Math 490 HW #2

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January 22, 2018

Question 1.

A box contains 7 balls numbered 1 through 7. Alice chooses two balls at random with replacement, and label the two observed values X and Y. Let V = X - 2Y and let W = |X - Y|.

a. Find the p.m.f. of V.

```
       -13  -12  -11  -10  -9   -8   -7   -6   -5   -4   -3   -2   -1
p.m.f. 1/49 1/49 2/49 2/49 3/49 3/49 4/49 3/49 4/49 3/49 4/49 3/49 4/49
       0    1    2    3    4    5
p.m.f. 3/49 3/49 2/49 2/49 1/49 1/49
```

b. Find E(V) and Var(V).

Since X & Y are independent events we can write:

$$E(X-2Y) = E(X) - 2E(Y),$$

$$Var(X-2Y) = Var(X) + 4Var(Y).$$

Furthermore, since E(X) = E(Y) and Var(X) = Var(Y) we have:

$$E(V) = -E(X),$$

$$Var(V) = 5Var(X).$$

Thus

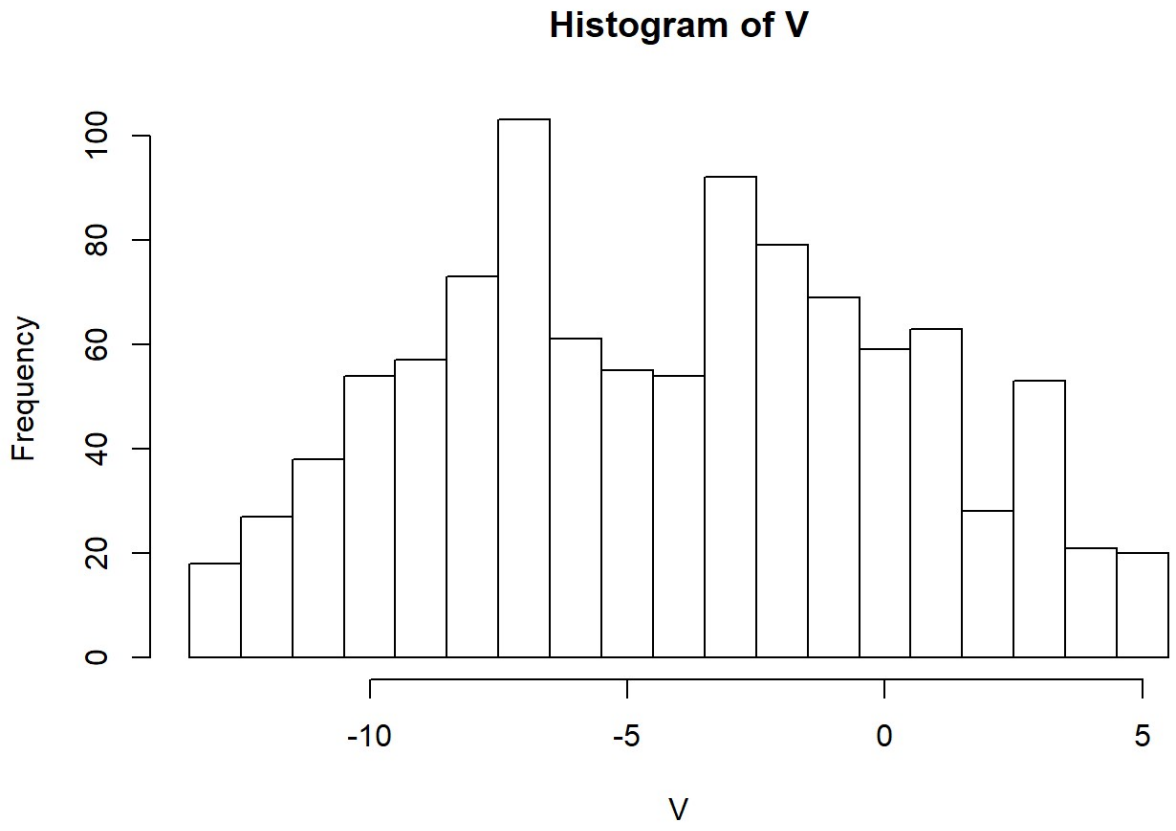
$$E(V) = -\frac{1+2+\cdots+7}{7} = -4,$$

and

$$Var(V) = 5\frac{(1-4)^2+(2-4)^2+\cdots+(7-4)^2]}{7} = 20.$$

c. Use R to sample 1024 values from V, make a histogram, and report the mean and variance of the observed values.

```
X <- sample(1:7, 1024, replace=T)
Y <- sample(1:7, 1024, replace=T)
V = X - 2*Y
hist(V, breaks=c(-13.5, -12.5, -11.5, -10.5, -9.5, -8.5, -7.5, -6.5, -5.5, -4.5, -3.
5, -2.5, -1.5, -0.5, 0.5, 1.5, 2.5, 3.5, 4.5, 5.5), prob=F)
```



The mean is:

```
[1] -4.132812
```

The variance is:

```
[1] 20.43982
```

d. Find the p.m.f. of W.

```
       0    1     2     3    4    5    6
p.m.f. 7/49 12/49 10/49 8/49 6/49 4/49 2/49
```

e. Find E(W) and Var(W).

We can compute E(W) using our table above:

$$E(W) = 0\left(\frac{7}{49}\right) + 1\left(\frac{12}{49}\right) + \cdots + 6\left(\frac{2}{49}\right).$$

This yields:

$$E(W) \approx 2.286.$$

We can compute Var(W) using our table and E(W):

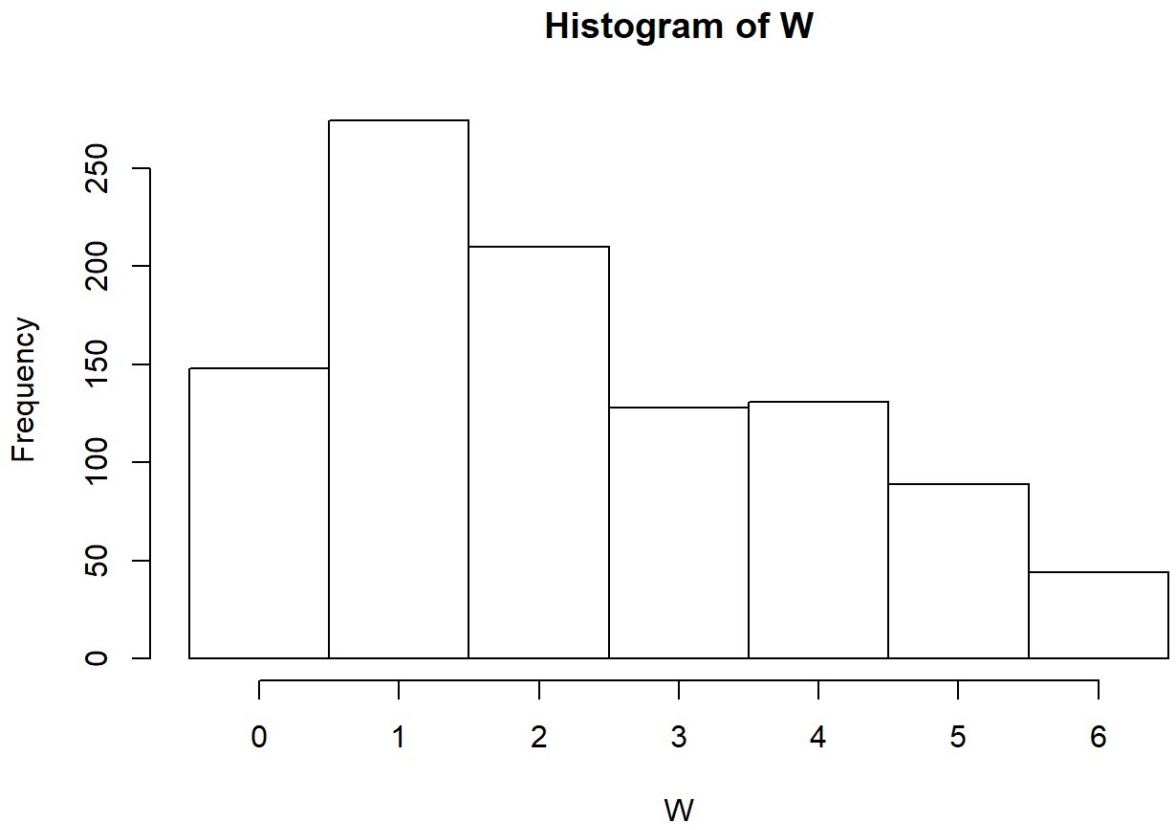
$$Var(W) = (0 - 2.286)^2\left(\frac{7}{49}\right) + (1 - 2.286)^2\left(\frac{12}{49}\right) + \cdots + (6 - 2.286)^2\left(\frac{2}{49}\right).$$

This yields:

$$Var(W) \approx 2.776.$$

f. Use R to sample 1024 values from W, make a histogram, and report the mean and variance of the observed values.

```
X <- sample(1:7, 1024, replace=T)
Y <- sample(1:7, 1024, replace=T)
W = abs(X - Y)
hist(W, breaks=c(-0.5, 0.5, 1.5, 2.5, 3.5, 4.5, 5.5, 6.5), prob=F)
```



The mean is:

```
[1] 2.256836
```

The variance is:

```
[1] 2.889005
```