Math 451 HW #24

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Question 1

You roll three fair dice. Let X be the minimum and Y be the median.

(a) Find $\mathbf{E[Y|X=3]}$.

Recall from our Homework #21 that we found the conditional probability of Y given $X = 3$ to be

$$f_{Y|X}(y|X=3) = \begin{cases} 10/37, & \text{if } y = 3; \\ 15/37, & \text{if } y = 4; \\ 9/37, & \text{if } y = 5; \\ 3/37, & \text{if } y = 6; \\ 0, & \text{otherwise.} \end{cases}$$

We can use this to calculate $E[Y|X=3]$ by:

$$\begin{aligned} E[Y|X=3] &= 3\left(\frac{10}{37}\right) + 4\left(\frac{15}{37}\right) + 5\left(\frac{9}{37}\right) + 6\left(\frac{3}{37}\right), \\ &= \frac{153}{37} \approx 4.135 \end{aligned}$$

(b) Find $\mathbf{Var[Y|X=3]}$.

We can calculate the variance directly by:

$$\begin{aligned} Var[y|X=3] &= \left(3 - \frac{153}{37}\right)^2\left(\frac{10}{37}\right) + \left(4 - \frac{153}{37}\right)^2\left(\frac{15}{37}\right) + \left(5 - \frac{153}{37}\right)^2\left(\frac{9}{37}\right) + \left(6 - \frac{153}{37}\right)^2\left(\frac{3}{37}\right), \\ &\approx 0.820. \end{aligned}$$

Question 2

Let X and Y be the life spans (in hours) of two electronic devices, and their joint p.d.f. is given below:

$$f_{X,Y}(x,y) = \begin{cases} 6e^{-x-2y}, & \text{if } 0 < x < y < \infty; \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $\mathbf{E[Y|X=3]}$.

Recall from Homework #20 that we found

$$f_{Y|X}(y, X=x) = \begin{cases} 2e^{2x-2y}, & \text{if } 0 < x < y < \infty; \\ 0, & \text{otherwise.} \end{cases}$$

To calculate $E[y|X = 3]$ we can take:

$$\int_3^\infty y f_{Y|X}(y|X = 3)dy = \int_3^\infty 2ye^{2(3)-2y}dy,$$

To solve this we use integration by parts:

$$= -ye^{6-2y}\Big|_3^\infty - \int_3^\infty -e^{6-2y}dy,$$

$$= 3 + \left(-\frac{1}{2}e^{6-2y}\right)\Big|_3^\infty$$

$$= 3.5.$$

Thus

$$E[y|X = 3] = 3.5.$$

(b) Find $\mathbf{Var[Y|X = 3]}$.

To calculate $Var[y|X = 3]$ we can use the fact that $Var[y|X = 3] = E[y^2|X = 3] - (E[y|X = 3])^2$. We first calculate $E[y^2|X = 3]$ by using a neat integration by parts trick that a classmate showed me which allows us to perform a series of integration by parts steps in a single step:

$$E[y^2|X = 3] = \int_3^\infty 2y^2e^{6-2y}dy,$$

$$= -\frac{1}{2}(2)y^2e^{6-2y}\Big|_3^\infty - \frac{1}{4}(4)ye^{6-2y}\Big|_3^\infty - \frac{1}{8}(4)e^{6-2y}\Big|_3^\infty,$$

$$= 9 + 3 + \frac{1}{2} = 12.5.$$

Now we take

$$Var[y|X = 3] = 12.5 - (3.5)^2 = \frac{1}{4}.$$