Math 451 HW #7

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Question 1.

My telephone rings 12 times each week, the calls being randomly distributed among the 7 days. What is the probability that I get at least one call each day?

Let $E_j = \{\text{at least 1 call on the jth day}\}$ and $E_j^c = A_j = \{\text{no calls on the jth day}\}$. In both cases we let j range from 1 to 7 to represent each day. We see that

$$\begin{aligned} Pr(\text{at least 1 call every day}) &= Pr(\cap_{i=1}^7 E_i), \\ &= 1 - Pr((\cap_{i=1}^7 E_i)^c), \\ &= 1 - Pr(\cup_{i=1}^7 A_i). \end{aligned}$$

Note that

$$Pr(A_j) = \left(\frac{6}{7}\right)^{12},$$

$$Pr(A_i \cap A_j) = \left(\frac{5}{7}\right)^{12},$$

and so on until we get to

$$Pr(\cap_{i=1}^7 A_i) = \left(\frac{1}{7}\right)^{12}.$$

The number of ways each of these events can occur is $\binom{7}{1}, \binom{7}{2}, \binom{7}{3} \dots \binom{7}{6}$. We can use the inclusion-exclusion principle to write

$$\begin{aligned} Pr(\cup_{i=1}^7 A_i) &= \sum_{j=1}^7 Pr(A_j) - \sum_{1 \le i < j \le 7} Pr(A_i \cap A_j) + \dots, \\ &= 1 - \binom{7}{1}\left(\frac{6}{7}\right)^{12} + \binom{7}{2}\left(\frac{5}{7}\right)^{12} - \dots + \binom{7}{6}\left(\frac{1}{7}\right)^{12} \approx 0.2285. \end{aligned}$$

Thus there is about a 23% chance I will receive at least one phone call every day.

Repeat the question for the case of receiving 16 calls in a week.

Note that our setup remains exactly the same, and we just need to update the specifics of our problem. We now note that

$$Pr(A_j) = \left(\frac{6}{7}\right)^{16},$$

$$Pr(A_i \cap A_j) = \left(\frac{5}{7}\right)^{16},$$

and so on until we get to

$$Pr(\cap_{i=1}^7 A_i) = \left(\frac{1}{7}\right)^{16}.$$

The number of ways of each of these events can occur remains unchanged, i.e. $\binom{7}{1}, \binom{7}{2}, \binom{7}{3} \ldots \binom{7}{6}$. Our application of the inclusion-exclusion principle yields

$$= 1 - \binom{7}{1}\left(\frac{6}{7}\right)^{16} + \binom{7}{2}\left(\frac{5}{7}\right)^{16} - \cdots + \binom{7}{6}\left(\frac{1}{7}\right)^{16} \approx 0.4977.$$

Thus there is about a 50% chance that I will receive at least one phone call every day.