Math 451 HW #5

Maxwell Levin

September 14, 2018

Question 1.

A closet contains n pairs of shoes. If $2r$ shoes are chosen at random (Such that $2r < n$), what is the probability that there will be no matching pair in the sample?

In thinking about this problem I found it helped to refer to Question 2 part (a) where we seek to find the probability of selecting no pairs of shoes with six individual shoe selections from 12 pairs of shoes. In this problem we have n pairs of shoes ($2n$ shoes total) and we must select $2r$ shoes. Since we do not want any pairs, we must select $2r$ distinct types of shoes from our collection of n distinct types of shoes and then for each type pick either the right shoe or the left shoe. We can do this in $\binom{n}{2r}2^{2r}$ ways. To get the probability we must divide by the total number of ways to pick $2r$ shoes from $2n$ distinct shoes. This probability is

$$\frac{\binom{n}{2r}2^{2r}}{\binom{2n}{2r}}.$$

Question 2.

In a closet there are 12 distinct pairs of shoes. If six shoes are selected at random, what is the probability of selecting:

(a) no complete pairs

Since there are 12 pairs of shoes there are 24 shoes total. This means that the number of ways of picking six shoes from this collection is $\binom{24}{6}$. Because we seek 6 distinct types of shoes, we have $\binom{12}{6}$ ways of picking the type of shoe. Within that we also have to select between a right shoe or a left shoe. Since we are picking this for 6 different types of shoes we have 2^6 ways of doing this. Thus the probability that we will select six shoes without a single pair is:

$$\frac{\binom{12}{6}2^6}{\binom{24}{6}}.$$

(b) exactly one complete pair

This time we see two shoes from one pair and 4 other distinct types of shoes. The number of ways that we can pick a single pair of shoes is $\binom{12}{1}$. After we pick this we have four remaining shoes that we require to be distinct. We have $\binom{11}{4}2^4$ ways of doing this. Thus the probability that we will pick exactly one complete pair of shoes is

$$\frac{\binom{12}{1}\binom{11}{4}2^4}{\binom{24}{6}}.$$

(c) exactly two complete pairs

Now we have two pairs of shoes so we first have to pick two distinct types of shoes from 12 types of shoes. We can do this in $\binom{12}{2}$ ways. Since we have already picked four shoes in two pairs, we now need two more

shoes that are of unique types. We can do this in $\binom{10}{2}2^2$ ways. Thus the probability that we will pick exactly two pairs is

$$\frac{\binom{12}{2}\binom{10}{2}2^2}{\binom{24}{6}}.$$

(d) exactly three complete pairs?

Finally, we deal with the case that we pick exactly three pairs of shoes. This means that all six shoes we pick will belong to a pair. Thus there are $\binom{12}{3}$ ways of doing this, i.e. the probability of picking three pairs of shoes is

$$\frac{\binom{12}{3}}{\binom{24}{6}}.$$