(a) Find all the independent scalar constraint equations when wheels B and D roll on N. Consequently solve these constraints for $\dot{q}_A$, ω_B, ω_D, v_y in terms of v_x and $\dot{q}_C$.

Result:

$\dot{q}_A = 0.5 v_x \tan(q_c)/L$

$\omega_B = v_x/R$

$\omega_D = v_x/(R\cos(q_c))$

$v_y = 0$

from MG:

Gotten from MG code.

$$\dot{q}_A = \frac{0.5 v_x \tan(q_c)}{L}$$

$$\omega_B = \frac{v_x}{R}$$

$$\omega_D = \frac{V_x}{R\cos(q_c)}$$

$$v_y = 0$$

(b) What is the number of degrees of freedom of this system when B and D roll on N?

DoF = # generalised speeds − # of constraints / kinematical relations

#speeds (8) − $\dot{x}, \dot{y}, v_x, v_y, \dot{q}_A, \dot{q}_c, \omega_B, \omega_D$

#constraints (6) −

- (2) Kinematic Relation b/n $\dot{x}, \dot{y}, v_x, v_y$
- (4) Rolling Constraint

(c) Find the dynamics equation for $\dot{v}_x$ and $\ddot{q}_A$ in matrix form as shown below. Report the elements of matrix N and M. You do not need to report R_1 and R_2.

Result:

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} T_B \\ T_{steer} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \dot{v}_x \\ \ddot{q}_A \end{bmatrix} + \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

- $N_{11} = \frac{1}{R}$ $\quad N_{21} = 0$

  $N_{12} = 0$ $\quad N_{22} = 1$

- $M_{11} = m + J/R^2 + M/\cos^2(q_c) + 1/4\, m_A(4 + \tan^2(q_c)) + J/(R^2\cos^2(q_c)) + 0.25(I^A_{zz} + 2K)\tan^2(q_c)/L^2$

  $M_{12} = \frac{1}{2} K \tan(q_c)/L, \quad M_{21} = \frac{1}{2} K \tan(q_c)/L, \quad M_{22} = K$

See MG code for confirmation

(d) Design a control system for the rear-wheel driving torque T_B and steering torque T_{steer} for the desired motion

$$(v_x)_{des} = \left(2\,\frac{m}{s^2}\right) * t, \qquad (q_C)_{des} = 15\left(\frac{\pi}{180}\right) * sin(t) \text{ rad} \tag{1}$$

The control system should regulate error in v_x and q_C, defined as $\tilde{v}_x = v_x - (v_x)_{des}$, and $\tilde{q}_C = q_C - (q_C)_{des}$, with

$$\frac{d\tilde{v}_x}{dt} + k_p\tilde{v}_x = 0, \qquad \frac{d^2\tilde{q}_C}{dt^2} + 2\zeta\omega_n\frac{d\tilde{q}_C}{dt} + \omega_n^2\tilde{q}_C = 0 \tag{2}$$

$\dot{\tilde{v}}_x + K_p\tilde{v}_x = 0 \qquad \ddot{\tilde{q}}_c + 2\zeta\omega_n\dot{\tilde{q}}_c + \omega_n^2\tilde{q}_c = 0$

$(\dot{v}_x - (\dot{v}_x)_{des}) + K_p(v_x - (v_x)_{des}) = 0 \qquad (\ddot{q}_C - (\ddot{q}_C)_{des}) + 2\zeta\omega_n(\dot{q}_C - (\dot{q}_C)_{des}) + \omega_n^2(q_C - (q_C)_{des})$

$\dot{v}_x = \dot{v}_{x,Des} + K_p(v_{x,des} - v_x) \qquad \ddot{q}_c = \ddot{q}_{c,des} + 2\zeta\omega_n(\dot{q}_{c,des} - \dot{q}_c) + \omega_n^2(q_{c,des} - q_c)$

Here, you may use the values $k_p = 2$, $\zeta = 1$ and $\omega_n = 2$ rad/s.
Knowing the system starts at rest, plot 20 seconds of x versus y (in meters). Also plot the time-histories of steering angle q_C (in degrees), drive torque T_B (Nm), and steering torque T_{steer} (Nm). Your plot for x versus y should match the given plot.

See plots in folder