Math 490 HW #19

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Question 1.

Use the method of solving a linear system to find the stationary distribution of the transition matrix in exercise 2.3 in Voss. Express your answer as a vector of fractions.

Voss provides us with the following transition matrix:

$$\mathbf{P} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{10} & \frac{9}{10} & 0 & 0 \\ \frac{1}{10} & 0 & \frac{9}{10} & 0 \\ \frac{1}{10} & 0 & 0 & \frac{9}{10} \end{bmatrix}$$

We can solve the linear system by taking the matrix product $\mathbf{vP}$ and setting it equal to $\mathbf{v}$, where $\mathbf{v}$ is the stationary distribution for the transition matrix $\mathbf{P}$. We do this manually first:

$$[a, b, c, d] \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{10} & \frac{9}{10} & 0 & 0 \\ \frac{1}{10} & 0 & \frac{9}{10} & 0 \\ \frac{1}{10} & 0 & 0 & \frac{9}{10} \end{bmatrix} = \left[\frac{2a}{3} + \frac{b}{10} + \frac{c}{10} + \frac{d}{10}, \frac{a}{3} + \frac{9b}{10}, \frac{9c}{10}, \frac{9d}{10}\right].$$

Since $\mathbf{v}$ is a stationary vector for $\mathbf{P}$, the resulting product should be equal to $\mathbf{v}$. Thus we have

$$a = \frac{2a}{3} + \frac{b}{10} + \frac{c}{10} + \frac{d}{10},$$
$$b = \frac{a}{3} + \frac{9b}{10},$$
$$c = \frac{9c}{10},$$
$$d = \frac{9d}{10},$$
$$a + b + c + d = 1.$$

We see that both c and d must be 0, so this leaves

$$a = \frac{2a}{3} + \frac{b}{10},$$
$$b = \frac{a}{3} + \frac{9b}{10},$$
$$a + b = 1.$$

This means that $a = \frac{3}{13}$ and $b = \frac{10}{13}$. Thus our stationary distribution is

$$\mathbf{v} = \left[\frac{3}{13}, \frac{10}{13}, 0, 0\right].$$

Question 2.

Use R to find the stationary distribution of the transition matrix in exercise 2.3.

We run the following code in R:

```
# Transition Matrix
P = matrix( c(2/3, 1/3, 0, 0, 1/10, 9/10, 0, 0, 1/10, 0, 9/10, 0, 1/10, 0, 0, 9/10),
            nrow=4, ncol=4, byrow=TRUE)

# Function that will compute the stationary vector for any
# row-oriented transition matrix
getEquilibrium = function(transition) {
  eigen_stuff = eigen(t(transition))
  eigen_vec1 = eigen_stuff$vectors[ , 1]
  library(MASS)
  fractions(abs(eigen_vec1) / sum(abs(eigen_vec1)))
}

getEquilibrium(P)
```

```
[1]  3/13 10/13     0     0
```

Thus we see that we've found the same stationary distribution as we found in the first question! This is good news.