1. **Analysis 1:** The dynamicist rotates about $\hat{a}_y$ and keeps the pole horizontal so $\hat{a}_x = \hat{b}_x$. The wheel rotates with angular velocity $\omega_C \hat{b}_x$ in B, where ω_C is positive (clockwise from the dynamicist's perspective).

a. **Guess:** Which way should you turn to make the wheel feel lighter? RIGHT or LEFT

b. **Demo:** Try it. Which way felt lighter? RIGHT or LEFT

c. †Using conservation of angular momentum, prove analytically (by hand) that $\dot{\omega}_C = 0$ (ω_C is constant).

d. **Solve:** Calculate T_B for $\omega_A = 0$ (dynamicist is stationary), $\omega_A = \pi \frac{rad}{s}$ (dynamicist turning left) and $\omega_A = -\pi \frac{rad}{s}$ (turning right). Solve for the following ratios.

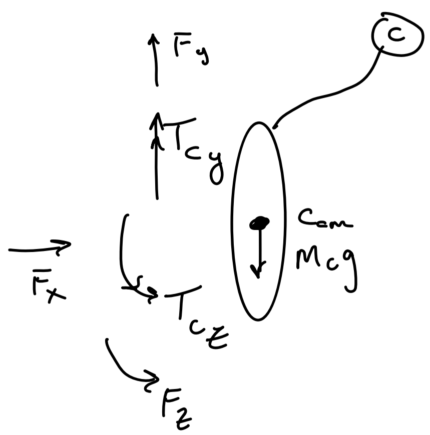
Result: (In terms of I_C, ω_A, ω_C, m^C, g, and L) and also provide a numerical value.

$$\frac{T_{B,left}}{T_{B,stationary}} = \frac{m^C g L - \omega_A \omega_C I_C}{m^C g L} = -0.51$$

$$\frac{T_{B,right}}{T_{B,stationary}} = \frac{m^C g L - \omega_A \omega_C I_C}{m^C g L} = 2.51$$

c. Road map

var	motion	dir	Sys		about	Eq
ω_c	rotate	$\hat{b}_x$	C	FBD	C_{cm}	$\left(\vec{M}^{C/C_{cm}} = \frac{{}^N d\, {}^N\vec{H}^{C/C_{cm}}}{dt} + \ldots\right) \cdot \hat{b}_x$



[1] $\vec{M}^{C/cm} \cdot \hat{b}_x = 0 = \hat{b}_x \cdot \frac{{}^N d\, {}^N\vec{H}^{C/C_{cm}}}{dt}$

$$\hat{b}_x \cdot \left(\frac{{}^N d\, {}^N\vec{H}^{C/C_{cm}}}{dt} = \underline{\underline{I}}^{C/C_{cm}} \cdot {}^N\alpha^C + {}^N\omega^C \times \left(\underline{\underline{I}}^{C/C_{cm}} \cdot {}^N\omega^C\right)\right)$$

$$\overset{\Rightarrow}{I}^{C/C_{cm}} \cdot {}^{N}\vec{\omega}^{C} = I_c(\underset{\overset{\uparrow}{{}^{N}\vec{\omega}^{C}\cdot\hat{b}_x}}{\ldots})\hat{b}_x + J_c(\ldots)\hat{b}_y + J_c(\ldots)\hat{b}_z$$

$$= (I_c - J_c)(\ldots)\hat{b}_x + J_c\underbrace{(\ldots\hat{b}_x + \ldots\hat{b}_y + \ldots\hat{b}_z)}_{{}^{N}\vec{\omega}^{C}}$$

$${}^{N}\vec{\omega}^{C} \times (\overset{\Rightarrow}{I}^{C/C_{cm}} \cdot {}^{N}\vec{\omega}^{C}) = {}^{N}\vec{\omega}^{C} \times (I_c - J_c)(\ldots)\hat{b}_x + \underbrace{\cancel{{}^{N}\vec{\omega}^{C} \times J_c\,{}^{N}\vec{\omega}^{C}}}_{0}$$

$$= {}^{N}\vec{\omega}^{C} \times (I_c - J_c)({}^{N}\vec{\omega}^{C}\cdot\hat{b}_x)\hat{b}_x \longrightarrow$$ after cross product, none in $\hat{b}_x$ direction

$$\hat{b}_x \cdot \left[{}^{N}\vec{\omega}^{C} \times (\overset{\Rightarrow}{I}^{C/C_{cm}} \cdot {}^{N}\vec{\omega}^{C})\right] = 0$$

$$\hat{b}_x \cdot \left(\underbrace{\cancel{\frac{{}^{N}d\,{}^{N}\vec{H}^{C/C_{cm}}}{dt}}}_{0} = \overset{\Rightarrow}{I}^{C/C_{cm}} \cdot {}^{N}\vec{\alpha}^{C} + \underbrace{\cancel{{}^{N}\vec{\omega}^{C} \times (\overset{\Rightarrow}{I}^{C/C_{cm}} \cdot {}^{N}\vec{\omega}^{C})}}_{0}\right)$$

*[1]

$$0 = \hat{b}_x \cdot \overset{\Rightarrow}{I}^{C/C_{cm}} \cdot {}^{N}\vec{\alpha}^{C}$$

$\overset{\Rightarrow}{I}^{C/C_{cm}}$ has $\emptyset$ products, so we only care about $\hat{b}_x$ component of ${}^{N}\vec{\alpha}^{C}$.

$$ {}^{N}\vec{\omega}^{A} = \omega_A \hat{a}_y $$

$$ {}^{A}\vec{\omega}^{B} = \dot{q}_B \hat{a}_z $$

$$ {}^{B}\vec{\omega}^{C} = \omega_c \hat{b}_x $$

$$ {}^{N}\vec{\omega}^{C} = \omega_A \hat{a}_y + \dot{q}_B \hat{a}_z + \omega_c \hat{b}_x $$

$$ \hat{a}_y = \hat{b}_y \cos q_B - \hat{b}_x \sin q_B $$

$$ {}^{N}\vec{\omega}^{C} = (\omega_c - \omega_A \sin q_B)\hat{b}_x + \omega_A \cos q_B \hat{b}_y + \dot{q}_B \hat{b}_z $$

$$ \hat{b}_x / \hat{c}_x \cdot \left[{}^{N}\vec{\alpha}^{C} = \frac{{}^{C}d}{dt}\left[(\omega_c - \omega_A \sin q_B)\hat{c}_x\right] + \frac{{}^{B}d}{dt}\left(\cancelto{0}{\omega_A \cos q_B \hat{b}_y + \dot{q}_B} \hat{b}_z\right) + \underset{\_\_\,\hat{b}_x}{{}^{C}\vec{\omega}^{B}} \times \underbrace{\left(\omega_A \cos q_B \hat{b}_y + \dot{q}_B \hat{b}_z\right)}_{\_\_\,\hat{b}_y \,+\, \_\_\,\hat{b}_z} \right] $$

$$ \hat{b}_x \cdot {}^{N}\vec{\alpha}^{C} = \dot{\omega}_c - \dot{\omega}_A \sin q_B - \omega_A \dot{q}_B \cos q_B $$

$$ 0 = \hat{b}_x \cdot \vec{\vec{I}}^{C/cm} \cdot {}^{N}\vec{\alpha}^{C} = I_c \, {}^{N}\alpha^{C} \cdot \hat{b}_x $$

$$ \hookrightarrow \dot{\omega}_c = \dot{\omega}_A \sin q_B - \omega_A \dot{q}_B \cos q_B $$

However: (Given)

- $\omega_A = \text{constant} \Rightarrow \dot{\omega}_A = 0$
- $q_B = 0 \Rightarrow \dot{q}_B = 0$

$$\boxed{\therefore \; \dot{\omega}_C = 0}$$

d. Done in MG Gyro...1.txt

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2. **Analysis 2:** The dynamicist does not rotate about $\hat{a}_y$, but rotates the pole at a constant specified rate $\dot{q}_B = \frac{\pi\, rad}{4\, s}$ about $\hat{a}_z$.

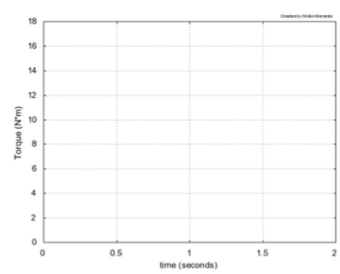
a. **Guess:** As you rotate the pole to lift the wheel, what direction does the wheel want to move?

RIGHT or LEFT

b. **Demo:** Try it. Which direction does the wheel want to move?

RIGHT or LEFT

c. **Solve:** Simulate 2 seconds of motion. Plot T_{Bz} and T_{By} vs time.
Is T_{Bz} constant? Yes/No. Is T_{By} constant? Yes/No.



See MG code:

Gyro... 2.txt