	0.22	**10**	**10.00**	0.24	**10**	**10.00**	0.48
BT10-RG250	**10**	**10.00**	0.11	**10**	**10.00**	0.08	**10**	**10.00**	0.11	**10**	**10.00**	0.06
H10-30	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H11-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H20-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H21-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01
H2-100	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01
H2-17	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H2-30	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01
H2-50	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01
H3-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H3-30	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H3-50	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01
H5-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H6-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H7-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H8-30	**5**	**5.00**	1.72	**5**	**5.00**	0.03	**5**	**5.00**	0.63	**5**	**5.00**	0.01
H9-30	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
SW-100-3-0d1-trial1	**61**	**61.00**	< 0.01	**61**	**61.00**	< 0.01	**61**	**61.00**	< 0.01	**61**	**61.00**	< 0.01
SW-100-3-0d2-trial1	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01
SW-100-3-0d2-trial3	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01
SW-100-4-0d1-trial1	**9**	**9.00**	0.13	**9**	**9.00**	0.01	**9**	**9.00**	0.20	**9**	**9.00**	0.01
SW-100-4-0d1-trial2	9	9.00	60.00	**8**	**8.00**	2.62	9	9.00	60.00	**8**	**8.00**	2.08
SW-100-4-0d1-trial3	**10**	**10.00**	0.23	**10**	**10.00**	0.01	**10**	**10.00**	0.18	**10**	**10.00**	0.01
SW-100-4-0d2-trial1	**8**	8.90	54.04	**8**	8.50	43.92	9	9.00	60.00	**8**	**8.30**	33.73
SW-100-4-0d2-trial2	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
SW-100-4-0d2-trial3	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
SW-100-4-0d3-trial1	**8**	**8.00**	8.08	**8**	**8.00**	0.95	**8**	**8.00**	11.43	**8**	**8.00**	0.82
SW-100-4-0d3-trial2	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
SW-100-4-0d3-trial3	**8**	**8.00**	7.51	**8**	**8.00**	0.25	**8**	**8.00**	14.29	**8**	**8.00**	0.36
SW-100-5-0d1-trial1	**9**	**9.00**	1.09	**9**	**9.00**	0.11	**9**	**9.00**	0.88	**9**	**9.00**	0.06
SW-100-5-0d1-trial2	**10**	**10.00**	0.07	**10**	**10.00**	< 0.01	**10**	**10.00**	0.09	**10**	**10.00**	< 0.01
SW-100-5-0d1-trial3	**12**	**12.00**	0.02	**12**	**12.00**	< 0.01	**12**	**12.00**	0.01	**12**	**12.00**	< 0.01
SW-100-5-0d2-trial1	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01
SW-100-5-0d2-trial2	**9**	**9.90**	56.23	**9**	**9.90**	55.94	10	10.00	60.00	10	10.00	60.00
SW-100-5-0d2-trial3	**9**	**9.00**	0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	0.01	**9**	**9.00**	< 0.01
SW-100-5-0d3-trial1	**8**	**8.00**	0.18	**8**	**8.00**	0.02	**8**	**8.00**	0.16	**8**	**8.00**	0.01
SW-100-5-0d3-trial2	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
SW-100-5-0d3-trial3	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
SW-100-6-0d1-trial1	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
SW-100-6-0d1-trial2	**8**	**8.00**	0.12	**8**	**8.00**	0.01	**8**	**8.00**	0.15	**8**	**8.00**	< 0.01
SW-100-6-0d1-trial3	**8**	**8.00**	0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	0.01	**8**	**8.00**	< 0.01
SW-100-6-0d2-trial1	**7**	7.60	47.60	**7**	**7.10**	16.68	**7**	7.90	58.18	**7**	**7.10**	20.26
SW-100-6-0d2-trial2	**7**	**7.00**	6.29	**7**	**7.00**	0.31	**7**	**7.00**	5.16	**7**	**7.00**	0.22
SW-100-6-0d2-trial3	**7**	7.70	54.48	**7**	**7.00**	4.93	**7**	7.30	38.60	**7**	**7.00**	8.78
SW-100-6-0d3-trial1	**7**	7.20	20.86	**7**	**7.00**	1.35	**7**	**7.00**	12.67	**7**	**7.00**	1.85
SW-100-6-0d3-trial2	**7**	**7.00**	8.34	**7**	**7.00**	0.49	**7**	**7.00**	3.61	**7**	**7.00**	0.43
SW-100-6-0d3-trial3	**7**	**7.00**	1.95	**7**	**7.00**	0.05	**7**	**7.00**	1.07	**7**	**7.00**	0.08

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Table 3.12 – continued from previous page

Instance	BP-0-SHCA			FRFS-0-SCHA			BP-2-SCHA			FRFS-2-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
SW-1000-3-0d2-trial1	**89**	**89.00**	0.01	**89**	**89.00**	0.01	**89**	**89.00**	0.01	**89**	**89.00**	< 0.01
SW-1000-3-0d2-trial2	**88**	**88.00**	0.01	**88**	**88.00**	< 0.01	**88**	**88.00**	0.01	**88**	**88.00**	< 0.01
SW-1000-3-0d3-trial2	**87**	**87.00**	0.01	**87**	**87.00**	< 0.01	**87**	**87.00**	0.01	**87**	**87.00**	< 0.01
SW-1000-4-0d1-trial1	17	17.00	60.00	**16**	**16.60**	46.55	17	17.00	60.00	**16**	16.80	51.20
SW-1000-4-0d1-trial2	**17**	17.70	49.42	**17**	**17.00**	0.38	**17**	17.70	47.98	**17**	**17.00**	0.19
SW-1000-4-0d1-trial3	18	18.00	60.00	**17**	17.50	43.10	18	18.00	60.00	**17**	**17.40**	43.24
SW-1000-4-0d2-trial1	**14**	**14.00**	0.60	**14**	**14.00**	0.01	**14**	**14.00**	0.65	**14**	**14.00**	0.02
SW-1000-4-0d2-trial2	**14**	**14.00**	12.72	**14**	**14.00**	0.27	**14**	14.10	27.06	**14**	**14.00**	0.36
SW-1000-4-0d2-trial3	**15**	**15.00**	0.15	**15**	**15.00**	0.01	**15**	**15.00**	0.23	**15**	**15.00**	0.01
SW-1000-4-0d3-trial1	**13**	**13.00**	0.02	**13**	**13.00**	< 0.01	**13**	**13.00**	0.02	**13**	**13.00**	< 0.01
SW-1000-4-0d3-trial3	**13**	**13.00**	0.03	**13**	**13.00**	< 0.01	**13**	**13.00**	0.03	**13**	**13.00**	0.01
SW-1000-5-0d1-trial1	17	17.00	60.00	17	17.00	60.00	17	17.00	60.00	**16**	**16.90**	55.37
SW-1000-5-0d1-trial2	**17**	**17.00**	3.21	**17**	**17.00**	0.12	**17**	**17.00**	3.53	**17**	**17.00**	0.06
SW-1000-5-0d1-trial3	**15**	15.60	46.39	**15**	**15.00**	0.72	**15**	15.80	52.09	**15**	**15.00**	0.36
SW-1000-5-0d2-trial1	15	15.00	60.00	**14**	**14.00**	1.57	15	15.00	60.00	**14**	**14.00**	1.41
SW-1000-5-0d2-trial2	**14**	**14.00**	11.45	**14**	**14.00**	0.06	**14**	**14.00**	8.03	**14**	**14.00**	0.07
SW-1000-5-0d2-trial3	**14**	**14.00**	0.04	**14**	**14.00**	0.01	**14**	**14.00**	0.03	**14**	**14.00**	0.01
SW-1000-5-0d3-trial1	**13**	**13.00**	2.35	**13**	**13.00**	0.04	**13**	**13.00**	1.89	**13**	**13.00**	0.04
SW-1000-5-0d3-trial2	**13**	**13.00**	0.61	**13**	**13.00**	0.02	**13**	**13.00**	0.41	**13**	**13.00**	0.02
SW-1000-5-0d3-trial3	**13**	13.90	54.71	**13**	**13.00**	4.67	**13**	13.90	54.51	**13**	**13.00**	1.85
SW-1000-6-0d1-trial1	**14**	**14.00**	10.82	**14**	**14.00**	0.36	**14**	**14.00**	8.05	**14**	**14.00**	0.28
SW-1000-6-0d1-trial2	14	14.00	60.00	**13**	13.20	18.83	**13**	13.90	54.22	**13**	**13.00**	22.81
SW-1000-6-0d1-trial3	**13**	**13.00**	4.35	**13**	**13.00**	0.07	**13**	**13.00**	1.82	**13**	**13.00**	0.04
SW-1000-6-0d2-trial1	**12**	**12.00**	0.17	**12**	**12.00**	0.01	**12**	**12.00**	0.15	**12**	**12.00**	0.01
SW-1000-6-0d2-trial2	**13**	**13.00**	0.01	**13**	**13.00**	< 0.01	**13**	**13.00**	0.01	**13**	**13.00**	< 0.01
SW-1000-6-0d2-trial3	**12**	**12.00**	0.58	**12**	**12.00**	0.02	**12**	**12.00**	0.40	**12**	**12.00**	0.02
SW-1000-6-0d3-trial1	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01
SW-1000-6-0d3-trial2	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01
SW-1000-6-0d3-trial3	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01
Avg. t (s)	9.84			5.34			9.75			5.53		
# Best (# Best Avg.)	103 (96)			108 (102)			103 (95)			108 (105)		

3.3.8 Experiments for comparison of BP-0, FRFS-0, FRFS-0-SCHA and MSF

Table 3.13 shows a comparison between the BP-0, FRFS-0, FRFS-0-SCHA, and MSF decoders in Harary Graphs, Small-world graphs, and synthetic instances. The results show in Table 3.13 indicates evolution of our algorithms. FRFS-0 decoder is 10% better than BP-0 for founding the best solution, and it's 1.33 times faster. When we compare the FRFS-0-SCHA decoder with the BP-0 decoder, the FRFS–SCHA is 3.17 times faster and found almost 20% more best values. Additionally, the number of best average solution values is 30% greater. On the other hand, the results of the MSF decoder are extremely powerless. We attribute this poor performance to the creation of forest/tree with several high degree vertices. We still try to mitigate this problem by adding as weight the index position of the edge concerning the vertex, but the results have not improved significantly.

Table 3.13: Comparison between BP-0, FRFS-0, FRFS-0-SCHA and MSF

Instance	BP-0			FRFS-0			FRFS-0-SCHA			MSF		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
BT05-RG050	**5**	5.30	27.06	**5**	5.30	26.97	**5**	**5.10**	12.35	6	6.00	60.00
BT05-RG075	**5**	**5.00**	2.53	**5**	**5.00**	2.33	**5**	**5.00**	2.60	**5**	5.80	55.62
BT05-RG100	**5**	**5.00**	0.04	**5**	**5.00**	0.03	**5**	**5.00**	0.02	6	6.00	60.00
BT05-RG150	**5**	**5.00**	0.03	**5**	**5.00**	0.02	**5**	**5.00**	0.01	6	6.00	60.00
BT05-RG200	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	6	6.00	60.00
BT05-RG250	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	6	6.00	60.00
BT06-RG050	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	0.32
BT06-RG075	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	1.33
BT06-RG100	**6**	**6.00**	4.81	**6**	**6.00**	3.45	**6**	**6.00**	4.23	7	7.00	60.00
BT06-RG150	**6**	**6.00**	0.02	**6**	**6.00**	0.01	**6**	**6.00**	0.02	7	7.00	60.00
BT06-RG200	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	0.01	7	7.20	60.00
BT06-RG250	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	7	7.00	60.00
BT07-RG050	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	9	9.30	60.00
BT07-RG075	8	8.00	60.00	**7**	**7.90**	58.00	8	8.00	60.00	9	9.00	60.00
BT07-RG100	**7**	**7.00**	2.78	**7**	**7.00**	2.48	**7**	**7.00**	4.76	9	9.00	60.00
BT07-RG150	**7**	**7.00**	0.07	**7**	**7.00**	0.05	**7**	**7.00**	0.09	9	9.20	60.00
BT07-RG200	**7**	**7.00**	0.01	**7**	**7.00**	0.01	**7**	**7.00**	0.01	9	9.30	60.00
BT07-RG250	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	**7**	**7.00**	< 0.01	9	9.90	60.00
BT08-RG050	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	11	11.40	60.00
BT08-RG075	9	9.00	60.00	**8**	**8.80**	54.27	**8**	8.90	56.97	12	12.30	60.00
BT08-RG100	**8**	**8.00**	8.44	**8**	8.10	27.02	**8**	8.30	35.68	12	12.50	60.00
BT08-RG150	**8**	**8.00**	0.31	**8**	**8.00**	0.12	**8**	**8.00**	0.18	12	12.60	60.00
BT08-RG200	**8**	**8.00**	0.02	**8**	**8.00**	0.03	**8**	**8.00**	0.04	12	12.50	60.00
BT08-RG250	**8**	**8.00**	< 0.01	**8**	**8.00**	0.01	**8**	**8.00**	0.01	12	12.90	60.00
BT09-RG050	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	15	15.80	60.00
BT09-RG075	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	16	16.00	60.00
BT09-RG100	**9**	**9.50**	45.48	**9**	9.70	44.03	**9**	9.70	44.93	15	16.00	60.00
BT09-RG150	**9**	**9.00**	1.18	**9**	**9.00**	1.13	**9**	**9.00**	1.49	16	17.20	60.00
BT09-RG200	**9**	**9.00**	0.07	**9**	**9.00**	0.08	**9**	**9.00**	0.09	16	17.10	60.00
BT09-RG250	**9**	**9.00**	0.02	**9**	**9.00**	0.02	**9**	**9.00**	0.03	17	17.40	60.00
BT10-RG050	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	18	19.70	60.00
BT10-RG075	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	**11**	**11.00**	< 0.01	20	20.60	60.00
BT10-RG100	**10**	**10.90**	58.50	11	11.00	60.00	11	11.00	60.00	21	21.80	60.00
BT10-RG150	**10**	**10.00**	3.28	**10**	**10.00**	4.18	**10**	**10.00**	4.90	19	20.20	60.00
BT10-RG200	**10**	**10.00**	0.19	**10**	**10.00**	0.19	**10**	**10.00**	0.22	19	21.20	60.00
BT10-RG250	**10**	**10.00**	0.10	**10**	**10.00**	0.08	**10**	**10.00**	0.08	21	21.80	60.00
H10-30	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	6	6.00	60.00
H11-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	7	7.00	60.00
H20-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	7	7.70	60.00
H21-50	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	**6**	**6.00**	< 0.01	7	7.00	60.00
H2-100	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01	**50**	**50.00**	< 0.01
H2-17	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H2-30	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01	**15**	**15.00**	< 0.01
H2-50	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01	**25**	**25.00**	< 0.01
H3-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H3-30	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
H3-50	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	< 0.01	**14**	**14.00**	0.06
H5-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H6-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H7-17	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01
H8-30	**5**	**5.00**	0.26	**5**	**5.00**	0.02	**5**	**5.00**	0.03	6	6.00	60.00
H9-30	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	**5**	**5.00**	< 0.01	6	6.00	60.00

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Table 3.13 – continued from previous page

Instance	BP-0			FRFS-0			FRFS-0-SCHA			MSF		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
SW-100-3-0d1-trial1	**61**	**61.00**	0.03	**61**	**61.00**	0.02	**61**	**61.00**	< 0.01	**61**	**61.00**	< 0.01
SW-100-3-0d2-trial1	**31**	**31.00**	0.01	**31**	**31.00**	0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01
SW-100-3-0d2-trial3	**31**	**31.00**	0.01	**31**	**31.00**	0.01	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01
SW-100-4-0d1-trial1	**9**	**9.00**	3.87	**9**	**9.00**	0.15	**9**	**9.00**	0.01	10	10.00	60.00
SW-100-4-0d1-trial2	**8**	8.80	51.08	**8**	8.20	27.92	**8**	**8.00**	2.62	9	9.70	60.00
SW-100-4-0d1-trial3	**10**	**10.00**	0.10	**10**	**10.00**	0.10	**10**	**10.00**	0.01	11	11.80	60.00
SW-100-4-0d2-trial1	9	9.00	60.00	**8**	8.60	46.54	**8**	**8.50**	43.92	9	9.90	60.00
SW-100-4-0d2-trial2	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	10	10.20	60.00
SW-100-4-0d2-trial3	**9**	**9.00**	0.01	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01	10	10.00	60.00
SW-100-4-0d3-trial1	**8**	8.10	9.95	**8**	**8.00**	2.16	**8**	**8.00**	0.95	9	9.70	60.00
SW-100-4-0d3-trial2	**8**	**8.00**	0.02	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	9	9.00	60.00
SW-100-4-0d3-trial3	**8**	8.20	26.27	**8**	**8.00**	1.61	**8**	**8.00**	0.25	9	9.90	60.00
SW-100-5-0d1-trial1	**9**	9.20	15.47	**9**	**9.00**	1.89	**9**	**9.00**	0.11	10	10.20	60.00
SW-100-5-0d1-trial2	**10**	**10.00**	1.17	**10**	**10.00**	0.04	**10**	**10.00**	< 0.01	11	11.40	60.00
SW-100-5-0d1-trial3	**12**	**12.00**	0.24	**12**	**12.00**	0.05	**12**	**12.00**	< 0.01	13	13.20	60.00
SW-100-5-0d2-trial1	**10**	**10.00**	0.01	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01	11	11.00	60.00
SW-100-5-0d2-trial2	10	10.00	60.00	10	10.00	60.00	**9**	**9.90**	55.94	11	11.40	60.00
SW-100-5-0d2-trial3	**9**	**9.00**	0.03	**9**	**9.00**	0.01	**9**	**9.00**	< 0.01	10	10.00	60.00
SW-100-5-0d3-trial1	**8**	**8.00**	0.43	**8**	**8.00**	0.09	**8**	**8.00**	0.02	9	9.10	60.00
SW-100-5-0d3-trial2	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	9	9.00	60.00
SW-100-5-0d3-trial3	**8**	**8.00**	0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	9	9.00	60.00
SW-100-6-0d1-trial1	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01	9	9.20	60.00
SW-100-6-0d1-trial2	**8**	**8.00**	0.16	**8**	**8.00**	0.02	**8**	**8.00**	0.01	9	9.90	60.00
SW-100-6-0d1-trial3	**8**	**8.00**	0.04	**8**	**8.00**	0.01	**8**	**8.00**	< 0.01	9	9.50	60.00
SW-100-6-0d2-trial1	**7**	7.80	52.07	**7**	7.20	20.38	**7**	**7.10**	16.68	9	9.00	60.00
SW-100-6-0d2-trial2	**7**	**7.00**	3.41	**7**	**7.00**	0.30	**7**	**7.00**	0.31	8	8.40	60.00
SW-100-6-0d2-trial3	**7**	7.60	45.90	**7**	**7.00**	5.53	**7**	**7.00**	4.93	9	9.00	60.00
SW-100-6-0d3-trial1	**7**	**7.00**	12.32	**7**	**7.00**	1.13	**7**	**7.00**	1.35	8	8.80	60.00
SW-100-6-0d3-trial2	**7**	**7.00**	4.01	**7**	**7.00**	0.75	**7**	**7.00**	0.49	8	8.70	60.00
SW-100-6-0d3-trial3	**7**	**7.00**	2.15	**7**	**7.00**	0.07	**7**	**7.00**	0.05	8	8.70	60.00
SW-1000-3-0d2-trial1	**89**	**89.00**	6.99	**89**	**89.00**	3.98	**89**	**89.00**	0.01	**89**	**89.00**	< 0.01
SW-1000-3-0d2-trial2	**88**	**88.00**	4.78	**88**	**88.00**	2.40	**88**	**88.00**	< 0.01	**88**	**88.00**	< 0.01
SW-1000-3-0d3-trial2	**87**	**87.00**	4.63	**87**	**87.00**	2.24	**87**	**87.00**	< 0.01	**87**	**87.00**	< 0.01
SW-1000-4-0d1-trial1	17	17.70	60.00	17	17.00	60.00	**16**	**16.60**	46.55	27	28.80	60.00
SW-1000-4-0d1-trial2	18	18.00	60.00	**17**	17.80	55.45	**17**	**17.00**	0.38	24	26.10	60.00
SW-1000-4-0d1-trial3	18	18.60	60.00	18	18.00	60.00	**17**	**17.50**	43.10	26	27.70	60.00
SW-1000-4-0d2-trial1	**14**	14.90	59.79	**14**	14.50	38.67	**14**	**14.00**	0.01	21	21.90	60.00
SW-1000-4-0d2-trial2	15	15.00	60.00	15	15.00	60.00	**14**	**14.00**	0.27	21	22.70	60.00
SW-1000-4-0d2-trial3	16	16.00	60.00	**15**	15.20	25.90	**15**	**15.00**	0.01	22	24.30	60.00
SW-1000-4-0d3-trial1	**13**	**13.00**	12.07	**13**	**13.00**	1.20	**13**	**13.00**	< 0.01	20	21.10	60.00
SW-1000-4-0d3-trial3	**13**	**13.00**	3.52	**13**	**13.00**	0.15	**13**	**13.00**	< 0.01	20	21.20	60.00
SW-1000-5-0d1-trial1	18	18.00	60.00	**17**	17.50	35.43	**17**	**17.00**	0.04	26	27.80	60.00
SW-1000-5-0d1-trial2	18	18.00	60.00	**17**	17.10	29.33	**17**	**17.00**	0.12	25	26.70	60.00
SW-1000-5-0d1-trial3	16	16.20	60.00	16	16.00	60.00	**15**	**15.00**	0.72	23	25.40	60.00
SW-1000-5-0d2-trial1	15	15.00	60.00	15	15.00	60.00	**14**	**14.00**	1.57	22	22.80	60.00
SW-1000-5-0d2-trial2	15	15.00	60.00	**14**	14.90	56.68	**14**	**14.00**	0.06	21	22.60	60.00
SW-1000-5-0d2-trial3	**14**	14.10	15.28	**14**	**14.00**	1.38	**14**	**14.00**	0.01	20	22.50	60.00
SW-1000-5-0d3-trial1	14	14.00	60.00	**13**	13.90	59.48	**13**	**13.00**	0.04	20	21.80	60.00
SW-1000-5-0d3-trial2	14	14.00	60.00	**13**	13.20	28.20	**13**	**13.00**	0.02	19	21.30	60.00
SW-1000-5-0d3-trial3	14	14.00	60.00	14	14.00	60.00	**13**	**13.00**	4.67	19	21.10	60.00
SW-1000-6-0d1-trial1	**14**	14.90	55.90	**14**	14.30	34.73	**14**	**14.00**	0.36	23	25.40	60.00
SW-1000-6-0d1-trial2	14	14.00	60.00	14	14.00	60.00	**13**	**13.20**	18.83	23	24.80	60.00
SW-1000-6-0d1-trial3	**13**	13.90	57.61	**13**	**13.00**	15.11	**13**	**13.00**	0.07	22	24.10	60.00

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Table 3.13 – continued from previous page

Instance	BP-0			FRFS-0			FRFS-0-SCHA			MSF		
	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)	Best	Avg.	t (s)
SW-1000-6-0d2-trial1	**12**	**12.00**	1.78	**12**	**12.00**	0.09	**12**	**12.00**	0.01	21	22.40	60.00
SW-1000-6-0d2-trial2	**13**	**13.00**	0.01	**13**	**13.00**	< 0.01	**13**	**13.00**	< 0.01	23	24.10	60.00
SW-1000-6-0d2-trial3	**12**	**12.00**	5.70	**12**	**12.00**	0.35	**12**	**12.00**	0.02	21	22.20	60.00
SW-1000-6-0d3-trial1	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	20	20.80	60.00
SW-1000-6-0d3-trial2	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	19	21.40	60.00
SW-1000-6-0d3-trial3	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01	20	22.40	60.00
Avg. t (s)	15.24			11.39			4.81			50.25		
# Best (# Best Avg.)	93 (82)			102 (87)			109 (106)			19 (18)		

3.3.9 Experiments for comparison of BRKGA-FRFS-0-SCHA and Hybrid BRKGA-ILP

Table 3.14 exhibits an initial comparison between the BRKGA-FRFS-0-SCHA and Hybrid BRKGA-ILP (using FRFS-0-SCHA decoder). The results in Table 3.14 indicate that the Hybrid BRKGA-ILP has been hopeful for graphs with up to 100 vertices. In this stage of the research, we have not yet tested this decoder in massive graphs (more than 100 vertices).

Table 3.14: Comparison between Hybrid BRKGA-ILP and BRKGA-FRFS-0-SCHA

Instance	Hybrid BRKGA-ILP			BRKGA-FRFS-0-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)
BT05-RG050	**5**	**5.00**	0.26	**5**	5.10	13.11
BT05-RG075	**5**	**5.00**	0.33	**5**	**5.00**	2.82
BT05-RG100	**5**	**5.00**	0.03	**5**	**5.00**	0.02
BT06-RG050	**6**	**6.00**	3.18	7	7.00	60.00
BT06-RG075	**6**	**6.00**	3.88	7	7.00	60.00
BT06-RG100	**6**	**6.00**	6.92	**6**	**6.00**	4.41
BT07-RG050	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
BT07-RG075	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
BT07-RG100	**7**	7.50	30.29	**7**	**7.00**	5.20
SW-100-3-0d1-trial1	**61**	**61.00**	< 0.01	**61**	**61.00**	< 0.01
SW-100-3-0d2-trial1	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01
SW-100-3-0d2-trial3	**31**	**31.00**	< 0.01	**31**	**31.00**	< 0.01
SW-100-4-0d1-trial1	**9**	**9.00**	0.01	**9**	**9.00**	0.01
SW-100-4-0d1-trial2	**8**	**8.00**	2.52	**8**	**8.00**	2.61
SW-100-4-0d1-trial3	**10**	**10.00**	0.01	**10**	**10.00**	0.01
SW-100-4-0d2-trial1	**8**	**8.00**	2.89	**8**	8.50	43.90
SW-100-4-0d2-trial2	**8**	**8.00**	8.34	9	9.00	60.00
SW-100-4-0d2-trial3	**9**	**9.00**	< 0.01	**9**	**9.00**	< 0.01
SW-100-4-0d3-trial1	**8**	**8.00**	1.85	**8**	**8.00**	0.96
SW-100-4-0d3-trial2	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
SW-100-4-0d3-trial3	**8**	**8.00**	0.56	**8**	**8.00**	0.30
SW-100-5-0d1-trial1	**9**	**9.00**	0.13	**9**	**9.00**	0.10
SW-100-5-0d1-trial2	**10**	**10.00**	< 0.01	**10**	**10.00**	< 0.01
SW-100-5-0d1-trial3	**12**	**12.00**	< 0.01	**12**	**12.00**	< 0.01
SW-100-5-0d2-trial1	**9**	**9.00**	2.98	10	10.00	60.00
SW-100-5-0d2-trial2	**9**	**9.00**	1.82	**9**	9.90	55.90

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Table 3.14 – continued from previous page

Instance	Hybrid BRKGA-ILP			BRKGA-FRFS-0-SCHA		
	Best	Avg.	t (s)	Best	Avg.	t (s)
SW-100-5-0d2-trial3	**8**	**8.00**	5.50	9	9.00	60.00
SW-100-5-0d3-trial1	**8**	**8.00**	0.03	**8**	**8.00**	0.02
SW-100-5-0d3-trial2	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
SW-100-5-0d3-trial3	**8**	**8.00**	< 0.01	**8**	**8.00**	< 0.01
SW-100-6-0d1-trial1	**7**	**7.40**	33.03	8	8.00	60.00
SW-100-6-0d1-trial2	**8**	**8.00**	0.01	**8**	**8.00**	0.01
SW-100-6-0d1-trial3	**7**	**7.00**	29.45	8	8.00	60.00
SW-100-6-0d2-trial1	**7**	**7.00**	8.39	**7**	7.10	17.58
SW-100-6-0d2-trial2	**7**	**7.00**	1.62	**7**	**7.00**	0.32
SW-100-6-0d2-trial3	**7**	**7.00**	7.73	**7**	**7.00**	5.10
SW-100-6-0d3-trial1	**7**	**7.00**	8.47	**7**	**7.00**	1.42
SW-100-6-0d3-trial2	**7**	**7.00**	3.87	**7**	**7.00**	0.50
SW-100-6-0d3-trial3	**7**	**7.00**	0.06	**7**	**7.00**	0.06
# Best (# Best Avg.)		39 (32)			32 (28)	

4

Final Considerations and Schedule

4.1 Final Considerations

In this work, we proposed some Biased Random-Key Genetic Algorithms (BRKGA) and a hybrid BRKGA for finding approximate solutions to the MINIMUM BROADCAST TIME problem. Computational experiments showed that the results obtained by our approach outperformed the state-of-the-art methods in both solution quality and CPU time. For all instances with known optimal value, the our best BRKGA either attained the optimal value or missed it by at most one broadcast step.

Additionally, we have also described a new lower bound procedure for the problem based on vertex distances in the input graph. The experimental results reveal that our approach increased several lower bounds and helped the Integer Linear Programming (ILP) model as well as our BRKGA to prove several previously unknown optima. To the best of our knowledge, this is the first time that major efforts were made to study, generate, and solve hard instances for the MBT. We have one article approved (Lima et al., 2020) and another under review (our first algorithm — BRKGA-BP).

We also plan to extend the proposed algorithm to solve related problems, such as the following MBT variants: (i) MBT with weighted edges/vertice (Harutyunyan and Kamali, 2008), (ii) MBT with with at most k transmissions (k-broadcast) (Lazard, 1992; Harutyunyan and Liestman, 2001), (iii) minimal k-broadcast network in which k-broadcasting can be completed in minimum time from any node (Lee and Ventura, 2001), (iv) apply our algorithms for solve real problems, such as swarm robotics (Al-Sarawi et al., 2017).

4.2 Schedule

The execution of these activities will follow the working schedule presented in Table 4.1, which are described below.

Table 4.1: Schedule

Activity	2020										2021	
	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb
Literature Review									X	X	X	X
Experiments									X	X	X	X
Literature Algorithms										X	X	X
Metaheuristic Algorithms										X	X	X
Hybrid Algorithm									X	X		
Publishing Paper									X	X	X	X
Thesis Writing										X	X	X
Exact Algorithms									X	X		
	2021										2022	
	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb
Literature Review	X											
Literature Algorithms	X											
Experiments	X											
Publishing Paper	X											
Thesis Presentation	X											

- **Literature review:** This step consists of reviewing the relevant literature. The process is continuous and lasts practically throughout all the master's period. It is very important that our reading is up to date the current works and state-of-the-art techniques that can be applied;
- **Metaheuristics**: This step, which lasts for all months, consists of the development, adaptation and execution of metaheuristic methods for variants MBT;
- **Hybrid algorithm**: We will test the proposed algorithm, make improvements and adapt to similar problems;
- **Experiments**: In the experiments step, we will compare our algorithms with literature algorithms;
- **Literature algorithms**: At any time a new algorithm for the MBT may appear, so we need to execute or implement and compare with ours;
- **Exact algorithms**: We will development exact algorithms (models) for the MBT variants;
- **Publishing of papers**: The publishing of articles in the relevant scientific media, such as journals, congresses, and conferences, will last throughout all the development of this work, and will occur whenever any relevant results are discovered;
- **Thesis writing**: The writing of the thesis is incremental, and will take place for almost the entire duration of this work;
- **Thesis presentation**: To Consolidate the results and the written thesis, this work will be presented in March 2021.

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