

Math 490 HW #18

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Question 1.

Use the transition matrix

$$P = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{10} & \frac{9}{10} & 0 & 0 \\ \frac{1}{10} & 0 & \frac{9}{10} & 0 \\ \frac{1}{10} & 0 & 0 & \frac{9}{10} \end{bmatrix}$$

and the initial distribution, $\mu = [\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}]$, to find the 10th-step transition matrix and the 15th-step distribution.

We can get the 10th-step transition matrix by running the following code in R:

Loading required package: expm

Warning: package 'expm' was built under R version 3.4.4

Loading required package: Matrix

Attaching package: 'expm'

The following object is masked from 'package:Matrix':

```
expm
# Transition Matrix
P = matrix( c(2/3, 1/3, 0, 0, 1/10, 9/10, 0, 0, 1/10, 0, 9/10, 0, 1/10, 0, 0, 9/10),
            nrow=4, ncol=4, byrow=TRUE)

# Initial distribution
mu = c(1/4, 1/4, 1/4, 1/4)

# The 10th-step transition matrix
P %>% 10
```

```
      [,1]      [,2]      [,3]      [,4]
[1,] 0.2333955 0.7666045 0.0000000 0.0000000
[2,] 0.2299814 0.7700186 0.0000000 0.0000000
[3,] 0.2299814 0.4213402 0.3486784 0.0000000
[4,] 0.2299814 0.4213402 0.0000000 0.3486784
```

We can now get the 15th-step distribution by running the following code in R:

```
# The 15th-step distribution
mu %>% ( P %>% 15 )
```

```
      [,1]      [,2]      [,3]      [,4]
[1,] 0.2307731 0.6662814 0.05147278 0.05147278
```

Question 2.

Use the transition matrix above and the initial distribution, $\pi = \left[\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{2}{5}\right]$, to find the 15th-step distribution.

We can find the 15th-step distribution by running the following code in R:

```
# Initial distribution;
pi = c(1/5, 1/5, 1/5, 2/5)

# The 15-th-step distribution
pi %*% ( P %^% 15 )
```

```
      [,1]      [,2]      [,3]      [,4]
[1,] 0.2307631 0.6457022 0.04117823 0.08235645
```