Math 451 HW #18

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Question 1

Let X be a $Gamma(\alpha, \beta)$ random variable. Compute the skewness measure:

$$\gamma_1 = E\left[\frac{(X-\mu_X)^3}{\sigma_X^3}\right]$$

Before we begin we note that

$$E[X] = \mu_X = \alpha\beta,$$
$$Var[X] = \sigma_X^2 = \alpha\beta^2,$$

and that

$$E[X^2] = (\alpha+1)\alpha\beta^2,$$
$$E[X^3] = (\alpha+2)(\alpha+1)\alpha\beta^3.$$

We now begin to compute the skewness measure:

$$\gamma_1 = \frac{1}{\sigma^2}E[(X-\mu)^3],$$

$$= \frac{1}{\sigma^3}E[X^3 - 3\mu X^2 + 3\mu^2 X - \mu^3],$$

$$= \frac{1}{\sigma^3}\left(E[X^3] - 3\mu E[X^2] + 3\mu^2 E[X] - \mu^3\right),$$

$$= \frac{1}{\sigma^3}\left[(\alpha+2)(\alpha+1)\alpha\beta^3 - 3(\alpha\beta)(\alpha+1)\alpha\beta^2 + 3(\alpha\beta)^2(\alpha\beta) - (\alpha\beta)^3\right],$$

$$= \frac{1}{\sigma^3}[\alpha^3\beta^3 + 3\alpha^2\beta^3 + 2\alpha\beta^3 - 3\alpha^3\beta^3 03\alpha^2\beta^3 + 2\alpha^3\beta^3],$$

$$= \frac{1}{(\alpha\beta)^{3/2}}[2\alpha\beta^3],$$

$$= \frac{2}{\sqrt{\alpha}}.$$

Thus we have computed our skewness measure:

$$\gamma_1 = \frac{2\sqrt{\alpha}}{\alpha}.$$

Question 2

A fair coin is tossed five times. Let X be the number of heads observed from the first three tosses, and let Y be the number of heads observed from the last three tosses.

	$X=0$	$X=1$	$X=2$	$X=3$
$Y=0$	$1/2^5$	$2/2^5$	$1/2^5$	0
$Y=1$	$2/2^5$	$5/2^5$	$4/2^5$	$1/2^5$
$Y=2$	$1/2^5$	$4/2^5$	$5/2^5$	$2/2^5$
$Y=3$	0	$1/2^5$	$2/2^5$	$1/2^5$

(a) Use a two-way table to show the joint distribution of *X* and *Y*.

Note that since there are only five tosses of the fair coin and X and Y measure the number of heads in the first three and last three tosses respectively, there is some overlap on toss number three. I kept this in mind as I thought about each entry in the following two-way table:

(b) Find the marginal distribution of *Y*.

We can find the marginal distribution of Y by summing up the elements in each row of our two-way table. Doing this we get the following marginal distribution of Y:

$$f_Y(y) = \begin{cases} 1/2^3, & \text{if } y = 0; \\ 3/2^3, & \text{if } y = 1; \\ 3/2^3, & \text{if } y = 2; \\ 1/2^3, & \text{if } y = 3; \\ 0, & \text{otherwise.} \end{cases}$$