Math 490 HW #12

Maxwell Levin

March 5, 2018

Question 1.

Use our class data to construct:

a. 95% confidence interval for the mean heart rate for the female group

We can grab our class data using R with the following code:

```r
our_data = read.table("math490.R", header=TRUE)
attach(our_data)
```

We can get a 95% confidence interval for the female group in R by simply asking R to run a t-test on our heartbeat data for females:

```r
f_hb = hb[gender == "f"]
t.test(f_hb)
```

```

    One Sample t-test

data:  f_hb
t = 26.018, df = 11, p-value = 3.129e-11
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 67.05348 79.44652
sample estimates:
mean of x
    73.25
```

Thus we see that a 95% confidence interval for the female group is the interval from 67.05 to 79.45. Additionally we also note that the mean heartbeat for females in Math 490 is 73.25 bpm.

b. 95% confidence interval for the mean heart rate for the male group

To do the same for the male group all we have to do is run the following code:

```r
m_hb = hb[gender == "m"]
t.test(m_hb)
```

```

    One Sample t-test

data:  m_hb
t = 17.418, df = 9, p-value = 3.062e-08
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 55.77497 72.42503
sample estimates:
mean of x
```

```
64.1
```

We see that the interval from 55.77 to 72.42 is a 95% confidence interval for the mean heartrate of the male group. We also note that the mean heartrate for the male group is 64.1 bpm.

Question 2.

Let $X \overset{d}{\sim} Normal(0,1)$. Use Monte Carlo estimation to obtain an estimate for $E(cos(X))$ to three digits of accuracy.

Recall from class that

$$I = E[cos(X)] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} cos(t) e^{\frac{-t^2}{2}} dt,$$

and that

$$\sigma^2 = E[cos(X)^2] - I^2.$$

We use Mathematica to find I and σ^2:

```
I = NIntegrate[Cos[t]*e^(-(t^2)/2)/(Sqrt[2*pi]), {t, -Infinity, Infinity}, 10]
Var = NIntegrate[(Cos[t]*e^(-(t^2)/2)/(Sqrt[2*pi]))^2, {t, -Infinity, Infinity}, 10] - I^2
```

Doing so, we see that

$$I \approx 0.6065,$$
$$\sigma^2 \approx 0.1998,$$
$$\sigma \approx 0.4470.$$

Now that we have σ and our error bound, $\epsilon = \pm 0.001$, we can calculate a value of N that will lead to a good estimate for I by using the following formula:

$$N \geq \left(\frac{1.96\sigma}{\epsilon}\right)^2,$$
$$N \geq \left(\frac{1.96(0.447)}{0.001}\right)^2 \approx 767,586.$$

Now we can use the Monte carlo Estimator

$$Z_N^{MC} = \frac{cos(X_1) + cos(X_2) + \cdots + cos(X_N)}{N} \approx I,$$

where $N = 800000$ to get an estimate for I accurate to 3 decimal places. To do this we run the following code in R:

```
zbar = function(sam, rep){
  ans = 0
  for (i in 1:rep) {
    s = 0
    for (j in 1:sam) {
      s = s + cos(rnorm(1))
```

```
    }
    s = s/sam
    ans = ans + s
  }
  ans/rep
}

zbar(800000, 100)
```

```
[1] 0.6065149
```

Success! We have used our calculations to create a Monte Carlo estimate for I accurate to 3 decimals places!